Evaluation of Shortest Path by using Breadth-First Algorithm under Neutrosophic Environment

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Abstract

This paper recommends and designs concepts for evaluating the shortest path (SP) for a connected network using a modified breadth-first search algorithm in an uncertain environment. Evaluating the SPs of a network is an essential and extensively encountered optimization problem. Here we develop a new method for determining the SP in a neutrosophic environment in which the arc lengths are uncertain. Here, we use the parameters as neutrosophic numbers, and the new methodology, i.e., canonical representation of neutrosophic numbers in a neutrosophic environment, is used to convert neutrosophic edge lengths to crisp edge lengths and enhance the traditional breadth-first search algorithm. We employ this concept to accommodate the ambiguities in subjective decisions and address these issues. Here we demonstrate that the proposed algorithm has good stability and high efficiency for evaluating the SP. Finally, a numerical example is provided to explain the suggested algorithm.

Keywords: Breadth-First Algorithm, Connected Network, Neutrosophic Numbers, Shortest Path Problem.

1 | Introduction

Lotfi Aliasker Zadeh first suggested a new thing that deals with uncertainty, i.e., fuzzy sets [1], which include membership degrees that are independent and uncertain. Later on, Smarandache established that the neutrosophic set (NS) is a field of philosophy that surveys the existence, provenance, and extent of uniformity as well as its relationship to different theoretical peaks, including parameters such as true (T), indeterminacy (I), and false membership (F) [2]. It has been established that when the inconsistencies of a set of vertices and edges are obtained in a network, a neutrosophic number can be used to evaluate the SPP. If the connection of nodes in a network is uncertain, the NS theory is an adequate principle for addressing real-world issues in which the arc length is considered the neutrosophic number (NN) for encountering SPP [3].

SPP is a fundamental issue that arises in many areas of science and technology. The SPP’s objective is to determine the minimum path distance between a given pair of nodes in a network [4]. The distance from end to end of an arc can signify actual quantities of uniqueness, for example, cost and time. The arc lengths between...
nodes in a connected network are typically accurately measured using traditional SPP, with precise numerical representations in specific conditions. However, in situations of ambiguity, neutrosophic numbers are employed.


Deli and Suba [25] solved various decision-making problems by applying the weighted mathematical operator with the SVN method. Basset et al. [26], [27] suggested a new concept based on managerial technology that allows decision-makers in a neutrosophic environment to select the most appropriate project. They also suggested a new method for assessing reliability and evaluating the extent of agreement between expert opinions in a NE. Kumar et al. [28] proposed a trapezoidal fuzzy neutrosophic environment algorithm for solving SPP. Broumi et al. [29] suggested using single-valued trapezoidal NN to evaluate the SP length of a network. Tan et al. [30] suggested a complete dynamic programming technique for evaluating the SPP. Broumi et al. [31] suggested a technique score function to evaluate the SPP. Goldfarb, D. et al.[32] previously suggested for finding shortest path where crisp number are arc lengths. But in this paper we propose a method for finding SPP in uncertain environment.

The subsequent portion of the research paper is organized as follows: Section-2 elucidates the motivation and contributions of this paper. Section-3 presents definitions of existing terminologies. In Section 4, novel algorithms utilizing the proposed score function are suggested. Section 5 provides a numerical example demonstrating the calculation of NSP in neutrosophic environments. Section 6 contrasts the shortest path across various networks and parameters, along with outlining the benefits of the proposed approach. The conclusion of the proposed methodology is detailed in Section 7.

2 | Motivation
• The primary aim of this research is to introduce a productive computational approach for SPP under Bredth's first algorithm that is adaptable to an ambiguous environment commonly faced in the procedure. The following are the paper's most significant contributions:
• We determined it by calculating the arc length on a neutrosophic network using the neutrosophic number FNSP as the vertex. Each vertex is assigned a neutrosophic number.
• Neutrosophic number NS is an extension of an intuitionistic fuzzy number.
• We introduce a novel algorithm designed to address the Shortest Path Problem (SPP) within an uncertain environment, which accurately computes the shortest path length between two specified nodes.
• In addition, we employ a selection sort algorithm to select the SP connected to the lowest rank.

3 | Preliminaries

In this part we show a general description of an Intuitionistic Fuzzy Set, neutrosophic set.

Definition 3.1 [2]: Assume set \( \tilde{X} \) is the universal set. An intuitionistic fuzzy set \( \tilde{A} \) in \( \tilde{X} \) is written in the form:
\[
\tilde{A} = \{ \tilde{x}, \mu_{\tilde{A}}(\tilde{x}), \nu_{\tilde{A}}(\tilde{x}) \} \quad (1)
\]
With \( \mu_{\tilde{A}}: \tilde{X} \rightarrow [0,1] \) and \( \nu_{\tilde{A}}: \tilde{X} \rightarrow [0,1] \) are the functions that define the degrees of membership non-membership of \( x \in \tilde{X} \) to \( \tilde{A} \in \tilde{X} \), respectively, and for every \( x \in \tilde{X} \), \( \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) \leq 1 \).

Definition 3.2: Let's suppose that \( \tilde{X} \) represents a set of spatial points (objects), where \( \tilde{x} \) denotes the corresponding generic elements in \( \tilde{X} \). Then, the element in the neutrosophic set \( \tilde{A} \) takes the following form:
\[
\tilde{A} = \{ < \tilde{x}: \tilde{\mu}_{\tilde{A}}(\tilde{x}), \tilde{\nu}_{\tilde{A}}(\tilde{x}), \tilde{F}_{\tilde{A}}(\tilde{x}) > \tilde{x} \in \tilde{X} \} \quad (2)
\]
Where three membership degree \( \tilde{\mu}_{\tilde{A}}, \tilde{\nu}_{\tilde{A}}, \tilde{F}_{\tilde{A}}: \tilde{X} \rightarrow [0^–, 1^+] \) where \( \tilde{T}, \tilde{I}, \tilde{F} \) represent the truth function, the indeterminacy function, and the falsity function, respectively.
\[
0^– \leq (\tilde{T}_{\tilde{A}}(\tilde{x}) + \tilde{I}_{\tilde{A}}(\tilde{x}) + \tilde{F}_{\tilde{A}}(\tilde{x})) \leq 3^+
\]
Now \( \tilde{T}_{\tilde{A}}(\tilde{x}), \tilde{I}_{\tilde{A}}(\tilde{x}), \tilde{F}_{\tilde{A}}(\tilde{x}) \) are representing subsets of the interval \([0^–, 1^+]\) makes it challenging to implement neutrosophic sets in real-world situations.

Definition 3.3: Canonical representation of neutrosophic number:

If \( \tilde{A} = (\tilde{p}, \tilde{q}, \tilde{r}) \) is neutrosophic number. Then the Canonical representation of neutrosophic number is defined as:
\[
C(\tilde{A}) = \frac{1}{6} (\tilde{p} + 4\tilde{q} + \tilde{r}) \quad (3)
\]
The canonical representation of neutrosophic numbers \( \tilde{A} = (\tilde{p}_1, \tilde{q}_1, \tilde{r}_1) \) and \( \tilde{B} = (\tilde{p}_2, \tilde{q}_2, \tilde{r}_2) \) is defined as follows.
\[
C(\tilde{A}) = \frac{1}{6} (\tilde{p}_1 + 4\tilde{q}_1 + \tilde{r}_1)
\]
\[ C(\tilde{B}) = \frac{1}{6}(\tilde{p}_2 + 4\tilde{q}_2 + \tilde{r}_2) \]

The operation of adding two neutrosophic numbers can be described as:

\[ C(\tilde{A} + \tilde{B}) = \frac{1}{6}(\tilde{p}_1 + 4\tilde{q}_1 + \tilde{r}_1) + \frac{1}{6}(\tilde{p}_2 + 4\tilde{q}_2 + \tilde{r}_2) \]

(4)

The operation of multiplication two neutrosophic numbers can be described as:

\[ C(\tilde{A} \times \tilde{B}) = \frac{1}{6}(\tilde{p}_1 + 4\tilde{q}_1 + \tilde{r}_1) \times \frac{1}{6}(\tilde{p}_2 + 4\tilde{q}_2 + \tilde{r}_2) \]

(5)

4 | Proposed Method

Consider a linked network with \( s \) as the initial node and \( e \) as the final node. The objective is to determine the SP between source nodes \( s \) and \( e \) with the parameters associated with the vertices are NNs.

4.1 | The Modified neutrosophic Breadth first search Algorithm (MNBFS)

Step 1: Take a graph \( G = (V, E) \) which is simple, weighted, and connected network with neutrosophic numbers as parameters for the edges. Let the source and destinations nodes be \( A \) and \( E \), respectively.

Step 2: Then, using Breadth's first algorithm, find a different possible path from source to destination.

Step 3: Using canonical representation, we exchange neutrosophic path length values to crisp path length values.

Step 4: Finally, after getting the total path from source to destination, we apply a selection sort algorithm to find the minimum rank of the path on a given network.
5 | Numerical Illustration

**Step 1:** Consider the graph $G = (V,E)$ as in the figure given below with the parameters associated with the edges are NNs and $A$ and $E$, respectively represent the source and destination nodes.

![Connected network](image)

**Figure 1:** Connected network.

<table>
<thead>
<tr>
<th>Arc Length</th>
<th>Edge value in neutrosophic number</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-B</td>
<td>$&lt;0.3, 0.6, 0.7&gt;$</td>
</tr>
<tr>
<td>A-C</td>
<td>$&lt;0.3, 0.5, 0.8&gt;$</td>
</tr>
<tr>
<td>B-C</td>
<td>$&lt;0.3, 0.4, 0.6&gt;$</td>
</tr>
<tr>
<td>B-F</td>
<td>$&lt;0.4, 0.6, 0.9&gt;$</td>
</tr>
<tr>
<td>C-D</td>
<td>$&lt;0.3, 0.6, 0.7&gt;$</td>
</tr>
<tr>
<td>C-F</td>
<td>$&lt;0.4, 0.7, 0.8&gt;$</td>
</tr>
<tr>
<td>D-E</td>
<td>$&lt;0.6, 0.7, 0.8&gt;$</td>
</tr>
<tr>
<td>F-E</td>
<td>$&lt;0.7, 0.8, 0.9&gt;$</td>
</tr>
</tbody>
</table>

**Table 1.** (Arc length as neutrosophic number).

**Step 2:** Now we apply BFS algorithm. Assume $A$ is a source node and $E$ is a Destination node.

**First step:**

- $A$ is the source node so it will be removed from the queue.
- $A$’s neighbors are $B$ and $C$, so it will be transited.
- $B$ and $C$ will be transited, which have not been transited previously.
- $A$ Pushed to the back of the line $B$ and $C$ will be identified as visited.
Second step:

- B is removed from the queue.
- The neighbors of B are C and F.
- C is disregarded as it has already been visited.
- F, which hasn't been explored previously, is now explored. It is then added to the queue and marked as visited.

Third step:

- C is taken out of the queue.
- The neighbors of C are D and F.
- F is skipped since it has already been visited.
- D, which hasn't been explored yet, is now explored. It is then added to the queue and marked as visited.

Fourth step:

- D is taken out of the queue.
- The neighbor of D is E.
- E, which hasn't been traversed before, is now traversed. It is then added to the queue and marked as visited.
Fifth step:

- F is removed from the queue.
- The neighbor of F is E.
- E, is ignored because it is identified as 'visited'
- So in conclusion the possible path is from initial to final node is in fig: 2

Figure 2.
Finally possible path from source to destination is in Table 2.

Table 2. (Possible path).

<table>
<thead>
<tr>
<th>Possible path</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-B-F-E</td>
</tr>
<tr>
<td>A-B-C-F-E</td>
</tr>
<tr>
<td>A-C-F-E</td>
</tr>
<tr>
<td>A-C-D-E</td>
</tr>
</tbody>
</table>

**Step 3:** Now by using canonical representation we find the Path distance

\[ A - B - F - E = (A - B) + (B - F) + (F - E) \]

\[ = \frac{1}{6} (0.3 + 4(0.6) + 0.7) + \frac{1}{6} (0.4 + 4(0.6) + 0.9) + \frac{1}{6} (0.7 + 4(0.8) + 0.9) \]

\[ = \frac{1}{6} (0.3 + 2.4 + 0.7) + \frac{1}{6} (0.4 + 2.4 + 0.9) + \frac{1}{6} (0.7 + 3.2 + 0.9) \]

\[ = \frac{1}{6} (3.4) + \frac{1}{6} (3.7) + \frac{1}{6} (4.8) \]

\[ = 0.56 + 0.61 + 0.80 \]

\[ = 1.97 \]

Path \( A - B - C - F - E = (A - B) + (B - C) + (C - F) + (F - E) \)

\[ = \frac{1}{6} (0.3 + 4(0.6) + 0.7) + \frac{1}{6} (0.3 + 4(0.4) + 0.6) + \frac{1}{6} (0.4 + 4(0.7) + 0.8) + \frac{1}{6} (0.7 + 4(0.8) + 0.9) \]

\[ = \frac{1}{6} (0.3 + 2.4 + 0.7) + \frac{1}{6} (0.3 + 1.6 + 0.6) + \frac{1}{6} (0.4 + 2.8 + 0.8) + \frac{1}{6} (0.7 + 3.2 + 0.9) \]

\[ = \frac{1}{6} (3.4) + \frac{1}{6} (2.5) + \frac{1}{6} (4.0) + \frac{1}{6} (4.8) \]

\[ = 0.56 + 0.41 + 0.66 + 0.80 \]

\[ = 2.43 \]

Path \( A - C - F - E = (A - C) + (C - F) + (F - E) \)

\[ = \frac{1}{6} (0.3 + 4(0.5) + 0.8) + \frac{1}{6} (0.4 + 4(0.7) + 0.8) + \frac{1}{6} (0.7 + 4(0.8) + 0.9) \]
Evaluation of Shortest Path by using Breadth-First Algorithm under Neutrosophic Environment

\[
= \frac{1}{6} (0.3 + 2.0 + 0.8) + \frac{1}{6} (0.4 + 2.8 + 0.8) + \frac{1}{6} (0.7 + 3.2 + 0.9) \\
= \frac{1}{6} (3.1) + \frac{1}{6} (4.0) + \frac{1}{6} (4.8) \\
= 0.51 + 0.66 + 0.80 \\
= 1.97
\]

Path \( A - C - D - E = (A - C) + (C - D) + (D - E) \)

\[
= \frac{1}{6} (0.3 + 4(0.5) + 0.8) + \frac{1}{6} (0.3 + 4(0.6) + 0.7) + \frac{1}{6} (0.6 + 4(0.7) + 0.8) \\
= \frac{1}{6} (0.3 + 2.0 + 0.8) + \frac{1}{6} (0.3 + 2.4 + 0.7) + \frac{1}{6} (0.6 + 2.8 + 0.8) \\
= \frac{1}{6} (3.1) + \frac{1}{6} (3.4) + \frac{1}{6} (4.2) \\
= 0.51 + 0.56 + 0.70 \\
= 1.77
\]

Step 4:
Input:

```python
# Selection sort in Python 3 compiler

def selectionSort(array, size):
    for step in range(size):
        min_idx = step
        for i in range(step + 1, size):
            # to sort in descending order, change > to < in this line
            # select the minimum element in each loop
            if array[i] < array[min_idx]:
                min_idx = i

    # put min at the correct position
    (array[step], array[min_idx]) = (array[min_idx], array[step])

data = [1.97, 2.43, 1.97, 1.77]
size = len(data)
selectionSort(data, size)
print('Sorted the shortest path : ')
print(data)
```
6 | Comparative Analysis between our Approach and the Existing Method

In this section, we compare our methodology with one other existing methodology and finally analyze our methodology for evaluating the shortest path based on the breadth first algorithm in a neutrosophic environment, which gives the optimal result. We discuss this in Table 3.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Shortest path</th>
<th>shortest path length</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;Shortest path problem within the framework of interval-valued neutrosophic settings [33]&quot;</td>
<td>A → C → D → E</td>
<td>[0.35,0.60],[0.01,0.04], [0.008,0.075]</td>
</tr>
<tr>
<td>Our proposed method</td>
<td>A → C → D → E</td>
<td>1.77</td>
</tr>
</tbody>
</table>

7 | Conclusions

In this paper, we determined the shortest path within a network operating in a neutrosophic environment, where edge weights neutrosophic number. The aforementioned paper elaborates on the benefits of employing neutrosophic numbers in NSP. The traditional breadth-first algorithm is modified to calculate the SP between a pair of given nodes. Here we illustrate the effectiveness of our suggested methodology by giving one numerical example. The most significant finding of this work is identifying an NSP algorithm in the neutrosophic environment that uses the NN as edge weights. The proposed algorithm is extremely effective for real-world application-oriented problems. In future work, one may explore different ways to resolve SPPs using and advancing our proposed algorithm.

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Author Contribution

All authors contributed equally to this work.

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**Data Availability**

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

**Conflicts of Interest**

The authors declare that there is no conflict of interest in the research.

**Ethical Approval**

This article does not contain any studies with human participants or animals performed by any of the authors.

**References**


