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Inverse Transformation to Generate Neutrosophic Random Variables Following Weibull and Geometric Distributions

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Abstract

In practical applications, we encounter many systems that cannot be studied directly, and the reason for this is due to the nature of the system or to the high cost. Therefore, we resort to the simulation process, which depends on conducting the study on systems similar to real systems, and then projecting these results, if they are appropriate, onto the system. The real system. The simulation process depends on generating a series of random numbers that follow a uniform probability distribution in the field [0, 1], then converting these random numbers into random variables that follow the probability distribution in which the system to be simulated operates. One of the most important conversion methods is the conversion method. The opposite. This method is used for probability distributions in which we can obtain a function inverse of its cumulative distribution function. In two previous researches, we used this method to generate neutrosophic random variables that follow a uniform distribution in the field [a, b] and the exponential distribution. In this research, we present a valuable study to clarify how to use this method. The method for generating neutrosophic random variables follows the Weibull distribution and the geometric distribution, based on what was presented in the classic study and in the research on neutrosophic random number generation.

Keywords: Neutrosophic Uniform Distribution, Simulation, Neutrosophic Random Number Generation, Weibull Distribution, Geometric Distribution.

1 | Introduction

Studies used in accordance with the concepts of classical logic have provided many methods that can be used to generate random variables that follow probability distributions, which have many uses in practical fields. One of these methods is the inverse transformation method, which can be used in probability distributions whose cumulative distribution function has an inverse function, but the results that we obtained are specific results that do not take into account the changes that may occur in the operating environment of the system to be simulated and to keep up with recent studies that have been presented using neutrosophic logic. Which included most branches of science [1-17], to obtain more accurate results, we presented in previous research a neutrosophical study to generate random numbers that follow a regular distribution with no specificity that can be enjoyed by either or both ends of the field [0,1]. Then we convert these random numbers into neutrosophic random variables that follow the probability distributions according to which the systems under study operate according to [18-26]. We used various methods in the conversion process, including the transformation method that was used to generate random variables that follow the regular distribution in the field [a, b] and the exponential distribution[19, 20], and given the great importance of the Weibull distribution

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and the geometric distribution, we present in this research a neutrosophical study of how to use the inverse transformation method to obtain random variables that follow them, which enables us to obtain more accurate simulation results that take into account all the conditions that the system's operating environment may experience to be simulated.

2 | Discussion

2.1 | Previous Studies

1. To generate classical random numbers that follow a uniform distribution in the interval $[0,1]$:[27]

Several methods can be used to obtain a series of classical random numbers.

$R_1, R_2, \ldots$ that follow a uniform distribution in the range $[0,1]$. In this research, we will use the mean square method defined according to the following relation:

$$R_{i+1} = Mid\left[R_i^2\right] ; i = 0,1,2,3, \tag{1}$$

Where Mid is the middle four ranks of R_i^2, and R_0 is chosen, i.e., a fractional random number composed of four ranks (called a seed) that does not contain zero in any of its four ranks.

2. To convert these random numbers into neutrosophic random numbers that follow a uniform distribution over the field $[0,1]$with the indeterminacy that can be enjoyed by either or both ends of the field: [18].

To convert the numbers resulting from (1) into neutrosophic random numbers that follow a uniform distribution over the field $[0,1]$, we distinguish the following forms for the field $[0,1]$ with the margin of indeterminacy δ where $\delta \in [0, m]$ and $0 < m < 1$.

- The first form: Indeterminacy at the minimum of the field, i.e., $[0 + \delta, 1]$. In this case, we substitute in the following relation:

$$NR_i = \frac{R_{i-}\delta}{1-\delta} \tag{2}$$

- The second form is indeterminacy at the upper limit of the field, i.e., $[0,1 + \delta]$. In this case, we substitute in the following relation:

$$NR_i = \frac{R_i}{1+\delta} \tag{3}$$

- The third form is indeterminacy in the upper and lower limits of the field, i.e., $[0 + \delta, 1 + \delta]$. In this case, we substitute in the following relation:

$$NR_i = R_i - \delta \tag{4}$$

3. Reverse conversion method: [27]

Using the sequence of random numbers $R_1, R_2, \ldots$ and the cumulative distribution function for the random variable, and since each of them is defined on the field $[0,1]$, we find that:

$$F(X) = R \tag{5}$$

$$\Longrightarrow X = F^{-1}(R) \tag{6}$$

2.2 | The Current Study

1. Generating neutrosophic random variables following the Weibull distribution:

The Weibull distribution is continuous. The Weibull distribution is defined by the following probability density function:

$$f(x) = \alpha\beta x^{\beta-1} e^{-\alpha x^{\beta}} \quad ; x \geq 0 \tag{7}$$

For $x > 0$, $\alpha > 0, \beta > 0$, the Weibull density function generates a family of probability density curves when the values of, α, β To generate random variables that follow it, we find the cumulative distribution function:

$$F(X) = \alpha\beta \int_{t=0}^{x} t^{\beta-1} e^{-\alpha t^{\beta}} dt$$

We make a change in the variable we impose:

$$y = \alpha t^{\beta} \Longrightarrow dy = \alpha\beta t^{\beta-1} dt$$

Substituting we find:

$$F(X) = \alpha\beta \int_{0}^{\alpha x^{\beta}} t^{\beta-1} e^{-y} \frac{dy}{\alpha\beta t^{\beta-1}} \Longrightarrow F(X) = 1 - e^{-\alpha x^{\beta}}$$

$$F(X) = 1 - e^{-\propto x^{\beta}}$$

According to relation (5) we find:

$$1 - R = e^{-\propto x^{\beta}}$$

R follows a uniform distribution in the domain $[0,1]$. Also, $1 - R$ follows a uniform distribution in the same domain, so:

$$R = e^{-\propto x^{\beta}} \Longrightarrow x = \left[-\frac{1}{\alpha} lnR\right]^{-\frac{1}{\beta}}$$

Therefore, to generate random variables that follow the Weibull distribution, we substitute the following relation:

$$x_i = \left[-\frac{1}{\alpha} lnR_i\right]^{-\frac{1}{\beta}} \tag{8}$$

2. Generating neutrosophic random variables following the Weibull distribution:

We know that to obtain neutrosophic random variables that follow a probability distribution based on a series of classical or neutrosophic random numbers, we distinguish three cases:

- The first case: Neutrosophic random numbers and the probability distribution are given in the classical form:

  In this case, the relation (8) is written as follows:

$$x_i = \left[-\frac{1}{\alpha} lnR_{iN}\right]^{-\frac{1}{\beta}} \tag{9}$$

Therefore, to generate neutrosophic random variables that follow the Weibull distribution

- We generate a series of random numbers that follow a uniform distribution in the range $[0,1]$, $R_1, R_2, \ldots$ using the relation (1).
- We convert these random numbers into neutrosophic random numbers by substituting them into one of the relations (2), (3), and (4).
- Substituting these resulting neutrosophic random numbers into the relation (9) we obtain what is required.

➢ The relation we get using the first form:

  We substitute the random numbers $R_1, R_2, \ldots$ into relation (2), and the result is substituted into relation (9) we get:

$$x_{iN} = \left[-\frac{1}{\alpha} lnR_{iN}\right]^{-\frac{1}{\beta}} = \left[-\frac{1}{\alpha} ln\left(\frac{R_i - \delta}{1-\delta}\right)\right]^{-\frac{1}{\beta}} \tag{10}$$

- The relation we get using the second form:

We substitute the random numbers $R_1, R_2, \ldots$ into relation (3), and the result is substituted into relation (9) we obtain:

$$x_{iN} = \left[-\frac{1}{\alpha} lnR_{iN}\right]^{-\frac{1}{\beta}} = \left[-\frac{1}{\alpha} ln\left(\frac{R_i}{1+\delta}\right)\right]^{-\frac{1}{\beta}} \tag{11}$$

- The relation we get using the third form:

We substitute the random numbers $R_1, R_2, \ldots$ into relation (4), and the result is substituted into relation (9) we get:

$$x_{iN} = \left[-\frac{1}{\alpha} lnR_{iN}\right]^{-\frac{1}{\beta}} = \left[-\frac{1}{\alpha} ln(R_i - \delta)\right]^{-\frac{1}{\beta}} \tag{12}$$

- In the second case: the random numbers are classical and the probability distribution is given in the neutrosophic form:

In this case, the relation (8) is written as follows:

$$x_i = \left[-\frac{1}{\alpha_N} lnR_i\right]^{-\frac{1}{\beta_N}}$$

Therefore, to generate neutrosophic random variables that follow the Weibull distribution

We generate a series of random numbers that follow a uniform distribution in the range $[0,1]$, $R_1, R_2, \ldots$, using the relation (1).

Substituting these resulting random numbers into the relation (9) we get:

$$x_{iN} = \left[-\frac{1}{\alpha_N} lnR_i\right]^{-\frac{1}{\beta_N}} \tag{13}$$

- The third case: Neutrosophic random numbers and the probability distribution are given in the neutrosophic form:

$$x_i = \left[-\frac{1}{\alpha_N} lnR_{iN}\right]^{-\frac{1}{\beta_N}} \tag{14}$$

Therefore, to generate neutrosophic random variables that follow the Weibull distribution.

We generate a series of random numbers that follow a uniform distribution in the range $[0,1]$, $R_1, R_2, \ldots$, using the relation (1).

We convert these random numbers into neutrosophic random numbers by substituting them into one of the relations (2), (3), and (4).

Substituting these resulting neutrosophic random numbers into the relation (14) we obtain what is required.

- The relation we get using the first form:

We substitute the random numbers $R_1, R_2, \ldots$ into relation (2), and the result is substituted into relation (9) we get:

$$x_{iN} = \left[-\frac{1}{\alpha} lnR_{iN}\right]^{-\frac{1}{\beta}} = \left[-\frac{1}{\alpha} ln\left(\frac{R_i - \delta}{1-\delta}\right)\right]^{-\frac{1}{\beta}} \tag{15}$$

➢ The relation we get using the second form:

We substitute the random numbers $R_1, R_2, \dots$ into relation (3), and the result is substituted into relation (9) we obtain:

$$x_{iN} = \left[-\frac{1}{\alpha_N} lnR_{iN}\right]^{-\frac{1}{\beta_N}} = \left[-\frac{1}{\alpha_N} ln\left(\frac{R_i}{1+\delta}\right)\right]^{-\frac{1}{\beta_N}} \tag{16}$$

➢ The relation we get using the third form:

We substitute the random numbers $R_1, R_2, \dots$ into relation (4), and the result is substituted into relation (9) we get:

$$x_{iN} = \left[-\frac{1}{\alpha_N} lnR_{iN}\right]^{-\frac{1}{\beta_N}} = \left[-\frac{1}{\alpha_N} ln(R_i - \delta)\right]^{-\frac{1}{\beta_N}} \tag{17}$$

3. Generating random variables that follow a geometric distribution:

A geometric distribution of discrete distributions. A random variable X, defined by the number of failures in a series of Bernoulli trials before the first success occurs, is said to be a geometric random variable. It has been used in the field of quality control and lag distributions in econometric models.

The following probability density function is known:

$$f(x) = pq^x \quad ; x = 0,1,2, \dots$$

p is defined as the probability of success in each Bernoulli trial, and it is $q = 1 - p$. The cumulative distribution function is given by the following relation:

$$F(x) = \sum_{k=0}^{x} pq^k$$

To generate random variables that follow it, we take advantage of the following relation:

$$1 - F(x) = q^{x+1}$$

and $\frac{[1-[F(x)]]}{q}$ has a range equal to one.

Using the inverse transformation method, we find:

$$R = q^x \Longrightarrow lnR = xlnq$$

Since x must be an integer, we choose x as the largest integer that satisfies the relation:

$$x \le \frac{lnR}{lnq}$$

So, to generate random variables that follow a geometric distribution, we substitute into the following relation:

$$x_i \le \frac{lnR_i}{lnq} \tag{18}$$

Where R_i are random numbers that follow a uniform distribution in the range $[0,1]$.

4. Generating neutrosophic random variables following a geometric distribution:

We know that to obtain neutrosophic random variables that follow a probability distribution based on a series of classical or neutrosophic random numbers, we distinguish three cases:

- The first case: Neutrosophic random numbers and the probability distribution are given in the classical form:

  In this case, the relation (81) is written as follows:

$$x_i \leq \frac{lnR_{iN}}{lnq} \tag{19}$$

Therefore, to generate neutrosophic random variables that follow a geometric distribution, we generate a series of random numbers that follow a uniform distribution in the range $[0,1]$, $R_1, R_2, \ldots$, using the relation (1).

We convert these random numbers into neutrosophic random numbers by substituting them into one of the relations (2), (3), (4)

Substituting these resulting neutrosophic random numbers into the relation (19) we obtain what is required.

- The relation we get using the first form:

We substitute the random numbers $R_1, R_2, \ldots$ into relation (2), and the result is substituted into relation (9) we get:

$$x_{iN} \leq \frac{lnR_{iN}}{lnq} = \frac{\ln(\frac{R_i-\delta}{1-\delta})}{lnq} \tag{20}$$

- The relation we get using the second form:

We substitute the random numbers $R_1, R_2, \ldots$ into relation (3), and the result is substituted into relation (9) we obtain:

$$x_{iN} \leq \frac{lnR_{iN}}{lnq} = \frac{\ln(\frac{R_i}{1+\delta})}{lnq} \tag{21}$$

- The relation we get using the third form:

We substitute the random numbers $R_1, R_2, \ldots$ into relation (4) and the result is substituted into relation (9) we get:

$$x_{iN} \leq \frac{lnR_{iN}}{lnq} = \frac{\ln(R_i-\delta\ )}{lnq} \tag{22}$$

- The second case: The random numbers are classical and the probability distribution is given in the neutrosophic form:

In this case, the relation (8) is written as follows:

$$x_i \leq \frac{lnR_i}{lnq_N}$$

Therefore, to generate neutrosophic random variables that follow the geometric distribution:

- We generate a series of random numbers that follow a uniform distribution in the range $[0,1]$, $R_1, R_2, \ldots$, using the relation (1).
- Substituting these resulting random numbers into the relation (9) we get:

$$x_{iN} \leq \frac{lnR_i}{lnq_N}$$

- The third case: Neutrosophic random numbers and the probability distribution are given in the neutrosophic form:

$$x_{iN} \leq \frac{lnR_{iN}}{lnq_N} \tag{24}$$

Therefore, to generate neutrosophic random variables that follow the Weibull distribution

- We generate a series of random numbers that follow a uniform distribution in the range $[0,1]$, $R_1, R_2, \ldots$, using the relation (1).
- We convert these random numbers into neutrosophic random numbers by substituting them into one of the relations (2), (3), and (4).
- Substituting these resulting neutrosophic random numbers into the relation (19) we obtain what is required.

➢ The relation we get using the first form:

We substitute the random numbers $R_1, R_2, \ldots$ into relation (2), and the result is substituted into relation (9) we get:

$$x_{iN} \leq \frac{lnR_{iN}}{lnq_N} = \frac{\ln(\frac{R_i - \delta}{1-\delta})}{lnq_N} \tag{25}$$

➢ The relation we get using the second form:

We substitute the random numbers $R_1, R_2, \ldots$ into relation (3), and the result is substituted into relation (9) we obtain:

$$x_{iN} \leq \frac{lnR_{iN}}{lnq_N} = \frac{\ln(\frac{R_i}{1+\delta})}{lnq_N} \tag{26}$$

➢ The relation we get using the third form:

We substitute the random numbers $R_1, R_2, \ldots$ into relation (4), and the result is substituted into relation (9) we get:

$$x_{iN} \leq \frac{lnR_{iN}}{lnq_N} = \frac{\ln(R_i - \delta)}{lnq_N} \tag{27}$$

3 | Practical Example

Starting from the seed $R_0 = 0.2151$, find two neutrosophic random variables.

They follow the Weibull distribution in the following two cases:

- $\alpha = 2 \;,\; \beta = 3$
- $\alpha_N \in \{2{,}4\} \;, \beta = 3$

The solution:

Starting from the seed $R_0 = 0.2151$ and using the mean square method we obtain the following two classical random numbers:

$$R_1 = 0.6268 \quad , R_2 = 0.2878$$

To obtain two Neutrosophic random variables following a Weibull distribution, we find:

- For the first case, the neutrosophic random numbers and the Weibull distribution are given in the classical form:

  We substitute the following data, $= 2 \;,\; \beta = 3 \;,\; R_1 = 0.6268 \;, R_2 = 0.2878 \;,\; \delta = [0{,}0.04]$. In relations (10), (11), (12) we find:

➢ From relation (10) we find:

$$x_{iN} = \left[-\frac{1}{\alpha} lnR_{iN}\right]^{-\frac{1}{\beta}} = \left[-\frac{1}{\alpha} ln\left(\frac{R_{i-}\delta}{1-\delta}\right)\right]^{-\frac{1}{\beta}} \qquad (10)$$

$$x_{1N} = \left[-\frac{1}{2} lnR_{1N}\right]^{-\frac{1}{3}} = \left[-\frac{1}{2} ln\left(\frac{0.6268-[0,0.04]}{1-[0,0.04]}\right)\right]^{-\frac{1}{3}} = \left[-\frac{1}{2} ln\left(\frac{[0.5868,0.6268]}{[0.96,1]}\right)\right]^{-\frac{1}{3}}$$

$$= \left[[0.2461,0.2336]\right]^{-\frac{1}{3}} = [1.5957,1.6237]$$

$$\Rightarrow x_{1N} \in [1.5957,1.6237]$$

$$x_{2N} = \left[-\frac{1}{2} lnR_{2N}\right]^{-\frac{1}{3}} = \left[-\frac{1}{2} ln\left(\frac{0.2878-[0,0.04]}{1-[0,0.04]}\right)\right]^{-\frac{1}{3}} = \left[-\frac{1}{2} ln\left(\frac{[0.2478,0.2878]}{[0.96,1]}\right)\right]^{-\frac{1}{3}}$$

$$= \left[[0.2581,0.2878]\right]^{-\frac{1}{3}} = [1.5146,1.5706]$$

$$\Rightarrow x_{2N} \in [1.5146,1.5706]$$

From relation (10) we get the following two neutrosophic random numbers:

$$x_{1N} \in [1.5957,1.6237] \quad , \quad x_{2N} \in [1.5146,1.5706]$$

- From relation (11) we find:

$$x_{iN} = \left[-\frac{1}{\alpha} lnR_{iN}\right]^{-\frac{1}{\beta}} = \left[-\frac{1}{\alpha} ln\left(\frac{R_i}{1+\delta}\right)\right]^{-\frac{1}{\beta}} \qquad (11)$$

$$x_{1N} = \left[-\frac{1}{2} lnR_{iN}\right]^{-\frac{1}{3}} = \left[-\frac{1}{2} ln\left(\frac{0.6268}{[0.96,1]}\right)\right]^{-\frac{1}{3}} = \left[-\frac{1}{2} ln[0.6268,0.6529]\right]^{-\frac{1}{3}} = [0.2336,0.2132]^{-\frac{1}{3}}$$

$$= [1.6739,1.6260]$$

$$\Rightarrow x_{1N} \in [1.6739,1.6260]$$

$$x_{2N} = \left[-\frac{1}{2} lnR_{2N}\right]^{-\frac{1}{3}} = \left[-\frac{1}{2} ln\left(\frac{0.2878}{[0.96,1]}\right)\right]^{-\frac{1}{3}} = \left[-\frac{1}{2} ln[0.2878,0.2998]\right]^{-\frac{1}{3}} = [0.6023,0.6227]^{-\frac{1}{3}}$$

$$= [1.1710,1.1841]$$

$$\Rightarrow x_{2N} \in [1.1710,1.1841]$$

From relation (11) we get the following two neutrosophic random numbers:

$$x_{1N} \in [1.6739,1.6260] \quad , \quad x_{2N} \in [1.1710,1.1841]$$

- From relation (12) we find:

$$x_{iN} = \left[-\frac{1}{\alpha} lnR_{iN}\right]^{-\frac{1}{\beta}} = \left[-\frac{1}{\alpha} ln(R_i - \delta\,)\right]^{-\frac{1}{\beta}} \qquad (12)$$

$$x_{1N} = \left[-\frac{1}{2} lnR_{iN}\right]^{-\frac{1}{3}} = \left[-\frac{1}{2} ln(0.6268-[0,0.04])\right]^{-\frac{1}{3}} = \left[-\frac{1}{2} ln([0.5868,0.6268])\right]^{-\frac{1}{3}}$$

$$= [0.2336,0.2665]^{-\frac{1}{3}} = [1.5539,1.6237]$$

$$\Rightarrow x_{2N} \in [1.5539,1.6237]$$

$$x_{2N} = \left[-\frac{1}{2} lnR_{2N}\right]^{-\frac{1}{3}} = \left[-\frac{1}{2} ln(0.2878 - [0,0.04])\right]^{-\frac{1}{3}} = \left[-\frac{1}{2} ln([0.2478,0.2878])\right]^{-\frac{1}{3}}$$
$$= [0.6227,0.6976]^{-\frac{1}{3}} = [1.1275,1.1710]$$

$$\Longrightarrow x_{2N} \in [1.1275,1.1710]$$

From relation (12) we get the following two neutrosophic random numbers:

$$x_{1N} \in [1.5539,1.6237] \ , \ x_{2N} \in [1.1275,1.1710]$$

In order to, $\alpha = 2$, $\beta = 3$, $R_1 = 0.6268$, $R_2 = 0.2878$, $\delta = [0,0.04]$. The two neutrosophic random numbers are:

$x_{1N} \in [1.5957,1.6237]$, $x_{2N} \in [1.5146,1.5706]$ or

$x_{1N} \in [1.6739,1.6260]$, $x_{2N} \in [1.1710,1.1841]$or

$x_{1N} \in [1.5539,1.6237]$, $x_{2N} \in [1.1275,1.1710]$

- For the second case, the classical random numbers and the Weibull distribution are given in the neutrosophic form:

We substitute the following data, $\alpha_N \in \{2,4\}$, $\beta = 3$, $R_1 = 0.6268$, $R_2 = 0.2878$

In relation (13) we find:

$$x_{iN} = \left[-\frac{1}{\alpha_N} lnR_i\right]^{-\frac{1}{\beta}} \quad (13)$$

$$x_{1N} = \left[-\frac{1}{\{2,4\}} \ln(0.6268)\right]^{-\frac{1}{3}}$$

$\alpha_N \in \{2,4\}$ So, for $\alpha_N = 2$ and $\beta = 3$ we find:

$$x_1 = \left[-\frac{1}{2} \ln(0.6268)\right]^{-\frac{1}{3}} = (0.2336)^{-\frac{1}{3}} = 1.6237$$

$$x_2 = \left[-\frac{1}{2} \ln(0.2878)\right]^{-\frac{1}{3}} = (0.6227)^{-\frac{1}{3}} = 1.1710$$

$\alpha_N \in \{2,4\}$ So, for $\alpha_N = 4$ and $\beta = 3$ we find:

$$x_1 = \left[-\frac{1}{4} \ln(0.6268)\right]^{-\frac{1}{3}} = (0.1168)^{-\frac{1}{3}} = 2.0457$$

$$x_2 = \left[-\frac{1}{4} \ln(0.2878)\right]^{-\frac{1}{3}} = (0.3114)^{-\frac{1}{3}} = 1.4753$$

From relation (13) we get the following two neutrosophic random numbers:

$$x_{1N} \in \{[1.2309,1.4384],[1.359,1.7106]\}$$

$$x_{2N} \in \{[1.07,1.1257],[1.1814,1.3387]\}$$

- For the third case, the random numbers are neutrosophic and the probability distribution is given in the neutrosophic form:

We substitute the following data: $\alpha_N \in \{2,4\}$, $\beta = 3, R_1 = 0.6268$, $R_2 = 0.2878$، $\delta = [0,0.04]$

In relations (15), (16), (17) we find:

➢ From relation (15) we find:

$$x_{iN} = \left[-\frac{1}{\alpha_N} lnR_{iN}\right]^{-\frac{1}{\beta}} = \left[-\frac{1}{\alpha_N} ln\left(\frac{R_{1-}\delta}{1-\delta}\right)\right]^{-\frac{1}{\beta}} \qquad (15)$$

$$x_{1N} = \left[-\frac{1}{\{2,4\}} lnR_{1N}\right]^{-\frac{1}{3}} = \left[-\frac{1}{\{2,4\}} ln\left(\frac{0.6268-[0,0.04]}{1-[0,0.04]}\right)\right]^{-\frac{1}{3}} = \left[-\frac{1}{\{2,4\}} ln\left(\frac{[0.5868,0.6268]}{[0.96,1]}\right)\right]^{-\frac{1}{3}}$$
$$= \left[-\frac{1}{\{2,4\}} ln([0.6113,0.6268])\right]^{-\frac{1}{3}}$$

$$x_{2N} = \left[-\frac{1}{\{2,4\}} lnR_{2N}\right]^{-\frac{1}{3}} = \left[-\frac{1}{\{2,4\}} ln\left(\frac{0.2878-[0,0.04]}{1-[0,0.04]}\right)\right]^{-\frac{1}{3}} = \left[-\frac{1}{\{2,4\}} ln([0.2581,0.2878])\right]^{-\frac{1}{3}}$$

$\alpha_N \in \{2,4\}$ So, for $\alpha_N = 2$ and $\beta = 3$ we find:

$$x_{1N} = \left[-\frac{1}{2} ln([0.6113,0.6268])\right]^{-\frac{1}{3}} = ([0.2461,0.2336])^{-\frac{1}{3}} = [1.5957,1.6237]$$
$$\Rightarrow x_{1N} \in [1.5957,1.6237]$$

$$x_{2N} = \left[-\frac{1}{2} ln([0.2581,0.2878])\right]^{-\frac{1}{3}} = ([0.2581,0.2878])^{-\frac{1}{3}} = [1.5146,1.5706]$$
$$\Rightarrow x_{2N} \in [1.5146,1.5706]$$

$\alpha_N \in \{2,4\}$ So, for $\alpha_N = 4$ and $\beta = 3$ we find:

$$x_{1N} = \left[-\frac{1}{4} ln([0.6113,0.6268])\right]^{-\frac{1}{3}} = [[0.1230,0.1168]]^{-\frac{1}{3}} = [2.0108,2.0457]$$
$$\Rightarrow x_{1N} \in [2.0108,2.0457]$$

$$x_{2N} = \left[-\frac{1}{4} ln([0.2581,0.2878])\right]^{-\frac{1}{3}} = ([0.3114,0.3386])^{-\frac{1}{3}} = [1.4347,1.4753]$$

$$\Rightarrow x_{2N} \in [1.4347,1.4753]$$

From relation (15) we get the following two neutrosophic random numbers:

$$x_{1N} \in \{[1.5957,1.6237],[2.0108,2.0457]\}$$

$$x_{2N} \in \{[1.5146,1.5706],[1.4347,1.4753]\}$$

➢ From relation (16) we find:

$$x_{iN} = \left[-\frac{1}{\alpha_N} lnR_{iN}\right]^{-\frac{1}{\beta_N}} = \left[-\frac{1}{\alpha_N} ln\left(\frac{R_i}{1+\delta}\right)\right]^{-\frac{1}{\beta_N}} \qquad (16)$$

$$x_{1N} = \left[-\frac{1}{\{2,4\}} lnR_{1N}\right]^{-\frac{1}{3}} = \left[-\frac{1}{\{2,4\}} ln\left(\frac{0.6268}{1-[0,0.04]}\right)\right]^{-\frac{1}{3}} = \left[-\frac{1}{\{2,4\}} ln\left(\frac{0.6268}{[0.96,1]}\right)\right]^{-\frac{1}{3}}$$
$$= \left[-\frac{1}{\{2,4\}} ln([0.6529,0.6268])\right]^{-\frac{1}{3}}$$

$$x_{2N} = \left[-\frac{1}{\{2,4\}} lnR_{2N}\right]^{-\frac{1}{3}} = \left[-\frac{1}{\{2,4\}} ln\left(\frac{0.2878}{1-[0,0.04]}\right)\right]^{-\frac{1}{3}}$$
$$= \left[-\frac{1}{\{2,4\}} ln\left(\frac{0.2878}{[0.96,1]}\right)\right]^{-\frac{1}{3}} \left[-\frac{1}{\{2,4\}} ln([0.2878,0.2998])\right]^{-\frac{1}{3}}$$

$\alpha_N \in \{2,4\}$ So, for $\alpha_N = 2$ and $\beta = 3$ we find:

$$x_{1N} = \left[-\frac{1}{2} lnR_{1N}\right]^{-\frac{1}{3}} = \left[-\frac{1}{2} ln\left(\frac{0.6268}{1-[0,0.04]}\right)\right]^{-\frac{1}{3}} = \left[-\frac{1}{2} ln\left(\frac{0.6268}{[0.96,1]}\right)\right]^{-\frac{1}{3}} = \left[-\frac{1}{2} ln([0.6529,0.6268])\right]^{-\frac{1}{3}}$$
$$= ([0.2132,0.2336])^{-\frac{1}{3}} = [1.6237,1.6739]$$

$$x_{2N} = \left[-\frac{1}{2} lnR_{2N}\right]^{-\frac{1}{3}} = \left[-\frac{1}{2} ln\left(\frac{0.2878}{1-[0,0.04]}\right)\right]^{-\frac{1}{3}} = \left[-\frac{1}{2} ln\left(\frac{0.2878}{[0.96,1]}\right)\right]^{-\frac{1}{3}} \left[-\frac{1}{2} ln([0.2878,0.2998])\right]^{-\frac{1}{3}}$$
$$= ([0.6023,0.6227])^{-\frac{1}{3}} = [1.1710,1.1841]$$

$\alpha_N \in \{2,4\}$ So, for $\alpha_N = 4$ and $\beta = 3$ we find:

$$x_{1N} = \left[-\frac{1}{4} lnR_{1N}\right]^{-\frac{1}{3}} = \left[-\frac{1}{4} ln\left(\frac{0.6268}{1-[0,0.04]}\right)\right]^{-\frac{1}{3}} = \left[-\frac{1}{4} ln\left(\frac{0.6268}{[0.96,1]}\right)\right]^{-\frac{1}{3}} = \left[-\frac{1}{4} ln([0.6529,0.6268])\right]^{-\frac{1}{3}}$$
$$= ([0.1066,0.1168])^{-\frac{1}{3}} = [2.1090,2.0457]$$

$$x_{2N} = \left[-\frac{1}{4} lnR_{2N}\right]^{-\frac{1}{3}} = \left[-\frac{1}{4} ln\left(\frac{0.2878}{1-[0,0.04]}\right)\right]^{-\frac{1}{3}} = \left[-\frac{1}{4} ln\left(\frac{0.2878}{[0.96,1]}\right)\right]^{-\frac{1}{3}} \left[-\frac{1}{4} ln([0.2878,0.2998])\right]^{-\frac{1}{3}}$$
$$= ([0.3012,0.3114])^{-\frac{1}{3}} = [1.4753,1.4918]$$

From relation (16) we get the following two neutrosophic random numbers:

$$x_{1N} \in \{[1.6237,1.6739], [2.1090,2.0457]\}$$

$$x_{2N} \in \{[1.1710,1.1841], [1.4753,1.4918]\}$$

- From relation (17) we find:

$$x_{iN} = \left[-\frac{1}{\alpha_N} lnR_{iN}\right]^{-\frac{1}{\beta_N}} = \left[-\frac{1}{\alpha_N} ln(R_i - \delta)\right]^{-\frac{1}{\beta_N}} \qquad (17)$$

$$x_{1N} = \left[-\frac{1}{\{2,4\}} lnR_{iN}\right]^{-\frac{1}{3}} = \left[-\frac{1}{\{2,4\}} ln(0.6268 - [0,0.04])\right]^{-\frac{1}{3}} = \left[-\frac{1}{\{2,4\}} ln([0.5868,0.6268])\right]^{-\frac{1}{3}}$$

$$x_{2N} = \left[-\frac{1}{\{2,4\}} lnR_{2N}\right]^{-\frac{1}{3}} = \left[-\frac{1}{\{2,4\}} ln(0.2878 - [0,0.04])\right]^{-\frac{1}{3}} = \left[-\frac{1}{\{2,4\}} ln([0.2478,0.2878])\right]^{-\frac{1}{3}}$$

$\alpha_N \in \{2,4\}$ So, for $\alpha_N = 2$ and $\beta = 3$ we find:

$$x_{1N} = \left[-\frac{1}{2} lnR_{iN}\right]^{-\frac{1}{3}} = \left[-\frac{1}{2} ln(0.6268 - [0,0.04])\right]^{-\frac{1}{3}} = \left[-\frac{1}{2} ln([0.5868,0.6268])\right]^{-\frac{1}{3}}$$
$$= ([0.2665,0.2336])^{-\frac{1}{3}} = [1.5539,1.6237]$$

$$x_{2N} = \left[-\frac{1}{2} lnR_{2N}\right]^{-\frac{1}{3}} = \left[-\frac{1}{2} ln(0.2878 - [0,0.04])\right]^{-\frac{1}{3}} = \left[-\frac{1}{2} ln([0.2478,0.2878])\right]^{-\frac{1}{3}}$$
$$= ([0.6227,0.6976])^{-\frac{1}{3}} = [1.1275,1.1710]$$

$\alpha_N \in \{2,4\}$ So, for $\alpha_N = 4$ and $\beta = 3$ we find:

$$x_{1N} = \left[-\frac{1}{4} lnR_{iN}\right]^{-\frac{1}{3}} = \left[-\frac{1}{4} ln(0.6268 - [0,0.04])\right]^{-\frac{1}{3}} = \left[-\frac{1}{4} ln([0.5868,0.6268])\right]^{-\frac{1}{3}}$$
$$= ([0.1333,0.1168])^{-\frac{1}{3}} = [1.9576,2.0457]$$

$$x_{2N} = \left[-\frac{1}{4} lnR_{2N}\right]^{-\frac{1}{3}} = \left[-\frac{1}{4} ln(0.2878 - [0,0.04])\right]^{-\frac{1}{3}} = \left[-\frac{1}{4} ln([0.2478,0.2878])\right]^{-\frac{1}{3}}$$
$$= ([0.3114,0.3488])^{-\frac{1}{3}} = [1.4206,1.4753]$$

From relation (17) we get the following two neutrosophic random numbers:

$$x_{1N} \in \{[1.5539,1.6237], [1.9576,2.0457]\}$$

$$x_{2N} \in \{[1.1275,1.1710], [1.4206,1.4753]\}$$

Note

We follow the same solution method if what is required is, starting from the seed $R_0 = 0.2151$, find two neutrosophic random variables that follow the two-case distribution in the following two cases:

- $p = 0.75$
- $p \in \{0.25,0.75\}$

4 | Conclusion and Results

In this research, we presented a neutrosophic study to generate random variables that follow the Weibull distribution and the geometric distribution using the inverse transformation method. The study included all cases that produce neutrosophic random variables that can be used to simulate systems that operate according to the geometric distribution and the Weibull distribution and give results through which decision-makers can make ideal decisions. For the functioning of these systems in all conditions that the work environment may experience.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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