HyperSoft Set Methods in Engineering

Journal Homepage: sciencesforce.com/hsse

HyperSoft Set Meth. Eng. Vol. 2 (2024) 72–82

Paper Type: Original Article

On the Generalization of Neutrosophic Set Operations: Testing Proofs by Examples

Adel Al-Odhari [1,2,*]

[1] Faculty of Education, Humanities and Applied Sciences (Khawlan), Yemen;

[2] Department of Foundations of Sciences, Faculty of Engineering, Sana'a University. Box:13509, Sana'a, Yemen; a.aleidhri@su.edu.ye.

Received: 14 Mar 2024 **Revised:** 31 May 2024 **Accepted:** 30 Jun 2024 **Published:** 03 Jul 2024

Abstract

This article aims to present more facts about neutrosophic set theory such as classical set theory. It includes; more connection between operations on the neutrosophic set theory of three types such as union, intersection, difference, and their generalization. further, the generalization of union, intersection, and complement with DE-Morgan's theorem. In addition, we studied the Cartesian Products and their properties. This work opens a new parallel path of a new neutrosophic structure of degree of membership functions that does not depend on intuitionistic fuzzy set and fuzzy set respectively.

Keywords: Neutrosophic Sets of Three Types; Operations on Neutrosophic Sets of Three Types; Cartesian Product on Neutrosophic Sets of Three Types.

1 | Introduction

In this article, we continue our work in [4, 6] for building the neutrosophic set theory for three types. Furthermore, the various facets of neutrosophic set problems related to the neutrosophic set of three types including the generalized union, intersection, complement, neutrosophic family set, Cartesian neutrosophic set, and their properties are highlighted.

The process of completing the construction of neutrosophic set theory will represent the opening of a new path for studying a different neutrosophic algebraic structure that is parallel to the path of neutrosophic structures which is based on the degree of membership functions. This work represents a modest contribution to the science of Neutrosophy, as it enhances our previous work in [1-3, 5, 7], for more information about Neutrosophy, neutrosophic sets, and neutrosophic logic, refer to [9-11] and for the classical set theory, indicate to [8, 12].

2 | Some Connections Between Neutrosophic Operations on Neutrosophic Sets of Three Types

In this section, some properties of our works in [4, 6] are extended. These properties illustrate the relationship between the neutrosophic union, intersection, and difference. Throughout this article, we use the terms

Corresponding Author: a.aleidhri@su.edu.ye

https://doi.org/10.61356/j.hsse.2024.2321

"Neutrosophic-hypothesis", "Neutrosophic-premise", and "Neu-conclusion" in short " "Neu-hypo", "NP", and "NC" respectively. The following theorems give us some relationship between neutrosophic union, intersections, and difference.

Theorem 2.1. Let $H_i^t[I\,]$, $N_i^t[I]$, and $M_i^t[I\,]$ be three neutrosophic sets of three types, where $i = 1,2,3$, then;

$H_i^t[I\,] \ominus \left(N_i^t[I] \cap M_i^t[I\,]\right) = \left(H_i^t[I\,] \ominus N_i^t[I]\right) \cup \left(H_i^t[I\,] \ominus M_i^t[I]\right)$,

$H_i^t[I\,] \ominus \left(N_i^t[I] \cup M_i^t[I\,]\right) = \left(H_i^t[I\,] \ominus N_i^t[I]\right) \cap \left(H_i^t[I\,] \ominus M_i^t[I]\right)$,

$H_i^t[I\,] \cap \left(N_i^t[I] \ominus M_i^t[I\,]\right) = \left(H_i^t[I\,] \cap N_i^t[I]\right) \ominus \left(H_i^t[I\,] \cap M_i^t[I\,]\right)$, and

$H_i^t[I\,] \cup \left(N_i^t[I] \ominus M_i^t[I\,]\right) = \left(H_i^t[I\,] \cup N_i^t[I]\right) \ominus \left(H_i^t[I\,] \cup M_i^t[I\,]\right)$.

Proof (1). Presume $x \in \left(H_i^t[I\,] \ominus \left(N_i^t[I] \cap M_i^t[I\,]\right)\right)$, "Neu-hypo"

$\Leftrightarrow x \in H_i^t[I\,] \wedge \left(x \notin \left(N_i^t[I] \cap M_i^t[I\,]\right)\right)$, "$NP_1$", from "Neu-hypo"

$\Leftrightarrow x \in H_i^t[I\,] \wedge \left(x \in \overbrace{\left(N_i^t[I] \cap M_i^t[I\,]\right)}^{c}\right)$, "$NP_2$", from "$NP_1$"

$\Leftrightarrow x \in H_i^t[I\,] \wedge \left(x \in \left(\overbrace{N_i^t[I]}^{c} \cup \overbrace{M_i^t[I\,]}^{c}\right)\right)$, "$NP_3$", from "$NP_2$"

$\Leftrightarrow x \in H_i^t[I\,] \wedge \left(x \in \overbrace{N_i^t[I]}^{c} \vee x \in \overbrace{M_i^t[I\,]}^{c}\right)$, "$NP_4$", from "$NP_3$"

$\Leftrightarrow \left(x \in H_i^t[I\,] \wedge x \in \overbrace{N_i^t[I]}^{c}\right) \vee \left(x \in H_i^t[I\,] \wedge x \in \overbrace{M_i^t[I\,]}^{c}\right)$, "$NP_5$", from "$NP_4$"

$\Leftrightarrow \left(x \in H_i^t[I\,] \wedge x \notin N_i^t[I]\right) \vee \left(x \in H_i^t[I\,] \wedge x \notin M_i^t[I\,]\right)$, "$NP_6$", from "$NP_5$"

$\Leftrightarrow \left(H_i^t[I\,] \ominus N_i^t[I]\right) \cup \left(H_i^t[I\,] \ominus M_i^t[I\,]\right)$, "$NP_7$", from "$NP_6$"

$\therefore H_i^t[I\,] \ominus \left(N_i^t[I] \cap M_i^t[I\,]\right) = \left(H_i^t[I\,] \ominus N_i^t[I]\right) \cup \left(H_i^t[I\,] \ominus M_i^t[I]\right)$■. The conclusion from "Neu-hypo" and "NP_7".

(2). By the same arguments.

(3). Suppose that $x \in \left(\left(H_i^t[I\,] \cap N_i^t[I]\right) \ominus \left(H_i^t[I\,] \cap M_i^t[I\,]\right)\right)$, "Neu-hypo"

$\Leftrightarrow x \in \left(H_i^t[I\,] \cap N_i^t[I]\right) \wedge x \notin \left(H_i^t[I\,] \cap M_i^t[I\,]\right)$, "$NP_1$", from "Neu-hypo"

$\Leftrightarrow x \in \left(H_i^t[I\,] \cap N_i^t[I]\right) \wedge x \in \left(\overbrace{H_i^t[I\,] \cap M_i^t[I\,]}^{c}\right)$, "$NP_2$", from "$NP_1$",

$\Leftrightarrow x \in \left(H_i^t[I\,] \cap N_i^t[I]\right) \wedge x \in \left(\overbrace{H_i^t[I\,]}^{c} \cup \overbrace{M_i^t[I\,]}^{c}\right)$, "$NP_3$", from "$NP_2$",

$\Leftrightarrow \left(x \in H_i^t[I\,] \wedge x \in N_i^t[I]\right) \wedge \left(x \in \overbrace{H_i^t[I\,]}^{c} \vee x \in \overbrace{M_i^t[I\,]}^{c}\right)$, "$NP_4$", from "$NP_3$",

$\Leftrightarrow \left(x \in H_i^t[I\,] \wedge x \in N_i^t[I]\right) \wedge \left(x \notin H_i^t[I\,] \vee x \notin M_i^t[I\,]\right)$, "$NP_5$", from "$NP_4$",

$\Leftrightarrow \left(x \in H_i^t[I\,] \wedge \left(x \notin H_i^t[I\,] \vee x \notin M_i^t[I\,]\right)\right) \wedge x \in N_i^t[I]$, "NP$_6$", from "NP$_5$",

$\Leftrightarrow \left(x \in H_i^t[I\,] \wedge x \notin H_i^t[I\,]\right) \vee \left(x \in H_i^t[I\,] \wedge x \notin M_i^t[I\,]\right) \wedge x \in N_i^t[I]$, "NP$_7$", from "NP$_6$",

$\Leftrightarrow \left(F_N \vee \left(x \in H_i^t[I\,] \wedge x \notin M_i^t[I\,]\right)\right) \wedge x \in N_i^t[I]$, "NP$_8$", from "NP$_7$",

$\Leftrightarrow \left(\left(x \in H_i^t[I\,] \wedge x \notin M_i^t[I\,]\right)\right) \wedge x \in N_i^t[I]$, "NP$_9$", from "NP$_8$",

$\Leftrightarrow x \in H_i^t[I\,] \wedge \left(x \notin M_i^t[I\,] \wedge x \in N_i^t[I]\right)$, "NP$_{10}$", from "NP$_9$",

$\Leftrightarrow x \in H_i^t[I\,] \wedge \left(x \in N_i^t[I] \wedge x \notin M_i^t[I\,]\right)$, "NP$_{11}$", from "NP$_{10}$",

$\Leftrightarrow x \in H_i^t[I\,] \wedge \left(x \in \left(N_i^t[I] \ominus M_i^t[I\,]\right)\right)$, "NP$_{12}$", from "NP$_{11}$",

$\Leftrightarrow x \in \left(H_i^t[I\,] \cap \left(N_i^t[I] \ominus M_i^t[I\,]\right)\right)$, "NP$_{13}$", from "NP$_{12}$",

$\therefore H_i^t[I\,] \cap \left(N_i^t[I] \ominus M_i^t[I\,]\right) = \left(H_i^t[I\,] \cap N_i^t[I]\right) \ominus \left(H_i^t[I\,] \cap M_i^t[I\,]\right)$ ■. The conclusion from "Neu-hypo" and "NP$_{13}$".

(4). It can be proven by a similar method.

Theorem 2.2 Let $H_i^t[I\,] \subset N_i^t[I\,] \subset U_i^t[I\,]$, for any $i = 1,2,3$, then;

1) $N_i^t[I\,] \ominus H_i^t[I\,] = N_i^t[I\,] \cap \left(X_i^t[I\,] \ominus H_i^t[I\,]\right)$, and
2) $\left(X_i^t[I\,] \ominus N_i^t[I\,]\right) \subseteq \left(X_i^t[I\,] \ominus H_i^t[I\,]\right)$.

Proof (1). Assume that $x \in \left(N_i^t[I\,] \ominus H_i^t[I\,]\right)$, "Neu-hypo"

$\Rightarrow \left(x \in N_i^t[I\,]\right) \wedge \left(x \notin H_i^t[I\,]\right)$, "$NP_1$", from "Neu-hypo"

$\Rightarrow \left(x \in N_i^t[I\,]\right)$, "$NP_2$", from NP$_1$",

$\Rightarrow \left(x \notin H_i^t[I\,]\right)$, "$NP_3$", from "$NP_1$",

$\Rightarrow x \in \left(X_i^t[I\,] \ominus H_i^t[I\,]\right)$, "$NP_4$", from "$NP_3$",

$\Rightarrow \left(x \in N_i^t[I\,]\right) \wedge x \in \left(X_i^t[I\,] \ominus H_i^t[I\,]\right)$, "$NP_5$", from "NP$_2$&NP$_4$",

$\Rightarrow x \in \left(N_i^t[I\,] \cap \left(X_i^t[I\,] \ominus H_i^t[I\,]\right)\right)$, "$NP_6$", from "$NP_5$",

$\Rightarrow N_i^t[I\,] \ominus H_i^t[I\,] \subset N_i^t[I\,] \cap \left(X_i^t[I\,] \ominus H_i^t[I\,]\right)$, " NC_1", from "neu-hypo"&"NP_6".

Conversely, Suppose that $x \in \left(N_i^t[I\,] \cap \left(X_i^t[I\,] \ominus H_i^t[I\,]\right)\right)$, "Neu-hypo"

$\Rightarrow \left(x \in N_i^t[I\,]\right) \wedge \left(x \in \left(X_i^t[I\,] \ominus H_i^t[I\,]\right)\right)$, "$NP_1$", from "Neu-hypo"

$\Rightarrow x \in N_i^t[I\,]$, "$NP_2$", from "$NP_1$",

$\Rightarrow x \in \left(X_i^t[I\,] \ominus H_i^t[I\,]\right)$, "$NP_3$", from "$NP_1$",

$\Rightarrow x \in X_i^t[I\,] \wedge x \notin H_i^t[I\,]$, "$NP_4$", from "$NP_3$",

$\Rightarrow x \notin H_i^t[I\,]$, "$NP_5$", from "$NP_4$",

$\Rightarrow x \in N_i^t[I\,] \wedge x \notin H_i^t[I\,]$, "$NP_6$", from "$NP_2$&$NP_5$",

$\Rightarrow x \in \left(N_i^t[I\,] \ominus H_i^t[I\,]\right)$, "$NP_7$", from "$NP_6$",

$\Rightarrow \left(N_i^t[I\,] \cap \left(X_i^t[I\,] \ominus H_i^t[I\,]\right)\right) \subset \left(N_i^t[I\,] \ominus H_i^t[I\,]\right)$, "$NC_2$", from "neu-hypo & NP_7".

$\therefore N_i^t[I\,] \ominus H_i^t[I\,] = N_i^t[I\,] \cap \left(X_i^t[I\,] \ominus H_i^t[I\,]\right)$■. From "$NC_1$& NC_2"

3 | Generalization of Neutrosophic Operations on Neutrosophic Sets of Three

In this section, the neutrosophic family of neutrosophic sets, indexed set, neutrosophic partition, finite general neutrosophic union, countable infinite neutrosophic union, finite general neutrosophic intersection, countable infinite neutrosophic intersection are defined.

Definition 3.1. Let $U_i^t[I\,]$ be any neutrosophic universal set, $i = 1,2,3,$ and $\mathbb{i} = \{1,2,3, \dots\}$. Define a neutrosophic set by: $\mathcal{F}_{\aleph} = \left\{\underbrace{H_i^t[I\,]}_{\alpha} : H_i^t[I\,] \subseteq U_i^t[I\,], \alpha \in \mathbb{i}\right\}$, where

- $\mathcal{F}_{\aleph}$ is called a neutrosophic family of neutrosophic sets,
- $\mathbb{i}$ is called an indexing set for the family, and
- $\underbrace{H_i^t[I\,]}_{\alpha}$ is called an indexed set.

Definition 3.2. Let $\wp_{\aleph} = \left\{\underbrace{H_i^t[I\,]}_{\alpha} : \alpha \in \mathbb{i}\right\}$ be a family of neutrosophic subsets of $H_i^t[I\,]$, for any $i = 1,2,3,$

We said that $\left\{\underbrace{H_i^t[I\,]}_{\alpha} : \alpha \in \mathbb{i}\right\}$ is a neutrosophic partition of $H_i^t[I\,]$, if satisfies the following conditions;

1) $\underbrace{H_i^t[I\,]}_{\alpha} \neq \emptyset_i^t[I\,]$, for any $i = 1,2,3,$ and $\forall \alpha \in \mathbb{i}$,
2) For each $\underbrace{H_i^t[I\,]}_{\alpha}$ and $\underbrace{H_i^t[I\,]}_{\beta}$, then either $\underbrace{H_i^t[I\,]}_{\alpha} = \underbrace{H_i^t[I\,]}_{\beta}$ or $\underbrace{H_i^t[I\,]}_{\alpha} \cap \underbrace{H_i^t[I\,]}_{\beta} = \emptyset$, $\forall i = 1,2,3,$
3) $H_i^t[I\,] = \cup \underbrace{H_i^t[I\,]}_{\alpha \in \mathbb{i}}$.

Example 3.1. Let $H = \{1,2,3,\}$ be a classical set, then the neutrosophic sets of three types are given by: $H_1^t[I\,] = \begin{Bmatrix} 1+1I, & 1+2I, & 1+3I, \\ 2+1I, & 2+2I, & 2+3I, \\ 3+1I, & 3+2I, & 3+3I, \end{Bmatrix}$, and $\underbrace{H_1^t[I\,]}_{1} = \{1+1I, 1+2I, 1+3I\}$

$\underbrace{H_1^t[I\,]}_{2} = \{2+1I, 2+2I, 2+3I\}$, and $\underbrace{H_1^t[I\,]}_{3} = \{3+1I, 3+2I, 3+3I\}$, we see that,

$\wp_{\aleph} = \left\{\underbrace{H_i^t[I\,]}_{\alpha} : \alpha \in \mathbb{i}\right\} = \left\{\underbrace{H_1^t[I\,]}_{1}, \underbrace{H_1^t[I\,]}_{2}, \underbrace{H_1^t[I\,]}_{3}\right\}$ is a neutrosophic partition of $H_1^t[I\,]$, because

$H_1^t[I\,] = \underbrace{H_1^t[I\,]}_{1} \cup \underbrace{H_1^t[I\,]}_{2} \cup \underbrace{H_1^t[I\,]}_{3\cup}, \underbrace{H_1^t[I\,]}_{1} \cap \underbrace{H_1^t[I\,]}_{2} = \emptyset, \underbrace{H_1^t[I\,]}_{1} \cap \underbrace{H_1^t[I\,]}_{3} = \emptyset$, and

$\underbrace{H_1^t[I]}_{2} \cap \underbrace{H_1^t[I]}_{3} = \emptyset$. While $\underbrace{H_1^t[I]}_{1} = \{1 + 1I, 2 + 2I, 3 + 3I\}$, $\underbrace{H_1^t[I]}_{2} = \{2 + 1I, 2 + 2I, 2 + 3I\}$, and $\underbrace{H_1^t[I]}_{3} = \{3 + 1I, 2 + 1I, 3 + 2I\}$, then $\wp_{\aleph} = \left\{\underbrace{H_i^t[I]}_{\alpha} : \alpha \in 𝕚\right\} = \left\{\underbrace{H_1^t[I]}_{1}, \underbrace{H_1^t[I]}_{2}, \underbrace{H_1^t[I]}_{3}\right\}$ is not a neutrosophic partition of $H_1^t[I]$, because $\underbrace{H_1^t[I]}_{1} \cap \underbrace{H_1^t[I]}_{2} = \{2 + 2I\} \neq \emptyset_1^t[I]$. In addition, if

$H_2^t[I] = \{hI \cup \{h\}: h \in H\} = \begin{Bmatrix} 1,1I \\ 2,2I \\ 3,3I \end{Bmatrix}$, then $\underbrace{H_2^t[I]}_{1} = \{1,1I\}$, $\underbrace{H_2^t[I]}_{2} = \{2,2I\}$, and $\underbrace{H_2^t[I]}_{3} = \{3,3I\}$

$\wp_{\aleph} = \left\{\underbrace{H_i^t[I]}_{\alpha} : \alpha \in 𝕚\right\} = \left\{\underbrace{H_2^t[I]}_{1}, \underbrace{H_2^t[I]}_{2}, \underbrace{H_2^t[I]}_{3}\right\}$ is a neutrosophic partition of $H_2^t[I]$, also the neutrosophic collection of the neutrosophic subset of $H_2^t[I]$ which is given by

$\wp_{\aleph} = \left\{\underbrace{H_i^t[I]}_{\alpha} : \alpha \in 𝕚\right\} = \left\{\underbrace{H_2^t[I]}_{1}, \underbrace{H_2^t[I]}_{2}, \underbrace{H_2^t[I]}_{3}, \underbrace{H_2^t[I]}_{4}, \underbrace{H_2^t[I]}_{5}\right\}$ is a neutrosophic partition of $H_2^t[I]$, where, $\underbrace{H_2^t[I]}_{1} = \{1\}$, $\underbrace{H_2^t[I]}_{2} = \{1I\}$, $\underbrace{H_2^t[I]}_{3} = \{2,2I\}$, $\underbrace{H_2^t[I]}_{4} = \{3\}$, and $\underbrace{H_2^t[I]}_{5} = \{3I\}$.

Definition 3.3. Let $\underbrace{H_i^t[I]}_{\alpha} : \alpha \in 𝕚$, where $𝕚 = \{1,2,3, \dots, n\}$ be a sequence of finite neutrosophic sets. Define the finite general neutrosophic union as follows:

$\bigcup_{\alpha=1}^{n} \underbrace{H_i^t[I]}_{\alpha\in𝕚} = \underbrace{H_i^t[I]}_{1} \cup \underbrace{H_i^t[I]}_{2} \cup \underbrace{H_i^t[I]}_{3} \cup \dots \underbrace{H_i^t[I]}_{n}$, where $i = 1,2,3$.

Definition 3.4. Let $\underbrace{H_i^t[I]}_{\alpha} : \alpha \in 𝕚$, where $𝕚 = \{1,2,3, \dots\}$ be a sequence of countable infinite neutrosophic sets. Define the countable infinite neutrosophic union as follows:

$\bigcup_{\alpha=1}^{\infty} \underbrace{H_i^t[I]}_{\alpha\in𝕚} = \underbrace{H_i^t[I]}_{1} \cup \underbrace{H_i^t[I]}_{2} \cup \underbrace{H_i^t[I]}_{3} \cup \dots \underbrace{H_i^t[I]}_{n} \cup \dots$, where $i = 1,2,3$.

Definition 3.5. Let $\left\{\underbrace{H_i^t[I]}_{\alpha} : \alpha \in 𝕚\right\}, i = 1,2,3$ be a neutrosophic family of indexed sets. Define arbitrary neutrosophic union as follows: $\bigcup \underbrace{H_i^t[I]}_{\alpha\in𝕚} = \left\{x: \exists\ \alpha \in 𝕚 \text{ such that } x \in \underbrace{H_i^t[I]}_{\alpha}\right\}$.

Theorem 3.1. Let $\left\{\underbrace{H_i^t[I]}_{\alpha\in𝕚}\right\}, i = 1,2,3$ be a neutrosophic family of indexed sets of three types, and $N_i^t[I]$ be neutrosophic sets of three types. If $\underbrace{H_i^t[I]}_{\alpha\in𝕚} \subset N_i^t[I], \forall\ \alpha \in 𝕚$, then $\bigcup \underbrace{H_i^t[I]}_{\alpha\in𝕚} \subset N_i^t[I]$.

Proof. Consider $\underbrace{H_i^t[I]}_{\alpha\in𝕚} \subset N_i^t[I], \forall\ \alpha \in 𝕚$. Suppose that $\boldsymbol{x} \in \bigcup \underbrace{H_i^t[I]}_{\alpha\in𝕚}$, "neu-hypo"

$\Rightarrow \exists\ \alpha \in 𝕚 \text{ such that } x \in H_i^t[I]$, for any $i = 1,2,3$, "NP_1", from "Neu-hypo"

$\Rightarrow \exists\ \alpha \in 𝕚 \text{ such that } x \in N_i^t[I]$, for any $i = 1,2,3$, "NP_2", from "NP_1&NP",

$\Rightarrow x \in N_i^t[I]$, for any $i = 1,2,3$, "NP_3", from "NP_2 ",

$\Rightarrow \bigcup \underbrace{H_i^t[I]}_{\alpha\in𝕚} \subset N_i^t[I]$, for any $i = 1,2,3$, "NC", from "NP_3"■.

Definition 3.6. Let $\underbrace{H_i^t[I\,]}_{\alpha}: \alpha \in 𝕚$, where $𝕚 = \{1,2,3,\dots,n\}$ be a sequence of finite neutrosophic sets. Define the finite general neutrosophic intersection as follows:

$\bigcap_{\alpha=1}^{n} \underbrace{H_i^t[I\,]}_{\alpha\in 𝕚} = \underbrace{H_i^t[I\,]}_{1} \cap \underbrace{H_i^t[I\,]}_{2} \cap \underbrace{H_i^t[I\,]}_{3} \cap \dots \underbrace{H_i^t[I\,]}_{n}$, where $i = 1,2,3$.

Definition 3.7. Let $\underbrace{H_i^t[I\,]}_{\alpha}: \alpha \in 𝕚$, where $𝕚 = \{1,2,3,\dots\}$ be a sequence of countable infinite neutrosophic sets. Define the countable infinite neutrosophic intersection as follows:

$\bigcap_{\alpha=1}^{\infty} \underbrace{H_i^t[I\,]}_{\alpha\in 𝕚} = \underbrace{H_i^t[I\,]}_{1} \cap \underbrace{H_i^t[I\,]}_{2} \cap \underbrace{H_i^t[I\,]}_{3} \cap \dots \underbrace{H_i^t[I\,]}_{n} \cap \dots$, where $i = 1,2,3$.

Definition 3.8. Let $\left\{\underbrace{H_i^t[I\,]}_{\alpha}: \alpha \in 𝕚\right\}, i = 1,2,3$ be a neutrosophic family of indexed sets. Define arbitrary neutrosophic intersection as follows: $\bigcap \underbrace{H_i^t[I\,]}_{\alpha\in 𝕚} = \left\{x: x \in \underbrace{H_i^t[I\,]}_{\alpha}, \forall\, \alpha \in 𝕚\right\}$.

Theorem 3.2. Let $\left\{\underbrace{H_i^t[I\,]}_{\alpha\in 𝕚}\right\}, i = 1,2,3$ be a neutrosophic family of indexed sets of three types, and $N_i^t[I\,]$ be neutrosophic sets of three types. If $N_i^t[I\,] \subset \underbrace{H_i^t[I\,]}_{\alpha\in 𝕚}, \forall\, \alpha \in 𝕚$, then

$N_i^t[I\,] \subset \bigcap \underbrace{H_i^t[I\,]}_{\alpha\in 𝕚}, \forall\, \alpha \in 𝕚$.

Proof. Suppose that $N_i^t[I\,] \subset \underbrace{H_i^t[I\,]}_{\alpha\in 𝕚}, \forall\, \alpha \in 𝕚$, and assume that $x \in N_i^t[I\,]$, "neu-hypo"

$\Rightarrow x \in H_i^t[I\,], \forall\, \alpha \in 𝕚$ for any , $i = 1,2,3$, "NP_1", from "Neu-hypo"

$\Rightarrow x \in \underbrace{H_i^t[I\,]}_{1}, \wedge\, x \in \underbrace{H_i^t[I\,]}_{2} \wedge \dots \wedge x \in \underbrace{H_i^t[I\,]}_{n} \wedge \dots, \forall\, \alpha \in 𝕚$ for any , $i = 1,2,3$, "NP_2", from "NP_1",

$\Rightarrow x \in \bigcap \underbrace{H_i^t[I\,]}_{\alpha\in 𝕚}, \forall\, \alpha \in 𝕚$. for any, $i = 1,2,3$, "NP_3", from "NP_2",

$\Rightarrow N_i^t[I\,] \subset \bigcap \underbrace{H_i^t[I\,]}_{\alpha\in 𝕚}, \forall\, \alpha \in 𝕚$. for any, $i = 1,2,3$, "NC", from NP_3 ■.

Theorem 3.3. Let $\left\{\underbrace{H_i^t[I\,]}_{\alpha\in 𝕚}\right\}, i = 1,2,3$ be a neutrosophic family of indexed sets of three types, then

$\overbrace{\left(\bigcup \underbrace{H_i^t[I\,]}_{\alpha\in 𝕚}\right)}^{c} = \bigcap \underbrace{\overbrace{H_i^t[I\,]}^{c}}_{\alpha\in 𝕚}$, and

$\overbrace{\left(\bigcap \underbrace{H_i^t[I\,]}_{\alpha\in 𝕚}\right)}^{c} = \bigcup \underbrace{\overbrace{H_i^t[I\,]}^{c}}_{\alpha\in 𝕚}, \forall\, \alpha \in 𝕚$. Where the components of complements are taken in $U_i^t[I\,]$.

Proof (1). Assume that $h \in \overbrace{\left(\bigcup \underbrace{H_i^t[I\,]}_{\alpha\in 𝕚}\right)}^{c}$, "neu-hypo",

$$\Rightarrow h \in \left(U_i^t[I] \ominus \left(\cup \underbrace{H_i^t[I]}_{\alpha\in\mathbb{i}} \right) \right), \text{"NP}_1\text{", from "neu-hypo",}$$

$$\Rightarrow h \in U_i^t[I] \wedge h \notin \left(\cup \underbrace{H_i^t[I]}_{\alpha\in\mathbb{i}} \right), \text{"NP}_2\text{", from "NP}_1\text{",}$$

$$\Rightarrow h \notin \left(\cup \underbrace{H_i^t[I]}_{\alpha\in\mathbb{i}} \right), \text{"NP}_3\text{", from "NP}_2\text{",}$$

$$\Rightarrow \ h \notin \underbrace{H_i^t[I]}_{\alpha\in\mathbb{i}}, \forall\, \alpha \in \mathbb{i}\,, \text{"NP}_4\text{", from "NP}_3\text{",}$$

$$\Rightarrow \ h \in \overbrace{\underbrace{H_i^t[I]}_{\alpha\in\mathbb{i}}}^{c}, \forall\, \alpha \in \mathbb{i}, \text{"NP}_5\text{", from "NP}_4\text{",}$$

$$\Rightarrow \ h \in \cap \overbrace{\underbrace{H_i^t[I]}_{\alpha\in\mathbb{i}}}^{c}, \forall\, \alpha \in \mathbb{i}\,, \text{"NP}_6\text{", from "NP}_5\text{",}$$

$$\Rightarrow \overbrace{\left(\cup \underbrace{H_i^t[I]}_{\alpha\in\mathbb{i}} \right)}^{c} \subset \cap \overbrace{\underbrace{H_i^t[I]}_{\alpha\in\mathbb{i}}}^{c}, \text{"N}C_1\text{", from "NP}_6\text{", conversely,}$$

Suppose that $h \in \cap \overbrace{\underbrace{H_i^t[I]}_{\alpha\in\mathbb{i}}}^{c}$, "neu-hypo",

$$\Rightarrow \ h \in \overbrace{\underbrace{H_i^t[I]}_{\alpha\in\mathbb{i}}}^{c}, \forall\, \alpha \in \mathbb{i}, \text{"NP}_1\text{", from "neu-hypo",}$$

$$\Rightarrow \ h \notin \ \underbrace{H_i^t[I]}_{\alpha\in\mathbb{i}}, \forall\, \alpha \in \mathbb{i}, \text{"NP}_2\text{", from "NP}_1\text{",}$$

$$\Rightarrow \ h \notin \cup \underbrace{H_i^t[I]}_{\alpha\in\mathbb{i}}, \forall\, \alpha \in \mathbb{i}, \text{"NP}_3\text{", from "NP}_2\text{",}$$

$$\Rightarrow \ h \in \cup \overbrace{\underbrace{H_i^t[I]}_{\alpha\in\mathbb{i}}}^{c}, \forall\, \alpha \in \mathbb{i}, \text{"NP}_4\text{", from "NP}_3\text{",}$$

$$\Rightarrow \cap \overbrace{\underbrace{H_i^t[I]}_{\alpha\in\mathbb{i}}}^{c} \ \subset \cup \overbrace{\underbrace{H_i^t[I]}_{\alpha\in\mathbb{i}}}^{c}, \text{"N}C_2\text{", from "NP}_4\text{", hence}$$

$$\overbrace{\left(\cup \underbrace{H_i^t[I]}_{\alpha\in\mathbb{i}} \right)}^{c} = \cap \overbrace{\underbrace{H_i^t[I]}_{\alpha\in\mathbb{i}}}^{c} \blacksquare, \text{ from "N}C_1\text{\&"N}C_2\text{".}$$

(2). By the same way.

Theorem 3.3. Let $\left\{ \underbrace{\mathrm{H}_\mathrm{i}^\mathrm{t}[\mathrm{I}]}_{\alpha\in\mathbb{i}} \right\}, \mathrm{i} = 1,2,3$ and $\left\{ \underbrace{\mathrm{N}_\mathrm{i}^\mathrm{t}[\mathrm{I}]}_{\beta\in\mathbb{j}} \right\}, \mathrm{i} = 1,2,3$ be a neutrosophic family of indexed sets of three types, then

$$\left(\cup \underbrace{H_i^t[I]}_{\alpha\in\mathbb{i}} \right) \cap \left(\cup \underbrace{N_i^t[I]}_{\beta\in\mathbb{j}} \right) = \cup_{(\alpha,\beta)\in\mathbb{i}\times\mathbb{j}} \left(\underbrace{H_i^t[I]}_{\alpha\in\mathbb{i}} \cap \underbrace{N_i^t[I]}_{\beta\in\mathbb{j}} \right), \textit{ and}$$

$$\left(\bigcap \underbrace{H_i^t[I\,]}_{\alpha\in\mathbb{i}}\right) \cup \left(\bigcap \underbrace{N_i^t[I\,]}_{\beta\in\mathbb{j}}\right) = \bigcap_{(\alpha,\beta)\in\mathbb{i}\times\mathbb{j}} \left(\underbrace{H_i^t[I\,]}_{\alpha\in\mathbb{i}} \cup \underbrace{N_i^t[I\,]}_{\beta\in\mathbb{j}}\right)$$

Proof (1). Assume that $h \in \left(\cup \underbrace{H_i^t[I\,]}_{\alpha\in\mathbb{i}}\right) \cap \left(\cup \underbrace{N_i^t[I\,]}_{\beta\in\mathbb{j}}\right)$, "neu-hypo",

$\Rightarrow h \in \left(\cup \underbrace{H_i^t[I\,]}_{\alpha\in\mathbb{i}}\right) \wedge h \in \left(\cup \underbrace{N_i^t[I\,]}_{\beta\in\mathbb{j}}\right)$, "$NP_1$", from "neu-hypo",

$\Rightarrow (\exists\ \alpha \in \mathbb{i} \text{ such that } h \in H_i^t[I\,]) \wedge (\exists\ \beta \in \mathbb{j} \text{ such that } h \in N_i^t[I\,])$, for any $i = 1,2,3$, "NP_2", from "NP_1",

$\Rightarrow (\exists\ (\alpha,\beta) \in \mathbb{i}\times\mathbb{j} \text{ such that } h \in H_i^t[I\,] \wedge N_i^t[I\,])$, for any $i = 1,2,3$, "NP_3", from "NP_2",

$\Rightarrow \left(\exists\ (\alpha,\beta) \in \mathbb{i}\times\mathbb{j} \text{ such that } h \in (H_i^t[I\,] \cap N_i^t[I\,])\right)$, for any $i = 1,2,3$, "NP_4", from "NP_3",

$\Rightarrow \left(h \in \cup_{(\alpha,\beta)\in\mathbb{i}\times\mathbb{j}}(H_i^t[I\,] \cap N_i^t[I\,])\right)$, for any $i = 1,2,3$, "NP_4", from "NP_3",

$\Rightarrow \left(\cup \underbrace{H_i^t[I\,]}_{\alpha\in\mathbb{i}}\right) \cap \left(\cup \underbrace{N_i^t[I\,]}_{\beta\in\mathbb{j}}\right) \subset \cup_{(\alpha,\beta)\in\mathbb{i}\times\mathbb{j}}(H_i^t[I\,] \cap N_i^t[I\,])$, "$NC_1$", from "$NP_4$", conversely,

Suppose that $h \in \cup_{(\alpha,\beta)\in\mathbb{i}\times\mathbb{j}}\left(\underbrace{H_i^t[I\,]}_{\alpha\in\mathbb{i}} \cap \underbrace{N_i^t[I\,]}_{\beta\in\mathbb{j}}\right)$, "neu-hypo",

$\Rightarrow \left(\exists\ (\alpha,\beta) \in \mathbb{i}\times\mathbb{j} \textit{ such that } h \in (H_i^t[I\,] \cap N_i^t[I\,])\right)$, *for any* $i = 1,2,3$, "NP_1", *from* "*neu*-hypo",

$\Rightarrow (\exists\ (\alpha,\beta) \in \mathbb{i}\times\mathbb{j} \text{ such that } h \in H_i^t[I\,] \wedge N_i^t[I\,])$, for any $i = 1,2,3$, "NP_2", from "NP_1",

$\Rightarrow (\exists\ \alpha \in \mathbb{i} \text{ such that } h \in H_i^t[I\,]) \wedge (\exists\ \beta \in \mathbb{j} \text{ such that } h \in N_i^t[I\,])$, for any $i = 1,2,3$, "NP_3", from "NP_2",

$\Rightarrow h \in \left(\cup \underbrace{H_i^t[I\,]}_{\alpha\in\mathbb{i}}\right) \wedge h \in \left(\cup \underbrace{N_i^t[I\,]}_{\beta\in\mathbb{j}}\right)$, "$NP_4$", from "$NP_3$",

$\Rightarrow h \in \left(\cup \underbrace{H_i^t[I\,]}_{\alpha\in\mathbb{i}}\right) \cap \left(\cup \underbrace{N_i^t[I\,]}_{\beta\in\mathbb{j}}\right)$, "$NP_5$", from "$NP_4$",

$\Rightarrow \cup_{(\alpha,\beta)\in\mathbb{i}\times\mathbb{j}}\left(\underbrace{H_i^t[I\,]}_{\alpha\in\mathbb{i}} \cap \underbrace{N_i^t[I\,]}_{\beta\in\mathbb{j}}\right) \subset \left(\cup \underbrace{H_i^t[I\,]}_{\alpha\in\mathbb{i}}\right) \cap \left(\cup \underbrace{N_i^t[I\,]}_{\beta\in\mathbb{j}}\right)$, "$NC_2$", from "$NP_5$", therefore

$\left(\cup \underbrace{H_i^t[I\,]}_{\alpha\in\mathbb{i}}\right) \cap \left(\cup \underbrace{N_i^t[I\,]}_{\beta\in\mathbb{j}}\right) = \cup_{(\alpha,\beta)\in\mathbb{i}\times\mathbb{j}}\left(\underbrace{H_i^t[I\,]}_{\alpha\in\mathbb{i}} \cap \underbrace{N_i^t[I\,]}_{\beta\in\mathbb{j}}\right)$ ∎.

(2). By the same arguments.

4 | Neutrosophic Cartesian Product on Three Types of Neutrosophic Sets and Their Properties

In this section, the neutrosophic family of neutrosophic sets, indexed set, neutrosophic partition, finite general neutrosophic union, countable infinite neutrosophic union, finite general neutrosophic intersection, countable infinite neutrosophic intersection are defined.

Definition 4.1. Let $H_i^t[I\,]$ and $N_i^t[I\,]$ be two neutrosophic sets of three types, for any $i = 1,2,3$. Define the neutrosophic order pair for two neutrosophic elements $h \in H_i^t[I\,]$, $n \in N_i^t[I\,]$ as following: $\langle h, n\rangle = \langle\, h_1 + h_2 I,\, n_1 + n_2 I\rangle$, for some $h_1, h_2 \in H$, $n_1, n_2 \in N$ and an indeterminacy I

$$= \Big\{\{\, h_1,\, h_2 I\}, \big\{\{\, h_1,\, h_2 I\}, \{\, n_1,\, n_2 I\}\big\}\Big\}.$$

The neutrosophic order pair $\langle h, n\rangle$ is a neutrosophic set of three types which related to $H_i^t[I\,]$ and $N_i^t[I\,]$ respectively.

Definition 4.2. Let $H_i^t[I\,]$ and $N_i^t[I\,]$ be two neutrosophic sets of three types, for any $i = 1,2,3$. The neutrosophic cartesian product denoted by $H_i^t[I\,] \times N_i^t[I\,]$, and defined by:

$H_i^t[I\,] \times N_i^t[I\,] = \{\langle h, n\rangle : h \in H_i^t[I\,] \wedge n \in N_i^t[I\,]\}$

$= \{\langle h, n\rangle : \exists\, h_1, h_2 \in H \wedge \exists\, n_1, n_2 \in N, h = h_1 + h_2 I, n = n_1 + n_2 I\}$, where I is an indeterminacy.

Theorem 4.1. Let $H_i^t[I\,]$ and $N_i^t[I\,]$ be two neutrosophic sets of three types, for any $i = 1,2,3$, and let $\langle h, n\rangle$ and $\langle h', n'\rangle$ be two neutrosophic order pairs belonging to $H_i^t[I\,] \times N_i^t[I\,]$. Then, $\langle h, n\rangle = \langle h', n'\rangle \Leftrightarrow h = h' \wedge n = n' \Leftrightarrow (\,h_1 = h_1' \wedge h_2 = h_2') \wedge (\,n_1 = n_1' \wedge n_2 = n_2')$.

Proof. Suppose that $(\,h_1 = h_1' \wedge h_2 = h_2') \wedge (\,n_1 = n_1' \wedge n_2 = n_2') \Rightarrow h = h' \wedge n = n'$,

Now, $\langle h, n\rangle = \langle\, h_1 + h_2 I,\, n_1 + n_2 I\rangle$,

$$= \Big\{\{\, h_1,\, h_2 I\}, \big\{\{\, h_1,\, h_2 I\}, \{\, n_1,\, n_2 I\}\big\}\Big\},$$

$$= \Big\{\{\, h_1',\, h_2' I\}, \big\{\{\, h_1',\, h_2' I\}, \{\, n_1',\, n_2' I\}\big\}\Big\},$$

$= \langle h', n'\rangle$. Conversely,

Assume that $\langle h, n\rangle = \langle h', n'\rangle \Rightarrow \langle\, h_1 + h_2 I,\, n_1 + n_2 I\rangle = \langle\, h_1' + h_2' I,\, n_1' + n_2' I\rangle$

$$\Rightarrow \Big\{\{\, h_1,\, h_2 I\}, \big\{\{\, h_1,\, h_2 I\}, \{\, n_1,\, n_2 I\}\big\}\Big\} = \Big\{\{\, h_1',\, h_2' I\}, \big\{\{\, h_1',\, h_2' I\}, \{\, n_1',\, n_2' I\}\big\}\Big\}.$$

$\because \{\, h_1,\, h_2 I\} \in \Big\{\{\, h_1,\, h_2 I\}, \big\{\{\, h_1,\, h_2 I\}, \{\, n_1,\, n_2 I\}\big\}\Big\}$

$\therefore \{\, h_1,\, h_2 I\} \in \Big\{\{\, h_1',\, h_2' I\}, \big\{\{\, h_1',\, h_2' I\}, \{\, n_1',\, n_2' I\}\big\}\Big\}$

$\therefore \{\, h_1,\, h_2 I\} = \{\, h_1',\, h_2' I\} \vee \{\, h_1,\, h_2 I\} =, \big\{\{\, h_1',\, h_2' I\}, \{\, n_1',\, n_2' I\}\big\}$

Case 1. If $\{\, h_1,\, h_2 I\} = \{\, h_1',\, h_2' I\} \Rightarrow \big\{\{\, h_1,\, h_2 I\}, \{\, n_1,\, n_2 I\}\big\} = \big\{\{\, h_1',\, h_2' I\}, \{\, n_1',\, n_2' I\}\big\}$

$\Rightarrow \{\, h_1,\, h_2 I\} = \{\, h_1',\, h_2' I\} \wedge \{\, n_1,\, n_2 I\} = \{\, n_1',\, n_2' I\}$

$\Rightarrow (\,h_1 = h_1' \wedge h_2 = h_2') \wedge (\,n_1 = n_1' \wedge n_2 = n_2')$

$\Rightarrow h = h' \wedge n = n'$

Case 2. If $\{\, h_1,\, h_2 I\} = \big\{\{\, h_1',\, h_2' I\}, \{\, n_1',\, n_2' I\}\big\} \Rightarrow \big\{\{\, h_1,\, h_2 I\}, \{\, n_1,\, n_2 I\}\big\} = \{\, h_1',\, h_2' I\}$.

$\Rightarrow \{\, h_1,\, h_2 I\} = \{\, h_1',\, h_2' I\} \wedge \{\, n_1,\, n_2 I\} = \{\, h_1',\, h_2' I\}$

$\because \{\, h_1,\, h_2 I\} = \{\, h_1',\, h_2' I\} \Rightarrow (\,h_1 = h_1' \wedge h_2 = h_2')$

$\because \{\, n_1,\, n_2 I\} = \{\, h_1',\, h_2' I\} \Rightarrow (\,n_1 = h_1' \wedge n_2 = h_2')$

$\Rightarrow (\,n_1 = h_1 \wedge n_2 = h_2)$

$\Rightarrow (\,n_1 = n_1' \wedge n_2 = n_2')$

$$\because (h_1 = h_1' \wedge h_2 = h_2') \wedge (n_1 = n_1' \wedge n_2 = n_2') \Longrightarrow h = h' \wedge n = n' \blacksquare.$$

Example 4.1. Let $H = \{a, b\}$ and $N = \{d\}$ be two classical sets, then neutrosophic set of type-1 is given by: $H_1^t[I\,] = \begin{Bmatrix} a + aI, & a + bI, \\ b + aI, & b + bI \end{Bmatrix}$, and $N_1^t[I\,] = \{d + dI\}$, then the neutrosophic cartesian product is given by:

$H_1^t[I\,] \times N_1^t[I\,] = \{\langle a + aI, d + dI\rangle, \langle a + bI, d + dI\rangle, \langle b + aI, d + dI\rangle, \langle b + bI, d + dI\rangle\}$, and

$N_1^t[I\,] \times H_1^t[I\,] = \{\langle d + dI, a + aI\rangle, \langle d + dI, a + bI\rangle, \langle d + dI, b + aI\rangle, \langle d + dI, b + bI\rangle\}$.

Note that. $H_1^t[I\,] \times N_1^t[I\,] \neq N_1^t[I\,] \times H_1^t[I\,]$, and

Observation. It is clear that from the definition and example the following properties

- $H_i^t[I\,] \times N_i^t[I\,] \neq N_i^t[I\,] \times H_i^t[I\,]$, for any $i = 1,2,3$,
- $\psi(H_1^t[I\,] \times N_1^t[I\,]) = \psi(H_1^t[I\,]) \times \psi(N_1^t[I\,])$,
- $H_i^t[I\,] \times \emptyset_i^t[I\,] = \emptyset_i^t[I\,]$

Theorem 4.2. Let $H_i^t[I\,]$, $N_i^t[I]$, and $M_i^t[I\,]$ be three neutrosophic sets of three types, where $i = 1,2,3$, then;

1. $H_i^t[I\,] \times (N_i^t[I] \cap M_i^t[I\,]) = (H_i^t[I\,] \times N_i^t[I]) \cap (H_i^t[I\,] \times M_i^t[I])$,
2. $H_i^t[I\,] \times (N_i^t[I] \cup M_i^t[I\,]) = (H_i^t[I\,] \times N_i^t[I]) \cup (H_i^t[I\,] \times M_i^t[I])$, and
3. $H_i^t[I\,] \times (N_i^t[I] \ominus M_i^t[I\,]) = (H_i^t[I\,] \times N_i^t[I]) \ominus (H_i^t[I\,] \times M_i^t[I])$,

Proof (1). Suppose that $\langle h, n\rangle \in \left(H_i^t[I\,] \times \left(N_i^t[I] \cap M_i^t[I\,]\right)\right)$, "neu-hypo"

$\Leftrightarrow h \in H_i^t[I\,] \wedge n \in \left(N_i^t[I] \cap M_i^t[I\,]\right)$, "N$P_1$" from "neu-hypo"

$\Leftrightarrow h \in H_i^t[I\,] \wedge \left(n \in N_i^t[I] \wedge n \in M_i^t[I\,]\right)$, "N$P_2$" from "N$P_1$"

$\Leftrightarrow \left(h \in H_i^t[I\,] \wedge h \in H_i^t[I\,]\right) \wedge \left(n \in N_i^t[I] \;\; \wedge n \in M_i^t[I\,]\right)$, "N$P_3$" from "N$P_2$"

$\Leftrightarrow \left(\left(h \in H_i^t[I\,] \wedge h \in H_i^t[I\,]\right) \wedge n \in N_i^t[I]\right) \wedge n \in M_i^t[I\,]$, "N$P_4$" from "N$P_3$"

$\Leftrightarrow \left(h \in H_i^t[I\,] \wedge \left(n \in N_i^t[I] \wedge h \in H_i^t[I\,]\right)\right) \wedge n \in M_i^t[I\,]$, "N$P_5$" from "N$P_4$"

$\Leftrightarrow \left(h \in H_i^t[I\,] \wedge \left(\wedge \;\; h \in H_i^t[I\,]\right)\right) \wedge n \in M_i^t[I\,]$, "N$P_6$" from "N$P_5$"

$\Leftrightarrow \left(h \in H_i^t[I\,] \wedge n \in N_i^t[I]\right)\left(n \in N_i^t[I] \wedge n \in M_i^t[I\,]\right)$

$\Leftrightarrow h \in H_i^t[I\,] \wedge \left(n \in N_i^t[I] \wedge n \in M_i^t[I\,]\right)\blacksquare$.

5 | Conclusions

In this paper, the previous work on neutrosophic set theory is extended. the relations between types of operation in the neutrosophic set theory are discussed with examples. In addition, the operations of union and intersection on neutrosophic set theory are generalized. Finally, the cartesian products and their properties are investigated.

Acknowledgments

The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Funding

This research did not receive any external funding.

Conflicts of Interest

The author declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Data Availability

There was no data used in the inquiry that was as stated in the article.

References

[1] Al-Odhari, A. M. (2023). A Review Study on Some Properties of The Structure of Neutrosophic Ring. Neutrosophic Sets and Systems, 54, 139-156. doi:DOI:10.5281/zenodo.7817736
[2] Al-Odhari, A. M. (2023). Some Algebraic Structure of Neutrosophic Matrices. Prospects for Applied Mathematics and data Analysis (PAMDA), 01(02), 37-44. doi:https://doi.org/10.54216/PAMDA.010204
[3] Al-Odhari, A. M. (2024). Axiomatic of Neutrosophic Groups. Sana'a University Journal of Applied Sciences and Technology, 2(2), 205-214. doi:https://doi.org/10.59628/jast.v2i2.793
[4] Al-Odhari, A. M. (2024). Basic Introduction of Neutrosophic Set Theory. to appear .
[5] Al-Odhari, A. M. (2024). Characteristics Neutrosophic Subgroups of Axiomatic Neutrosophic Groups. Neutrosophic Optimization and Intelligent Systems, 3, 32-40. doi:10.61356/j.nois.2024.3265
[6] Al-Odhari, A. M. (2024). Some Aspects of Neutrosophic Set Theory. To apear .
[7] Al-Odhari, A. M. (2024). The Computations of Algebraic Structure of Neutrosophic Determinants. Sana'a University Journal of Applied Sciences and Technology, 2(1), 41-52. doi:10.59628/jast.v2i1.691
[8] PINTER, C. C. (2014). A Book of SET THEORY. Mineola, New York: DOVER PUBLICATIONS, INC. Retrieved from www.doverpublications.com
[9] Smarandache, F. (2005). Neutrosophic Set, A Generalization of The Intuitionistic Fuzzy Sets. Inter. J. Pure Appl. Math, 24, 287-297.
[10] Smarandache, F. (2007). A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability and Statistics.
[11] Smarandache, F. (2015). Symbolic Neutrosophic Theory. Bruxelles, Belgium: EuropaNova asbl. Retrieved from https://fs.unm.edu/SymbolicNeutrosophicTheory.pdf
[12] Weiss, W. A. (2008). AN INTRODUCTION TO SET THEORY. university of Tornoto. Retrieved from https://www.math.toronto.edu/weiss/set_theory.pdf