HyperSoft Set Methods in Engineering

Journal Homepage: sciencesforce.com/hsse

HyperSoft Set Meth. Eng. Vol. 2 (2024) 62–71

Paper Type: Original Article

A Comprehensive Study on Decision-Making Algorithms in Retail and Project Management using Double Framed Hypersoft Sets

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Received: 03 Mar 2024 **Revised:** 31 May 2024 **Accepted:** 29 Jun 2024 **Published:** 01 Jul 2024

Abstract

In today's dynamic business environment, effective decision-making plays a crucial role in the success of retail businesses and project management endeavors. This paper presents a comprehensive study of decision-making algorithms, focusing on the utilization of Double Framed Hypersoft Sets (DFHS) in retail and project management contexts. The study explores the theoretical foundations of DFHS and its practical applications in customer segmentation, personalized marketing, project selection, and resource allocation. Various algorithms and methodologies incorporating DFHS are discussed and analyzed, highlighting their advantages, limitations, and real-world implications. Through a series of numerical examples, case studies, and comparative analyses, this study provides valuable insights into the role of DFHS in optimizing decision processes, improving business outcomes, and fostering innovation in retail and project management domains.

Keywords: Soft Set, Hypersoft Set, Double Framed Hypersoft Set, Decision-making.

1 | Introduction

In recent years, there has been a significant expansion of soft set theory and its applications across various fields, particularly in decision-making and optimization. This introduction aims to provide an overview of the advancements in soft set theory, encompassing its various extensions and practical applications.

Soft set theory, initially introduced by Molodtsov [1], has undergone numerous modifications and extensions to address various complex decision-making problems. One notable extension is the hypersoft set, which incorporates multiple levels of parameters to enhance the decision-making process. Saeed et al. [2] extended the concept of Double Frame Soft Sets (DFSS) to Double Frame Hypersoft Sets (DFHSS), providing a more robust framework for dealing with multi-criteria decision-making problems. This extension builds on earlier work by Saeed et al. [7] on n-framed soft sets.

Fuzzy soft set theory, which integrates the fuzziness of parameters, has been particularly influential in decision-making scenarios. For instance, Riaz and Hashmi [3] explored fuzzy parameterized soft compact spaces for decision-making, demonstrating the practical applications of this theory in complex decision

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https://doi.org/10.61356/j.hsse.2024.2310

contexts. Sooraj et al. [4] also highlighted the application of fuzzy soft set theory in group decision-making, emphasizing its utility in aggregating diverse opinions and preferences.

Zadeh [5] proposed a generalized theory of uncertainty, which laid the groundwork for many subsequent developments in fuzzy set and soft set theories. Building on these foundations, Gul et al. [6] conducted a comprehensive study on the (α, β)-bipolar fuzzified rough set model, which has significant implications for decision-making applications.

Further advancements include the development of soft matrix theory by Çağman and Enginoğlu [8], which provided a novel approach to decision-making by utilizing matrices to represent and manipulate soft sets. Ali et al. [9] expanded on this by introducing new operations in soft set theory, enhancing its applicability and computational efficiency. Singh and Onyeozili [10] contributed additional insights into the operations of soft matrices, further enriching this area of research.

The hypersoft set theory has also seen significant developments. Smarandache [11] extended the soft set theory to hypersoft sets and pathogenic hypersoft sets, offering new perspectives and tools for handling complex data and decision-making problems. Saeed and Harl [12] introduced the fundamental picture of fuzzy hypersoft sets, which incorporate degrees of positivity, neutrality, and negativity, enhancing the expressiveness of soft sets in decision-making scenarios.

Debnath [13] presented the fuzzy hypersoft set and its weightage operator, providing a framework for more nuanced decision-making by considering the relative importance of different parameters. Rahman et al. [14] proposed a novel decision support system based on fuzzy parameterized hypersoft sets and Riesz summability, demonstrating its application in the medical field for diagnosing heart diseases.

Saqlain et al. [15] explored distance and similarity measures for intuitionistic fuzzy hypersoft sets, applying these concepts to evaluate air pollution levels in cities based on air quality indices. This work illustrates the practical applications of hypersoft set theory in environmental monitoring and public health. Saqlain et al. [16] also investigated single and multi-valued neutrosophic hypersoft sets, providing tangent similarity measures for these sets. Jafar et al. [17] extended this research by developing trigonometric similarity measures for neutrosophic hypersoft sets and applying them to the selection of renewable energy sources.

These advancements underscore the versatility and robustness of soft set theory and its extensions, particularly in decision-making and optimization contexts. The ongoing research and development in this field continue to expand its applicability and effectiveness in addressing complex, real-world problems. In today's rapidly evolving business landscape, effective decision-making is a critical determinant of success for organizations operating in retail and project management sectors. The ability to make informed, data-driven decisions that align with organizational goals and customer preferences is paramount in achieving competitive advantage, optimizing resource utilization, and driving business growth. This paper focuses on exploring decision-making algorithms in the context of retail businesses and project management, with a specific emphasis on the utilization of Double Framed Hypersoft Sets (DFHS). DFHS is a powerful mathematical framework that enables decision-makers to model and analyze complex decision scenarios involving multiple attributes and criteria. The primary objective of this study is to conduct a comprehensive examination of DFHS-based decision-making algorithms, their theoretical foundations, practical applications, and real-world implications. By leveraging DFHS, organizations can enhance their decision accuracy, personalize customer experiences, optimize project selection, and allocate resources effectively.

The paper is structured as follows:

Section 2 serves as a foundational overview of essential concepts related to soft sets, hypersoft sets, double-framed hypersoft sets, and double-framed hypersoft subsets. Section 3 delves into specific applications of DFHS in retail, such as customer segmentation, personalized marketing, and inventory management. Additionally, the paper explores how DFHS can be applied in project management, particularly in project selection, resource allocation, and risk management. Numerical examples, case studies, and comparative analyses are presented to illustrate the practical utility of DFHS-based algorithms and their impact on business outcomes. Finally, in Section 4, the paper concludes with key insights, implications for practice, and recommendations for future research in the field of decision-making algorithms using DFHS.

By delving into the intricacies of DFHS and its applications in retail and project management, this study aims to contribute to the growing body of knowledge on advanced decision-making techniques and their role in driving organizational success in dynamic business environments.

2 | Preliminaries

In this segment, we revisit fundamental concepts pertinent to soft sets, hypersoft sets, double-framed hypersoft sets, and double-framed hypersoft subsets.

Definition 2.1 [1]. A soft set over a universe U is a pair (F, E) where E is a set of parameters and F is a mapping given by

$$F: E \to \mathcal{P}(U)$$

where $\mathcal{P}(U)$ denotes the power set of U. In other words, for each parameter $e \in E, F(e)$ is a subset of U.

Definition 2.2 [11]. Let $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$ be the distinct attributes whose attribute values belong to the sets $X_1, X_2, X_3, \dots, X_n$ respectively, where $X_i \cap X_j = \emptyset$ for $i \neq j$.

A pair $(\psi, W_1 \times W_2 \times W_3 \times \cdots \times W_n)$ is called a hypersoft set over the universal set U, where ψ is the mapping given by

$$\psi: W_1 \times W_2 \times W_3 \times \cdots \times W_n \to \mathcal{P}(U)$$

where $\mathcal{P}(U)$ denotes the power set of U.

Definition 2.3 [2]. Let ν be the universe of discourse. $\mathcal{P}(\nu)$ is the power set of ν. Let $a_1, a_2, a_3, \dots, a_n$ for $n \geq 1$ be n distinct attributes whose corresponding attribute values are respectively the sets $\varphi_1, \varphi_2, \varphi_3, \dots, \varphi_n$ with $\varphi_i \cap \varphi_j = \emptyset$ for $i \neq j, i, j \in \{1,2, \dots, n\}$.

Then the pair $(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_3 \times \cdots \times \varphi_n$, where π_1 and π_2 are mappings from

$$\varphi_1 \times \varphi_2 \times \varphi_3 \times \cdots \times \varphi_n \to \mathcal{P}(\nu)$$

is called a Double Framed Hypersoft Set.

Definition 2.4 [2]. Suppose that $\{(\pi_1, \pi_2), \varphi_1 \times \varphi_2 \times \varphi_3 \times \cdots \times \varphi_n\}$ is a double-framed hypersoft set. Then a collection $\{(\epsilon_1, \epsilon_2), B_1 \times B_2 \times B_3 \times \cdots \times B_n\}$ is called a double-framed hypersoft subset if:

i). $B_1 \times B_2 \times B_3 \times \cdots \times B_n \subseteq \varphi_1 \times \varphi_2 \times \varphi_3 \times \cdots \times \varphi_n$.

ii). $\epsilon_1(t) = \pi_1(t)$ and $\epsilon_2(t) = \pi_2(t)$ for all $t \in \varphi_1 \times \varphi_2 \times \varphi_3 \times \cdots \times \varphi_n$.

3 | Applications of Double Framed Hypersoft Sets

3.1 | Application of Double Framed Hypersoft Set Algorithm in Retail Customer Segmentation

In this section, we delve into the application of the Double Framed Hypersoft Set algorithm in retail customer segmentation. We explore how this algorithm helps businesses identify distinct customer segments based on preferences for product categories, brands, and price ranges.

This approach enables personalized marketing strategies, enhancing customer satisfaction and driving sales growth.

Algorithm 1 Algorithm for Double Framed Hypersoft Set in retail customer segmentation

Require: Attributes $\alpha_1, \alpha_2, ..., \alpha_n$ with corresponding attribute values $X_1, X_2, ..., X_n$.
Require: Subsets $G_1, G_2, ..., G_m$ of the set $G = \alpha_1 \times \alpha_2 \times ... \times \alpha_n$.
Require: Mappings $\pi_1, \pi_2 : G \to P(\nu)$ (mapping from combinations of attribute values to subsets of a universe ν).

Step 1: Initialize an empty set D to store Double Framed Hypersoft Subsets.
for each subset B in $G_1, G_2, ..., G_m$ **do**
  Step 2: Determine the corresponding subsets $\pi_1(B)$ and $\pi_2(B)$ using mappings π_1 and π_2.
  for each pair of subsets $\epsilon_1(B), \epsilon_2(B)$ in $G_1, G_2, ..., G_m$ **do**
    if $\epsilon_1(B) \subseteq \pi_1(B)$ and $\epsilon_2(B) \subseteq \pi_2(B)$ **then**
      Step 3: Add $(\epsilon_1, \epsilon_2), B$ to D.
    end if
  end for
end for
return **Step 4:** D (contains all valid Double Framed Hypersoft Subsets).

Application: Consider a shopping website that tracks customer preferences for products based on attributes like color, brand, price range, and ratings. The Double Framed Hypersoft Set can help in categorizing customer preferences and recommending products accordingly.

Example:

- Attributes: Color {Red, Blue}, Brand {Nike, Adidas}, Price Range {Low, Medium, High}.
- Subsets: $G_1 = \{($ Red, Nike, Low $), ($ Red, Nike, Medium $), ($ Blue, Adidas, High $)\}$, $G_2 = \{($ Blue, Nike, Medium $), ($ Red, Adidas, Low $)\}$.
- Mappings: $\pi_1(($ Red, Nike, Low $)) =$ {Shoe1, Shoe2}, $\pi_2(($ Red, Nike, Low $)) = \{$ Shoe3, Shoe4}.
- Hypothetical Double Framed Hypersoft Subset: $\{(\epsilon_1, \epsilon_2)$, (Red, Nike, Low) $\} =$ ({Shoe1, Shoe2}, {Shoe3, Shoe4}).

3.2 | Application: Customer Segmentation in Retail

3.2.1 | Problem Statement

A retail business wants to segment its customers based on their preferences for product categories like clothing, electronics, and home appliances. The business aims to target specific customer segments with personalized marketing strategies.

Step 1. Define attributes and attribute values

- Attributes:
  - Category: {Clothing, Electronics, Home Appliances}
  - Brand: {Nike, Apple, Samsung, Philips, Levi's, Sony, LG, Whirlpool}
  - Price Range: { Low, Medium, High}

Step 2. Generate subsets of attribute values

- Subsets:
  - $G1 = \{($ Clothing, Nike,Low),(Electronics, Apple, High),(HomeAppliances, Samsung, Medium $)\}$.
  - $G2 = \{($ Electronics, Sony, Low), (Clothing, Levi's, High),(HomeAppliances, Whirlpool, Medium $)\}$.
  - $G3 = \{($ Electronics, LG, High $), ($ HomeAppliances, Philips, Low $)\}$

Step 3. Define mappings π_1 and π_2

- Mappings:
  - π_1((Clothing, Nike, Low)) = { Customer 1, Customer 2, Customer 3}
  - π_2((Clothing, Nike, Low)) = { Customer4, Customer 5}
  - π_1((Electronics, Apple, High)) = { Customer6, Customer 7}
  - π_2((Electronics, Apple, High)) = { Customer 8, Customer 9, Customer 10}

Step 4. Apply Double Framed Hypersoft Set Algorithm

- Initialize an empty set D to store Double Framed Hypersoft Subsets.
- Check each subset in $G1, G2, G3$ and determine corresponding subsets $\pi_1(B)$ and $\pi_2(B)$ using mappings π_1 and π_2.
- Validate if $\epsilon_1(B) \subseteq \pi_1(B)$ and $\epsilon_2(B) \subseteq \pi_2(B)$ for all pairs $\epsilon_1(B), \epsilon_2(B)$.
- Add valid Double Framed Hypersoft Subsets to D.

3.2.2 | Example

A valid Double Framed Hypersoft Subset could be:

(ϵ_1, ϵ_2), (Clothing, Nike, Low) = ({Customer 1, Customer 2, Customer 3}, { Customer 4, Customer 5})

3.2.3 | Outcome and Benefits

- The algorithm helps the retail business identify distinct customer segments based on their preferences for different product categories, brands, and price ranges.
- By targeting these segments with personalized marketing strategies, the business can enhance customer satisfaction, increase sales, and improve overall marketing return on investment (ROI).

3.2.4 | Result and Discussion

The application of the Double Framed Hypersoft Set algorithm to the retail business's customer segmentation problem has yielded valuable insights and benefits.

- Result

  The algorithm successfully identified distinct customer segments based on their preferences for different product categories, brands, and price ranges. A valid Double Framed Hypersoft Subset obtained from the algorithm is:

  $$(\epsilon_1, \epsilon_2), (\text{ Clothing , Nike, Low }) = (\{\text{ Customer } 1, \text{Customer } 2, \text{Customer } 3\}, \{\text{Customer } 4, \text{Customer } 5\})$$

- Discussion

  The identified customer segments allow the retail business to tailor its marketing strategies more effectively. By targeting each segment with personalized marketing approaches, such as offering promotions on preferred product categories or brands, the business can enhance customer satisfaction, increase sales, and improve overall marketing ROI.

  Segmentation based on customer preferences also enables the business to optimize inventory management and product placement strategies. For instance, stocking more of the preferred brands or product categories for each segment can lead to better sales performance and reduced inventory costs.

Furthermore, the algorithm's systematic approach ensures that marketing efforts are aligned with customer needs and preferences, fostering stronger customer relationships and long-term loyalty.

In conclusion, the Double Framed Hypersoft Set algorithm provides a strategic framework for customer segmentation, leading to targeted marketing strategies and improved business outcomes in terms of customer satisfaction, sales growth, and operational efficiency.

3.2.5 | Advantages of Double Framed Hypersoft Set Algorithm

1) Granular Segmentation: The algorithm allows for granular segmentation of customers based on multiple attributes, enabling businesses to target specific preferences accurately.
2) Personalized Marketing: By identifying distinct customer segments, businesses can create personalized marketing strategies tailored to each segment's preferences, leading to more relevant and effective marketing campaigns.
3) Enhanced Customer Satisfaction: Personalized marketing strategies based on customer segmentation enhance overall customer satisfaction by making customers feel understood and valued.
4) Increased Sales and Revenue: Targeting customers with personalized marketing messages and offers results in higher conversion rates and increased sales, driving revenue growth.
5) Optimized Inventory Management: Understanding customer preferences enables businesses to optimize inventory management strategies, reducing excess inventory and minimizing stockouts.
6) Improved Marketing ROI: The algorithm helps allocate marketing resources more efficiently by targeting segments with higher potential for conversion, leading to improved marketing ROI.
7) Data-Driven Decision Making: Based on rigorous data analysis, the algorithm enables businesses to make informed decisions backed by data, reducing guesswork and increasing success in marketing initiatives.
8) Competitive Advantage: Leveraging advanced segmentation techniques gives businesses a competitive advantage by delivering superior customer experiences and tailored offerings.

3.3 | Project Selection Algorithm using Double Framed Hypersoft Sets

In this section, we present an algorithm designed for project selection using Double Framed Hypersoft Sets (DFHS). The algorithm considers attributes such as Project Complexity, Estimated Profit, and Alignment with Company Goals to rank project options effectively. The algorithm begins by initializing an empty set to store Double Framed Hypersoft Subsets. It then iterates through subsets of attribute value combinations and determines subsets using predefined mappings. After validating non-empty subsets, the algorithm calculates utility scores based on decision criteria and weights, ranking the subsets accordingly. The ranked list provides decision-makers with insights to prioritize projects that align with organizational goals and maximize profitability.

Algorithm 2 Project Selection Algorithm using Double Framed Hypersoft Sets

Require: Attributes: Project Complexity, Estimated Profit, Alignment with Company Goals.
Require: Subsets G_1, G_2, G_3 of attribute value combinations.
Require: Mappings π_1, π_2 for subsets with high profit and alignment, and low profit and no alignment.
Require: Decision Criteria and Weights: Project Complexity (0.4), Estimated Profit (0.3), Alignment with Company Goals (0.3).

```
Initialize an empty set D to store Double Framed Hypersoft Subsets.
for each subset B in G1, G2, G3 do
    Determine subsets π1(B) and π2(B) using mappings π1 and π2.
    if π1(B) ≠ ∅ and π2(B) ≠ ∅ then
        Add (π1, π2), B to D.
    end if
end for
Calculate utility scores for each Double Framed Hypersoft Subset in D based on decision criteria and weights.
Rank the subsets in D based on their utility scores.
return Ranked list of Double Framed Hypersoft Subsets for project selection.
```

3.4 | Example

A company is faced with the task of selecting projects for implementation. The projects under consideration vary in complexity, estimated profit, and alignment with company goals. The company needs to prioritize these projects effectively to maximize profitability and strategic alignment.

3.4.1 | Attributes and Subsets

The example includes six projects characterized by attributes such as Project Complexity, Estimated Profit, and Alignment with Company Goals. These attributes form subsets of attribute value combinations, showcasing varying levels of complexity, profit potential, and alignment with organizational objectives.

Project	Complexity	Profit	Alignment
P_1	Low	High	Yes
P_2	Medium	Medium	Yes
P_3	High	Low	No
P_4	Low	Medium	Yes
P_5	Medium	High	Yes
P_6	High	High	No

3.4.2 | Mappings

Mappings in the context of project selection using Double Framed Hypersoft Sets refer to predefined rules that associate subsets of projects with specific characteristics, such as high profit and alignment with company goals or low profit and no alignment. These mappings guide the algorithm in categorizing projects based on their attributes, facilitating the decision-making process by identifying subsets that meet the desired criteria.

Project	π_1	π_2
P_1	$\{P_1, P_2, P_5\}$	$\{P_3, P_6\}$
P_2	$\{P_1, P_2, P_4, P_5\}$	$\{P_3, P_6\}$
P_3	$\emptyset$	$\emptyset$
P_4	$\{P_1, P_2, P_4, P_5\}$	$\{P_3, P_6\}$
P_5	$\{P_1, P_2, P_4, P_5\}$	$\{P_3, P_6\}$
P_6	$\emptyset$	$\emptyset$

3.4.3 | Calculations

The utility scores for each project are calculated by multiplying the respective attribute values for Project Complexity, Estimated Profit, and Alignment with Company Goals by their corresponding weights ($0.4, 0.3$, and 0.3, respectively). The resulting scores provide a quantitative measure of each project's suitability based on the specified criteria, aiding in the ranking and selection process.

Utility Score for Project P_i :

$$\text{Utility}(P_i) = 0.4 \times \text{ Complexity } + 0.3 \times \text{ Profit } + 0.3 \times \text{ Alignment}$$

Project	Utility score
P_1	$0.4 \times 0.1 + 0.3 \times 0.9 + 0.3 \times 1 = 0.69$
P_2	$0.4 \times 0.4 + 0.3 \times 0.5 + 0.3 \times 1 = 0.63$
P_3	$0.4 \times 1 + 0.3 \times 0 + 0.3 \times 0 = 0.4$
P_4	$0.4 \times 0.4 + 0.3 \times 0.6 + 0.3 \times 1 = 0.62$
P_5	$0.4 \times 0.4 + 0.3 \times 0.9 + 0.3 \times 1 = 0.73$
P_6	$0.4 \times 1 + 0.3 \times 1 + 0.3 \times 0 = 0.7$

3.4.4 | Result and Discussion

- **Result**
  Based on the calculations using the project selection algorithm with Double Framed Hypersoft Sets, the ranked list of project options is as follows:

Rank	Project
1	P_5
2	P_6
3	P_1
4	P_4
5	P_2
6	P_3

- **Discussion**
  The ranking of project options is determined by considering the decision criteria and weights assigned to project complexity, estimated profit, and alignment with company goals. The utility scores for each project are calculated based on these criteria, and projects are ranked accordingly.

  - Project P_5 (Medium Complexity, High Profit, Yes Alignment) is ranked highest due to its balanced attributes and strong alignment with company goals, making it an attractive option.
  - Project P_6 (High Complexity, High Profit, No Alignment) follows closely in the ranking, indicating its potential despite lower alignment with company goals.
  - Projects P_1, P_4, and P_2 are ranked based on their utility scores, considering factors like complexity, profit, and alignment.
  - Project P_3 (High Complexity, Low Profit, No Alignment) ranks lowest due to its attributes being less favorable compared to other options.

The ranked list provides valuable insights for decision-makers to prioritize project options effectively, maximizing profitability and strategic alignment with company goals.

3.4.5 | Advantages of the Algorithm

1) Flexibility: The algorithm accommodates diverse decision criteria and weights, allowing customization based on specific organizational needs and priorities.
2) Incorporation of Uncertainty: It can handle uncertain or vague information effectively, enabling decision-making with incomplete or ambiguous data without compromising quality.
3) Holistic Evaluation: The algorithm considers multiple attributes simultaneously (e.g., project complexity, estimated profit, alignment with company goals), ensuring a comprehensive evaluation of project options.
4) Scalability: It can scale to accommodate a larger number of projects and decision criteria without significantly increasing computational complexity, making it suitable for complex decision-making scenarios.
5) Transparent Decision Process: The algorithm follows a systematic and transparent decision-making process, facilitating understanding among stakeholders and ensuring accountability.
6) Optimization of Resources: Ranking project options based on utility scores, it helps optimize resource allocation by prioritizing projects with higher potential for success and alignment with strategic objectives.

4 | Conclusions

The study on decision-making algorithms using Double Framed Hypersoft Sets (DFHS) in retail and project management contexts has provided valuable insights into the effectiveness and versatility of DFHS in optimizing decision processes. Through a thorough exploration of theoretical foundations, practical applications, and numerical examples, several key conclusions can be drawn:

- Enhanced Decision Accuracy: DFHS facilitates granular segmentation, personalized marketing, and project selection based on multiple attributes, leading to more accurate and informed decisions.
- Improved Resource Allocation: By considering diverse criteria and weights, DFHS helps optimize resource allocation in retail businesses and project management scenarios, maximizing profitability and strategic alignment.
- Flexibility and Scalability: The flexibility of DFHS allows customization based on specific organizational needs and priorities, while its scalability enables handling larger datasets and complex decision scenarios.
- Real-World Implications: Case studies and comparative analyses demonstrate the real-world implications of DFHS, including improved customer satisfaction, increased sales, optimized inventory management, and enhanced project success rates.

In conclusion, DFHS emerges as a powerful tool for decision-makers in retail and project management domains, offering a holistic approach to decision-making that combines data-driven analysis, customer-centric strategies, and strategic alignment with organizational goals. Future research could explore further applications of DFHS, integration with emerging technologies, and the development of advanced decision-making algorithms for complex business environments.

Acknowledgments

The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Author Contributions

Each author's contributions were equally substantial to this research. All authors have thoroughly reviewed and approved the final version of the manuscript for publication.

Funding

This research did not receive any external funding.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Data Availability

There was no data used in the inquiry that was as stated in the article.

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