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Examples of Fuzzy Sets, Hyperfuzzy Sets, and SuperHyperfuzzy Sets in Climate Change and the Proposal of Several New Concepts

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Abstract

A fuzzy set generalizes classical set theory by assigning each element a membership value within $[0, 1]$, allowing for the representation of partial or uncertain membership. It is well established that fuzzy sets can be further extended to Hyperfuzzy sets and SuperHyperfuzzy sets. However, as these concepts have been introduced only recently, their practical applications remain largely unexplored.
In this paper, we investigate potential applications of Fuzzy sets, Hyperfuzzy sets, and SuperHyperfuzzy sets in the context of climate change and environmental factors. Additionally, we introduce the Forest Fuzzy Set, Forest Hyperfuzzy Set, and Forest SuperHyperfuzzy Set, and explore their possible applications. Furthermore, Hyperfuzzy sets and SuperHyperfuzzy sets are known to be associated with Hyperstructure and SuperHyperstructure concepts. While our primary focus is on the aforementioned extensions, we also provide a brief discussion on TreeStructure and ForestStructure by considering their definitions in the context of Hyperstructure and SuperHyperstructure.

Keywords: Set Theory, SuperhyperFuzzy set, Fuzzy Set, HyperFuzzy set, Hyperstructure, Superhyperstructure

1 | Fuzzy, HyperFuzzy, and n-SuperHyperFuzzy Sets

To address uncertainty, vagueness, and imprecision in decision-making, various set-theoretic models have been proposed and refined over time. The concept of Fuzzy Sets, introduced by Zadeh, laid the foundation for this field and remains a cornerstone of uncertainty modeling [92, 94, 95, 97, 96, 93, 94, 99, 98]. This paper focuses on Fuzzy Sets and their hierarchical extensions, including the HyperFuzzy Set [56, 84, 33] and the n-SuperHyperFuzzy Set [31, 40, 41, 33]. These advanced frameworks are deeply connected to the concept of the powerset and higher-order powersets, which play a crucial role in their definitions and applications. The following sections provide formal definitions and detailed discussions of these theoretical frameworks.

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Definition 1 (Base Set)**.** A *base set* S is the foundational set from which more complex structures, such as powersets and hyperstructures, are derived. It is formally defined as:

$$S = \{x \mid x \text{ is an element within a specified domain}\}.$$

All elements in constructs like $\mathcal{P}(S)$ or $\mathcal{P}_n(S)$ originate from elements of S.

Definition 2 (Powerset)**.** [32, 69] The *powerset* of a set S, denoted $\mathcal{P}(S)$, is the collection of all possible subsets of S, including both the empty set and S itself. Formally, it is defined as:

$$\mathcal{P}(S) = \{A \mid A \subseteq S\}.$$

Definition 3 (n-th Powerset)**.** (cf.[79, 32, 30, 29, 73])

The n-th powerset of a set H, denoted $P_n(H)$, is defined iteratively, starting with the standard powerset. The recursive definition is given by:

$$P_1(H) = P(H), \quad P_{n+1}(H) = P(P_n(H)), \quad \text{for } n \geq 1.$$

Similarly, the n-th non-empty powerset, denoted $P_n^*(H)$, is defined recursively as:

$$P_1^*(H) = P^*(H), \quad P_{n+1}^*(H) = P^*(P_n^*(H)),$$

where $P^*(H)$ represents the powerset of H with the empty set removed.

Definition 4 (Fuzzy Set)**.** [92, 97] A *fuzzy set* τ in a non-empty universe Y is a mapping $\tau : Y \to [0,1]$. A *fuzzy relation* on Y is a fuzzy subset δ in $Y \times Y$. If τ is a fuzzy set in Y and δ is a fuzzy relation on Y, then δ is called a *fuzzy relation on* τ if

$$\delta(y,z) \leq \min\{\tau(y), \tau(z)\} \quad \text{for all } y, z \in Y.$$

Definition 5 (HyperFuzzy Set)**.** [56, 60, 9, 84, 50, 77] Let X be a non-empty set. A mapping $\tilde{\mu} : X \to \tilde{P}([0,1])$ is called a *hyperfuzzy set* over X, where $\tilde{P}([0,1])$ denotes the family of all non-empty subsets of the interval $[0,1]$.

Definition 6 (n-SuperHyperFuzzy Set)**.** [35, 42, 33] Let X be a non-empty set, and let $n \geq 0$ be an integer. An *n-SuperHyperFuzzy Set* is a mapping:

$$\tilde{\mu}_n : \tilde{\mathcal{P}}_n^*(X) \to \tilde{\mathcal{P}}([0,1]),$$

where:

- $\tilde{\mathcal{P}}_n^*(X)$ denotes the family of all non-empty elements of the n-th PowerSet $\mathcal{P}_n^*(X)$, recursively defined as:

$$\mathcal{P}_0^*(X) = X, \quad \mathcal{P}_1^*(X) = \mathcal{P}(X), \quad \mathcal{P}_n^*(X) = \mathcal{P}(\mathcal{P}_{n-1}^*(X)), \text{ for } n \geq 2,$$

  with $\tilde{\mathcal{P}}_n^*(X) = \mathcal{P}_n^*(X) \setminus \{\emptyset\}$.

- $\tilde{\mathcal{P}}([0,1])$ denotes the family of all non-empty subsets of the interval $[0,1]$.

Structural Properties:

(1) Each element $A \in \tilde{\mathcal{P}}_n^*(X)$ is a non-empty subset within the n-th PowerSet hierarchy of X.

(2) The mapping $\tilde{\mu}_n$ assigns to each $A \in \tilde{\mathcal{P}}_n^*(X)$ a non-empty subset $\tilde{\mu}_n(A) \subseteq [0,1]$, representing the degrees of membership associated with the subset A.

Special Cases:

- If $n = 0$, $\tilde{\mathcal{P}}_0^*(X) = X$, and the structure reduces to a standard fuzzy set:

$$\tilde{\mu}_0 : X \to [0,1].$$

- If $n = 1$, $\tilde{\mathcal{P}}_1^*(X) = \tilde{\mathcal{P}}(X)$, and the structure represents a SuperHyperFuzzy Set:

$$\tilde{\mu}_1 : \tilde{\mathcal{P}}(X) \to \tilde{\mathcal{P}}([0,1]).$$

- For $n \geq 2$, the structure extends recursively to higher-order fuzzy relationships:

$$\tilde{\mu}_n : \tilde{\mathcal{P}}_n^*(X) \to \tilde{\mathcal{P}}([0,1]).$$

The n-SuperHyperFuzzy Set generalizes fuzzy sets by incorporating hierarchical and recursive membership degrees, enabling the modeling of uncertainty at multiple levels within the n-th PowerSet hierarchy of X.

A TreeFuzzy Set is a generalization of the Fuzzy Set concept using a Tree structure.

Definition 7. [33] A *TreeFuzzy Set* F is a mapping:

$$F : P(\text{Tree}(A)) \to [0,1]^U,$$

where $P(\text{Tree}(A))$ denotes the power set of the set of all nodes and leaves in $\text{Tree}(A)$, and $[0,1]^U$ denotes the set of all fuzzy subsets of U.

For each attribute combination $S \in P(\text{Tree}(A))$, $F(S)$ is a membership function $\mu_S : U \to [0,1]$, assigning to each element $x \in U$ a degree of membership with respect to the attribute combination S.

The ForestFuzzy Set is a concept that applies the idea of the ForestSoft Set to the framework of Fuzzy Sets. The definition is provided below.

Definition 8 (ForestFuzzy Set)**.** [34] Let $\{F_t\}_{t\in T}$ be a collection of TreeFuzzy Sets, where each

$$F_t : P\big(\text{Tree}(A^{(t)})\big) \to [0,1]^U.$$

Form the *forest*

$$\text{Forest}(\{A^{(t)}\}) \;=\; \bigsqcup_{t\in T} \text{Tree}(A^{(t)}).$$

A *ForestFuzzy Set* is a mapping

$$\mathbf{F} : P\big(\text{Forest}(\{A^{(t)}\})\big) \;\longrightarrow\; [0,1]^U$$

defined by: for each $X \subseteq \text{Forest}(\{A^{(t)}\})$ and each $x \in U$,

$$\mathbf{F}(X)(x) \;=\; \max_{\substack{t\in T\\ X\cap \text{Tree}(A^{(t)})\neq\emptyset}} F_t\big(X \cap \text{Tree}(A^{(t)})\big)(x).$$

2 | Results: Forest Hyperfuzzy Set and Forest SuperHyperfuzzy Set

This section defines the Forest Hyperfuzzy Set and the Forest SuperHyperfuzzy Set as extensions of the Forest Fuzzy Set.

Definition 9 (Forest Hyperfuzzy Set)**.** Let $\{\tilde{F}_t\}_{t\in T}$ be a collection of *Tree Hyperfuzzy Sets*, where each

$$\tilde{F}_t \;:\; P\big(\text{Tree}(A^{(t)})\big) \;\longrightarrow\; \tilde{P}([0,1]),$$

and $\tilde{P}([0,1])$ denotes the family of all non-empty subsets of $[0,1]$. Define

$$\text{Forest}(\{A^{(t)}\}) \;=\; \bigsqcup_{t\in T} \text{Tree}(A^{(t)}).$$

A *Forest Hyperfuzzy Set* is a mapping

$$\tilde{\mathbf{F}} \;:\; P\big(\text{Forest}(\{A^{(t)}\})\big) \;\longrightarrow\; \tilde{P}([0,1])$$

defined by, for each $X \subseteq \text{Forest}(\{A^{(t)}\})$,

$$\tilde{\mathbf{F}}(X) \;=\; \bigcup_{\substack{t\in T\\ X\cap \text{Tree}(A^{(t)})\neq\emptyset}} \tilde{F}_t\big(X \cap \text{Tree}(A^{(t)})\big).$$

In words, $\tilde{\mathbf{F}}(X)$ is obtained by *taking the set-theoretic union* of the membership-subsets provided by each $\tilde{F}_t$ for the portion of X that lies in the corresponding tree $\text{Tree}(A^{(t)})$.

Theorem 10 (Forest Hyperfuzzy Set Generalizes ForestFuzzy Set)**.** *Any ForestFuzzy Set can be seen as a special case of a Forest Hyperfuzzy Set.*

Proof: **Step 1. (ForestFuzzy Set Setup)** Recall a ForestFuzzy Set

$$\mathbf{F} \;:\; P\big(\mathrm{Forest}(\{A^{(t)}\})\big) \;\longrightarrow\; [0,1]^U$$

is defined by

$$\mathbf{F}(X)(x) \;=\; \max_{\substack{t\in T\\ X\cap \mathrm{Tree}(A^{(t)})\neq\emptyset}} F_t\big(X\cap \mathrm{Tree}(A^{(t)})\big)(x),$$

where each F_t is a "tree-fuzzy" mapping into $[0,1]^U$.

Step 2. (Embedding into Hyperfuzzy Notation) We convert each real-valued membership $F_t(Y)(x) \in [0,1]$ into a *singleton subset* of $[0,1]$. Define

$$\tilde{F}_t(Y) \;:=\; \{\, F_t(Y)(x) \,\}, \quad \text{for all } x \in U.$$

In other words, instead of reporting a single membership value in $[0,1]$, we report a *one-element subset* of $[0,1]$. This is a legitimate hyperfuzzy assignment, since each image is now a non-empty subset of $[0,1]$.

Step 3. (Forest Hyperfuzzy Construction) Under this embedding, we define a Forest Hyperfuzzy Set $\tilde{\mathbf{F}}$ via

$$\tilde{\mathbf{F}}(X) \;=\; \bigcup_{\substack{t\in T\\ X\cap \mathrm{Tree}(A^{(t)})\neq\emptyset}} \tilde{F}_t\big(X\cap \mathrm{Tree}(A^{(t)})\big).$$

Because each $\tilde{F}_t$ returns a singleton in $[0,1]$, the set-theoretic union in the hyperfuzzy sense corresponds (at each $x \in U$) to a finite or countable union of singletons.

Step 4. (Equivalence to ForestFuzzy Membership) For each $X \subseteq \mathrm{Forest}(\{A^{(t)}\})$ and each $x \in U$:

$$\tilde{\mathbf{F}}(X) \;=\; \bigcup_{t:X\cap\mathrm{Tree}(A^{(t)})\neq\emptyset} \{\, F_t\big(X\cap \mathrm{Tree}(A^{(t)})\big)(x) \,\}.$$

Since a union of singletons is just a set of possible membership values, one can recover the numerical membership by taking, for instance, its supremum or maximum. Indeed, in typical ForestFuzzy definitions, we use max across the relevant F_t. Thus, if we adopt the natural correspondence

$$\bigcup_{i=1}^{k}\{\, a_i \,\} \;\longmapsto\; \max_{1\le i\le k} a_i,$$

then we see that $\tilde{\mathbf{F}}(X)$ recovers exactly the same membership values as $\mathbf{F}(X)$.

Hence, every ForestFuzzy Set is a special case of a Forest Hyperfuzzy Set in which all hyperfuzzy images are singletons. This completes the proof. □

Theorem 11 (Forest Hyperfuzzy Set Generalizes Hyperfuzzy Set)**.** *Any Hyperfuzzy Set is a special case of a Forest Hyperfuzzy Set.*

Proof: **Step 1. (Single "Tree" Case)** Consider a Hyperfuzzy Set $\tilde{\mu} : X \to \tilde{P}([0,1])$ on some non-empty base set X. We can regard X as a single "tree" $\mathrm{Tree}(A^{(t^*)})$ with $T = \{t^*\}$.

Step 2. (Constructing the Forest) Define

$$\mathrm{Forest}(\{A^{(t^*)}\}) \;=\; \mathrm{Tree}(A^{(t^*)}) \;\cong\; X.$$

Thus, the forest contains exactly one tree.

Step 3. (Defining a Single Tree Hyperfuzzy Set) Let $\tilde{F}_{t^*}$ be given by

$$\tilde{F}_{t^*}(Y) \;=\; \tilde{\mu}(Y), \quad \text{for each } Y \subseteq X.$$

Here we simply identify each subset Y of X with the same subset Y of $\mathrm{Tree}(A^{(t^*)})$.

Step 4. (The Resulting Forest Hyperfuzzy Set) Because $T = \{t^*\}$, a Forest Hyperfuzzy Set $\tilde{\mathbf{F}}$ on $\mathrm{Forest}(\{A^{(t^*)}\})$ is given by

$$\tilde{\mathbf{F}}(X') \;=\; \bigcup_{\substack{t\in\{t^*\}\\ X'\cap\mathrm{Tree}(A^{(t)})\neq\emptyset}} \tilde{F}_t\big(X'\cap \mathrm{Tree}(A^{(t)})\big).$$

But there is only one t^*, so

$$\tilde{\mathbf{F}}(X') \;=\; \tilde{F}_{t^*}(X') \;=\; \tilde{\mu}(X').$$

Hence, $\tilde{\mathbf{F}}$ coincides exactly with the original Hyperfuzzy Set $\tilde{\mu}$.

Therefore, any Hyperfuzzy Set is a special case of a Forest Hyperfuzzy Set with exactly one tree in the forest. □

Definition 12 (Forest n-SuperHyperfuzzy Set)**.** Let $\{\tilde{F}_t^n\}_{t\in T}$ be a collection of *Tree n-SuperHyperfuzzy Sets*, where each

$$\tilde{F}_t^n : \tilde{\mathcal{P}}_n^*(\mathrm{Tree}(A^{(t)})) \;\longrightarrow\; \tilde{\mathcal{P}}([0,1]).$$

Define

$$\mathrm{Forest}(\{A^{(t)}\}) \;=\; \bigsqcup_{t\in T} \mathrm{Tree}(A^{(t)}),$$

and consider the *n-th non-empty powerset* $\tilde{\mathcal{P}}_n^*(\mathrm{Forest}(\{A^{(t)}\}))$. A *Forest n-SuperHyperfuzzy Set* is a mapping

$$\tilde{\mathbf{F}}_n \;:\; \tilde{\mathcal{P}}_n^*(\mathrm{Forest}(\{A^{(t)}\})) \;\longrightarrow\; \tilde{\mathcal{P}}([0,1])$$

defined by, for each $A \in \tilde{\mathcal{P}}_n^*(\mathrm{Forest}(\{A^{(t)}\}))$,

$$\tilde{\mathbf{F}}_n(A) \;=\; \bigcup_{\substack{t\in T \\ A\,\cap\,\tilde{\mathcal{P}}_n^*(\mathrm{Tree}(A^{(t)}))\neq\emptyset}} \tilde{F}_t^n\Big(A \,\cap\, \tilde{\mathcal{P}}_n^*(\mathrm{Tree}(A^{(t)}))\Big).$$

In other words, we first restrict A to whichever part lies in the "tree" $\mathrm{Tree}(A^{(t)})$ at the n-th powerset level, then collect the corresponding hyperfuzzy-subset from $\tilde{F}_t^n$, and finally take the union of all such results over all $t \in T$ for which the intersection is non-empty.

Theorem 13 (Forest n-SuperHyperfuzzy Set Generalizes Forest Hyperfuzzy Set)**.** *If $n = 1$, then any Forest 1-SuperHyperfuzzy Set is a Forest Hyperfuzzy Set.*

Proof: **Step 1. (Recalling Definitions)** By Definition 12, a Forest 1-SuperHyperfuzzy Set is

$$\tilde{\mathbf{F}}_1 : \tilde{\mathcal{P}}_1^*(\mathrm{Forest}(\{A^{(t)}\})) \;\longrightarrow\; \tilde{\mathcal{P}}([0,1]),$$

where $\tilde{\mathcal{P}}_1^*(\cdot)$ is simply the non-empty subsets of the original forest. But

$$\tilde{\mathcal{P}}_1^*(\mathrm{Forest}(\{A^{(t)}\})) \;=\; P\big(\mathrm{Forest}(\{A^{(t)}\})\big) \setminus \{\emptyset\},$$

and each $\tilde{F}_t^1$ is a Tree 1-SuperHyperfuzzy Set on $\mathrm{Tree}(A^{(t)})$, which again corresponds to

$$\tilde{F}_t^1 \;:\; P\big(\mathrm{Tree}(A^{(t)})\big) \setminus \{\emptyset\} \;\longrightarrow\; \tilde{\mathcal{P}}([0,1]).$$

Step 2. (Coincidence with Forest Hyperfuzzy) For any non-empty $X \subseteq \mathrm{Forest}(\{A^{(t)}\})$,

$$\tilde{\mathbf{F}}_1(X) \;=\; \bigcup_{\substack{t\in T \\ X\cap\mathrm{Tree}(A^{(t)})\neq\emptyset}} \tilde{F}_t^1(X \cap \mathrm{Tree}(A^{(t)})).$$

This is exactly the same union-based construction used in Definition 9 for a Forest Hyperfuzzy Set. Thus, when $n = 1$, $\tilde{\mathbf{F}}_1$ is just a Forest Hyperfuzzy Set (possibly ignoring the empty set if required by notation).

Hence, setting $n = 1$ recovers the Forest Hyperfuzzy structure exactly, and so the class of Forest 1-SuperHyperfuzzy Sets *contains* the class of Forest Hyperfuzzy Sets. □

Theorem 14 (Forest n-SuperHyperfuzzy Set Generalizes n-SuperHyperfuzzy Set)**.** *Any n-SuperHyperfuzzy Set is a special case of a Forest n-SuperHyperfuzzy Set.*

Proof: **Step 1. (Single "Tree" Equivalence)** Let $\tilde{\mu}_n : \tilde{\mathcal{P}}_n^*(X) \to \tilde{\mathcal{P}}([0,1])$ be an n-SuperHyperfuzzy Set on a non-empty set X. We can view X as a single "tree" in the sense that

$$\mathrm{Tree}(A^{(t^*)}) \;\cong\; X, \quad T = \{\, t^* \}.$$

Then

$$\mathrm{Forest}(\{A^{(t^*)}\}) \;=\; \mathrm{Tree}(A^{(t^*)}) \;\cong\; X.$$

Step 2. (Defining the Single Tree n-SuperHyperfuzzy Set) Define

$$\tilde{F}_{t^*}^n : \; \tilde{\mathcal{P}}_n^*(\mathrm{Tree}(A^{(t^*)})) \;\longrightarrow\; \tilde{\mathcal{P}}([0,1])$$

by the direct correspondence

$$\tilde{F}_{t^*}^n(A) \;=\; \tilde{\mu}_n(A), \quad \text{for each } A \in \tilde{\mathcal{P}}_n^*(X).$$

Step 3. (Constructing the Forest n-SuperHyperfuzzy Set) The Forest n-SuperHyperfuzzy Set $\tilde{\mathbf{F}}_n$ defined on $\mathrm{Forest}(\{A^{(t^*)}\})$ becomes

$$\tilde{\mathbf{F}}_n(A') \;=\; \bigcup_{\substack{t \in \{t^*\} \\ A' \cap \tilde{\mathcal{P}}_n^*(\mathrm{Tree}(A^{(t)})) \neq \emptyset}} \tilde{F}_t^n(A' \cap \tilde{\mathcal{P}}_n^*(\mathrm{Tree}(A^{(t)}))).$$

Because t^* is the *only* element in T,

$$\tilde{\mathbf{F}}_n(A') \;=\; \tilde{F}_{t^*}^n(A') \;=\; \tilde{\mu}_n(A'),$$

as soon as A' is a valid element in $\tilde{\mathcal{P}}_n^*(\mathrm{Tree}(A^{(t^*)})) \cong \tilde{\mathcal{P}}_n^*(X)$.

Thus, the structure $\tilde{\mathbf{F}}_n$ is precisely the original n-SuperHyperfuzzy Set $\tilde{\mu}_n$. Consequently, any n-SuperHyperfuzzy Set on a set X is a trivial one-tree example of a Forest n-SuperHyperfuzzy Set. □

3 | Some Examples of Climate Change Applications

In this section, we examine various examples of Climate Change applications for each type of Fuzzy Set. Note that climate change refers to long-term shifts in temperature, precipitation, and weather patterns, primarily caused by human activities like fossil fuel burning (cf.[90, 53, 8, 6, 64]).

Example 15 (Fuzzy Set Example: CO_2 emissions)**. CO_2 emissions** refer to the release of carbon dioxide into the atmosphere from fossil fuel combustion, industrial activities, deforestation, and natural processes, contributing to climate change ([28, 62, 83, 7]). Let X be the set of all countries (or regions). We are interested in assessing each country's annual CO_2 emissions intensity on a scale from 0 to 1.

Suppose we have a maximum emission value $\mathrm{CO2}_{\max}$ observed globally (e.g., from historical data). For each country $x \in X$, let $\mathrm{CO2}(x)$ denote its current emission level in metric tons (MT) of CO_2 per year.

Define a fuzzy set $\tau : X \to [0,1]$ by

$$\tau(x) \;=\; \frac{\mathrm{CO2}(x)}{\mathrm{CO2}_{\max}},$$

capped at 1 if the fraction exceeds 1. Thus, $\tau(x)$ measures "the degree to which country x has a high CO_2 emission level," relative to the global maximum:

$$\tau(x) \;=\; \min\Bigl\{\frac{\mathrm{CO2}(x)}{\mathrm{CO2}_{\max}},\; 1\Bigr\}.$$

If $\tau(x) \approx 1$, then x is essentially at the maximum emission level. If $\tau(x) \approx 0$, then x emits very little CO_2 compared to the global maximum.

Example 16 (Fuzzy Set Example: Sea-Level Rise Vulnerability)**. Sea-level rise (SLR)** refers to the increase in global ocean levels due to climate change, driven by ice sheet melting, glacier retreat, and thermal expansion of seawater (cf.[14, 61, 13, 17]). Let X be a collection of coastal cities. We wish to assign a *vulnerability score* in $[0,1]$ to each city based on predicted sea-level rise (SLR).

Assume we have a maximum projected sea-level rise $\mathrm{SLR}_{\max}$ (e.g., 1 meter) for the most extreme case over a given timeframe. For each city $x \in X$, let $\mathrm{SLR}(x)$ be the predicted sea-level rise (in meters) at that location.

Define a fuzzy set $\tau : X \to [0,1]$ by

$$\tau(x) \;=\; \frac{\mathrm{SLR}(x)}{\mathrm{SLR}_{\max}},$$

capped at 1 if the fraction exceeds 1. Thus, $\tau(x)$ indicates how close x is to the worst-case scenario:

$$\tau(x) \;=\; \min\Bigl\{\frac{\mathrm{SLR}(x)}{\mathrm{SLR}_{\max}},\, 1\Bigr\}.$$

A city with $\tau(x) \approx 1$ is highly vulnerable to sea-level rise, whereas a city with $\tau(x) \approx 0$ faces minimal risk.

Example 17 (Hyperfuzzy set Example: CO_2 emissions)**.** Let X again be the set of countries. However, we now have *multiple* climate scenarios or data sources (e.g., different IPCC scenarios: RCP 2.6[15], RCP 4.5[16], RCP 6.0[67], RCP 8.5[68]) that yield varying estimates of CO_2 emissions for each country. Instead of assigning a single number in $[0,1]$ to each country, we might assign a *range* or a set of plausible membership values.

For each country $x \in X$, let

$$\mathrm{CO2}_{\mathrm{scenario}}(x) \;=\; \{\, \mathrm{CO2}_s(x) \;\mid\; s \in \mathrm{Scenarios}\},$$

be the set of predicted emissions under different scenarios. Let $\mathrm{CO2}_{\max}$ be an upper bound across *all* scenarios.

Define a hyperfuzzy set $\tilde{\mu} : X \to \tilde{P}([0,1])$ by

$$\tilde{\mu}(x) \;=\; \Bigl\{\, \min\Bigl\{\frac{\mathrm{CO2}_s(x)}{\mathrm{CO2}_{\max}},\, 1\Bigr\} \;\Bigm|\; s \in \mathrm{Scenarios}\Bigr\}.$$

Hence, for each x, $\tilde{\mu}(x)$ is a *set of membership values* in $[0,1]$. Some scenario might predict high emissions (leading to membership near 1), while another might predict lower emissions (leading to membership near 0.3, for example). In this way, each country's membership in "high CO_2 emission" is represented as a *set of possible values* rather than a single value.

Example 18 (Hyperfuzzy Set Example: Ocean Acidification Projections)**. Ocean Acidification Projections** estimate future ocean pH decreases due to CO_2 absorption, impacting marine ecosystems, coral reefs, shellfish, and biodiversity, based on climate models and emission scenarios (cf.[86, 88, 24]). Let X be a set of marine regions (e.g., different areas of the ocean). Future pH levels of ocean water can vary significantly depending on the climate model or scenario considered. Instead of assigning one pH-based membership value, we may wish to represent a *range* of plausible acidification levels.

Denote by $\mathrm{pH}(r,s)$ the predicted ocean pH of region $r \in X$ under scenario $s \in S$, where S is the set of available climate scenarios. Lower pH indicates higher acidity (i.e., more acidification). Let $\mathrm{pH}_{\min}$ be the minimum pH (most acidic) observed across *all* regions and scenarios, and $\mathrm{pH}_{\max}$ be the maximum pH (least acidic).

Define a hyperfuzzy set $\tilde{\mu} : X \to \tilde{P}([0,1])$ by mapping each region r to a *non-empty subset* of $[0,1]$. For each scenario s, we normalize pH into the interval $[0,1]$ by

$$\mathrm{norm\_pH}(r,s) \;=\; \frac{\mathrm{pH}(r,s) - \mathrm{pH}_{\min}}{\mathrm{pH}_{\max} - \mathrm{pH}_{\min}},$$

so that 0 corresponds to the most acidic pH and 1 corresponds to the least acidic pH. Then:

$$\tilde{\mu}(r) \;=\; \bigl\{\mathrm{norm\_pH}(r,s) \;\bigm|\; s \in S\bigr\}.$$

A region r with a large spread in $\tilde{\mu}(r)$ has higher uncertainty in its acidification projections. A region with nearly constant norm_pH across scenarios would have a narrower set of values in $\tilde{\mu}(r)$.

Example 19 (n-SuperHyperfuzzy Set)**.** We may want to group countries into *subsets* (e.g., by continent or economic alliance) and then assess the membership of these *subsets of countries* in a fuzzy property, such as collective high emission risk. An n-SuperHyperfuzzy Set allows membership assignments not just for singletons (countries) but for subsets of countries, subsets of subsets, and so on, up to the n-th powerset.

Let X be the set of countries. Define the subsets in the second-level powerset $\mathcal{P}(X)$ as groupings like:

$$A_1 = \{\text{countries in Europe}\}, \quad A_2 = \{\text{countries in Asia}\}, \quad ...$$

Potentially, we can then form subsets of these subsets (like alliances that straddle continents), up to level n.

Let

$$\tilde{\mu}_n : \tilde{\mathcal{P}}^*_n(X) \longrightarrow \tilde{\mathcal{P}}([0,1])$$

be an n-SuperHyperfuzzy Set. For any $A \in \tilde{\mathcal{P}}^*_n(X)$, we assign

$$\tilde{\mu}_n(A) = \Big\{ m \in [0,1] \;\Big|\; m \text{ is a plausible membership level for the subset } A, \Big\}.$$

Concretely, for a second-level subset $A \subseteq \mathcal{P}(X)$, $\tilde{\mu}_n(A)$ might incorporate aggregated emission data, historical climate vulnerability, or other risk factors to determine how strongly A (as a group) is "responsible for high emissions" or "affected by climate risk." Since there may be multiple data sources, each subset-level membership is a *set* of possible membership values, capturing multiple lines of evidence or scenario assumptions.

For instance, if A is a set of countries spanning both developed and developing nations, $\tilde{\mu}_n(A)$ might be wide (e.g., $\{0.4, 0.75, 0.8\}$) if there is strong disagreement across climate models or across historical data adjustments.

Example 20 (n-SuperHyperfuzzy Set Example: Hierarchical Grouping of Extreme Weather Events)**. Extreme Weather Events** are severe, unpredictable climate phenomena, including hurricanes, heatwaves, floods, and droughts, intensified by climate change, causing significant environmental, economic, and human impacts (cf.[54, 20, 87, 85]). Consider a set X of weather event records (e.g., hurricanes[51], typhoons[52], floods) over a certain period. We may want to assign "risk membership" not only to individual events but also to *subsets* or *clusters* of these events (e.g., hurricanes that form in a specific ocean basin during consecutive years). This might continue up to higher-order subsets of subsets, capturing complex interdependencies.

Let $\tilde{\mathcal{P}}^*_n(X)$ denote the n-th non-empty powerset hierarchy on X. For example, when $n = 2$, we consider subsets of $\mathcal{P}(X)$, i.e., sets of sets of events.

Define an n-SuperHyperfuzzy Set

$$\tilde{\mu}_n : \tilde{\mathcal{P}}^*_n(X) \longrightarrow \tilde{\mathcal{P}}([0,1]).$$

For any $A \in \tilde{\mathcal{P}}^*_n(X)$,

$$\tilde{\mu}_n(A) = \{ \alpha \in [0,1] \mid \alpha \text{ is a plausible collective risk level for this hierarchy of weather events } A\}.$$

Concretely, if A represents a cluster of floods within a specific river basin plus a cluster of hurricanes over the same timeframe, $\tilde{\mu}_n(A)$ might combine meteorological data, seasonal risk factors, and historical severity indexes to assign a *set* of possible membership degrees. This captures multiple models or multiple weighting schemes for how these events could collectively exacerbate climate impact.

Example 21 (Forest Fuzzy Set: CO_2 emissions)**.** A forest structure can be used when we want to model multiple "trees" of climate data. For example, one tree could represent temperature-related data, another could represent precipitation patterns, and another could represent CO_2 emissions. We can then form a *forest* by taking the disjoint union of these trees, each capturing a different aspect of climate.

Let

$$\text{Tree}(A^{(1)}) \quad \text{be the temperature tree,} \quad \text{Tree}(A^{(2)}) \quad \text{the precipitation tree,} \quad \text{Tree}(A^{(3)}) \quad \text{the CO}_2\text{-emission tree.}$$

Each tree might contain nodes (or sub-variables) such as "urban vs. rural temperature" or "annual vs. seasonal precipitation levels," etc. The *forest* is

$$\text{Forest}(\{A^{(1)}, A^{(2)}, A^{(3)}\}) = \text{Tree}(A^{(1)}) \sqcup \text{Tree}(A^{(2)}) \sqcup \text{Tree}(A^{(3)}).$$

Suppose we define a "TreeFuzzy Set" F_1 for temperature risk, F_2 for precipitation anomalies, and F_3 for emission intensity. Each F_t returns a function in $[0,1]^U$ for subsets of the corresponding tree. Then, by the ForestFuzzy construction,

$$\mathbf{F}(X) = \max_{\substack{t \in \{1,2,3\} \\ X \cap \text{Tree}(A^{(t)}) \neq \emptyset}} F_t(X \cap \text{Tree}(A^{(t)})),$$

we aggregate the membership degrees across these trees. For a given subset X of the forest, if X contains nodes from both the temperature and precipitation trees, we take the maximum membership contribution from each relevant tree. This can be used, for example, to classify how strongly a collection of climate indicators suggests a high-risk region.

Example 22 (ForestFuzzy Set Example: Integrating Temperature Anomalies and Deforestation Rates)**.** Temperature Anomalies refer to deviations from long-term average temperatures, indicating climate change trends, caused by greenhouse gas emissions, deforestation, and natural climate variability (cf.[26, 65, 11, 27]). Deforestation Rates measure the speed of forest loss due to logging, agriculture, and urbanization, impacting biodiversity, carbon storage, and contributing to global climate change (cf.[70, 59, 72]). We can create two different "trees" of climate information: one tree for temperature anomalies and one tree for deforestation metrics (e.g., forest loss rate, forest fragmentation). By uniting these trees into a single forest, we can define a ForestFuzzy Set to assess how strongly any subset of data from these two domains indicates climate stress.

$$\mathrm{Tree}(A^{(1)}) = \text{Tree of temperature anomalies (spatial and temporal nodes)},$$
$$\mathrm{Tree}(A^{(2)}) = \text{Tree of deforestation metrics (regional forest cover, loss rate over time)},$$
$$\mathrm{Forest}\big(\{A^{(1)}, A^{(2)}\}\big) \;=\; \mathrm{Tree}(A^{(1)}) \;\sqcup\; \mathrm{Tree}(A^{(2)}).$$

Let F_1 be a TreeFuzzy Set assigning a membership in $[0,1]^U$ to subsets of the temperature tree, and F_2 similarly for the deforestation tree.

$$\mathbf{F}(X) \;=\; \max_{\substack{t\in\{1,2\}\\ X\cap \mathrm{Tree}(A^{(t)})\neq\emptyset}} F_t\big(X \cap \mathrm{Tree}(A^{(t)})\big).$$

For any subset X of the forest (which may contain nodes from both temperature and deforestation data), the membership value in $\mathbf{F}(X)$ is the maximum of the fuzzy memberships provided by F_1 and F_2. This might be interpreted as the degree of overall climate stress indicated by the data points in X.

Example 23 (Forest Hyperfuzzy Set: CO_2 emissions)**.** We combine the idea of a *forest* of climate variables (e.g., temperature, precipitation, CO_2 emission) with the notion of hyperfuzzy membership (sets of values instead of single values). Different models might give different ranges for temperature change, precipitation change, or emissions. We want to unify these uncertainties across multiple trees.

Let $\tilde{F}_1$ be a Tree Hyperfuzzy Set capturing uncertain temperature data, $\tilde{F}_2$ be a Tree Hyperfuzzy Set capturing uncertain precipitation data, $\tilde{F}_3$ be a Tree Hyperfuzzy Set capturing uncertain emissions data. Then

$$\mathrm{Forest}\big(\{A^{(1)}, A^{(2)}, A^{(3)}\}\big) \;=\; \mathrm{Tree}(A^{(1)}) \;\sqcup\; \mathrm{Tree}(A^{(2)}) \;\sqcup\; \mathrm{Tree}(A^{(3)}).$$

A Forest Hyperfuzzy Set $\tilde{\mathbf{F}} : P(\mathrm{Forest}) \to \tilde{P}([0,1])$ is given by

$$\tilde{\mathbf{F}}(X) \;=\; \bigcup_{t\in\{1,2,3\}} \tilde{F}_t\big(X \cap \mathrm{Tree}(A^{(t)})\big).$$

Thus, for each subset X of the forest, we gather the *set of membership values* from each tree's hyperfuzzy assignment and then take the union. This might represent, for instance, a union of plausible risk levels (in $[0,1]$) for a combined climate impact, drawn from each uncertain data model.

Example 24 (Forest Hyperfuzzy Set Example: Uncertain Data for Drought and Biodiversity Loss)**.** Suppose we have two climate-related trees: one for *drought severity* data (cf.[23, 19, 89]) and one for *biodiversity loss* indicators (cf.[12, 91, 55]). Each tree is hyperfuzzy, meaning that for any subset of the tree, we assign a *set of possible membership values* rather than a single value.

Let

$$\tilde{F}_1 : P\big(\mathrm{Tree}(A^{(1)})\big) \to \tilde{P}([0,1])$$

capture the uncertain degrees of "high drought risk," and similarly

$$\tilde{F}_2 : P\big(\mathrm{Tree}(A^{(2)})\big) \to \tilde{P}([0,1])$$

capture the uncertain degrees of "critical biodiversity loss." The forest is

$$\mathrm{Forest}(\{A^{(1)}, A^{(2)}\}) \;=\; \mathrm{Tree}(A^{(1)}) \;\sqcup\; \mathrm{Tree}(A^{(2)}).$$

A Forest Hyperfuzzy Set $\tilde{\mathbf{F}} : P(\mathrm{Forest}) \to \tilde{P}([0,1])$ is defined by

$$\tilde{\mathbf{F}}(X) \;=\; \tilde{F}_1(X \cap \mathrm{Tree}(A^{(1)})) \;\cup\; \tilde{F}_2(X \cap \mathrm{Tree}(A^{(2)})).$$

For a subset X that mixes drought data and biodiversity data, $\tilde{\mathbf{F}}(X)$ unifies the *set of membership values* from $\tilde{F}_1$ and $\tilde{F}_2$. If one scenario indicates severe drought (near membership 1) but uncertain biodiversity impact (0.3 to 0.6), and another scenario indicates moderate drought (0.5) but severe biodiversity impact (0.8), the union might yield a fairly wide range in $[0.3, 1]$. This represents uncertainty in how strongly X suggests a combined climate risk.

Example 25 (Forest n-SuperHyperfuzzy Set)**.** We now extend the forest-based perspective to an n-layer hierarchical setting, where subsets of climate data can themselves be grouped, and so on, up to the n-th power. This is useful if we have multi-layered structures of climate metrics (e.g., local vs. regional vs. global data sets, or short-term vs. medium-term vs. long-term projections), and each layer can contain uncertain membership values.

We define $\tilde{F}_t^n$ on each $\mathrm{Tree}(A^{(t)})$ to be a Tree n-SuperHyperfuzzy Set. That is, $\tilde{F}_t^n$ assigns to each element of $\tilde{\mathcal{P}}_n^*(\mathrm{Tree}(A^{(t)}))$ a non-empty subset of $[0,1]$.

By Definition 12, the Forest n-SuperHyperfuzzy Set $\tilde{\mathbf{F}}_n$ is

$$\tilde{\mathbf{F}}_n : \tilde{\mathcal{P}}_n^*(\mathrm{Forest}(\{A^{(t)}\})) \;\longrightarrow\; \tilde{\mathcal{P}}([0,1]),$$

$$\tilde{\mathbf{F}}_n(A) \;=\; \bigcup_{\substack{t \in \{1,2,3\} \\ A \cap \tilde{\mathcal{P}}_n^*(\mathrm{Tree}(A^{(t)})) \neq \emptyset}} \tilde{F}_t^n\Big(A \;\cap\; \tilde{\mathcal{P}}_n^*(\mathrm{Tree}(A^{(t)}))\Big).$$

For example, an element A at level n could be a subset-of-subset-of-subset of climate metrics (e.g., certain temperature anomalies combined with specific precipitation anomalies over certain years). $\tilde{\mathbf{F}}_n(A)$ would then unify the *sets* of membership degrees from each tree's n-SuperHyperfuzzy assignment. This structure can capture advanced hierarchical uncertainty, such as nested climate models (local, regional, global) or nested time scales (short-, mid-, long-term) across multiple types of indicators (temperature, precipitation, emissions).

If $n = 1$, we recover a Forest Hyperfuzzy Set. If there is only one tree in the forest, we recover the n-SuperHyperfuzzy Set from standard definitions. Thus, these forest-based higher-order frameworks allow modeling of extremely rich, multi-layered climate uncertainties in a single unifying scheme.

Example 26 (Forest n-SuperHyperfuzzy Example: Multi-Layered Heatwaves and Air Quality Indicators)**.** Imagine building three "trees" of climate data: one for *heatwave frequencies* (cf.[66, 22]) in different regions and seasons, one for *air quality* indicators [10, 63, 25] (e.g., $\mathrm{PM}_{2.5}$[100, 49], ozone[18]), and one for *water scarcity* metrics (cf.[57, 58, 71]). Each tree is assigned an n-SuperHyperfuzzy structure, reflecting hierarchical subsets (e.g., multiple subregions, multiple time frames, multiple pollutant thresholds).

$$\tilde{F}_{(1)}^n : \tilde{\mathcal{P}}_n^*(\mathrm{Tree}(A^{(1)})) \to \tilde{\mathcal{P}}([0,1]), \quad \text{(heatwaves)}$$

$$\tilde{F}_{(2)}^n : \tilde{\mathcal{P}}_n^*(\mathrm{Tree}(A^{(2)})) \to \tilde{\mathcal{P}}([0,1]), \quad \text{(air quality)}$$

$$\tilde{F}_{(3)}^n : \tilde{\mathcal{P}}_n^*(\mathrm{Tree}(A^{(3)})) \to \tilde{\mathcal{P}}([0,1]), \quad \text{(water scarcity)}$$

Then the forest is

$$\mathrm{Forest}(\{A^{(1)}, A^{(2)}, A^{(3)}\}) \;=\; \mathrm{Tree}(A^{(1)}) \;\sqcup\; \mathrm{Tree}(A^{(2)}) \;\sqcup\; \mathrm{Tree}(A^{(3)}).$$

We define

$$\tilde{\mathbf{F}}_n : \tilde{\mathcal{P}}_n^*(\mathrm{Forest}(\{A^{(1)}, A^{(2)}, A^{(3)}\})) \;\longrightarrow\; \tilde{\mathcal{P}}([0,1]),$$

so that, for $A \in \tilde{\mathcal{P}}^*_n(\text{Forest})$,

$$\tilde{\mathbf{F}}_n(A) \;=\; \bigcup_{t\in\{1,2,3\}} \tilde{F}^n_{(t)}\Big(A \cap \tilde{\mathcal{P}}^*_n(\text{Tree}(A^{(t)}))\Big).$$

In practical terms, if A is a complex hierarchical subset that includes certain clusters of heatwave data, certain clusters of air-quality metrics, and certain subsets of water-scarcity indicators, then $\tilde{\mathbf{F}}_n(A)$ merges the *sets of membership values* from each tree's n-SuperHyperfuzzy assignment. This allows for integrated, multi-level climate risk assessment across these three distinct but interrelated dimensions.

4 | Additional Results: Tree and Forest Structures and Their Hyperextensions

At the beginning of this paper, we briefly discussed the concepts of Hyperstructure and SuperHyperstructure. In this section, as additional results, we introduce several hierarchical structures and their hyperextensions. Specifically, we define:

- the *Tree Structure*,
- the *Forest Structure*,
- the *Forest-Hyperstructure*, and
- the *Forest-n-SuperHyperstructure*.

Definition 27 (Tree Structure)**.** Let A be a non-empty set of attributes. A *tree structure* is a pair

$$T = (V, E),$$

where:

(1) V is a non-empty set (the set of nodes).

(2) $E \subseteq V \times V$ is a set of directed edges representing the parent-child relation.

(3) There exists a unique element $r \in V$ (the root) such that for every $v \in V$, there exists a unique simple path from r to v.

(4) Every node $v \in V \setminus \{r\}$ has exactly one parent; that is, there exists a unique $u \in V$ with $(u, v) \in E$.

Definition 28 (Forest Structure)**.** A *forest structure* is a collection of disjoint tree structures. Formally, let

$$\{T_i = (V_i, E_i) \mid i \in I\}$$

be a family of tree structures such that $V_i \cap V_j = \emptyset$ for all $i \neq j$. Then the *forest* is defined as

$$F = \bigsqcup_{i\in I} V_i,$$

with the edge set being the union

$$E_F = \bigcup_{i\in I} E_i.$$

In other words, a forest is simply a disjoint union of trees.

Definition 29 (Forest-Hyperstructure)**.** Let F be a forest structure as defined above and let R be a non-empty set (for example, $R = [0, 1]$ or any other range of interest). A *forest-hyperstructure* is a mapping

$$H : \mathcal{P}(F) \to \mathcal{P}^*(R),$$

where $\mathcal{P}(F)$ is the powerset of F (i.e., the set of all subsets of F) and $\mathcal{P}^*(R)$ denotes the family of all non-empty subsets of R. For each subset $X \subseteq F$, $H(X)$ is interpreted as the hyper-assignment (or hyper-membership value) associated with X.

Definition 30 (Forest-n-SuperHyperstructure)**.** Let F be a forest structure. Define the n-th non-empty powerset of F recursively as follows:

$$\mathcal{P}_0^*(F) = F, \quad \mathcal{P}_1^*(F) = \mathcal{P}(F) \setminus \{\emptyset\},$$

and for $n \geq 2$,

$$\mathcal{P}_n^*(F) = \mathcal{P}(\mathcal{P}_{n-1}^*(F)) \setminus \{\emptyset\}.$$

A *forest-n-superhyperstructure* is a mapping

$$S_n : \mathcal{P}_n^*(F) \to \mathcal{P}^*(R),$$

which assigns to each non-empty element A of the n-th non-empty powerset of F a non-empty subset of R. This construction generalizes the notion of hyperstructures by incorporating hierarchical (recursive) assignments.

Theorem 31 (A Tree Is a Forest)**.** *Any tree structure $T = (V, E)$ can be viewed as a forest structure with a single tree.*

Proof: Let $T = (V, E)$ be a tree. Define the forest F as the disjoint union of trees with the index set $I = \{1\}$ by setting $T_1 = T$. Then

$$F = V,$$

and the edge set of the forest is simply E. Hence, T is trivially a forest structure consisting of a single tree. □

Theorem 32 (Restriction of a Forest-Hyperstructure)**.** *Let $H : \mathcal{P}(F) \to \mathcal{P}^*(R)$ be a forest-hyperstructure defined on a forest F, and let $T \subseteq F$ be one of its trees. Then the restriction $H|_T : \mathcal{P}(T) \to \mathcal{P}^*(R)$ defines a hyperstructure on T.*

Proof: Since $T \subseteq F$ and $\mathcal{P}(T) \subseteq \mathcal{P}(F)$, for every $X \subseteq T$ the hyper-assignment $H(X)$ is already defined. Therefore, $H|_T$ is simply the restriction of H to subsets of T, satisfying all the properties of a hyperstructure on T (i.e., mapping each non-empty subset $X \subseteq T$ to a non-empty subset $H(X) \subseteq R$). Thus, $H|_T$ is a hyperstructure on T. □

Theorem 33 (Embedding of Forest-Hyperstructure into Forest-n-Superhyperstructure)**.** *Any forest-hyperstructure $H : \mathcal{P}(F) \to \mathcal{P}^*(R)$ is a special case of a forest-n-superhyperstructure for $n = 1$. That is, if we define*

$$S_1 : \mathcal{P}_1^*(F) \to \mathcal{P}^*(R)$$

by $S_1(X) = H(X)$ for every non-empty $X \subseteq F$, then S_1 is a forest-1-superhyperstructure.

Proof: By definition, the 1-th non-empty powerset of F is

$$\mathcal{P}_1^*(F) = \mathcal{P}(F) \setminus \{\emptyset\}.$$

The mapping H assigns to each $X \in \mathcal{P}_1^*(F)$ a non-empty subset $H(X) \subseteq R$. Therefore, setting $S_1(X) = H(X)$ for all $X \in \mathcal{P}_1^*(F)$ exactly matches the definition of a forest-1-superhyperstructure. Thus, any forest-hyperstructure is embedded in the framework of forest-n-superhyperstructures for $n = 1$. □

5 | Additional Results: GraphicFuzzy Set and ClusterFuzzy Set

In this section, we propose two new types of fuzzy sets: the GraphicFuzzy Set and the ClusterFuzzy Set. The following provides their formal definitions along with related theorems.

Definition 34 (Graph and Its Power Set)**.** (cf.[21]) Let $G = (V, E)$ be a finite graph, where V is a set of vertices (representing attributes) and $E \subseteq V \times V$ is a set of edges (representing relationships among these attributes). A *subgraph* H of G is a graph $H = (V_H, E_H)$ with $V_H \subseteq V$ and $E_H \subseteq E \cap (V_H \times V_H)$. The *power set of the graph* G, denoted by $\mathcal{P}(G)$, is defined as

$$\mathcal{P}(G) = \{ H \mid H \text{ is a subgraph of } G \}.$$

Definition 35 (GraphicFuzzy Set)**.** Let U be a universe of discourse and let $G = (V, E)$ be a graph representing a set of attributes and their interrelationships. A *GraphicFuzzy Set* is a mapping

$$F : \mathcal{P}(G) \to \mathcal{F}(U),$$

where $\mathcal{F}(U)$ denotes the collection of all fuzzy sets on U (i.e. functions $\mu : U \to [0, 1]$). For each subgraph $H \in \mathcal{P}(G)$, the mapping $F(H) : U \to [0, 1]$ assigns to each $x \in U$ a membership degree $F(H)(x)$ representing the extent to which x possesses the combined attributes described by the subgraph H.

A common method to aggregate attributes is to define

$$F(H)(x) = \min_{v \in V(H)} f(v)(x),$$

where $f(v) : U \to [0, 1]$ is a basic fuzzy membership function corresponding to the attribute v.

Example 36 (GraphicFuzzy Set on Personal Attributes)**.** Let

$$U = \{\text{Alice, Bob, Charlie, Diana}\}$$

be a set of individuals. Consider a graph

$$G = (V, E)$$

with

$$V = \{\text{Smart, Friendly, Athletic}\}.$$

Assume the following basic fuzzy membership functions:

$$\begin{aligned}
f(\text{Smart})(\text{Alice}) &= 0.8, \quad f(\text{Smart})(\text{Bob}) = 0.6, \\
f(\text{Smart})(\text{Charlie}) &= 0.9, \quad f(\text{Smart})(\text{Diana}) = 0.7, \\
f(\text{Friendly})(\text{Alice}) &= 0.7, \quad f(\text{Friendly})(\text{Bob}) = 0.8, \\
f(\text{Friendly})(\text{Charlie}) &= 0.6, \quad f(\text{Friendly})(\text{Diana}) = 0.9, \\
f(\text{Athletic})(\text{Alice}) &= 0.6, \quad f(\text{Athletic})(\text{Bob}) = 0.7, \\
f(\text{Athletic})(\text{Charlie}) &= 0.8, \quad f(\text{Athletic})(\text{Diana}) = 0.65.
\end{aligned}$$

Define a GraphicFuzzy Set $F : \mathcal{P}(G) \to \mathcal{F}(U)$ by:

- For the subgraph H_1 containing only the vertex {Smart}:
$$F(H_1)(x) = f(\text{Smart})(x).$$
- For the subgraph H_2 containing only the vertex {Friendly}:
$$F(H_2)(x) = f(\text{Friendly})(x).$$
- For the subgraph H_3 with vertices $\{\text{Smart}, \text{Friendly}\}$ (assuming the edge connecting them is present), define
$$F(H_3)(x) = \min\{f(\text{Smart})(x),\ f(\text{Friendly})(x)\}.$$
Hence, for $x = \text{Alice}$, $F(H_3)(\text{Alice}) = \min\{0.8,\ 0.7\} = 0.7$.
- For the entire graph G (with all three vertices), one may define:
$$F(G)(x) = \min\{f(\text{Smart})(x),\ f(\text{Friendly})(x),\ f(\text{Athletic})(x)\}.$$
For example, $F(G)(\text{Bob}) = \min\{0.6,\ 0.8,\ 0.7\} = 0.6$.

Theorem 37 (Monotonicity of GraphicFuzzy Set)**.** *Let $H_1, H_2 \in \mathcal{P}(G)$ be two subgraphs such that $H_1 \subseteq H_2$ (i.e., $V(H_1) \subseteq V(H_2)$). If the aggregation is defined via the minimum operator as*

$$F(H)(x) = \min_{v \in V(H)} f(v)(x),$$

then for all $x \in U$,

$$F(H_2)(x) \leq F(H_1)(x).$$

Proof: Since $H_1 \subseteq H_2$, we have $V(H_1) \subseteq V(H_2)$. Thus,

$$F(H_1)(x) = \min_{v \in V(H_1)} f(v)(x)$$

and

$$F(H_2)(x) = \min_{v \in V(H_2)} f(v)(x).$$

Because the minimum taken over a larger set is less than or equal to the minimum taken over a subset, we obtain

$$\min_{v \in V(H_2)} f(v)(x) \leq \min_{v \in V(H_1)} f(v)(x),$$

i.e., $F(H_2)(x) \leq F(H_1)(x)$ for all $x \in U$. □

Definition 38 (ClusterFuzzy Set)**.** Let $\{F_i\}_{i \in I}$ be a finite family of fuzzy sets over a universe U, where each F_i is a mapping

$$F_i : U \to [0, 1],$$

such that for each $x \in U$, $F_i(x)$ represents the degree of membership of x with respect to the i-th fuzzy evaluation. Assume that the index set I is partitioned into disjoint clusters $\{C_j\}_{j \in J}$, where for each cluster $C \subseteq I$ the fuzzy sets in $\{F_i \mid i \in C\}$ are to be aggregated. Then a *ClusterFuzzy Set* is defined as a mapping

$$G : \{C_j \mid j \in J\} \to \mathcal{F}(U),$$

where for each cluster C and each $x \in U$,

$$G(C)(x) = \max_{i \in C} F_i(x).$$

This aggregation operator is chosen to reflect the union of fuzzy information from multiple sources.

Example 39 (ClusterFuzzy Set: Evaluating Product Quality)**.** Let

$$U = \{\text{Product1, Product2, Product3}\}$$

be a set of products. Suppose three independent quality assessments are given by fuzzy sets:

$$F_1 : U \to [0, 1], \quad F_1(\text{Product1}) = 0.8,\ F_1(\text{Product2}) = 0.6,\ F_1(\text{Product3}) = 0.7,$$

$$F_2 : U \to [0, 1], \quad F_2(\text{Product1}) = 0.85,\ F_2(\text{Product2}) = 0.65,\ F_2(\text{Product3}) = 0.75,$$

$$F_3 : U \to [0, 1], \quad F_3(\text{Product1}) = 0.8,\ F_3(\text{Product2}) = 0.7,\ F_3(\text{Product3}) = 0.72.$$

Let the index set be $I = \{1, 2, 3\}$ and form a single cluster $C_1 = \{1, 2, 3\}$. Then the ClusterFuzzy Set G is defined by

$$G(C_1)(x) = \max\{F_1(x),\ F_2(x),\ F_3(x)\}.$$

For instance, for $x = \text{Product1}$,

$$G(C_1)(\text{Product1}) = \max\{0.8,\ 0.85,\ 0.8\} = 0.85.$$

Similarly, for $x = \text{Product2}$,

$$G(C_1)(\text{Product2}) = \max\{0.6,\ 0.65,\ 0.7\} = 0.7.$$

Example 40 (ClusterFuzzy Set: Student Performance Evaluation)**.** Let

$$U = \{\text{Student1, Student2, Student3, Student4}\}$$

be a set of students. Assume two teachers provide evaluations in the form of fuzzy sets:

$$F_1 : U \to [0, 1], \quad F_1(\text{Student1}) = 0.9,\ F_1(\text{Student2}) = 0.75,\ F_1(\text{Student3}) = 0.8,\ F_1(\text{Student4}) = 0.85,$$

$$F_2 : U \to [0, 1], \quad F_2(\text{Student1}) = 0.88,\ F_2(\text{Student2}) = 0.8,\ F_2(\text{Student3}) = 0.82,\ F_2(\text{Student4}) = 0.9.$$

Let the index set be $I = \{1, 2\}$ and define a cluster $C_1 = \{1, 2\}$. Then the ClusterFuzzy Set G is given by

$$G(C_1)(x) = \max\{F_1(x),\ F_2(x)\} \quad \text{for all } x \in U.$$

For example, for $x = \text{Student2}$:

$$G(C_1)(\text{Student2}) = \max\{0.75,\ 0.8\} = 0.8.$$

Theorem 41 (Idempotence of Cluster Aggregation)**.** *Suppose that for every $i \in C \subseteq I$, the fuzzy set F_i is identical; that is, $F_i(x) = \mu(x)$ for all $x \in U$ and for all $i \in C$. Then, for every $x \in U$,*

$$G(C)(x) = \mu(x).$$

Proof: Since $F_i(x) = \mu(x)$ for all $i \in C$, it follows that

$$\max_{i \in C} F_i(x) = \max_{i \in C} \mu(x) = \mu(x).$$

Thus, $G(C)(x) = \mu(x)$ for every $x \in U$, proving the claim. □

Theorem 42 (Monotonicity of ClusterFuzzy Set Aggregation)**.** *Let C_1 and C_2 be clusters such that $C_1 \subseteq C_2 \subseteq I$. Then, for every $x \in U$,*

$$\max_{i \in C_1} F_i(x) \leq \max_{i \in C_2} F_i(x).$$

Proof: Since $C_1 \subseteq C_2$, every fuzzy value $F_i(x)$ for $i \in C_1$ is also included in the set $\{F_i(x) \mid i \in C_2\}$. Therefore,

$$\max_{i \in C_1} F_i(x) \leq \max_{i \in C_2} F_i(x),$$

for all $x \in U$. □

6 | Future Tasks

This section briefly outlines future research directions. Several extensions of Fuzzy Sets, such as Intuitionistic Fuzzy Sets [2, 5, 3, 1, 4], Neutrosophic Sets[74, 75, 80, 76, 82], Hyperneutrosophic Sets[77, 44, 45, 47, 48, 46, 43], and Plithogenic Sets[38, 78, 36, 39, 37, 81], are well known. A key future task is to explore the application of these advanced set-theoretic frameworks to the study of climate change.

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The datasets generated and/or analyzed during the current study are not publicly available due to privacy-preserving constraints but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in this research.

Ethical Approval

This article does not contain any studies involving human participants or animals conducted by any of the authors.

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