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A New Neutrosophic Algebraic Structure II

Riad K. Al-Hamido [1,2,*]

[1] Faculty of Science, AlFurat University, Deir-ez-Zor, Syrian Arab Republic.

[2] Cordoba private University, Alqamishli branch, Syrian Arab Republic; riad-hamido1983@hotmail.com.

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Abstract

A new approach of “neutrosophic group” is defined in the new work, this new important concept will open new horizons and direction in front of researchers to study "new neutrosophic algebraic" such that neutrosophic ring, neutrosophic module, and neutrosophic vector space which are different from old one. In this paper, we define “new neutrosophic group” and “ new neutrosophic subgroup “ in a new way that is different from the previous versions and we study some properties of this “ new neutrosophic group “. Also, we discuss the relationship of the “new neutrosophic group” with other “classical groups “and prove some results in the context of the new neutrosophic group. Finally, we show a property that holds in the new neutrosophic group which does not hold in the “ neutrosophic group “, the important one is that a “ neutrosophic group “ is not a “ classical group”, but a “new neutrosophic group “ is “ classical group”. Moreover, we studied the “neutrosophic symbolic Turiyam group “and “new neutrosophic symbolic Turiyam group” and studied its basic properties.

Keywords: New Neutrosophic Group, New Neutrosophic Subgroup, Neutrosophic Symbolic Turiyam Group.

1 | Introduction

Since F. Smarandache introduced the "neutrosophic" in [1,2] "neutrosophic" studied in many sciences, such as algebra by introducing neutrosophic group which played a basic role in studying many neutrosophic algebraic structures as a neutrosophic ring, neutrosophic module, and neutrosophic vector space.

In recent years, Agboola et al. studied the concept of “ neutrosophic group “ and “ neutrosophic ring “ in [3, 4], They also, In 2015, introduced the concept of “ refined neutrosophic algebraic structures ” [5], and presented “ refined neutrosophic groups”. Also, Adeleke et al. in [6, 7] in 2020, studied several refined concepts such as “refined neutrosophic rings”, “refined neutrosophic ideals” and “refined neutrosophic homomorphisms” in detail.

Also, many researchers study applications of “ neutrosophic group “ in topology, such as R. Al-Hamido, in 2021, [8] studied “ neutrosophic bi-topological groups ”, and investigated its basic properties.

Before this study, Q. Imran, et al. in [9] studied several types of “neutrosophic topological groups” and introduced their basic properties. Also, Sumathi et al. in [10] introduced the concept of “neutrosophic topological groups”.

Also, P. K. Singh, et al. in [11] defined the concept of a “symbolic Turiyam ring “as an application of the “Turiyam symbolic set” [12, 13] and as a “new generalization” of “neutrosophic rings”.

Corresponding Author: riad-hamido1983@alfuratuniv.edu.sy

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Also, R. Al-Hamido, in [14] defined "new neutrosophic algebraic structures" as "new neutrosophic groupoid" and "new neutrosophic monoid".

The "neutrosophic group" is not "a group" and also has many properties that hold in the "classical group", and that do not hold in "neutrosophic group theory". So we think about defining a "new neutrosophic group" which is different from the "neutrosophic group".

In this paper, we defined and studied a "new neutrosophic group" and a "new neutrosophic subgroup "for the first time. This " new neutrosophic group " opens the door to re-defining many " neutrosophic algebraic structures ", such as the "new neutrosophic ring", and it may contribute to solving some open problems that the " neutrosophic group" could not solve.

Also, we studied the basic properties of this "new neutrosophic algebraic structure".

2 | Preliminaries

Remark 2.1: the neutrosophic element I where I is an "indeterminate" and (I) is such that ($I^2 = I$).

Definition 2.2. [10] If (Ģ, ∗) is any group, the "neutrosophic group" is generated by (I) and (Ģ) under (∗) denoted by Ð(Ģ)={< Ģ ∪I>,∗}.

Theorem 2. 3. [10] If (Ģ, ∗) be a group, then Ð(Ģ) = {⟨ Ģ ∪ I ⟩, ∗} be the "neutrosophic group".

i. Ð(Ģ) is not a group (in general).

ii. Ð(Ģ) contains a" group ".

Definition 2.4. [11] If R be a "ring", we define the "symbolic Turiyam ring" (STR) as $T_R=\{a+bT+cF+dI+eY;a,b,c,d,e\in R\}$.

- In case R is a "field" then T_R is called "symbolic Turiyam field "(STF).

Definition 2.5. [14] If (Ģ,∗) be an "groupoid" and let Ģ(I)={α+βI : α, β ∈ Ģ}, And ' ⚹̌ ' be a "binary operation" on Ģ(I) defined as :

$$(\alpha+\beta I) \check{\ast} (c+s I)=(\alpha\ast c)+(\beta\ast s)I \quad \forall\, \alpha,\beta,c,s\in Ģ$$

Then: $(Ģ(I),\check{\ast})$ is a "groupoid", called it "new neutrosophic groupoid".

Definition 2.6. [14] If (Ģ,∗) be an "semigroup" and let Ģ(I)={α+βI : α, β ∈ Ģ}, And ' ⚹̌ ' be a" binary operation" on Ģ(I) defined as:

$$(\alpha+\beta I) \check{\ast} (c+s I)=(\alpha\ast c)+(\beta\ast s)I \quad \forall\, \alpha,\beta,c,s\in Ģ$$

Then: $(Ģ(I).\check{\ast})$ is a "semigroup", called it "new neutrosophic semigroup".

3 | New Neutrosophic Group

In this part, we defined "new neutrosophic group" and "new neutrosophic subgroup" and studied its "basic properties".

Theorem 3.1: If (Ģ,∗) be an "group" and let Ģ(I)={ α+βI : α, β ∈ Ģ }, And ' ⚹̌ ' be a "binary operation" on Ģ(I) defined as follows:

$$(\alpha+\beta I) \check{\ast} (c+s I)=(\alpha\ast c)+(\beta\ast s)I \quad \forall\, \alpha,\beta,c,s\in Ģ$$

Then: $(Ģ(I).\check{\ast})$ is a "group" we called it "new neutrosophic group".

Proof:

i. $\forall\ (\alpha+\beta I)\check{\ast}(c+sI)\in Ģ(I)$ then $(\alpha+\beta I)\check{\ast}(c+sI)=(\alpha\ast c)+(\beta\ast s)I\in Ģ(I)$ implies that Ģ is closed under ⚹̌ .

ii. $\forall\ (\alpha+\beta I), (c+sI), (e+fI) \in Ģ(I)$ then $[(\alpha+\beta I) \check{*} (c+sI)] \check{*} (e+fI) =$

$[(\alpha+\beta I) + (c+sI)] \check{*} (e+fI) = [(\alpha * c) * e] + [(\beta * s) * f]I$ (Since $*$ such that associative Law)

$$= [\alpha * (c * e)] + [\beta * (s * f)]I = (\alpha+\beta I) \check{*} [(c * e) + (s * f)I]$$

$$= (\alpha+\beta I) \check{*} [(c+sI) \check{*} (e+fI)]$$

(Associative law).

i. for every $(c+sI) \in Ģ(I)$There exists an identity element $(e+eI) \in Ģ(I)$ such that

$$(c+sI) \check{*} (e+eI) = (c * e) + (s * e)I \ = (c+sI)$$

$$(e+eI) \check{*} (c+sI) = (e * c) + (e * s)I \ = (c+sI)$$

Therefore$(c+sI) \check{*} (e+eI) = (e+eI) \check{*} (c+sI) = (c+sI)$.

ii. for every $(a+bI) \in Ģ(I)$ there exists an element $(a^{-1}+b^{-1}I) \in Ģ(I)$ such that

$$(c+sI) \check{*} (c^{-1}+s^{-1}I) = (c * c^{-1}) + (s * s^{-1})I = e+eI$$

$$(c^{-1}+s^{-1}I) \check{*} (c+sI) = (c^{-1} * c) + (s^{-1} * s)I = e+eI$$

Therefore $(c+sI) \check{*} (c^{-1}+s^{-1}I) = (c^{-1}+s^{-1}I) \check{*} (c+sI) = e+eI$

(The existence of inverse in Ģ(I)).

If (Ģ,$*$) be an "group" and let Ģ(I)={ $\alpha+\beta I$: $\alpha, \beta \in$ Ģ }, And ' $\check{*}$ ' be a "binary operation" on Ģ(I) defined as follows:

$$(\alpha+\beta I) \check{*} (c+sI) = (\alpha * c) + (\beta * s)I \quad \forall\ \alpha, \beta, c, s \in Ģ$$

Then: $(Ģ(I).\check{*})$ is a "group" we called it "new neutrosophic group".

Definition 3.2: If (Ģ,$*$) be an "group" and let Ģ(I)={ $\alpha+\beta I$: $\alpha, \beta \in$ Ģ }, And ' $\check{*}$ ' be a "binary operation" on Ģ(I) defined as follows:

$$(\alpha+\beta I) \check{*} (c+sI) = (\alpha * c) + (\beta * s)I \quad \forall\ \alpha, \beta, c, s \in Ģ$$

Then: $(Ģ(I).\check{*})$ is a "group" we called it "new neutrosophic group".

Example 3.3: If (R,+) be an "group" and let Ģ(I)={ $\alpha+\beta I$: $\alpha, \beta \in R$ }, And ' $\check{*}$ ' be a "binary operation" on Ģ(I) defined as the following:

$$(\alpha+\beta I) \check{*} (c+sI) = (\alpha * c) + (\beta * s)I \quad \forall\ \alpha, \beta, c, s \in R.$$

Then: $(R(I).\check{*})$ is a new neutrosophic group

Example 3.4: Let $\bar{\mathcal{R}} = \mathcal{R}$ -{0} then $(\bar{\mathcal{R}},.)$ be an "group" and let Ģ(I)={ $\alpha+\beta I$: $\alpha, \beta \in \bar{\mathcal{R}}$ }, And ' $\check{*}$ ' be a "binary operation" on Ģ(I) defined as:

$$(\alpha+\beta I) \check{*} (c+sI) = (\alpha * c) + (\beta * s)I \quad \forall\ \alpha, \beta, c, s \in \bar{\mathcal{R}}$$

Then: $(\bar{\mathcal{R}}(I).\check{*})$ is a "new neutrosophic group".

Remark 3.5: We know that a "neutrosophic group" is not a "group", but a "new neutrosophic group" is a "group".

Neutrosophic group $\nrightarrow$ group

The new neutrosophic group $\longrightarrow$ group

Definition 3.6: If $(Ģ(I),.)$ be a "new neutrosophic group" then if $\divideontimes$ be a "commutative binary operation" on Ģ(I) Then: $(Ģ(I).\divideontimes)$ is called the" commutative new neutrosophic group".

Example 3.7: Let $\bar{\mathcal{R}}= \mathcal{R}$ -{0} then $(\bar{\mathcal{R}},.)$ be an "group" and let Ģ(I)={ $\alpha+\beta I : \alpha, \beta \in \bar{\mathcal{R}}$ }, And ' $\divideontimes$ ' be a "binary operation" on Ģ(I) defined as:

$$(\alpha + \beta I) \divideontimes (c + \mathfrak{s}I) = (\alpha * c) + (\beta * \mathfrak{s})I \quad \forall\, \alpha, \beta, c, \mathfrak{s} \in \bar{\mathcal{R}}$$

Then: $(\bar{\mathcal{R}}(I).\divideontimes)$ is a "commutative new neutrosophic group", because

$$(\alpha + \beta I) \divideontimes (c + \mathfrak{s}I) = (\alpha. c) + (\beta. \mathfrak{s})I\ \forall\, \alpha, \beta, c, \mathfrak{s} \in \bar{\mathcal{R}}.$$

- If (Ģ,*) is a "commutative group", what about $(Ģ(I).\divideontimes)$. The following remark answer

Remark 3.8: If (Ģ,*) is a " commutative group ", then $(Ģ(I).\divideontimes)$ is a "commutative new neutrosophic group".

Proof:

Since (Ģ,*) is a "commutative group" and ' $\divideontimes$ ' be a "binary operation" on Ģ(I) defined as :

$$(\alpha + \beta I) \divideontimes (c + \mathfrak{s}I) = (\alpha * c) + (\beta * \mathfrak{s})I \quad \forall\, \alpha, \beta, c, \mathfrak{s} \in \bar{\mathcal{R}}$$

Then: $(Ģ(I).\divideontimes)$ is a "commutative new neutrosophic group", because

$$(\alpha + \beta I) \divideontimes (c + \mathfrak{s}I) = (\alpha * c) + (\beta * \mathfrak{s})I = (c * \alpha) + (\mathfrak{s} * \beta)I = (c + \mathfrak{s}I) \divideontimes (\alpha + \beta I) \forall\, \forall\, \alpha, \beta, c, \mathfrak{s} \in \bar{\mathcal{R}}$$

Definition 3.9: A subset $(M.\divideontimes)$ of a "new neutrosophic group" $(Ģ(I).\divideontimes)$ is said to be a "new neutrosophic subgroup" of Ģ(I) if $(M.\divideontimes)$ is also a" new neutrosophic group".

Example 3.10: Let $\bar{\mathcal{R}}= \mathcal{R}$ -{0} then $(\bar{\mathcal{R}},.)$ be an "group" and let Ģ(I)={ $\alpha+\beta I : \alpha, \beta \in \bar{\mathcal{R}}$ }, And ' $\divideontimes$ ' be a "binary operation" on Ģ(I) defined as:

$$(\alpha + \beta I) \divideontimes (c + \mathfrak{s}I) = (\alpha. c) + (\beta. \mathfrak{s})I \quad \forall\, \alpha, \beta, c, \mathfrak{s} \in \bar{\mathcal{R}}$$

Then: $(\bar{\mathcal{R}}(I).\divideontimes)$ is a "new neutrosophic group", because

$$(\alpha + \beta I) \divideontimes (c + \mathfrak{s}I) = (\alpha. c) + (\beta. \mathfrak{s})I\ \forall\, \alpha, \beta, c, \mathfrak{s} \in \bar{\mathcal{R}}.$$

Let Ŋ ={-1,1} Then : $(Ŋ(I).\divideontimes)$ is a" new neutrosophic subgroup".

Theorem 3.11: Let $(Ģ(I).\divideontimes)$ is a "new neutrosophic group" and $Ŋ(I)$ is a "subset" of $Ģ(I)$ then:

$(Ŋ(I).\divideontimes)$ is "a new neutrosophic subgroup" of $(Ģ(I).\divideontimes)$ if it satisfied:

1) a+bI, c+dI∈Ɗ(I) then a+bI $\divideontimes$ c+dI ∈Ɗ(I).
2) a+bI∈Ɗ(I) then $a^{-1} + b^{-1}I$∈Ɗ(I).

Proof:

i. from 1) ∀ a+bI, c+dI∈Ŋ(I) then a+bI ⁎̆ c+dI ∈Ŋ(I).

implies Ŋ(I) is closed under⁎̆ .

ii. for all $a + bI$∈Ŋ(I) (from 2)) then $a^{-1} + b^{-1}I$∈Ŋ(I), then

(a+bI) ⁎̆ $(a^{-1} + b^{-1}I) = e + ei$ ∈Ŋ(I)and (a+bI) ⁎̆ $(e + ei) = a + ei = (e + ei)$ ⁎̆ $(a + bi)$

iii. for all $a + bI$∈Ŋ(I) (from 2)) then $a^{-1} + b^{-1}I$∈Ŋ(I) and (a+bI) ⁎̆ $(a^{-1} + b^{-1}I) = e + eI = (a^{-1} + b^{-1}I)$ ⁎̆ $(a + bI)$
(The existence of inverse in Ģ(I)).
Therefore(Ŋ(I).⁎̆) is "a new neutrosophic subgroup" of (Ģ(I).⁎̆).

Theorem 3.12: Let (Ģ(I),⁎̆) is "a new neutrosophic group" and Ŋ(I) is a subset of Ģ(I) then:

(Ŋ(I),⁎̆) is "a new neutrosophic subgroup" of (Ģ(I),⁎̆) if it satisfied:

∀ $a + bI.$ $c + dI$∈Ŋ(I) then (a+bI) ⁎̆ $(c^{-1} + d^{-1}I)$ ∈Ŋ(I).

Proof:

i. For all $a + Bi$∈Ŋ(I), since $a + Bi$∈Ŋ(I) then e+eI= $a + bI$ ⁎̆ $(a^{-1} + b^{-1}I)$∈Ŋ(I). Therefore (e+eI) ⁎̆ $(a^{-1} + b^{-1}I) = (a^{-1} + b^{-1}I)$∈Ŋ(I).

Therefore for all $a + bI$∈Ŋ(I) then $(a^{-1} + b^{-1}I)$∈Ŋ(I).

ii. ∀ $a + bi, c + di$∈Ŋ(I) then $(a + bi)$ ⁎̆ $(c + di)$∈Ŋ(I).since $(c + di)$∈Ŋ(I) then $c^{-1} + d^{-1}I$∈Ŋ(I), therefore $(a + bi)$ ⁎̆ $(c^{-1} + d^{-1}I)^{-1} = (a + bi)$ ⁎̆ $(c + di)$∈Ŋ(I)

by i) and ii) and theorem 3.11 (Ŋ(I).⁎̆) is "a new neutrosophic subgroup" of (Ģ(I).⁎̆).

Theorem 3.13: If (Ŋ, ∗) is a "subgroup" of (Ģ. ∗) then:

A subset (Ŋ(I).⁎̆) is said to be "a new neutrosophic subgroup" of (Ģ(I).⁎̆).

Proof:

Since (Ŋ. ∗) is a "subgroup" of (Ģ. ∗) then (Ŋ. ∗) is also a "group", therefore (Ŋ(I).⁎̆) is a "new neutrosophic group", so (Ŋ(I).⁎̆) is "a new neutrosophic subgroup" of (Ģ(I).⁎̆).

Theorem 3.14: If a subset Ŋ of the "group" (Ģ. ∗) such that:

1) a,b∈N then a∗b∈Ŋ.

2) ∀ a∈Ŋ then a^{-1}∈Ŋ.

Then: (Ŋ(I).⁎̆) is "a new neutrosophic subgroup" of (Ģ(I).⁎̆).

Proof:

i. From 1) ∀ a,b∈Ŋ then a∗b∈Ŋ.

Implies that Ŋ is closed under ∗.

ii. for all a∈Ŋ (From 2) then a^{-1}∈Ŋ, then $a*a^{-1} = e \in$ Ŋ and $a*e = e*a = a$.

iv. for all a∈Ŋ (from 2)) then a^{-1}∈Ŋ.
(The existence of inverse).

Therefore $(Ŋ.*)$ is "subgroup of the group" $(Ģ.*)$, therefore$(Ŋ(I).\check{*})$ is "a new neutrosophic subgroup" of $(Ģ(I).\check{*})$.

Example 3.15: Let $\breve{Q}$=Q-{0} then ($\breve{Q}$,.) be an "group" and let Ģ(I)={ α+βI : α, β ∈ $\breve{Q}$ }, And ' $\check{*}$ ' be a "binary operation" on Ģ(I) defined as the following:

$$(\alpha + \beta I) \check{*} (c + s I) = (\alpha . c) + (\beta . s) I \ \forall\ \alpha, \beta, c, s \in \breve{Q}.$$

Then: $(\breve{Q}(I).\check{*})$ is a "new neutrosophic group".

Let N={-1,1} Then(Ŋ,.) be an "subgroup" and therefore $(Ŋ(I).\check{*})$ is a "neutrosophic subgroup".

Example 3.16: Let (R,+) be an "group" and let Ģ(I)={ α+βI : α, β ∈ R }, And ' $\check{*}$ ' be a" binary operation" on Ģ(I) defined as:

$$(\alpha + \beta I) \check{*} (c + s I) = (\alpha + c) + (\beta + s) I \ \forall\ \alpha, \beta, c, s \in R$$

Then: $(R(I).\check{*})$ is a "new neutrosophic group".

We now That (Z,+) be an subgroup of (R,+) and therefore $(Z(I),\check{*})$ is a "neutrosophic subgroup" of $(R(I).\check{*})$.

Theorem 3.17: If A subset Ŋ of the "group" $(Ģ.*)$ such that:

∀ a,b∈Ŋ then a∗b^{-1}∈Ŋ.

then:

$(Ŋ(I).\check{*})$ is a "neutrosophic subgroup" of $(Ģ(I).\check{*})$.

Proof.

i. for all a∈Ŋ, since a∈Ŋ then e=a∗ a^{-1}∈Ŋ. Therefore e∗ a^{-1}= a^{-1}∈Ŋ

Therefore ∀ a∈Ŋ then a^{-1}∈Ŋ

ii. ∀ a,b∈Ŋ then a∗b∈Ŋ.since b∈Ŋ then b^{-1}∈Ŋ,therefore $a*(b^{-1})^{-1} = a*b \in Ŋ$

by i) and ii) and theorem3 $(Ŋ.*)$ is a subgroup of $(Ģ.*)$.

Therefore $(Ŋ(I).\check{*})$ is a "new neutrosophic subgroup" of $(Ģ(I).\check{*})$.

Theorem 3.18: If $(Ŋ.*)$ is a "subgroup" of $(M.*)$ and $(M.*)$ is a "subgroup" of $(Ģ.*)$then: $(Ŋ(I).\check{*})$ is a "new neutrosophic subgroup" of $(Ģ(I).\check{*})$

Proof:

Since $(Ŋ.*)$ is a "subgroup" of $(M.*)$ and $(M.*)$ is a "subgroup" of $(Ģ.*)$then: $(Ŋ.*)$ is a subgroup of $(Ģ.*)$. Therefore $(Ŋ(I).\check{*})$ is a new neutrosophic subgroup of $(Ģ(I).\check{*})$.

Theorem 3.19: If $(N(I).\check{*})$ is a new neutrosophic subgroup of $(M(I).\check{*})$ and $(M(I).\check{*})$ is a "new neutrosophic subgroup" of $(Ģ(I).\check{*})$then: $(Ŋ(I).\check{*})$ is a "new neutrosophic subgroup" of $(Ģ(I).\check{*})$

Proof:

Follow from theorem 3.18.

4 | A New Neutrosophic Symbolic Turiyam Group

In this section, we studied the neutrosophic symbolic Turiyam group and the new neutrosophic symbolic Turiyam group and studied its important properties.

Definition 4.1: Let (Ģ,*) be a "group", we define the "neutrosophic symbolic Turiyam group" (NSTĢ) as : ($T_{Ģ}$,*) where $T_{Ģ}=\{a+bT+cF+dI+gY; a,b,c,d,g \in R\}$.

Theorem 4.2: Let (Ģ,∗) be an "group" and let $T_{Ģ} = \{ a+bT+cF+dI+gY; a,b,c,d,g \in Ģ \}$, and ' $\breve{\#}$ ' be a "binary operation" on $T_{Ģ}$ defined as the following:

$$(a+bT+cF+dI+gY)\breve{\#}(\grave{a}+\grave{b}T+\grave{c}F+\grave{d}I+\grave{g}Y) = (a*\grave{a})+(b*\grave{b})T+(c*\grave{c})F+(d*\grave{d})I+(g*\grave{g})Y \quad \forall\, a,b,c,d,g,\grave{a},\grave{b},\grave{c},\grave{d},\grave{g} \in Ģ$$

Then: $(T_{Ģ}, \breve{\#})$ is a group we called it "new neutrosophic symbolic Turiyam group".

Proof:

i. $\forall\,(a+bT+cF+dI+gY),(\grave{a}+\grave{b}T+\grave{c}F+\grave{d}I+\grave{g}Y) \in T_{Ģ}$ then $(a+bT+cF+dI+gY)\breve{\#}(\grave{a}+\grave{b}T+\grave{c}F+\grave{d}I+\grave{g}Y) = (a*\grave{a})+(b*\grave{b})T+(c*\grave{c})F+(d*\grave{d})I+(g*\grave{g})Y \in T_{Ģ}$ implies that $T_{Ģ}$ is closed under $\breve{\#}$.

ii. $\forall\,(a+bT+cF+dI+gY),(\grave{a}+\grave{b}T+\grave{c}F+\grave{d}I+\grave{g}Y),(\ddot{a}+\ddot{b}T+\ddot{c}F+\ddot{d}I+\ddot{g}Y) \in T_{Ģ}$ then: $(a+bT+cF+dI+gY)\breve{\#}\left((\grave{a}+\grave{b}T+\grave{c}F+\grave{d}I+\grave{g}Y)\breve{\#}(\ddot{a}+\ddot{b}T+\ddot{c}F+\ddot{d}I+\ddot{g}Y)\right) = (a+bT+cF+dI+gY)\breve{\#}\left((\grave{a}*\ddot{a})+(\grave{b}*\ddot{b})T+(\grave{c}*\ddot{c})F+(\grave{d}*\ddot{d})I+(\grave{g}*\ddot{g})Y\right) = a*(\grave{a}*\ddot{a})+b*(\grave{b}*\ddot{b})T+c*(\grave{c}*\ddot{c})F+d*(\grave{d}*\ddot{d})I+g*(\grave{g}*\ddot{g})Y \in T_{Ģ}$ (since ∗ such that associative Law)

$$= (a*\grave{a})*\ddot{a}+[(b*\grave{b})*\ddot{b}]T+[(c*\grave{c})*\ddot{c}]F+[(d*\grave{d})*\ddot{d}]I+[(g*\grave{g})*\ddot{g}]Y$$

$$= \left((a*\grave{a})+(b*\grave{b})T+((c*\grave{c}))F+\left((d*\grave{d})\right)I+((g*\grave{g}))Y\right)\breve{\#}(\ddot{a}+\ddot{b}T+\ddot{c}F+\ddot{d}I+\ddot{g}Y)$$

$= \left((a+bT+cF+dI+gY)\breve{\#}(\grave{a}+\grave{b}T+\grave{c}F+\grave{d}I+\grave{g}Y)\right)\breve{\#}(\ddot{a}+\ddot{b}T+\ddot{c}F+\ddot{d}I+\ddot{g}Y)$ (associative law).

iii. for every $(a+bT+cF+dI+gY) \in T_{Ģ}$ There exists an identity element $(e+eT+eF+eI+eY) \in T_{Ģ}$ such that

$(a+bT+cF+dI+gY)\breve{\#}(e+eT+eF+eI+eY) = (a*e)+(b*e)T+((c*e))F+((d*e))I+((g*e))Y = (a+bT+cF+dI+gY)$.

In the same way, we see that:

$$(e+eT+eF+eI+eY)\breve{\#}(a+bT+cF+dI+gY) = (a+bT+cF+dI+gY).$$

Therefore $(a+bT+cF+dI+gY)\breve{\#}(e+eT+eF+eI+eY) = (e+eT+eF+eI+eY)\breve{\#}(a+bT+cF+dI+gY) = a+bT+cF+dI+gY$.

iv. for every $x = (a+bT+cF+dI+gY) \in T_{Ģ}$ there exists an element $x^{-1} = (a^{-1}+b^{-1}T+c^{-1}F+d^{-1}I+g^{-1}Y) \in T_{Ģ}$

such that $x\breve{\#}x^{-1} = x^{-1}\breve{\#}x = (e+eT+eF+eI+eY)$

(the existence of inverse in $T_{Ģ}$).

Definition 4.3: Let (Ģ,∗) be an "group" and let $T_{Ģ}$ = { $a+bT+cF+dI+gY;a,b,c,d,g \in Ģ$ }, and ' $\breve{\#}$ ' be a "binary operation" on $T_{Ģ}$ defined as:

$$(a + bT + cF + dI + gY)\breve{\#}(\grave{a} + \grave{b}T + \grave{c}F + \grave{d}I + \grave{g}Y) = (a * \grave{a}) + (b * \grave{b})T + (c * \grave{c})F + (d * \grave{d})I + (g * \grave{g})Y \ \ \forall\, a, b, c, d, g, \grave{a}, \grave{b}, \grave{c}, \grave{d}, \grave{g} \in Ģ$$

Then: $(T_{Ģ}, \breve{\#})$ is a "group" we called it "new neutrosophic symbolic Turiyam group".

Example 4.4: Let (R,+) be an group and let $T_{Ģ}$ = { $a+bT+cF+dI+gY;a,b,c,d,g \in Ģ$ }, and ' $\breve{\#}$ ' be a " binary operation" on T_{R} defined as :

$$(a + bT + cF + dI + gY)\breve{\#}(\grave{a} + \grave{b}T + \grave{c}F + \grave{d}I + \grave{g}Y) = (a + \grave{a}) + (b + \grave{b})T + (c + \grave{c})F + (d + \grave{d})I + (g + \grave{g})Y \ \ \forall\, a, b, c, d, g, \grave{a}, \grave{b}, \grave{c}, \grave{d}, \grave{g} \in R$$

Then: $(T_{R}. \breve{\#})$ is a group we called it "new neutrosophic symbolic Turiyam group".

5 | Conclusion

In this paper, we have defined new algebraic structures such as "new neutrosophic groups" and "new neutrosophic subgroups". Finally, the "new neutrosophic group" is just the beginning of a "new neutrosophic algebraic structure" and we have studied a few ideas only, it will be necessary to carry out "more theoretical research" to establish a "general framework" for the practical application. In the future, we will study special elements in this new neutrosophic group and subgroup.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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