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On $\delta^* g\alpha$-in Terms of $\mathcal{M}$-Structure Spaces of Continuous and Irresolute Functions

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Abstract

The paper is an introduction to Minimal structure spaces and their properties. The extension of indiscrete topology is known as minimal structure. Indiscrete topology contains only an empty set and a universal set. The minimal structure contains an empty set, a universal set and it may also contain any subset of universal set but it should satisfy the first axiom of topology. We introduce the terms of Minimal delta star g alpha closed sets and also study a new class of functions namely Minimal delta star g alpha continuous and Minimal delta star g alpha irresolute function.

Keywords: Minimal Delta Star g Alpha Closed Sets, Minimal Delta Star g Alpha Continuous, Minimal Delta Star g Alpha Irresolute Function.

1 | Introduction

Veliko [13], Mashhour et al. [11], Levine [9], Njastad [12] were introduced δ-closed (briefly δ-C) sets, pre-open (briefly pre-O) sets, semi-open (briefly semi-O)sets, α-open (briefly α-O) sets respectively. Levine [10] introduced the concept of generalized closed (briefly g-C) sets and studied their basic properties. Popa and Noiri [1] introduced the concept of minimal structure (briefly $\mathcal{M}$-structure) and also they introduced the notions of $min^{\mathbb{S}}$-open (briefly $min^{\mathbb{S}}$-O) sets and $min^{\mathbb{S}}$-closed (briefly $min^{\mathbb{S}}$-C) sets and characterize those sets using $min^{\mathbb{S}}$-closure and $min^{\mathbb{S}}$-interior respectively. They introduced the notion of $\mathcal{M}$-$CONT$ functions defined between minimal structures.

Csaszar [2] introduced the concept of generalized topology and the concept of minimal structure. In 2011, Iuadong S et al. [14] introduced the notion of the generalized topology and minimal structure spaces and also they studied some properties of closed sets on the space. V. Kokilavani [7] introduced the concept of $min^{\mathbb{S}}\delta$-closed (briefly $min^{\mathbb{S}}\delta$-C) set in $\mathcal{M}$-structure.

We are going to use a term of a new class of $\mathcal{M}$-structure set called $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$-set and also we have to introduce $Min^{\mathbb{S}}_{\delta^* g\alpha - CONT}$ and $Min^{\mathbb{S}}_{\delta^* g\alpha - IRST}$-functions.

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2 | Preliminaries

In this section, we introduce the $\mathcal{M}$-structure and also its relation to them.

Definition 2.1. [3] Let $\mathbb{S}$ be a non-empty set and let $min^{\mathbb{S}} \subseteq P(\mathbb{S})$ denotes the power of $\mathbb{S}$ where $min^{\mathbb{S}}$ is an $\mathcal{M}$-structure (or a minimal structure) on $\mathbb{S}$, if φ and $\mathbb{S}$ belong to $min^{\mathbb{S}}$. The members of the minimal structure $min^{\mathbb{S}}$ are called $min^{\mathbb{S}}$-O set and the pair $(\mathbb{S}, min^{\mathbb{S}})$ is called an m-space. The complement of $min^{\mathbb{S}}$-O set is said to be $min^{\mathbb{S}}$-C set.

Definition 2.2. [3] Let $\mathbb{S}$ be a non-empty set and let $min^{\mathbb{S}}$ is an $\mathcal{M}$-structure (or a minimal structure) on $\mathbb{S}$. For a subset J of $\mathbb{S}$, $min^{\mathbb{S}}$-closure of J and $min^{\mathbb{S}}$-interior of J is defined as follows:

$min^{\mathbb{S}}cl(J) = \cap \{F: J \subseteq F, \mathbb{S} - F \in min^{\mathbb{S}}\}$.

$min^{\mathbb{S}}int(J) = \cup \{F: K \subseteq J, K \in min^{\mathbb{S}}\}$.

Lemma 2.1. [3] Let $\mathbb{S}$ be a non-empty set and let $min^{\mathbb{S}}$ is an $\mathcal{M}$-structure (or a minimal structure) on $\mathbb{S}$. For subsets J and I of $\mathbb{S}$, the following properties hold:

a) $min^{\mathbb{S}}cl(\mathbb{S} - J) = \mathbb{S} - min^{\mathbb{S}}int(J)\ and\ min^{\mathbb{S}}int(\mathbb{S} - J) = \mathbb{S} - min^{\mathbb{S}}cl(J)$.

b) If $\mathbb{S} - J \in min^{\mathbb{S}}$, then $min^{\mathbb{S}}cl(J) = J\ and\ if\ J \in min^{\mathbb{S}}$ then $min^{\mathbb{S}}int(J) = J$.

c) $min^{\mathbb{S}}cl(\varphi) = \varphi, min^{\mathbb{S}}cl(\mathbb{S}) = \mathbb{S}\ and\ min^{\mathbb{S}}int(\varphi) = \varphi, min^{\mathbb{S}}int(\mathbb{S}) = \mathbb{S}$.

d) If $J \subseteq I$ then $min^{\mathbb{S}}cl(J) \subseteq min^{\mathbb{S}}cl(I)\ and\ min^{\mathbb{S}}int(J) \subseteq min^{\mathbb{S}}int(I)$.

e) $J \subseteq min^{\mathbb{S}}cl(J)\ and\ min^{\mathbb{S}}int(J) \subseteq J$.

f) $min^{\mathbb{S}}cl(min^{\mathbb{S}}cl(J)) = min^{\mathbb{S}}cl(J)\ and\ min^{\mathbb{S}}int(min^{\mathbb{S}}int(J)) = min^{\mathbb{S}}int(J)$.

g) $min^{\mathbb{S}}_{int(J\cap I)} \subseteq (min^{\mathbb{S}}_{int(J)}) \cap (min^{\mathbb{S}}_{int(I)}) and\ (min^{\mathbb{S}}_{int(J)}) \cup (min^{\mathbb{S}}_{int(I)}) \subseteq min^{\mathbb{S}}_{int(J\cup I)}$.

h) $min^{\mathbb{S}}_{cl(J\cup I)} \subseteq (min^{\mathbb{S}}_{cl(J)}) \cup (min^{\mathbb{S}}_{cl(I)}) and (min^{\mathbb{S}}_{cl(J\cap I)}) \subseteq (min^{\mathbb{S}}_{cl(J)}) \cap (min^{\mathbb{S}}_{cl(I)})$.

Lemma 2.2. [1] Let $(\mathbb{S}, min^{\mathbb{S}})$ be an m-space and J be a non-empty set of $\mathbb{S}$. Then $x \in min^{\mathbb{S}}cl(J)$ if and only if $K \cap J \neq \varphi$ for every $K \in min^{\mathbb{S}}$ containing x.

Definition 2.3. [5] Let $\mathbb{S}$ be a non-empty set and $min^{\mathbb{S}}$ is an $\mathcal{M}$-structure (or a minimal structure) on $\mathbb{S}$. For a subset J of $\mathbb{S}$, pre-closure of J and pre-interior of J are defined as follows:

$min^{\mathbb{S}} - pcl(\mathbb{S} - J) = \mathbb{S} - (min^{\mathbb{S}} - pInt(J))$.

$min^{\mathbb{S}} - pInt(\mathbb{S} - J) = \mathbb{S} - (min^{\mathbb{S}} - pcl(J))$.

Definition 2.4. [5] Let $\mathbb{S}$ be a non-empty set and $min^{\mathbb{S}}$ is an $\mathcal{M}$-structure (or a minimal structure) on $\mathbb{S}$. For a subset J of $\mathbb{S}$, semi-closure of J and semi-interior of J are defined as follows:

$min^{\mathbb{S}} - scl(\mathbb{S} - J) = \mathbb{S} - (min^{\mathbb{S}} - sInt(J))$.

$min^{\mathbb{S}} - sInt(\mathbb{S} - J) = \mathbb{S} - (min^{\mathbb{S}} - scl(J))$.

Definition 2.5. [6] Let $\mathbb{S}$ be a non-empty set and $min^{\mathbb{S}}$ is an $\mathcal{M}$-structure (or a minimal structure) on $\mathbb{S}$. For a subset J of $\mathbb{S}$, αm-closure of J and αm-interior of J are defined as follows:

$\alpha min^{\mathbb{S}}cl(J) = \cap \{F: J \subseteq F, F\ is\ \alpha m - C\ in\ \mathbb{S}\}$

$\alpha min^{\mathbb{S}}int(J) = \cup \{F: K \subseteq J, K\ is\ \alpha m - O\ in\ \mathbb{S}\}$

Definition 2.6. A subset J of an m-space $(\mathbb{S}, min^{\mathbb{S}})$ is said to be i). $min^{\mathbb{S}}$-semi O set [5] if $J \subseteq min^{\mathbb{S}}cl\left(min^{\mathbb{S}}int(J)\right)$.

ii). $min^{\mathbb{S}}$-pre O set [5] if $J \subseteq min^{\mathbb{S}}int\left(min^{\mathbb{S}}cl(J)\right)$.

iii). $\alpha min^{\mathbb{S}}$- O set [6] if $J \subseteq min^{\mathbb{S}}int(min^{\mathbb{S}}cl(min^{\mathbb{S}}int(J)))$.

iv). $min^{\mathbb{S}}$-regular O set [4] if $J = min^{\mathbb{S}}int\left(min^{\mathbb{S}}cl(J)\right)$.

The complement of a $min^{\mathbb{S}}$-semi O (resp. $min^{\mathbb{S}}$-pre O, $\alpha min^{\mathbb{S}}$-O, $min^{\mathbb{X}}$-regular O) set is called $min^{\mathbb{S}}$-semi C (resp. $min^{\mathbb{S}}$-pre C, $\alpha min^{\mathbb{S}}$-C, $min^{\mathbb{S}}$-regular C).

Definition 2.7. [7] The $min^{\mathbb{S}}\delta$-interior of a subset J of $\mathbb{S}$ is the union of all $min^{\mathbb{S}}$- regular O set of $\mathbb{S}$ contained in J and is denoted by $min^{\mathbb{S}}int_{\delta}(J)$. The subset J is called $min^{\mathbb{S}}\delta$-O if $J = min^{\mathbb{S}}int_{\delta}(J)$, i.e. a set is $min^{\mathbb{S}}\delta$-O if it is the union of $min^{\mathbb{S}}$-regular O sets. The complement of a $min^{\mathbb{S}}\delta$-O is called $min^{\mathbb{S}}\delta$-C. Alternatively, a set $(\mathbb{S}, min^{\mathbb{S}})$ is called $min^{\mathbb{S}}\delta$-C if $J = min^{\mathbb{S}}cl_{\delta}(A)$, where $min^{\mathbb{S}}cl_{\delta}(J) = \left\{x \in \mathbb{S}: min^{\mathbb{S}}int\left(min^{\mathbb{S}}cl(K)\right) \neq \varphi, K \in min^{\mathbb{S}} \text{ and } x \in K\right\}$.

Definition 2.8. A subset J of an m-space$(\mathbb{S}, min^{\mathbb{S}})$ is called

i). A $min^{\mathbb{S}}$- generalized-C (briefly $min^{\mathbb{S}}$-g-C) set [4] if $min^{\mathbb{S}}cl(J) \subseteq$ K whenever $J \subseteq K$ and K belong to $min^{\mathbb{S}}$.

ii). A $min^{\mathbb{S}}$- generalized semi-C (briefly $min^{\mathbb{S}}$-gs-C) set [4] if $min^{\mathbb{S}}scl(J) \subseteq$ K whenever $J \subseteq K$ and K belong to $min^{\mathbb{S}}$.

iii). A $min^{\mathbb{S}}$- α generalized-C (briefly $min^{\mathbb{S}}$-αg-C) set [7] if $min^{\mathbb{S}}\alpha cl(J) \subseteq K$ whenever $J \subseteq K$ and K is $min^{\mathbb{S}}$-O set in $(\mathbb{S}, min^{\mathbb{S}})$.

iv). A $min^{\mathbb{S}}$- generalized pre-C (briefly $min^{\mathbb{S}}$-gp-C) set [4] if $min^{\mathbb{S}}pcl(J) \subseteq K$ whenever $J \subseteq K$ and K belong to $min^{\mathbb{S}}$.

The complement of a $min^{\mathbb{S}}$-g-C(resp. $min^{\mathbb{S}}$-gs-C, $min^{\mathbb{S}}$-αg-C, $min^{\mathbb{S}}$-gp-C) set is called $min^{\mathbb{S}}$-g-O (resp. $min^{\mathbb{S}}$-gs-O, $min^{\mathbb{S}}$-αg-O, $min^{\mathbb{S}}$-gp-O).

Definition 2.9. A function $f: \left(\mathbb{S}, min^{\mathbb{S}}\right) \longrightarrow \left(\mathbb{R}, min^{\mathbb{R}}\right)$ is called

i). g-$\left(min^{\mathbb{S}}, min^{\mathbb{R}}\right)$ $CONT$ [4] if $f^{-1}(O)$ is $min^{\mathbb{S}}$-g-C in $\left(\mathbb{S}, min^{\mathbb{S}}\right)$ for every $min^{\mathbb{R}}$-C in $\left(\mathbb{R}, min^{\mathbb{R}}\right)$.

ii). gs-$\left(min^{\mathbb{S}}, min^{\mathbb{R}}\right)$ $CONT$ [4] if $f^{-1}(O)$ is $min^{\mathbb{S}}$-gs-C in $\left(\mathbb{S}, min^{\mathbb{S}}\right)$ for every $min^{\mathbb{R}}$-C in $\left(\mathbb{R}, min^{\mathbb{R}}\right)$.

iii). A $min^{\mathbb{S}}\alpha$ generalized $CONT$ [8] if $f^{-1}(O)$ is $\alpha\, min^{\mathbb{S}}$-$C$ in $\left(\mathbb{S}, min^{\mathbb{S}}\right)$ for every $min^{\mathbb{R}}$-C in $\left(\mathbb{R}, min^{\mathbb{R}}\right)$.

iv). gp-$\left(min^{\mathbb{S}}, min^{\mathbb{R}}\right)$ $CONT$ [4] if $f^{-1}(O)$ is $min^{\mathbb{S}}$-gp-C in $\left(\mathbb{S}, min^{\mathbb{S}}\right)$ for every $min^{\mathbb{R}}$-C in $\left(\mathbb{R}, min^{\mathbb{R}}\right)$.

3 | Properties of $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$-Sets in $\mathcal{M}$ structure Spaces

Definition 3.1. A subset J of an m-space $\left(\mathbb{S}, min^{\mathbb{S}}\right)$ is said to be

i). $min^{\mathbb{S}}_{semi-preO}$-set if $J \subseteq min^{\mathbb{S}}cl(min^{\mathbb{S}}int(min^{\mathbb{S}}cl(J)))$

ii). $min^{\mathbb{S}}_{b-O}$-set if $J \subseteq min^{\mathbb{S}}int(cl(J)) \cup min^{\mathbb{S}}cl(int(J))$

Definition 3.2. A subset J of an m-space $(\mathbb{S}, min^{\mathbb{S}})$ is called

i). $A\ min^{\mathbb{S}}_{\delta generalized-C}$ (briefly $min^{\mathbb{S}}_{\delta g-C}$)-set if $min^{\mathbb{S}}cl_{\delta}(J) \subseteq U$ then $J \subseteq K$ where K is $min^{\mathbb{S}}$-O set in $(\mathbb{S}, min^{\mathbb{S}})$.

ii). $A\ min^{\mathbb{S}}_{generalized\ semi\ pre-C}$ (briefly $min^{\mathbb{S}}_{gsp-C}$) -set if $min^{\mathbb{S}}spcl(J) \subseteq K$ then $J \subseteq K$ where K is $min^{\mathbb{S}}$-O set in $(\mathbb{S}, min^{\mathbb{S}})$.

iii). $A\ min^{\mathbb{S}}_{*generalized\ \alpha-C}$ (briefly $min^{\mathbb{S}}_{*g\alpha-C}$) -set if $min^{\mathbb{S}}cl(J) \subseteq K$ then $J \subseteq K$ where K is $min^{\mathbb{S}}_{g\alpha-O}$ set in $(\mathbb{S}, min^{\mathbb{S}})$.

iv). $A\ min^{\mathbb{S}}_{generalized\ \delta-C}$ (briefly $min^{\mathbb{X}}_{g\delta-C}$) -set if $min^{\mathbb{S}}cl(J) \subseteq K$ then $J \subseteq K$ where K is $min^{\mathbb{S}}_{\delta-O}$ set in $(\mathbb{S}, min^{\mathbb{S}})$.

v). $A\ min^{\mathbb{S}}_{\delta\ generalized*-C}$ (briefly $min^{\mathbb{S}}_{\delta g*-C}$)-set if $min^{\mathbb{S}}cl_{\delta}(J) \subseteq K$ then $J \subseteq K$ where K is $min^{\mathbb{S}}_{\delta-O}$ set in $(\mathbb{S}, min^{\mathbb{S}})$.

vi). $A\ min^{\mathbb{S}}_{generalized\ \delta\ semi-C}$ (briefly $min^{\mathbb{S}}_{g\delta s-C}$)-set if $min^{\mathbb{S}}scl(J) \subseteq K$ then $J \subseteq K$ where K is $min^{\mathbb{S}}_{\delta-O}$ set in $(\mathbb{S}, min^{\mathbb{S}})$.

vii). $A\ min^{\mathbb{S}}_{\delta\ generalized\ b-C}$ (briefly $min^{\mathbb{S}}_{\delta gb-C}$)-set if $min^{\mathbb{S}}bcl(J) \subseteq K$ then $J \subseteq K$ where K is $min^{\mathbb{S}}_{\delta-O}$ set in $(\mathbb{S}, min^{\mathbb{S}})$.

Definition 3.3. A function $f: (\mathbb{S}, min^{\mathbb{S}}) \longrightarrow (\mathbb{R}, min^{\mathbb{R}})$ is called

i). A $min^{\mathbb{S}}_{\delta\ generalized-CONT}$ (briefly $min^{\mathbb{S}}_{\delta g-CONT}$) if $f^{-1}(E)$ is $min^{\mathbb{S}}_{\delta g-C}$ in $(\mathbb{S}, min^{\mathbb{S}})$ for every $min^{\mathbb{S}}$-C in $(\mathbb{R}, min^{\mathbb{R}})$.

ii). A $min^{\mathbb{S}}_{generalized\ semi\ pre-CONT}$ (briefly $min^{\mathbb{S}}_{gsp-CONT}$) if $f^{-1}(E)$ is $min^{\mathbb{S}}_{gsp-C}$ in $(\mathbb{S}, min^{\mathbb{S}})$ for every $min^{\mathbb{S}}$-C in $(\mathbb{R}, min^{\mathbb{R}})$.

iii). A $min^{\mathbb{S}}_{generalized\ \delta-CONT}$ (briefly $min^{\mathbb{S}}_{g\delta-CONT}$) if $f^{-1}(E)$ is $min^{\mathbb{S}}_{g\delta-C}$ in $(\mathbb{S}, min^{\mathbb{S}})$ for every $min^{\mathbb{S}}$-C in $(\mathbb{R}, min^{\mathbb{R}})$.

iv). A $min^{\mathbb{S}}_{\delta\ generalized*-CONT}$ (briefly $min^{\mathbb{S}}_{\delta g*-CONT}$) if $f^{-1}(E)$ is $min^{\mathbb{S}}_{\delta g*-C}$ in $(\mathbb{S}, min^{\mathbb{S}})$ for every $min^{\mathbb{S}}$-C in $(\mathbb{R}, min^{\mathbb{R}})$.

v). A $min^{\mathbb{S}}_{generalized\ \delta\ semi-CONT}$ (briefly $min^{\mathbb{S}}_{g\delta s-CONT}$) if $f^{-1}(E)$ is $min^{\mathbb{S}}_{g\delta s-C}$ in $(\mathbb{S}, min^{\mathbb{S}})$ for every $min^{\mathbb{S}}$-C in $(\mathbb{R}, min^{\mathbb{R}})$.

vi). A $min^{\mathbb{S}}_{\delta\ generalized\ pre-CONT}$ (briefly $min^{\mathbb{S}}_{\delta gp-CONT}$) if $f^{-1}(E)$ is $min^{\mathbb{S}}_{\delta gp-C}$ in $(\mathbb{S}, min^{\mathbb{S}})$ for every $min^{\mathbb{S}}$-C in $(\mathbb{R}, min^{\mathbb{R}})$.

vii). A $min^{\mathbb{S}}_{\delta\ generalized\ b-CONT}$ (briefly $min^{\mathbb{S}}_{\delta gb-CONT}$) if $f^{-1}(E)$ is $min^{\mathbb{S}}_{\delta gb-C}$ in $(\mathbb{S}, min^{\mathbb{S}})$ for every $min^{\mathbb{S}}$-C in $(\mathbb{R}, min^{\mathbb{R}})$.

Theorem 3.1. Every $min^{\mathbb{S}}$-O set is $min^{\mathbb{S}}_{*g\alpha-O}$ set.

Proof. Let J be $min^{\mathbb{S}}$-O set in $\mathbb{S}$, then $\mathbb{S} - J$ is $min^{\mathbb{S}}$-C set. Therefore $min^{\mathbb{S}}cl(\mathbb{S} - J) = \mathbb{S} - J \subseteq \mathbb{S}$ whenever $\mathbb{S} - J \subseteq \mathbb{S}$ and $\mathbb{S}$ is $min^{\mathbb{S}}_{g\alpha-O}$ implies $\mathbb{S} - J$ is $min^{\mathbb{S}}_{*g\alpha-C}$ set.

Theorem 3.2. Every $min^{\mathbb{S}}\delta\text{-}C$ set is $min^{\mathbb{S}}_{*g\alpha-C}$ set.

Proof. Let J be $min^{\mathbb{S}}\delta\text{-}C$ set and K be any $min^{\mathbb{S}}_{g\alpha-O}$ set containing J. Since J is $min^{\mathbb{S}}\delta\text{-}C$, $min^{\mathbb{S}}cl_{\delta}(J) = J$. Therefore $min^{\mathbb{S}}cl_{\delta}(J) \subseteq J \subseteq K$. We know that $min^{\mathbb{S}}cl(J) \subseteq min^{\mathbb{S}}cl_{\delta}(J) \subseteq K$. Hence J is $min^{\mathbb{S}}_{*g\alpha-C}$ set.

Definition 3.4. A subset J of an m-space $(\mathbb{S}, min^{\mathbb{S}})$ is called $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ if $min^{\mathbb{S}}cl_{\delta}(J) \subseteq K$ whenever $J \subseteq K$ and K is a $min^{\mathbb{S}}_{*g\alpha-O}$ set in $(\mathbb{S}, min^{\mathbb{S}})$.

Theorem 3.3. Every $min^{\mathbb{S}}\delta\text{-}C$ set is $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ set but the converse is not true.

Proof. Let $J \subseteq K$ and K is $min^{\mathbb{S}}_{*g\alpha-O}$ set. Since J is $min^{\mathbb{S}}\delta\text{-}C$, $min^{\mathbb{S}}cl_{\delta}(J) = J$, then $min^{\mathbb{S}}cl_{\delta}(J) \subseteq K$. Therefore J is $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ set.

Example 1. Let $\mathbb{S} = \{h, i, k\}, min^{\mathbb{S}}\text{-}O = \{\varphi, \mathbb{S}, \{h\}, \{k\}, \{h, k\}\}$; *Here* $\{k\}$ is $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ but not $min^{\mathbb{S}}\delta\text{-}C$ in $(\mathbb{S}, min^{\mathbb{S}})$.

Theorem 3.4. Every $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ set is $min^{\mathbb{S}}\text{-}gs\text{-}C$ set but the converse is not true.

Proof. Let $J \subseteq K$ and K is $min^{\mathbb{S}}_{*g\alpha-O}$ set. Since every $min^{\mathbb{S}}\text{-}O$ set is $min^{\mathbb{S}}_{*g\alpha-O}$ set [by theorem 3.1], then K is $min^{\mathbb{S}}_{*g\alpha-O}$ set. Since J is $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ set, then $min^{\mathbb{S}}cl_{\delta}(J) \subseteq K$. But $min^{\mathbb{S}}scl(J) \subseteq min^{\mathbb{S}}cl_{\delta}(J)$, then $min^{\mathbb{S}}scl(J) \subseteq K$. Therefore J is $min^{\mathbb{S}}\text{-}gs\text{-}C$ set.

Example 2. Let $\mathbb{S} = \{h, i, k\}, min^{\mathbb{S}}\text{-}O = \{\varphi, \mathbb{S}, \{h\}, \{h, i\}\}$; *Here* $\{i\}$ is $min^{\mathbb{S}}\text{-}gs\text{-}C$ but not $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ in $(\mathbb{S}, min^{\mathbb{S}})$.

Theorem 3.5. Every $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ set is $min^{\mathbb{S}}\text{-}\alpha g\text{-}C$ set but the converse is not true.

Proof. Let $J \subseteq K$ and K is $min^{\mathbb{S}}\text{-}O$ set. Since every $min^{\mathbb{S}}\text{-}O$ set is $min^{\mathbb{S}}_{*g\alpha-O}$ set, then K is $min^{\mathbb{S}}_{*g\alpha-O}$ set. Since J is $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$, then $min^{\mathbb{S}}cl_{\delta}(J) \subseteq K$. But $min^{\mathbb{S}}\alpha cl(J) \subseteq min^{\mathbb{S}}cl_{\delta}(J)$ set, then $min^{\mathbb{S}}\alpha cl(J) \subseteq K$. Therefore J is $min^{\mathbb{S}}\text{-}\alpha g\text{-}C$ set.

Example 3. Let $\mathbb{S} = \{h, i, k\}, min^{\mathbb{S}}\text{-}O = \{\varphi, \mathbb{S}, \{k\}, \{h, k\}\}$; *Here* $\{h\}$ is $min^{\mathbb{S}}\text{-}\alpha g\text{-}C$ set but not $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ in $(\mathbb{S}, min^{\mathbb{S}})$.

Theorem 3.6. Every $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ set is $min^{\mathbb{S}}\text{-}gsp\text{-}C$ set but the converse is not true.

Proof. Let $J \subseteq K$ and K is $min^{\mathbb{S}}\text{-}O$ set. Since every $min^{\mathbb{S}}\text{-}O$ set is $min^{\mathbb{S}}_{*g\alpha-O}$ set, then K is $min^{\mathbb{S}}_{*g\alpha-O}$ set. Since J is $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ set, then $min^{\mathbb{S}}cl_{\delta}(J) \subseteq K$. But $min^{\mathbb{S}}spcl(J) \subseteq min^{\mathbb{S}}cl_{\delta}(J)$, then $min^{\mathbb{S}}spcl(J) \subseteq K$. Therefore J is $min^{\mathbb{S}}\text{-}gsp\text{-}C$ set.

Example 4. Let $\mathbb{S} = \{h, i, k\}, min^{\mathbb{S}}\text{-}O = \{\varphi, \mathbb{S}, \{h, i\}\}$; *Here* $\{h\}$ is $min^{\mathbb{S}}\text{-}gsp\text{-}C$ set but not $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ in $(\mathbb{S}, min^{\mathbb{S}})$.

Theorem 3.7. Every $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ set is $min^{\mathbb{S}}\text{-}gp\text{-}C$ set but the converse is not true.

Proof. Let $J \subseteq K$ and K is $min^{\mathbb{S}}\text{-}O$ set. Since every $min^{\mathbb{S}}\text{-}O$ set is $min^{\mathbb{S}}_{*g\alpha-O}$ set, then K is $min^{\mathbb{S}}_{*g\alpha-O}$ set. Since J is $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$, then $min^{\mathbb{S}}cl_{\delta}(J) \subseteq K$. But $min^{\mathbb{S}}pcl(J) \subseteq min^{\mathbb{S}}cl_{\delta}(J)$, then $min^{\mathbb{S}}pcl(J) \subseteq K$. Therefore J is $min^{\mathbb{S}}\text{-}gp\text{-}C$ set.

Example 5. Let $\mathbb{S} = \{h, i, k\}, min^{\mathbb{S}}\text{-}O = \{\varphi, \mathbb{S}, \{i, k\}\}$; *Here* $\{i\}$ is $min^{\mathbb{S}}\text{-}gp\text{-}\mathrm{C}$ but not $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ in $(\mathbb{S}, min^{\mathbb{S}})$.

Theorem 3.8. Every $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ set is $min^{\mathbb{S}}_{\delta gp-C}$ set but the converse is not true.

Proof. Let $J \subseteq K$ and K is $min^{\mathbb{S}}\delta\text{-}O$ set. Since every $min^{\mathbb{S}}\delta\text{-}O$ set is $min^{\mathbb{S}}_{*g\alpha-O}$, then K is $min^{\mathbb{S}}_{*g\alpha-O}$ set. Since J is $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ set, then $min^{\mathbb{S}}cl_{\delta}(J) \subseteq K$. But $min^{\mathbb{S}}pcl(J) \subseteq min^{\mathbb{S}}cl_{\delta}(J)$, then $min^{\mathbb{S}}pcl(J) \subseteq K$. Therefore J is $min^{\mathbb{S}}_{\delta gp-C}$ set.

Example 6. Let $\mathbb{S} = \{h, i, k\}$, $min^{\mathbb{S}}\text{-}O = \{\varphi, \mathbb{S}, \{h\}, \{i\}, \{h, i\}, \{i, k\}\}$; $Here\ \{h, i\}$ is $min^{\mathbb{S}}_{\delta gp-C}$ but not $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ in $(\mathbb{S}, min^{\mathbb{S}})$.

Theorem 3.9. Every $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ set is $min^{\mathbb{S}}_{g\delta-C}$ set but the converse is not true.

Proof. Let $J \subseteq K$ and K is $min^{\mathbb{S}}\delta\text{-}O$ set. Since every $min^{\mathbb{S}}\delta\text{-}O$ set is $min^{\mathbb{S}}_{*g\alpha-O}$, then K is $min^{\mathbb{S}}_{*g\alpha-O}$ set. Since J is $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ set, then $min^{\mathbb{S}}cl_{\delta}(J) \subseteq K$. But $min^{\mathbb{S}}cl(J) \subseteq min^{\mathbb{S}}cl_{\delta}(J)$, then $min^{\mathbb{S}}cl(J) \subseteq K$. Therefore J is $min^{\mathbb{S}}_{g\delta-C}$ set.

Example 7. Let $\mathbb{S} = \{h, i, k\}$, $min^{\mathbb{S}}\text{-}O = \{\varphi, \mathbb{S}, \{i, k\}\}$; $Here\ \{k\}$ is $min^{\mathbb{S}}_{g\delta-C}$ set but not $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ in $(\mathbb{S}, min^{\mathbb{S}})$.

Theorem 3.10. Every $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ set is $min^{\mathbb{S}}_{g\delta*-C}$ set but the converse is not true.

Proof. Let $J \subseteq K$ and K is $min^{\mathbb{S}}\delta\text{-}O$ set. Since every $min^{\mathbb{S}}\delta\text{-}O$ set is $min^{\mathbb{S}}_{*g\alpha-O}$, then K is $min^{\mathbb{S}}_{*g\alpha-O}$-set. Since J is $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$, then $min^{\mathbb{S}}cl_{\delta}(J) \subseteq K$. Therefore J is $min^{\mathbb{S}}_{g\delta*-C}$-set.

Example 8. Let $\mathbb{S} = \{h, i, k\}$, $min^{\mathbb{S}}\text{-}O = \{\varphi, \mathbb{S}, \{i\}\}$; $Here\ \{i\}$ is $min^{\mathbb{S}}_{g\delta*-C}$ but not $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ in $(\mathbb{S}, min^{\mathbb{S}})$.

Theorem 3.11. Every $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ set is $min^{\mathbb{S}}_{g\delta s-C}$ set but the converse is not true.

Proof. Let $J \subseteq K$ and K is $min^{\mathbb{S}}\delta\text{-}O$ set. Since every $min^{\mathbb{S}}\delta\text{-}O$ set is $min^{\mathbb{S}}_{*g\alpha-O}$, then K is $min^{\mathbb{S}}_{*g\alpha-O}$-set. Since J is $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$, then $min^{\mathbb{S}}cl_{\delta}(J) \subseteq K$. But $min^{\mathbb{S}}scl(J) \subseteq min^{\mathbb{S}}cl_{\delta}(J)$, then $min^{\mathbb{S}}scl(J) \subseteq K$. Therefore J is $min^{\mathbb{S}}_{g\delta s-C}$ set.

Example 9. Let $\mathbb{S} = \{h, i, k\}$, $min^{\mathbb{S}}\text{-}O = \{\varphi, \mathbb{S}, \{k\}, \{h, k\}\}$; $Here\ \{h, k\}$ is $min^{\mathbb{S}}_{g\delta s-C}$ but not $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ in $(\mathbb{S}, min^{\mathbb{S}})$.

Theorem 3.12. Every $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ -set is $min^{\mathbb{S}}_{\delta gb-C}$ set but the converse is not true.

Proof. Let $J \subseteq K$ and K is $min^{\mathbb{S}}\delta\text{-}O$ set. Since every $min^{\mathbb{S}}\delta\text{-}O$ set is $min^{\mathbb{S}}_{*g\alpha-O}$, then K is $min^{\mathbb{S}}_{*g\alpha-O}$-set. Since J is $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$, then $min^{\mathbb{S}}cl_{\delta}(J) \subseteq K$. But $min^{\mathbb{S}}bcl(J) \subseteq min^{\mathbb{S}}cl_{\delta}(J)$, then $min^{\mathbb{S}}bcl(J) \subseteq K$. Therefore J is $min^{\mathbb{S}}_{\delta gb-C}$-set.

Example 10. Let $\mathbb{S} = \{h, i, k\}$, $min^{\mathbb{S}}\text{-}O = \{\varphi, \mathbb{S}, \{h, i\}\}$; $Here\ \{h, i\}$ is $min^{\mathbb{S}}_{\delta gb-C}$ but not $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ in $(\mathbb{S}, min^{\mathbb{S}})$.

Theorem 3.13. The finite union of $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$-sets is $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ set.

Proof. Let $\{J_i\ /\ i = 1,2,3 \ldots n\}$ be a finite class of $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ subsets of an m-space $(\mathbb{S}, min^{\mathbb{S}})$. Then for each $min^{\mathbb{S}}_{*g\alpha-O}$ set K_i in $\mathbb{S}$ containing $J_i, cl_{\delta}(J_i) \subseteq K_i, i \in \{1,2,3 \ldots . n\}$. Hence $\cup_i\ J_i \subseteq \cup_i\ K_i = E$. Since the arbitrary union of $min^{\mathbb{S}}_{*g\alpha-O}$ sets in $(\mathbb{S}, min^{\mathbb{S}})$ is also $min^{\mathbb{S}}_{*g\alpha-O}$-set in $(\mathbb{S}, min^{\mathbb{S}}), E$ is $min^{\mathbb{S}}_{*g\alpha-O}$ in $(\mathbb{S}, min^{\mathbb{S}})$. Also $\cup_i\ cl_{\delta}(J_i) = cl_{\delta}(\cup_i J_i) \subseteq E$. Therefore $\cup_i J_i$ is $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ in $(\mathbb{S}, min^{\mathbb{S}})$.

Remark 3.1. The intersection of any two $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ in $(\mathbb{S}, min^{\mathbb{S}})$ need not be $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ in $(\mathbb{S}, min^{\mathbb{S}})$, it can be shown through an example.

Example 11. Let $\mathbb{S} = \{h, i, k, j\},, min^{\mathbb{S}}\text{-}O = \{\varphi, \mathbb{S}, \{h\}, \{i\}, \{h, i\}\}$; *Here* $\{h, i\}$ *and* $\{h, k\}$ are $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ sets but their intersection $\{h\}$ is not $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ set.

Theorem 3.14. Let J be a $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ of $(\mathbb{S}, min^{\mathbb{S}})$, then $min^{\mathbb{S}} cl_\delta(J) - J$ does not contain a non-empty $min^{\mathbb{S}}_{* g\alpha - C}$ set.

Proof. Suppose that J is $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$, let F be a $min^{\mathbb{S}}_{* g\alpha - C}$ set contained in $min^{\mathbb{S}} cl_\delta(J) - J$. Now F^c is $min^{\mathbb{S}}_{* g\alpha - O}$set of $(\mathbb{S}, min^{\mathbb{S}})$ such that $J \subseteq F^c$. Since J is $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$set of $(\mathbb{S}, min^{\mathbb{S}})$, then $min^{\mathbb{S}} cl_\delta(J) \subseteq F^c$. Thus $F \subseteq min^{\mathbb{S}} cl_\delta((J))^c$. Also $F \subseteq min^{\mathbb{S}} cl_\delta(J) - J$. Therefore $F \subseteq min^{\mathbb{S}}(cl_\delta(J)) \subset\cap min^{\mathbb{S}}(cl_\delta(J)) = \varphi$. Hence $F = \varphi$.

Theorem 3.15. If J is $min^{\mathbb{S}}_{* g\alpha - O}$ and $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ subset of $(\mathbb{S}, min^{\mathbb{S}})$ then J is a $min^{\mathbb{S}}\delta\text{-}C$ subset of $(\mathbb{S}, min^{\mathbb{S}})$.

Proof. Since J is $min^{\mathbb{S}}_{* g\alpha - O}$ and $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$, $min^{\mathbb{S}}(cl_\delta(J)) \subseteq J$. Hence J is $min^{\mathbb{S}}\delta\text{-}C$.

Theorem 3.16. The intersection of a $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ set and a $min^{\mathbb{S}}\delta\text{-}C$ set is always $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$.

Proof. Let J be $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ set and let F be $min^{\mathbb{S}}\delta\text{-}C$. If K is an $min^{\mathbb{S}}_{* g\alpha - O}$-set with $J \cap F \subseteq K$, then $J \subseteq K \cap F^c$ and so $min^{\mathbb{S}} cl_\delta(J) \subseteq K \cap F^c$. Now $min^{\mathbb{S}} cl_\delta(\mathrm{J} \cap \mathrm{F}) \subseteq min^{\mathbb{S}} cl_\delta(\mathrm{J}) \cap \mathrm{F} \subseteq K$. Hence $J \cap F$ is $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ set.

Theorem 3.17. If J is a $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ set in an m-space $(\mathbb{S}, min^{\mathbb{S}})$ and $J \subseteq I \subseteq min^{\mathbb{S}} cl_\delta(J)$, then I is also a $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ set.

Proof. Let K be a $min^{\mathbb{S}}_{* g\alpha - O}$ set of $(\mathbb{S}, min^{\mathbb{S}})$ such that $I \subseteq min^{\mathbb{S}} cl_\delta(J)$, then $J \subseteq K$. Since J is $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$set, $min^{\mathbb{S}} cl_\delta(J) \subseteq K$. Also since $I \subseteq min^{\mathbb{S}} cl_\delta(J)$, $min^{\mathbb{S}} cl_\delta(I) \subseteq min^{\mathbb{S}} cl_\delta(cl_\delta(J) = min^{\mathbb{S}} cl_\delta(J) \subseteq K$ implies $min^{\mathbb{S}} cl_\delta(I) \subseteq K$. Therefore I is also a $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ set.

Theorem 3.18. Let J be $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ of $(\mathbb{S}, min^{\mathbb{S}})$, then J is $min^{\mathbb{S}}\delta\text{-}C$ iff $min^{\mathbb{S}} cl_\delta(J) - J$ is $min^{\mathbb{S}}_{* g\alpha - C}$ set.

Proof. Necessity. Let J be a $min^{\mathbb{S}}\delta\text{-}C$ subset of $\mathbb{S}$. Then $min^{\mathbb{S}} cl_\delta(J) - J$ and so $min^{\mathbb{S}} cl_\delta(J) - J = \varphi$ which is $min^{\mathbb{S}}_{* g\alpha - C}$ set.

Sufficiency. Since J is $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$, by theorem 3.14, $min^{\mathbb{S}} cl_\delta(J) - J$ does not contain a non-empty $min^{\mathbb{S}}_{* g\alpha - C}$ -set. But $min^{\mathbb{S}} cl_\delta(J) - J = \varphi$. That is $min^{\mathbb{S}} cl_\delta(J) = J$. Hence J is $min^{\mathbb{S}}\delta\text{-}C$ set.

4 | $Min^{\mathbb{S}}_{\delta^* g\alpha - CONT}$ Functions in $\mathcal{M}$Structure Spaces

Definition 4.1. A function $f : (\mathbb{S}, min^{\mathbb{S}}) \longrightarrow (\mathbb{R}, min^{\mathbb{R}})$ is said to be a $Min^{\mathbb{S}}_{\text{delta star}\, g\alpha - CONT}$ (briefly $Min^{\mathbb{S}}_{\delta^* g\alpha - CONT}$) if $f^{-1}(E)$ is $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ in $(\mathbb{S}, min^{\mathbb{S}})$ for every $min^{\mathbb{S}}\text{-}C$ in $(\mathbb{R}, min^{\mathbb{R}})$.

Theorem 4.1. Every $Min^{\mathbb{X}}_{\delta^* g\alpha - CONT}$ is $min^{\mathbb{X}}\text{-}gs\text{-}CONT$ (resp. $min^{\mathbb{S}}\text{-}\alpha g\text{-}CONT$, $min^{\mathbb{S}}\text{-}gsp\text{-}CONT$, $min^{\mathbb{X}}\text{-}gp\text{-}CONT$) but the converse is not true.

Proof. Let E be a $min^{\mathbb{S}}\text{-}C$ set in $(\mathbb{R}, min^{\mathbb{R}})$. Since f is $Min^{\mathbb{S}}_{\delta^* g\alpha - CONT}$ map. $f^{-1}(E)$ is $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ in $(\mathbb{S}, min^{\mathbb{S}})$. Since every $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ set is $min^{\mathbb{S}}\text{-}gs\text{-}C$ (resp $min^{\mathbb{S}}\text{-}\alpha g\text{-}C$, $min^{\mathbb{S}}\text{-}gsp\text{-}C$, $min^{\mathbb{S}}\text{-}gp\text{-}C$), therefore $f^{-1}(E)$ is $min^{\mathbb{S}}\text{-}gs\text{-}C$ (resp $min^{\mathbb{S}}\text{-}\alpha g\text{-}C$, $min^{\mathbb{S}}\text{-}gsp\text{-}C$, $min^{\mathbb{S}}\text{-}gp\text{-}C$) in $(\mathbb{S}, min^{\mathbb{S}})$. Hence f is $min^{\mathbb{S}}\text{-}gs\text{-}CONT$ (resp. $min^{\mathbb{S}}\text{-}\alpha g\text{-}CONT$, $min^{\mathbb{S}}\text{-}gsp\text{-}CONT$, $min^{\mathbb{S}}\text{-}gp\text{-}CONT$).

Example 12. Let $\mathbb{S} = \mathbb{R} = \{h, i, k\}, min^{\mathbb{S}}\text{-}O = \{\varphi, \mathbb{S}, \{h\}, \{i\}, \{h, i\}\}; min^{\mathbb{R}}\text{-}O = \{\varphi, \mathbb{R}, \{i\}, \{h, k\}\}$

Def $f : (\mathbb{S}, min^{\mathbb{S}}) \longrightarrow (\mathbb{R}, min^{\mathbb{R}})$ where $f(h) = h, f(i) = i, f(k) = k,$

$min^{\mathbb{S}} gsC(\mathbb{S}, min^{\mathbb{S}}) = \{\varphi, \mathbb{S}, \{h\}, \{i\}, \{k\}, \{i,k\}, \{h,k\}\};$

$min^{\mathbb{S}} gspC(\mathbb{S}, min^{\mathbb{S}}) = \{\varphi, \mathbb{S}, \{h\}, \{i\}, \{k\}, \{i,k\}, \{h,k\}\};$

$Min^{\mathbb{S}}_{\delta^* g\alpha C}(\mathbb{S}, min^{\mathbb{S}}) = \{\varphi, \mathbb{S}, \{k\}, \{h,k\}, \{i,k\}\};$

Here $f^{-1}[\{\mathrm{i}\}] = \{\mathrm{i}\}$ is not $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ in $(\mathbb{S}, min^{\mathbb{S}})$, therefore it is $min^{\mathbb{S}}$-gs-$CONT$, $min^{\mathbb{S}}$-gsp-$CONT$ but not $Min^{\mathbb{S}}_{\delta^* g\alpha - CONT}$.

Example 13. Let $\mathbb{S} = \mathbb{R} = \{h, i, k\}, min^{\mathbb{S}}\text{-}O = \{\varphi, \mathbb{S}, \{k\}, \{h,k\}\}; min^{\mathbb{R}}\text{-}O = \{\varphi, \mathbb{R}, \{i,k\}\}$

Def $f : (\mathbb{S}, min^{\mathbb{S}}) \longrightarrow (\mathbb{R}, min^{\mathbb{R}})$ where $f(h) = h, f(i) = i, f(k) = k,$

$min^{\mathbb{S}} gpC(\mathbb{S}, min^{\mathbb{S}}) = \{\varphi, \mathbb{S}, \{h\}, \{i\}, \{k\}, \{i,k\}, \{h,k\}\};$

$Min^{\mathbb{S}}_{\delta^* g\alpha C}(\mathbb{S}, min^{\mathbb{S}}) = \{\varphi, \mathbb{S}, \{h\}, \{i\}, \{h,i\}, \{i,k\}\}$

Here $f^{-1}[\{\mathrm{h}\}] = \{\mathrm{h}\}$ is not $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ in $(\mathbb{S}, min^{\mathbb{S}})$, therefore it is $min^{\mathbb{S}}$-gp-$CONT$ but not $Min^{\mathbb{S}}_{\delta^* g\alpha - CONT}$.

Example 14. Let $\mathbb{S} = \mathbb{R} = \{h, i, k\}, min^{\mathbb{S}}\text{-}O = \{\varphi, \mathbb{S}, \{h\}, \{h,i\}\}; min^{\mathbb{R}}\text{-}O = \{\varphi, \mathbb{R}, \{i\}, \{k\}, \{h,k\}\}$

Def $f : (\mathbb{S}, min^{\mathbb{S}}) \longrightarrow (\mathbb{R}, min^{\mathbb{R}})$ where $f(h) = h, f(i) = i, f(k) = k,$

$min^{\mathbb{S}} \alpha gC(\mathbb{S}, min^{\mathbb{S}}) = \{\varphi, \mathbb{S}, \{i\}, \{k\}, \{i,k\}, \{h,k\}\};$

$Min^{\mathbb{S}}_{\delta^* g\alpha C}(\mathbb{S}, min^{\mathbb{S}}) = \{\varphi, \mathbb{S}, \{k\}, \{h,k\}, \{i,k\}\};$

Here $f^{-1}[\{\mathrm{i}\}] = \{\mathrm{i}\}$ is not $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ in $(\mathbb{S}, min^{\mathbb{S}})$, therefore it is $min^{\mathbb{S}}$-αg-$CONT$ but not $Min^{\mathbb{S}}_{\delta^* g\alpha - CONT}$.

Theorem 4.2. Every $Min^{\mathbb{S}}_{\delta^* g\alpha - CONT}$ is $min^{\mathbb{S}}_{\delta g - CONT}$(resp $min^{\mathbb{S}}_{\delta gp - CONT}, min^{\mathbb{S}}_{\delta g* - CONT}, min^{\mathbb{S}}_{g\delta s - CONT}, min^{\mathbb{S}}_{\delta gb - CONT}$) but the converse is not true.

Proof. Let E be a $min^{\mathbb{S}}$-C set in $(\mathbb{R}, min^{\mathbb{R}})$. Since f is $Min^{\mathbb{S}}_{\delta^* g\alpha - CONT}$ map. $f^{-1}(E)$ is $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ in $(\mathbb{S}, min^{\mathbb{S}})$. Since every $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$-set is $min^{\mathbb{S}}_{\delta g - C}$ (resp $min^{\mathbb{S}}_{\delta gp - C}$, $min^{\mathbb{S}}_{\delta g* - C}, min^{\mathbb{S}}_{g\delta s - C}, min^{\mathbb{S}}_{\delta gb - C}$), therefore $f^{-1}(E)$ is$min^{\mathbb{S}}_{\delta g - C}$ (resp $min^{\mathbb{S}}_{\delta gp - C}, min^{\mathbb{S}}_{\delta g* - C}, min^{\mathbb{S}}_{g\delta s - C}, min^{\mathbb{S}}_{\delta gb - C}$) in $(\mathbb{S}, min^{\mathbb{S}})$. Hence f is $min^{\mathbb{S}}_{\delta g - CONT}$(resp $min^{\mathbb{S}}_{\delta gp - CONT}, min^{\mathbb{S}}_{\delta g* - CONT}, min^{\mathbb{S}}_{g\delta s - CONT}, min^{\mathbb{S}}_{\delta gb - CONT}$).

Example 15. Let $\mathbb{S} = \mathbb{R} = \{h, i, k\}, min^{\mathbb{S}}\text{-}O = \{\varphi, \mathbb{S}, \{i\}\}; min^{\mathbb{R}}\text{-}O = \{\varphi, \mathbb{R}, \{h\}, \{i\}, \{h,i\}, \{h,k\}\}$

Def $f : (\mathbb{S}, min^{\mathbb{S}}) \longrightarrow (\mathbb{R}, min^{\mathbb{R}})$ where $f(h) = h, f(i) = i, f(k) = k,$

$min^{\mathbb{S}}_{\delta gbC}(\mathbb{S}, min^{\mathbb{S}}) = \{\varphi, \mathbb{S}, \{h\}, \{i\}, \{k\}, \{h,i\}, \{i,k\}, \{h,k\}\};$

$min^{\mathbb{S}}_{g\delta sC}(\mathbb{S}, min^{\mathbb{S}}) = \{\varphi, \mathbb{S}, \{h\}, \{i\}, \{k\}, \{h,i\}, \{i,k\}, \{h,k\}\};$

$Min^{\mathbb{S}}_{\delta^* g\alpha C}(\mathbb{S}, min^{\mathbb{S}}) = \{\varphi, \mathbb{S}, \{h\}, \{k\}, \{h,i\}, \{i,k\}, \{h,k\}\};$

Here $f^{-1}[\{\mathrm{i}\}] = \{\mathrm{i}\}$ is not $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ in $(\mathbb{S}, min^{\mathbb{S}})$, therefore it is $min^{\mathbb{S}}_{\delta gb - CONT}, min^{\mathbb{S}}_{g\delta s - CONT}$ but not $Min^{\mathbb{S}}_{\delta^* g\alpha - CONT}$.

Example 16. Let $\mathbb{S} = \mathbb{R} = \{h, i, k\}, min^{\mathbb{S}}\text{-}O = \{\varphi, \mathbb{S}, \{h\}\}; min^{\mathbb{R}}\text{-}O = \{\varphi, \mathbb{R}, \{h\}, \{i\}, \{i,k\}\}$

Def $f:(\mathbb{S}, min^{\mathbb{S}}) \longrightarrow (\mathbb{R}, min^{\mathbb{R}})$ where $f(h) = h, f(i) = i, f(k) = k,$

$min^{\mathbb{S}}_{\delta gC}(\mathbb{S}, min^{\mathbb{S}}) = \{\varphi, \mathbb{S}, \{h\}, \{i\}, \{k\}, \{h, i\}, \{i, k\}, \{h, k\}\};$

$min^{\mathbb{S}}_{\delta gpC}(\mathbb{S}, min^{\mathbb{S}}) = \{\varphi, \mathbb{S}, \{h\}, \{i\}, \{k\}, \{h, i\}, \{i, k\}, \{h, k\}\};$

$min^{\mathbb{S}}_{\delta g*C}(\mathbb{S}, min^{\mathbb{S}}) = \{\varphi, \mathbb{S}, \{h\}, \{i\}, \{k\}, \{h, i\}, \{i, k\}, \{h, k\}\};$

$Min^{\mathbb{S}}_{\delta^* g\alpha C}(\mathbb{S}, min^{\mathbb{S}}) = \{\varphi, \mathbb{S}, \{i\}, \{k\}, \{h, i\}, \{i, k\}, \{h, k\}\};$

Here $f^{-1}[\{h\}] = \{h\}$ is not $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ in $(\mathbb{S}, min^{\mathbb{S}})$, therefore it is $min^{\mathbb{S}}_{\delta g - CONT}$, $min^{\mathbb{S}}_{\delta gp - CONT}$, $min^{\mathbb{S}}_{\delta g* - CONT}$ but not $Min^{\mathbb{S}}_{\delta^* g\alpha - CONT}$.

5 | $Min^{\mathbb{S}}_{\delta^* g\alpha - IRST}$ Functions in $\mathcal{M}$Structure Spaces

The authors introduce the following definition.

Definition 5.1. A function $f:(\mathbb{S}, min^{\mathbb{S}}) \longrightarrow (\mathbb{R}, min^{\mathbb{R}})$ is said to be a $Min^{\mathbb{S}}_{\text{delta star}\, g\alpha - IRST}$ (briefly $Min^{\mathbb{S}}_{\delta^* g\alpha - IRST}$) if $f^{-1}(E)$ is $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ in $(\mathbb{S}, min^{\mathbb{S}})$ for every $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ in $(\mathbb{R}, min^{\mathbb{R}})$.

Theorem 5.1. Let $f:(\mathbb{S}, min^{\mathbb{S}}) \longrightarrow (\mathbb{R}, min^{\mathbb{R}})$ and $g:(\mathbb{R}, min^{\mathbb{R}}) \longrightarrow (\mathbb{P}, min^{\mathbb{P}})$ be any two functions, then

i). $g^\circ f:(\mathbb{S}, min^{\mathbb{S}}) \longrightarrow (\mathbb{P}, min^{\mathbb{P}})$ is $Min^{\mathbb{S}}_{\delta^* g\alpha - CONT}$ if g is $min^{\mathbb{S}}$-$CONT$ and f is $Min^{\mathbb{S}}_{\delta^* g\alpha - CONT}$.

ii). $g^\circ f:(\mathbb{S}, min^{\mathbb{S}}) \longrightarrow (\mathbb{P}, min^{\mathbb{P}})$ is $Min^{\mathbb{S}}_{\delta^* g\alpha - IRST}$ if both g and f is $Min^{\mathbb{S}}_{\delta^* g\alpha - IRST}$.

iii). $g^\circ f:(\mathbb{S}, min^{\mathbb{S}}) \longrightarrow (\mathbb{P}, min^{\mathbb{P}})$ is $Min^{\mathbb{S}}_{\delta^* g\alpha - CONT}$ if g is $Min^{\mathbb{S}}_{\delta^* g\alpha - CONT}$ and f is $Min^{\mathbb{S}}_{\delta^* g\alpha - IRST}$.

Proof.

i). Let v be a $min^{\mathbb{S}}$-C set in $(\mathbb{P}, min^{\mathbb{P}})$. Since g is $min^{\mathbb{S}}$-$CONT$, $g^{-1}(v)$ is $min^{\mathbb{S}}$-C in $(\mathbb{R}, min^{\mathbb{R}})$. Since f is $Min^{\mathbb{S}}_{\delta^* g\alpha - CONT}$, $f^{-1}(g^{-1}(v)) = (g^\circ f)^{-1}(v)$ is $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ in $(\mathbb{S}, min^{\mathbb{S}})$, therefore $g^\circ f$ is $Min^{\mathbb{S}}_{\delta^* g\alpha - CONT}$.

ii). Let v be a $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ set in $(\mathbb{P}, min^{\mathbb{P}})$. Since g is $Min^{\mathbb{S}}_{\delta^* g\alpha - IRST}$, $g^{-1}(v)$ is $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ in $(\mathbb{R}, min^{\mathbb{R}})$. Since f is $Min^{\mathbb{S}}_{\delta^* g\alpha - IRST}$, $f^{-1}(g^{-1}(v)) = (g^\circ f)^{-1}(v)$ is $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ in $(\mathbb{S}, min^{\mathbb{S}})$, therefore $g^\circ f$ is $Min^{\mathbb{S}}_{\delta^* g\alpha - IRST}$.

iii). Let v be a $min^{\mathbb{S}}$-C set in $(\mathbb{P}, min^{\mathbb{P}})$. Since g is $Min^{\mathbb{S}}_{\delta^* g\alpha - CONT}$, $g^{-1}(v)$ is $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ in $(\mathbb{R}, min^{\mathbb{R}})$. Since f is $Min^{\mathbb{S}}_{\delta^* g\alpha - IRST}$, $f^{-1}(g^{-1}(v)) = (g^\circ f)^{-1}(v)$ is $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ in $(\mathbb{S}, min^{\mathbb{S}})$, therefore $g^\circ f$ is $Min^{\mathbb{S}}_{\delta^* g\alpha - CONT}$.

Theorem 5.2. Let $f:(\mathbb{S}, min^{\mathbb{S}}) \longrightarrow (\mathbb{R}, min^{\mathbb{R}})$ be a surjective, $min^{\mathbb{S}}_{*g\alpha - \mathrm{IRST}}$ and $min^{\mathbb{S}}_{\delta - \mathrm{C}}$ map. Then $f(J)$ is $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ set of $(\mathbb{R}, min^{\mathbb{R}})$ for every $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ set of $(\mathbb{S}, min^{\mathbb{S}})$.

Proof. Let J be a $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ set of $(\mathbb{S}, min^{\mathbb{S}})$. Let K be a $min^{\mathbb{S}}_{*g\alpha - \mathrm{O}}$ set of $(\mathbb{R}, min^{\mathbb{R}})$ such that $f(J) \subseteq K$. Since f is surjective and $min^{\mathbb{S}}_{*g\alpha - \mathrm{IRST}}$, $f^{-1}(K)$ is $min^{\mathbb{S}}_{*g\alpha - \mathrm{O}}$ set in $(\mathbb{S}, min^{\mathbb{S}})$. Since $J \subseteq f^{-1}(K)$ and J is $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ set of $(\mathbb{S}, min^{\mathbb{S}})$, $min^{\mathbb{S}} cl_\delta(J) \subseteq f^{-1}(K)$. Then $f[min^{\mathbb{S}} cl_\delta(J)] = min^{\mathbb{S}} cl_\delta\left[f\left(min^{\mathbb{S}} cl_\delta(J)\right)\right]$. This implies $min^{\mathbb{S}} cl_\delta[f(J)] \subseteq min^{\mathbb{S}} cl_\delta[f(min^{\mathbb{S}} cl_\delta(J)] = f[min^{\mathbb{S}} cl_\delta(J)] \subseteq K$, Therefore $f(J)$ is a $Min^{\mathbb{S}}_{\delta^* g\alpha - C}$ set of $(\mathbb{R}, min^{\mathbb{R}})$.

6 | Conclusion

This article defined $Min^{\mathbb{S}}_{\delta^* g\alpha-C}$ set in Minimal structure spaces and some of their properties were discussed. Also $Min^{\mathbb{S}}_{\delta^* g\alpha-CONT}$, $Min^{\mathbb{S}}_{\delta^* g\alpha-IRST}$ functions were introduced and their properties. In the future, this work will be extended to neutrosophic topological spaces.

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Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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