HyperSoft Set Methods in Engineering

SCIENCES FORCE
International Academic Publisher

Journal Homepage: sciencesforce.com/hsse

HyperSoft Set Meth. Eng. Vol. 2 (2024) 18–27

Paper Type: Original Article

Extension of Double Frame Soft Set to Double Frame Hypersoft Set (DFSS to DFHSS)

Muhammad Saeed [1,*], **Hafiz Inam ul Haq** [1] **and Mubashir Ali** [1]

[1] Department of Mathematics, University of Management and Technology Lahore, 54770 Pakistan.
Emails: muhammad.saeed@umt.edu.pk; f2021109017@umt.edu.pk; v31553@umt.edu.pk.

Received: 12 Feb 2024 **Revised**: 09 May 2024 **Accepted**: 10 Jun 2024 **Published**: 13 Jun 2024

Abstract

This paper provides the double framed hyper soft set (DFHSS) by considering different problems that contains multiple functions against same parameters or different parameters. This paper may help us to solve two different kinds of problems under a single definition this may help the reader to makes the decision very easily. We will use the two different types of notion like double framed soft set (DFSS) and hyper soft set and convert it to (DFHSS). In this we will use different aggregation operation including union, intersection, not set, compliment and compliment.

Keywords: Soft Set, Hypersoft Set, Double Frame Soft Set, Compliment of a Double Framed Hyper Soft Set, Extensions of DFHSS, Applications of DFHSS.

1 | Introduction

The concept of fuzzy set [1-3], has proven to be a powerful tool for handling uncertainty [4] in various fields such as decision making, data analysis, and pattern recognition [5]. Theoretical development of a cubic Pythagorean fuzzy soft set and its application in multi-attribute decision-making [6]. A soft set [7,8] is characterized by a parameterized family of subsets of a universal set, providing a flexible mathematical [9] framework to address problems where traditional methods may fall short due to the presence of vague or imprecise information [10]. In a double frame soft set [11], two mappings are employed to associate each parameter with a subset of the universal set, allowing for a more nuanced representation of the relationships among parameters [12-13]. The double frame soft set [14] extends this idea by considering two frames of reference simultaneously [15-16], thereby enhancing the capability to model complex situations where interactions between different attributes must be considered [17]. Smarandache [18] proposed the hypersoft set in 2018, which is a new set structure anew from the product of divided attributes to the set on a universal set of attributes. A Novel Fuzzy Parameterized Fuzzy Hypersoft Set [19-20] and Riesz Summability Approach Based Decision Support System for Diagnosis of Heart Diseases where the health status of a patient depends on multiple factors such as age, immunity level, and treatment type [21]. The distance and similarity measures for intuitionistic hypersoft sets, neutrosophic hypersoft sets, and trigonometric similarity measures are proposed by [22-24], considering truthiness, incertitude, and in-determinacy.

Corresponding Author: muhammad.saeed@umt.edu.pk
https://doi.org/10.61356/j.hsse.2024.2293

The transition from double-frame soft sets to double-frame hypersoft sets involves the introduction of hyperparameters and the construction of mappings that consider multiple layers of interaction. This process enhances the descriptive power of the model and enables the handling of more complex datasets. The formal definition of a double frame hyper soft set involves a universal set ν and a collection of parameters $\{\varphi_1, \varphi_2, \dots, \varphi_n\}$. Each parameter φ_i is associated with a set of values, and mappings $\pi_1, \pi_2, \dots, \pi_k$ are defined to map these parameter combinations to subsets of ν. The pair $(\pi_1, \pi_2, \dots, \pi_k; \varphi_1, \varphi_2, \dots, \varphi_n)$ forms the double frame hyper soft set, capturing the multi-layered relationships among parameters. This extension of the double frame soft set to the double frame hyper soft set thus provides a more powerful and flexible approach to modeling and analyzing complex systems. It opens new possibilities for research and application in various fields, offering a robust tool for dealing with high-dimensional and interdependent data. Future work may explore the development of efficient algorithms for manipulating these sets and applying them to real-world problems, further validating their utility and effectiveness.

2 | Preliminaries

Definition 1: (Soft Set) A pair (W, X) is said to be a soft set over a universe ν where W is a mapping defined by $W: G \to P(\nu)$.

In other words, a soft set over universe ν is a collection of attributes of subsets of the universe ν. For $\kappa \in X, W(\kappa)$ may be acknowledged as the set of κ-approximate elements of the soft set (W, X).

Definition 2: (Double frame soft set) A double frame pair $(\varphi, \psi; W)$ is called a double frame soft set over a universe ν, where φ and ψ are mappings from A to $P(\nu)$.

Definition 3: (Hypersoft Set) Let $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$ be the distinct attributes whose attribute values belong to the sets $X_1, X_2, X_3, \dots, X_n$ respectively, where $X_i \cap X_j = \emptyset$ for $i \neq j$.

A pair $(\psi, W_1 \times W_2 \times W_3 \times \dots \times W_n)$ is called a hyper soft set over the universal set U, where ψ is the mapping given by $\psi: W_1 \times W_2 \times W_3 \times \dots \times W_n \to P(U)$.

3 | Operation of Double Framed Hypersoft Set

Definition 4: (Double Framed Hypersoft Set) Let ν be the universe of discourse. $P(\nu)$ is the power set of ν. Let $a_1, a_2, a_3, \dots, a_n$ for $n \geq 1$ be n distinct attributes whose corresponding attribute values are respectively the sets $\varphi_1, \varphi_2, \varphi_3, \dots, \varphi_n$ with $\varphi_i \cap \varphi_j = \emptyset$ for $i \neq j, i, j \in \{1,2, \dots, n\}$.

Then the pair $(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_3 \times \dots \times \varphi_n$, where π_1 and π_2 are mappings from $\varphi_1 \times \varphi_2 \times \varphi_3 \times \dots \times \varphi_n \to P(\nu)$.

Definition 5: (Double Framed Hypersoft Subset) Suppose that $\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_3 \times \dots \times \varphi_n\}$ is a double framed hyper soft set. Then a collection $\{(\epsilon_1, \epsilon_2), B_1 \times B_2 \times B_3 \times \dots \times B_n\}$ is called a double framed hypersoft subset if:

$B_1 \times B_2 \times B_3 \times \dots \times B_n \subseteq \varphi_1 \times \varphi_2 \times \varphi_3 \times \dots \times \varphi_n$ 2. $\epsilon_1(t) = \pi_1(t)$ and $\epsilon_2(t) = \pi_2(t)$ for all $t \in \varphi_1 \times \varphi_2 \times \varphi_3 \times \dots \times \varphi_n$

Example 1: Let $\nu = \{y_1, y_2, y_3, y_4\}$ be the collection of cars and $M = \{y_1, y_3\}, N = \{y_2, y_4\}$ be its subsets. Let $\varphi_1 =$ color, $\varphi_2 =$ company, $\varphi_3 =$ mileage, $\varphi_4 =$ price where:

$$\varphi_1 = \{ \text{red, black, blue, white} \}$$
$$\varphi_2 = \{ \text{Mehran, Corolla, Honda, Suzuki} \}$$
$$\varphi_3 = \{15 \text{ km/h}, 20 \text{ km/h}, 25 \text{ km/h}, 18 \text{ km/h}\}$$
$$\varphi_4 = \{ \text{9lac, 30lac, 32lac, 18lac} \}$$

Then a mapping $\pi_1: \varphi_1 \times \varphi_2 \times \varphi_3 \times \varphi_4 \to P(\nu)$ is defined as follows:

$$\pi_1(\text{ red , Corolla, } 20\text{ km/h}, 32\text{lac}) = \{y_1, y_3\}$$
$$\pi_2(\text{ red , Corolla, } 20\text{ km/h}, 32\text{lac}) = \{y_2, y_4\}\psi$$

Then $\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_3 \times \varphi_4\}$ is called a Framed Hypersoft Set.

Now consider:

$$B_1 = \text{ Color } = \{\text{ Red , Black }\}$$
$$B_2 = \text{ Company } = \{\text{ Corolla, Honda, Suzuki }\}$$
$$B_3 = \text{ Mileage } = \{18\text{ km/h}, 15\text{ km/h}, 20\text{ km/h}\}$$
$$B_4 = \text{ Price } = \{30\text{ lac, 32lac, 18lac }\}$$

Then $\epsilon: B_1 \times B_2 \times B_3 \times B_4 \rightarrow P(\nu)$ is defined as follows:

$$\epsilon_1(\text{ Black , Honda, } 18\text{ km/h}, 30lac) = \{y_1\}$$
$$\epsilon_2(\text{ Black , Honda, } 18\text{ km/h}, 30lac) = \{y_4\}$$

Then $\{(\epsilon_1, \epsilon_2), B_1 \times B_2 \times B_3 \times B_4\}$ is called a double framed hypersoft subset.

Definition 6: (Double Framed Hyper Null Soft Set)

Let $\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_3 \times \cdots \times \varphi_n\}$ be a double framed hypersoft set.

Then it is called a double framed null hypersoft set if $\pi_1(t) = \emptyset$ and $\pi_2(t) = \emptyset$ for all $t \in \varphi_1 \times \varphi_2 \times \varphi_3 \times \varphi_4$

Definition 7: (Union of Double Framed Hypersoft Set)

Let $\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_3 \times \cdots \times \varphi_n\}$ and $\{(\epsilon_1, \epsilon_2), B_1 \times B_2 \times B_3 \times \cdots \times B_n\}$ be double framed hyper soft sets.

Then their union is defined as.

$\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_3 \times \cdots \times \varphi_n\} \cup \{(\epsilon_1, \epsilon_2), B_1 \times B_2 \times B_3 \times \cdots \times B_n\} = \{(\xi_1, \xi_2), (\varphi_1 \times \varphi_2 \times \varphi_3 \times \cdots \times \varphi_n) \cup (B_1 \times B_2 \times B_3 \times \cdots \times B_n)\}$ where $\{(\xi_1, \xi_2)\}$ is defined as follows:

$(\xi_1, \xi_2)(t) = (\pi_1, \pi_2)(t)$ for all $t \in \varphi_1 \times \varphi_2 \times \varphi_3 \times \cdots \times \varphi_n$
$(\xi_1, \xi_2)(t) = (\epsilon_1, \epsilon_2)(t)$ for all $t \in B_1 \times B_2 \times B_3 \times \cdots \times B_n$
$(\xi_1, \xi_2)(t) = (\pi_1, \pi_2)(t) \cup (\epsilon_1, \epsilon_2)(t)$ for all $t \in (\varphi_1 \times \varphi_2 \times \varphi_3 \times \cdots \times \varphi_n) \cup (B_1 \times B_2 \times B_3 \times \cdots \times B_n)$

Example 2: Let $\nu = \{x_1, x_2, x_3, x_4, x_5\}$ be a collection of houses. Now let:

$$\varphi_1 = \text{ color } = \{\text{ white, yellow, blue }\}$$
$$\varphi_2 = \text{ area } = \{\text{ Lahore, Karachi, Islamabad }\}$$
$$\varphi_3 = \text{ cost } = \{\text{ 3olac , } 40\text{ lac, 50lac, 6olac }\}$$

Then a mapping $\pi_1: \varphi_1 \times \varphi_2 \times \varphi_3 \rightarrow P(\nu)$ is defined as:

$$\pi_1(\text{ white, Lahore, } 50\text{ lac }) = \{x_1, x_2\}$$

Similarly, a mapping $\pi_2: \varphi_1 \times \varphi_2 \times \varphi_3 \rightarrow P(\nu)$ is defined as:

$$\pi_2(\text{ white, Lahore, } 50\text{ lac }) = \{x_1, x_2\}$$

Then $\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_3\}$ is called a double framed hyper soft set. Now consider:

$$B_1 = \text{ surrounding } = \{\text{ green, dusty, clean }\}$$
$$B_2 = \text{ structure } = \{\text{ 2floor, } 3\text{ floor, } 1\text{ floor }\}$$

Then a mapping $\epsilon_1: B_1 \times B_2 \rightarrow P(\nu)$ is defined as:

$$\epsilon_1(\text{ green, } 2 \text{ floor }) = \{x_1, x_3\}$$

Similarly, a mapping $\epsilon_2: B_1 \times B_2 \rightarrow P(\nu)$ is defined as:

$$\epsilon_2(\text{ green, 2floor }) = \{x_3\}$$

Now, their union is defined as:

$$\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_3\} \cup \{(\epsilon_1, \epsilon_2), B_1 \times B_2\} = \{(\xi_1, \xi_2), (\varphi_1 \times \varphi_2 \times \varphi_3) \cup (B_1 \times B_2)\}$$

where $\{(\xi_1, \xi_2)\}$ is defined as follows:

$$\xi_1(\text{ white, Lahore, } 50 \text{ lac, green, 2floor }) = \{x_1, x_2, x_3\}$$
$$\xi_2(\text{ white, Lahore, 50lac, green, 2floor }) = \{x_2, x_3\}$$

Then the pair $\{(\xi_1, \xi_2), (\varphi_1 \times \varphi_2 \times \varphi_3) \cup (B_1 \times B_2)\}$ is the union of double framed hyper soft sets.

Definition 8: Let $\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_3 \times \cdots \times \varphi_n\}$ and $\{(\epsilon_1, \epsilon_2), B_1 \times B_2 \times B_3 \times \cdots \times B_n\}$ be double framed hyper soft sets.

Their intersection is defined as follows:

$\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_3 \times \cdots \times \varphi_n\} \cap \{(\epsilon_1, \epsilon_2), B_1 \times B_2 \times B_3 \times \cdots \times B_n\} = \{(\xi_1, \xi_2), (\varphi_1 \times \varphi_2 \times \varphi_3 \times \cdots \times \varphi_n) \cap (B_1 \times B_2 \times B_3 \times \cdots \times B_n)\}$ where $\{(\xi_1, \xi_2)\}$ is defined as follows:

$(\xi_1, \xi_2)(t) = (\pi_1, \pi_2)(t)$ for all $t \in \varphi_1 \times \varphi_2 \times \varphi_3 \times \cdots \times \varphi_n$
$(\xi_1, \xi_2)(t) = (\epsilon_1, \epsilon_2)(t)$ for all $t \in B_1 \times B_2 \times B_3 \times \cdots \times B_n$
$(\xi_1, \xi_2)(t) = (\pi_1, \pi_2)(t) \cap (\epsilon_1, \epsilon_2)(t)$ for all $t \in (\varphi_1 \times \varphi_2 \times \varphi_3 \times \cdots \times \varphi_n) \cap (B_1 \times B_2 \times B_3 \times \cdots \times B_n)$

Example 3: Let $\nu = \{M_1, M_2, M_3, M_4, M_5\}$ be a collection of mobile phones. Let:

$$\varphi_1 = \text{ company } = \{\text{ Samsung, } i \text{ Phone, Oppo }\}$$
$$\varphi_2 = \text{ specification (RAM, ROM) } = \{(3,32)GB, (4,64)GB, (6,128)GB\}$$
$$\varphi_3 = \text{ color } = \{\text{ black, golden, grey }\}$$

Then a mapping $\pi_1: \varphi_1 \times \varphi_2 \times \varphi_3 \rightarrow P(\nu)$ is defined as:

$$\pi_1(\text{ Samsung, golden, } (4,64)GB) = \{x_1, x_2\}$$

Similarly, a mapping $\pi_2: \varphi_1 \times \varphi_2 \times \varphi_3 \rightarrow P(\nu)$ is defined as:

$$\pi_2(\text{ Samsung, golden , } (4,64)GB) = \{x_3, x_5\}$$

Then $\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_3\}$ is called a double framed hyper soft set. Now consider:

$$B_1 = \text{ Battery } = \{4000mAh, 5000mAh, 6000mAh\}$$
$$B_2 = \text{ Camera } = \{48MP, 60MP, 40MP\}$$
$$B_3 = \text{ Price } = \{60{,}000, 48{,}000, 108{,}000\}$$

Then a mapping $\epsilon_1: B_1 \times B_2 \times B_3 \rightarrow P(\nu)$ is defined as:

$$\epsilon_1(5000mAh, 60MP, 60{,}000) = \{x_2, x_4\}$$

Similarly, a mapping $\epsilon_2: B_1 \times B_2 \times B_3 \rightarrow P(\nu)$ is defined as:

$$\epsilon_2(5000mAh, 60MP, 60{,}000) = \{x_4, x_5\}$$

Then $\{(\epsilon_1, \epsilon_2), B_1 \times B_2 \times B_3\}$ is another double framed hyper soft set.

Their intersection is defined as:

$$\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_3\} \cap \{(\epsilon_1, \epsilon_2), B_1 \times B_2 \times B_3\} = \{(\xi_1, \xi_2), (\varphi_1 \times \varphi_2 \times \varphi_3) \cap (B_1 \times B_2 \times B_3)\}$$

where $\{(\xi_1, \xi_2)\}$ is defined as follows:

$$\xi_1(\text{ Samsung, golden, } (4{,}64)GB, 5000mAh, 60MP, 60{,}000) = \{x_2\}$$
$$\xi_2(\text{ Samsung, golden, } (4{,}64)GB, 5000mAh, 60MP, 60{,}000) = \{x_5\}$$

Then the pair $\{(\xi_1, \xi_2), (\varphi_1 \times \varphi_2 \times \varphi_3) \cap (B_1 \times B_2 \times B_3)\}$ is the intersection of double framed hyper soft sets.

3.1 | Complement of a Double Framed Hyper Soft Set

The complement of a double framed hyper soft set $\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \cdots \times \varphi_n\}$ is written $\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \cdots \times \varphi_n\}^c$ and is denoted by $\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \cdots \times \varphi_n\}^c = \{(\pi_1^c, \pi_2^c), \sim \varphi_1 \times \varphi_2 \cdots \times \varphi_n\}$ s.t $\pi^c \to \varphi_1 \times \varphi_2 \cdots \times \varphi_n \to p(\eta)$

Then:

$\pi_1^c(t) = \eta - \pi_1(t)$

$t \in \sim (\varphi_1 \times \varphi_2 \cdots \times \varphi_n)$

and

$\pi_2^c(t) = \eta - \pi_2(t)$

$t \in \sim (\varphi_1 \times \varphi_2 \cdots \times \varphi_n)$

Example 4: Let $\eta = \{x_1, x_2, x_3, x_4, x_5\}$ be a universe of discourse let

$\varphi_1 =$ Company $=$ { Samsung, iPhone, oppo }

$\varphi_2 =$ Specification (Ram, Rom) $= \{(3{,}32)GB, (4{,}64)GB, (6{,}128)GB\}$

$\varphi_3 =$ Color $=$ {black, golden, grey}

then

$\{\pi_1$: (iPhone, Golden, $(4{,}64)GB)\} = \{x_1, x_3\}$

and

$\{\pi_2$: (iPhone, Golden, $(4{,}64)GB)\} = \{x_2, x_3\}$

then $\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_3\}$ is called a double framed soft set. Then their complement is. $\{(\pi_1, \pi_2)^c, \varphi_1 \times \varphi_2 \times \varphi_3\} = \{(\pi_1^c, \pi_2^c), \sim (\varphi_1 \times \varphi_2 \times \varphi_3)\}$ is written as $\{\pi_1^c$: ($\sim$ iphone,$\sim$ Golden,$\sim$ $(4{,}64)GB)\} = \{x_2, x_4, x_5\}$

$\{\pi_2^c$: ($\sim$ iphone,$\sim$ Golden,$\sim$ $(4{,}64)GB)\} = \{x_1, x_4, x_5\}$ is called a compliment of the double framed hyper soft set.

3.2 | Relative Compliment of a Double Framed Hyper Soft Set

The relative complement of double framed hyper soft set $\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_n\}$ is denoted by $\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_3\}^r$ and is written as:

$\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_n\}^r = \{(\pi_1^r, \pi_2^r), \varphi_1 \times \varphi_2 \times \varphi_n\}$

Example 5: Let $\eta = \{x_1, x_2, x_3, x_4\}$ be the universe of discourse then let

$\varphi_1 =$ Books $=$ (math, phy, ioo, chem)

$\varphi_2 =$ Syllabus $=$ (17 chap , 9 chap, 6 chap, 8 chap)

$\varphi_3 =$ time $=$ (1 weak, 3 weak , 7 weak)

$\varphi_4 =$ marks $= (40,60,90)$

then

$\pi_1 : \varphi_1 \times \varphi_2 \times \varphi_3 \times \varphi_4 \to P(\eta)$

$\{\pi_1 :$ (math , 17 chap , 7 weak , 90)$\} = \{x_2, x_3, x_4\}$

$\{\pi_1 :$ (math, 17chap , 7 weak , 90)$\} = \{x_1, x_2\}$

then $\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_n\}$ is a double framed hyper soft set then their relative

$\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_3 \times \varphi_4\}^r = \{(\pi_1^r, \pi_2^r), \varphi_1 \times \varphi_2 \times \varphi_3 \times \varphi_4\}$

$\{\pi_1^r :$ (math, 17chap , 7 weak, 90)$\} = \{x_1\}$

$\{\pi_1^r :$ (math, 17chap, 7 weak , 90)$\} = \{x_3, x_4\}$ is called relative compliment.

3.3 | Proposition 1

1.1: the union of any two double framed hypersoft sets is commutative.

i.e. $\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_3 \cdots \times \varphi_n\} \cup \{(\varepsilon_1, \varepsilon_2), (B_1 \times B_2 \times B_3 \cdots B_n\}$

$= \{(\varepsilon_1, \varepsilon_2), (B_1 \times B_2 \times B_3 \cdots B_4\} \cup \{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_3 \cdots \times \varphi_n\}$

1.2: the union of any two double framed is associated.

3.4 | Proposition 2

2.1: the intersection of any two double framed hypersoft sets is not commutative if $\varphi_1 \times \varphi_2 \times \varphi_3 \cdots \times \varphi_n = B_1 \times B_2 \times B_3 \cdots B_n$ then there is a possibility that is commutative.

2.2: the intersection of any two double framed hypersoft sets is associative.

3.5 | Proposition 3

let η be the universe of course and $\varphi_1 \times \varphi_2 \times \varphi_3 \cdots \times \varphi_n and B_1 \times B_2 \times B_3 \cdots B_n$ be the set of parameters further let $\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_3 \cdots \times \varphi_n\}$ and $\{(\varepsilon_1, \varepsilon_2), (B_1 \times B_2 \times B_3 \cdots B_n\}$ be two double framed hyper soft sets then

3.5.1 3.1

$\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_3 \cdots \times \varphi_n\} \cup \{(\varepsilon_1, \varepsilon_2), (B_1 \times B_2 \times B_3 \cdots B_n\}^c$

$\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_3 \cdots \times \varphi_n\}^c \cap \{(\varepsilon_1, \varepsilon_2), (B_1 \times B_2 \times B_3 \cdots B_n\}^c$

3.5.2 3.1

$\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_3 \cdots \times \varphi_n\} \cap \{(\varepsilon_1, \varepsilon_2), (B_1 \times B_2 \times B_3 \cdots B_n\}^c$

$\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_3 \cdots \times \varphi_n\}^c \cup \{(\varepsilon_1, \varepsilon_2), (B_1 \times B_2 \times B_3 \cdots B_n\}^c$

3.5.3 Proof 3.1

L.H.S

$[\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_3 \cdots \times \varphi_n\} \cup \{(\varepsilon_1, \varepsilon_2), (B_1 \times B_2 \times B_3 \cdots B_n\}]^c$

$\{(\pi_1, \pi_2) \cup (\varepsilon_1, \varepsilon_2) : \varphi_1 \times \varphi_2 \times \varphi_3 \cdots \times \varphi_n \cup B_1 \times B_2 \times B_3 \cdots B_n\}^c$ by definition of relative compliment

$\{(\pi_1 \cup \varepsilon_1)^c, (\pi_2 \cup \varepsilon_2)^c : \varphi_1 \times \varphi_2 \times \varphi_3 \cdots \times \varphi_n \cup B_1 \times B_2 \times B_3 \cdots B_n\}$

$[\{(\pi_1^c, \pi_2^c), \varphi_1 \times \varphi_2 \times \varphi_3 \cdots \times \varphi_n\} \cap \{(\varepsilon_1^c, \varepsilon_2^c), (B_1 \times B_2 \times B_3 \cdots B_n\}]$

$[\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_3 \cdots \times \varphi_n\}^c \cup \{(\varepsilon_1, \varepsilon_2), (B_1 \times B_2 \times B_3 \cdots B_n\}]^c$

= R.H.S

3.6 | Difference of Double Framed Hyper Soft Set

Let $\{(\pi_1, \pi_2); \varphi_1 \times \varphi_2 \times \varphi_3 \times \cdots \times \varphi_n\}$ and $\{(\epsilon_1, \epsilon_2); B_1 \times B_2 \times B_3 \times \cdots \times B_n\}$ be two double framed soft sets.

The difference is defined as: $\{(\pi_1, \pi_2); \varphi_1 \times \varphi_2 \times \varphi_3 \times \cdots \times \varphi_n\} \setminus \{(\epsilon_1, \epsilon_2); B_1 \times B_2 \times B_3 \times \cdots \times B_n\} = \{(\xi_1, \xi_2); (\varphi_1 \times \varphi_2 \times \varphi_3 \times \cdots \times \varphi_n) \cup (B_1 \times B_2 \times B_3 \times \cdots \times B_n)\}$

where $\{(\xi_1, \xi_2)\}$ is defined as follows:

$(\xi_1, \xi_2)(t) = (\pi_1, \pi_2)(t)$ for all $t \in \varphi_1 \times \varphi_2 \times \varphi_3 \times \cdots \times \varphi_n$
$(\xi_1, \xi_2)(t) = (\epsilon_1, \epsilon_2)(t)$ for all $t \in B_1 \times B_2 \times B_3 \times \cdots \times B_n$
$(\xi_1, \xi_2)(t) = (\pi_1, \pi_2)(t) \setminus (\epsilon_1, \epsilon_2)(t)$ for all $t \in (\varphi_1 \times \varphi_2 \times \varphi_3 \times \cdots \times \varphi_n) \cap (B_1 \times B_2 \times B_3 \times \cdots \times B_n)$

4 | Extension of DFHSS

Definition 4: (Double Framed Hypersoft Set) Let ν be the universe of discourse. $P(\nu)$ is the power set of ν. Let $a_1, a_2, a_3, \dots, a_n$ for $n \geq 1$ be n distinct attributes whose corresponding attribute values are respectively the sets $\varphi_1, \varphi_2, \varphi_3, \dots, \varphi_n$ with $\varphi_i \cap \varphi_j = \emptyset$ for $i \neq j, i, j \in \{1,2, \dots, n\}$.

4.1 | Tripled Framed Hyper Soft Set

Let ν be the course of the universe. Consider the following mappings:

$$\pi_1 : \varphi_1 \times \varphi_2 \times \varphi_3 \times \varphi_4 \to \mathcal{P}(\nu)$$
$$\pi_2 : \varphi_1 \times \varphi_2 \times \varphi_3 \times \varphi_4 \to \mathcal{P}(\nu)$$
$$\pi_3 : \varphi_1 \times \varphi_2 \times \varphi_3 \times \varphi_4 \to \mathcal{P}(\nu)$$

Then, the pair $\{(\pi_1, \pi_2, \pi_3), \varphi_1 \times \varphi_2 \times \varphi_3 \times \cdots \times \varphi_n\}$ is called a tripled framed hyper soft set.

4.2 | N-Framed Hyper Soft Set

Let ν be the universe of discourse and let $\varphi_1, \varphi_2, \dots, \varphi_n$ be the set of parameters.

The mapping

$\pi_i : \varphi_i \to \mathcal{P}(\nu)$ is called an N-framed Hyper Soft Set.

The pair

$\{(\pi_1, \pi_2, \dots, \pi_n), \varphi_1, \varphi_2, \dots, \varphi_n\}$ is called an N-framed Hyper Soft Set.

Example 6: Let $\varphi = \{30°\text{C}, 26°\text{C}, 40°\text{C}, 45°\text{C}, 35°\text{C}\}$ be the set of temperatures in the world.

Let

$\varphi_1 =$ { Pakistan, India, America, Australia, England }
$\varphi_2 =$ { January, Feb, March, April, May, June, July, Aug, Sep, Oct, Nov, Dec },
$\varphi_3 =$ { Cloudy, Sunny, Rainy }
$\varphi_4 =$ { Hilly, Groundy, Ocean }

Then the mapping

$\pi_i : \varphi_1 \times \varphi_2 \times \varphi_3 \times \varphi_4 \to \mathcal{P}(\nu)$ is such that

$$\{\pi_1(\text{ Pakistan, April, Sunny, Groundy })\} = \{26°\mathrm{C}, 30°\mathrm{C}\},$$
$$\{\pi_2(\text{ Pakistan, April, Sunny, Groundy })\} = \{26°\mathrm{C}\},$$
$$\{\pi_3(\text{ Pakistan, April, Sunny, Groundy })\} = \{40°\mathrm{C}\},$$
$$\{\pi_4(\text{ Pakistan, April, Sunny, Groundy })\} = \{30°\mathrm{C}\},$$
$$\{\pi_5(\text{ Pakistan, April, Sunny, Groundy })\} = \{35°\mathrm{C}\},$$
$$\{\pi_6(\text{ Pakistan, April, Sunny, Groundy })\} = \{30°\mathrm{C}, 35°\mathrm{C}\}$$
$$\{\pi_7(\text{ Pakistan, April, Sunny, Groundy })\} = \{35°\mathrm{C}, 40°\mathrm{C}\}$$
$$\{\pi_8(\text{ Pakistan, April, Sunny, Groundy })\} = \{30°\mathrm{C}, 40°\mathrm{C}\}$$

Then, the pair

$\{(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7, \pi_8), (\varphi_1, \varphi_2, \varphi_3, \varphi_4)\}$ is called an **N**-Framed Hyper Soft Set.

5 | Application of Double Framed Hyper Soft Set

Let $\nu = \{$ Recovery, Death $\}$ be the stability of a Dengue patient after treatment. The patient is treated by a civil and private hospital in Pakistan. Let the parameters for treatment be:

$$\begin{aligned} \text{Patient } &= \{\text{ Children, Young, Old }\} \\ \text{Immunity\_Level } &= \{\text{ Strong, Weak }\} \\ \text{Health\_Condition } &= \{\text{ Mild, Severe, Complicated }\} \\ \text{Dosage } &= \{\text{ Plenty of water and rest, I/V Fluids, Blood transfusion }\}. \end{aligned}$$

Now, let π_1 represent the treatment by a private hospital and π_2 represent the treatment by a civil hospital. We have:

$$\pi_1(\text{ Children, Weak, Severe, I/V Fluids }) = \{\text{ Recovery }\}$$
$$\pi_2(\text{ Children, Weak, Severe, I/V Fluids }) = \{\text{ Death }\}$$

Then, the pair

$\{(\pi_1, \pi_2), (\text{ Patient, Immunity\_Level, Health\_Condition, Dosage })\}$ is called a Double Framed Hyper Soft Set.

6 | Conclusions and Future Work

The extension of double frame soft sets to double frame hyper soft sets provides a robust framework for handling complex decision-making problems involving multiple attributes and their interrelationships. This extension enhances the ability to model real-world scenarios with greater precision, offering a flexible and comprehensive approach to data analysis. The introduction of hypersoft sets allows for the representation of more intricate structures, capturing the nuances of interactions among parameters in various domains Future research could focus on developing efficient algorithms for manipulating double frame hyper soft sets, including intersection, union, and difference operations. Additionally, applying this extended framework to diverse fields such as medical diagnosis, financial analysis, and environmental monitoring could further validate its effectiveness. Exploring the integration of fuzzy logic with double frame hyper soft sets may also yield new insights and methodologies for dealing with uncertainty and imprecision in data.

Acknowledgments

The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Author Contribution

All authors contributed equally to this work.

Funding

This research has no funding source.

Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

References

[1] M. Riaz and M. Raza Hashmi, "Fuzzy parameterized fuzzy soft compact spaces with decision-making," Punjab University Journal of Mathematics, vol. 50, no. 2, 2020.

[2] T. Sooraj, R. Mohanty, and B. Tripathy, "Fuzzy soft set theory and its application in group decision making," in Advanced Computing and Communication Technologies: Proceedings of the 9th ICACCT, 2015, pp. 171-178, Springer, 2016.

[3] M. Saeed, M. H. Saeed, R. Shafaqat, S. Sessa, U. Ishtiaq, and F. Di Martino, "A theoretical development of cubic pythagorean fuzzy soft set with its application in multi-attribute decision making," Symmetry, vol. 14, no. 12, p. 2639, 2022.

[4] L. A. Zadeh, "Toward a generalized theory of uncertainty (gtu)-an outline," Information sciences, vol. 172, no. 1-2, pp. 1-40, 2005.

[5] R. Gul, M. Shabir, and M. Naeem, "A comprehensive study on (α, β)-bipolar fuzzified rough set model based on bipolar fuzzy preference relation and corresponding decision-making applications," Computational and Applied Mathematics, vol. 42, no. 7, p. 310, 2023.

[6] M. Imtiaz, M. Saqlain, and M. Saeed, "TOPSIS for multi-criteria decision making in octagonal intuitionistic fuzzy environment by using accuracy function," Journal of new theory, no. 31, pp. 32-40, 2020.

[7] P. Maji, A. R. Roy, and R. Biswas, "An application of soft sets in a decision-making problem,"

[8] M. Saeed, M. El-Ghoneimy, and H. Kamal, "An enhanced fuzzy logic optimization technique based on user mobility for lte handover," in 2017 34th national radio science conference (NRSC), pp. 230-237, IEEE, 2017.

[9] M. Saeed, M. R. Ahmad, M. Saqlain, and M. Riaz, "Rudiments of n-framed soft sets," Punjab University Journal of Mathematics, vol. 52, no. 5, 2020.

[10] N. Çağman and S. Enginoğlu, "Soft matrix theory and its decision making," Computers E Mathematics with Applications, vol. 59, no. 10, pp. 3308-3314, 2010.

[11] M. I. Ali, F. Feng, X. Liu, W. K. Min, and M. Shabir, "On some new operations in soft set theory," Computers & Mathematics with Applications, vol. 57, no. 9, pp. 1547-1553, 2009.

[12] D. Singh and I. Onyeozili, "Notes on soft matrices operations," ARPN Journal of science and technology, vol. 2, no. 9, pp. 861-869, 2012.

[13] M. Saeed, U. Ali, J. Ali, and F. Dayan, "Fuzzy soft relative method and its application in decision making problem: Fuzzy soft relative method and its application in decision making problem," Proceedings of the Pakistan Academy of Sciences: A. Physical and Computational Sciences, vol. 57, no. 1, pp. 21-30, 2020.

[14] M. Saeed, A. Mehmood, and M. Arslan, "Multipolar interval-valued fuzzy set with application of similarity measures and multi-person TOPSIS technique," Punjab University Journal of Mathematics, vol. 53, no. 10,2021.

[15] A. Bhaumik, S. K. Roy, and G. W. Weber, "Multi-objective linguistic-neutrosophic matrix game and its applications to tourism management," J. Dyn. Games, vol. 8, no. 2, pp. 101-118, 2021.

[16] A. Lima, E. S. Palmeira, B. Bedregal, and H. Bustince, "Multidimensional fuzzy sets," IEEE Transactions on Fuzzy Systems, vol. 29, no. 8, pp. 2195-2208, 2020.

[17] A. Shahil-Feroz, H. Yasmin, S. Saleem, Z. Bhutta, and E. Seto, "Remote moderated usability testing of a mobile phone app for remote monitoring of pregnant women at high risk of preeclampsia in karachi, pakistan," in Informatics, vol. 10, p. 79, MDPI, 2023.

[18] F. Smarandache, "Extension of Soft Set to Hypersoft Set, and then to Plithogenic Hypersoft Set," Neutrosophic Sets and System, vol. 22, pp. 168-170, 2018.

[19] M. Saeed and M. I. Harl, "Fundamentals of picture fuzzy hypersoft set with application," Neutrosophic Sets and Systems, vol. 53, no. 1, p. 24, 2023.

[20] S. Debnath, "Fuzzy hypersoft sets and its weightage operator for decision making," Journal of Fuzzy Extension and Applications, vol. 2, pp. 163-170, 2021. doi: 10.22105/jfea.2021.275132.1083

[21] A. U. Rahman, M. Saeed, M. A. Mohammed, M. M. Jaber, and B. Garcia-Zapirain, "A novel fuzzy parameterized fuzzy hypersoft set and riesz summability approach-based decision support system for diagnosis of heart diseases," Diagnostics, vol. 12, no. 7, p. 1546, 2022.

[22] M. Saqlain, M. Riaz, R. Imran, and F. Jarad, "Distance and similarity measures of intuitionistic fuzzy hypersoft sets with application: Evaluation of air pollution in cities based on air quality index," AIMS Mathematics, vol. 8, no. 3, pp. 6880-6899, 2023. https://doi.org/10.3934/math.2023348

[23] M. Saqlain, S. Moin, M. N. Jafar, M. Saeed, S. Broumi, "Single and multi-valued neutrosophic hypersoft set and tangent similarity measure of single valued neutrosophic hypersoft sets," Neutrosophic Sets and Syst., vol. 32, pp. 317-329, 2020.

[24] M. N. Jafar, M. Saeed, M. Saqlain, M. S. Yang, "Trigonometric similarity measures for neutrosophic hypersoft sets with application to renewable energy source selection," IEEE Access, vol. 9, pp. 129178-129187, 2021. https://doi.org/10.1109/ACCESS.2021.3112721