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Enhancing Set-Theoretic Research Methods with Neutrosophic Sets

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Abstract

This study employed the integration of neutrosophic set theory with set-theoretic methods and Qualitative Comparative Analysis (QCA) to examine intricate social phenomena. Participants' opinions and attitudes were captured using neutrosophic Likert scales, which reflected elements of truth, indeterminacy, and falsity. This proposal is illustrated in a case study facilitated a more comprehensive and subtle examination, emphasizing the significance of variables such as motivation, instructional excellence, and educational resources in achieving academic accomplishment. The results of the necessary condition analysis and set coincidence analysis indicated that motivation and teaching quality have the highest individual impact. Nevertheless, the convergence of scholarly resources, drive, and instructional excellence markedly augmented the likelihood of achieving academic success, underscoring the significance of considering multiple factors collectively. The study proposes that future research should investigate additional sets associated with neutrosophic approaches, such as plithogenic sets, and employ more intricate neutrosophic scales to better capture the complexity of data. This approach provides a comprehensive and precise viewpoint for comprehending cause-and-effect relationships in social phenomena, offering a valuable instrument for research and policy development.

Keywords: Set-Theoretic Methods; Qualitative Comparative Analysis; fsQCA; Neutrosophic Sets.

1 | Introduction

Set-theoretic methods reflect a positivist approach to knowledge, as they are used to forecast and elucidate real-world occurrences by discerning patterns and causal connections among the elements [1]. Ragin initially introduced set-theoretic research methods in the late 1980s, which were primarily founded on crisp sets [2]. Set-theoretic methods, which focus on the antecedent conditions shared by instances of an outcome, are rooted in John Stuart Mill's method of agreement [3]. The set-theoretic perspective employs the concept of set membership to determine whether a case can be characterized by a particular concept or not. Within the context of set-theoretic methods, the process of concept formation carries a distinct meaning compared to traditional measurement theory. It primarily revolves around determining whether a particular case can be classified as belonging to a concept (or set) or not. The act of assigning set membership is also referred to as calibration [4].

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Qualitative Comparative Analysis (QCA) [5] is a well-established method based on set theory. During the initial discussions of QCA in the 1980s and 1990s, it was only applicable to crisp sets, which means that a decision had to be made about whether a case belongs to a set or not. The necessity for "dichotomization" has prompted significant criticism of crisp-set QCA.

Fuzzy Set Qualitative Comparative Analysis (fsQCA) [6] is an advanced analytical technique used to explore the causal relationships between different conditions and outcomes within complex social phenomena. Unlike traditional quantitative methods that often rely on large datasets and assume linear relationships, fsQCA allows researchers to analyze data sets with a more nuanced and qualitative approach, capturing the complexity and richness of real-world scenarios.

There is a connection between fuzzy sets and neutrosophic Likert scales [7] in the way they handle imprecision and uncertainty. Neutrosophic sets and, thus, neutrosophic Likert scales enable an even broader representation that incorporates indeterminacy as a core component, whereas fuzzy sets enable the representation and manipulation of data that are not exactly specified. For survey responses in which participants' opinions not only vary across a spectrum (as accommodated by fuzzy sets) but also may include a degree of indecision or neutrality that is difficult to capture using traditional fuzzy logic or crisp Likert scales, neutrosophic Likert scales are, therefore, particularly well suited.

In this paper, we incorporate neutrosophic set theory into set-theoretic methods and Qualitative Comparative Analysis (QCA), integrating indeterminacy. We present a method for developing single-valued neutrosophic sets [8] from questionnaires applied to social groups to set-theoretic methods.

2 | Preliminaries

2.1 | Complexity Theory and Causality and Neutrosophic Sets

Complexity theory suggests that the relationships between variables are not always straightforward and can exhibit non-linear patterns [9]. This means that the same cause might lead to different effects depending on the context. Three key principles provide insight into this theory: conjunction, equifinality, and causal asymmetry [10]:

- Conjunction refers to the idea that antecedent conditions work together cooperatively to produce an outcome, rather than acting independently to explain variance.
- Equifinality implies that a system can achieve the same end state through multiple different initial conditions and pathways.
- Causal asymmetry indicates that the presence of an outcome might be driven by certain conditions, yet the absence of those conditions does not necessarily lead to the absence of the outcome. For instance, consider a restaurant with exceptional food quality. This high food quality might attract many diners. However, the same restaurant might also have low patronage due to its poor location or inadequate parking facilities. Conversely, a restaurant with mediocre food might still attract many customers if it has excellent service, a prime location, or unique entertainment options. This demonstrates that the relationship between conditions (food quality, location, service) and the outcome (number of diners) is not straightforward or fixed.

These principles highlight that the relationship between conditions and outcomes is complex and not fixed.

Neutrosophy can extend the notion of complex causality by incorporating the inherent indeterminacy and uncertainty in social phenomena [11]. Neutrosophic set theory, with its ability to handle indeterminacy, provides a more nuanced approach to understanding these complex relationships.

2.2 | Neutrosophic Liker Scales

Surveys utilizing neutrosophic Likert scales [12, 13] effectively measure the diversity of opinions and their influence on public policy and social discourse, capturing areas of consensus, disagreement, and ambivalence.

Next, we introduce the fundamental definitions and concepts related to neutrosophic sets and single-valued neutrosophic sets.

Definition 1. [14] Let U be a discourse universe. $N=\{(x, T(x), I(x), F(x)): x \in U\}$ is a neutrosophic set, denoted by a truth-membership function, $TN : U \rightarrow]0-, 1+[$; an indeterminacy-membership function, $IN : U \rightarrow]0-, 1+[$; and a falsity-membership function, $FN : U \rightarrow]0-, 1+[$.

Single-valued neutrosophic sets provide a way to represent and analyze possible elements in the discourse universe U

Definition 2. [15] Let U be a discourse universe. A single-valued neutrosophic set is defined as $N=\{(x, T(x), I(x), F(x)): x \in U\}$, which is identified by a truth-membership function, $TN : U \rightarrow [0, 1]$; indeterminacy-membership function, $IN : U \rightarrow [0, 1]$; and falsity-membership function, $FN : U \rightarrow [0, 1]$, with $0 \leq TN(x)+IN(x)+FN(x) \leq 3$

Using neutrosophic scales with single-valued neutrosophic sets, responses are categorized based on the total of the True, Indeterminate, and False components as follows:

- T+I+F<1: Incomplete
- T+I+F=1: Complete
- T+I+F>1: Contradictory

These values are obtained because, in many cases, opinions are incomplete or contradictory. This classification is one of the advantages of using neutrosophic methods, as it allows for a more nuanced understanding of varying degrees of truth, indeterminacy, and falsity in responses.

3 | Proposed Framework

To illustrate the detailed process to follow, Figure 1 is presented. This figure shows a flowchart that describes the key steps in the implementation model up to the analysis. Each step of the diagram is explained with a brief description, thus facilitating the understanding of the entire process.

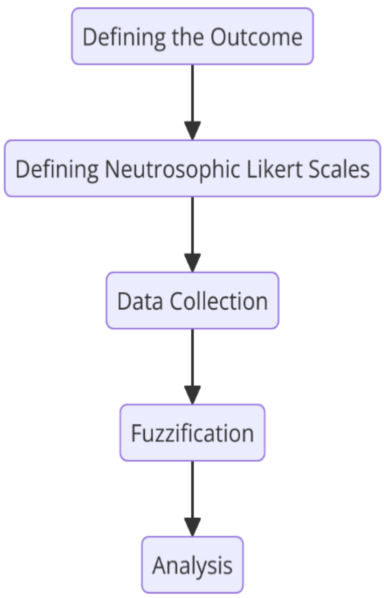

Figure 1. Proposed framework.

i). Defining the Outcome: Identify and clearly define the specific phenomenon, event, or condition that is the focus of the investigation.

ii). Defining Neutrosophic Likert Scales: Develop Neutrosophic Likert Scales to measure the outcome and variables. Unlike traditional Likert scales that measure responses on a fixed scale (e.g., 1 to 5), Neutrosophic Likert Scales incorporate elements of truth, indeterminacy, and falsity. Each response on the scale is represented by a triple (T, I, F), where T is the degree of truth, I is the degree of indeterminacy, and F is the degree of falsity. This allows for a more nuanced capture of respondent opinions and attitudes.

iii). Data Collection: Gather relevant data on the cases in terms of various indicators or measures related to the outcome. Ensure that the data collected are comprehensive and accurately reflect the variables being studied. Use the Neutrosophic Likert Scales in questionnaires and surveys to gather data on the outcome and relevant variables. This approach provides a richer dataset that captures the complexity of respondent opinions and attitudes.

iv). Fuzzification: Neutrosophic sets are converted to equivalent fuzzy membership sets used in this case in line with [16].

Let $AN = \{x, (TA(x), IA(x), FA(x)) : x \in X\}$ be an NS. Its equivalent fuzzy membership set is defined as $AF = \{(x, \mu A(x)) : x \in X\}$, where $\mu A(x) = s\left((TA(x), IA(x), FA(x)), (1,0,0)\right)$. So, using the equation of similarity proposed,

$$\mu A(x) = 1 - \frac{1}{2}[(1 - T_A(x)) + \max\{I_A(x), F_A(x)\}] \quad (1)$$

As the range of the similarity measure function is the unit interval [0,1], $\mu A(x) \in [0,1]$ for all $x \in X$. Hence, the membership function of the derived fuzzy set belongs to [0,1] and thus it satisfies the property of a membership function of a fuzzy set (FS).

v). Analysis: Conduct fsQCA to identify which combinations of factors or conditions are associated with the presence or degree of the outcome. For fsQCA program for Windows is used for data processing [17, 18].

The validity of the configuration is assessed by measuring the consistency and coverage values. Consistency is the measure of how reliably the set of pathways produces the desired outcome. Coverage pertains to the degree to which the outcome is elucidated by this arrangement of pathways[19]:

$$\text{Consistency } (Y_i \leq X_i) = \frac{\sum \min(X_i, Y_i)}{\sum Y_i} \quad (2)$$

$$\text{Coverage } (Y_i \leq X_i) = \frac{\sum \min(X_i, Y_i)}{\sum X_i} \quad (3)$$

where:

X_i is the membership value of case i in the set of causal conditions.

Y_i is the membership value of case iii in the outcome set.

Both are utilized during the comparative analysis to measure established relationships between individual conditions, combinations of conditions, configurations of pathways, and the result. Typically, values that exceed 0.8 are regarded as strong [19].

4 | Case Study

The defined outcome is the perception of Academic Success (SUCCESS). A Likert scale is developed, represented as single-valued neutrosophic sets. The study also considers other variables: Academic Resources

(REC), Motivation (MOT), and Quality of Teaching (CAL). A survey was conducted with a group of 12 Software Engineering students at the University of Guayaquil (see Table 1).

Table 1. Survey data.

Case	REC	MOT	CAL	SUCCESS
1	(0.9,0.9, 0.2)	(0.6, 1, 0.5)	(0.3, 0.7, 0.3)	(0.8, 0.6, 0.7)
2	(0.5, 0.5,0.5)	(1,1,1)	(0.5,0.2,0.5)	(0.6, 0.6, 0.7)
3	(0.8, 0.7, 0.4)	(0.7, 0.9, 0.5)	(0.8, 0.5, 0.5)	(0.8, 0.5, 0.5)
4	(1,1,0)	(0.8,0.8,0)	(1,0.9,0.3)	(0.7, 1, 0.9)
5	(1,0.5,0)	(1,0.5,1)	(1,0.5,1)	(0.9, 0.6, 0.1)
6	(0.9, 0.9, 0.9)	(0.9, 0.9, 0.9)	(0.9, 0.9, 0.9)	(0.9, 0.9, 0.9)
7	(0.2, 0.5, 0.8)	(1,0,0)	(0.5, 0.5, 0.5)	(0.8, 0.5, 0.2)
8	(1, 0.9, 0.1)	(0.9, 0.9, 0.1)	(0.9, 0.9, 0.1)	(0.9, 0.9, 0.1)
9	(1,1,0)	(0.8, 0.8, 0)	(1,0,0)	(0.9, 0, 0)
10	(0.7, 1, 0.2)	(0.9, 0.4, 0)	(0.6, 0.9, 0.1)	(1,0,0)
11	(0.4, 0.7, 0.2)	(0.3, 0.9, 0.4)	(0.8, 0.4, 0.6)	(0.4, 0.8, 0.3)
12	(0.6, 1, 0.6)	(0.6, 0.5, 0.2)	(0.2, 0.5, 0.7)	(1,0,1)

The fuzzification process is developed using Eq. (1) (see Table 2).

Table 2. Fuzzified values.

Case	REC	MOT	CAL	SUCCESS
1	0.50	0.30	0.30	0.55
2	0.50	0.50	0.50	0.45
3	0.55	0.40	0.65	0.65
4	0.50	0.50	0.55	0.35
5	0.75	0.50	0.50	0.65
6	0.50	0.50	0.50	0.50
7	0.20	1.00	0.50	0.65
8	0.55	0.50	0.50	0.50
9	0.50	0.50	1.00	0.95
10	0.35	0.75	0.35	1.00
11	0.35	0.20	0.60	0.30
12	0.30	0.55	0.25	0.50

A necessary condition analysis is performed to test consistency and coverage (Table 3).

Table 3. Necessary condition analysis.

Conditions tested	Consistency	Coverage
REC	0.730496	0.927928
MOT	0.794326	0.903226
CAL	0.794326	0.903226
REC+MOT+CAL	0.957447	0.854430

According to the analysis of essential factors for success, the results show that the individual factors of REC, MOT, and CAL have a high occurrence rate, with each being present in around 90% of successful cases. This implies that these factors are frequently linked to favorable results. Nevertheless, their consistencies are marginally lower, with REC at 73%, and both MOT and CAL at around 79%, suggesting that although they are often present, they are not conclusive indicators of success on their own. The aggregate condition (REC + MOT + CAL) exhibits a significantly high level of consistency at 95%, highlighting its robust predictive capability when all three factors are simultaneously present. However, the coverage of this combined condition decreases to 85%, indicating that while the combination of REC, MOT, and CAL is a strong indicator of success, it does not explain all instances of success. Hence, the combination of REC, MOT, and CAL not only holds individual importance but also greatly increases the probability of success, emphasizing the synergistic impact of these factors in attaining the intended result. A set coincidence analysis is performed (see Table 4).

Table 4. Set coincidence analysis.

Conditions	Coincidence
REC, MOT, CAL	0.594937
REC, MOT	0.678571
REC, CAL	0.740741
MOT, CAL	0.675676

The Set Coincidence procedure was utilized to assess the intersection of key variables: Academic Resources (REC), Motivation (MOT), and Quality of Teaching (CAL). The findings demonstrate that the level of overlap between REC and MOT is 0.678571, between REC and CAL is 0.740741, and between MOT and CAL is 0.675676. The combined overlap of REC, MOT, and CAL is precisely 0.594937. These findings indicate that although each variable is strongly associated with academic success, their combined impact exhibits a robust predictive ability, emphasizing the significance of considering multiple interacting factors in attaining academic achievement. The results of the set superset analysis are shown in Table 5.

Table 5. Subset/Superset analysis results.

Terms	Consistency	Coverage	Combined
REC*MOT*CAL	0.957447	0.638298	0.794931
REC*MOT	0.957895	0.645390	0.799335
REC*CAL	0.950000	0.673759	0.812578
MOT*CAL	0.960000	0.680851	0.821001
REC	0.927928	0.730496	0.841773
MOT	0.903226	0.794326	0.873243
CAL	0.903226	0.794326	0.873243

The results of the subset/superset analysis indicate that motivation (MOT) and quality of teaching (CAL) are the strongest and most relevant conditions for predicting academic success individually. The combination of these two conditions is also very effective. Although the combination of the three conditions (REC, MOT, and CAL) has a high consistency, its coverage is smaller, suggesting that while it is a powerful combination, it is not the only way to achieve academic success. Academic resources (ACRs), both individually and in combination with other conditions, also play an important role, but not as dominant as MOT and CAL.

5 | Conclusions

This study demonstrates that the integration of set-theoretical methods with neutrosophic approaches enables a more sophisticated and comprehensive examination of intricate social phenomena. Neutrosophic Likert scales were employed to more accurately capture participants' opinions and attitudes, encompassing aspects of truth, indeterminacy, and falsehood. The findings from the necessary condition analysis, set matching, and subset/superset analysis indicate that motivation and quality of teaching are the most significant factors in predicting academic success, both independently and when considered together. Nevertheless, it was observed that the amalgamation of scholarly resources, drive, and excellence in instruction substantially enhances the likelihood of achievement, underscoring the synergistic significance of these factors. It is recommended that future research investigates sets associated with neutrosophic approaches, such as plithogenic sets, and employs more intricate neutrosophic scales to better capture the complexity of data.

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Author Contributions

All authors contributed equally to this work.

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Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Data Availability

There was no data used in the inquiry that was as stated in the article.

References

[1] Farrugia, B. (2019). WASP (write a scientific paper): An introduction to set-theoretic methods and qualitative comparative analysis. Early Human Development, 133, 43-47.

[2] Ragin, C. C., Mayer, S. E., & Drass, K. A. (1984). Assessing discrimination: A Boolean approach. American sociological review, 221-234.

[3] Mill, J. S. (1904). A System of Logic, Rationcinative and Inductive: Being a Connected View of the Principles of Evidence, and the Methods of Scientific Investigation. Harper.

[4] Wagemann, C. (2017). Qualitative Comparative Analysis (QCA) and set theory. In Oxford Research Encyclopedia of Politics.

[5] Drass, K., & Ragin, C. C. (1986). QCA: A microcomputer package for qualitative comparative analysis of social data. Center for urban affairs and policy research, Northwestern University.

[6] Ragin, C. C., Drass, K. A., & Davey, S. (2006). Fuzzy-set/qualitative comparative analysis 2.0. Tucson, Arizona: Department of Sociology, University of Arizona, 23(6), 1949-1955.

[7] Kandasamy, I., Kandasamy, W. V., Obbineni, J. M., & Smarandache, F. (2020). Indeterminate Likert scale: feedback based on neutrosophy, its distance measures and clustering algorithm. Soft Computing, 24, 7459-7468.

[8] Pramanik, S. (2022). Single-valued neutrosophic set: An overview. Transdisciplinarity, 563-608.

[9] Misangyi, V. F., Greckhamer, T., Furnari, S., Fiss, P. C., Crilly, D., & Aguilera, R. (2017). Embracing causal complexity: The emergence of a neo-configurational perspective. Journal of management, 43(1), 255-282.

[10] Furnari, S., Crilly, D., Misangyi, V. F., Greckhamer, T., Fiss, P. C., & Aguilera, R. V. (2021). Capturing causal complexity: Heuristics for configurational theorizing. Academy of Management Review, 46(4), 778-799.

[11] Al-Subhi, S. H. S., Pupo, I. P., Vacacela, R. G., Pérez, P. Y. P., & Vázquez, M. Y. L. (2018). A new neutrosophic cognitive map with neutrosophic sets on connections, application in project management. Infinite Study.

[12] Leonor, M. M., Easud, G. S., & Fernando, P. P. (2022). Indeterminate Likert Scale in Social Sciences Research. International Journal of Neutrosophic Science (IJNS), 19(1).

[13] Vázquez, M. Y. L., Ricardo, J. E., Hernández, N. B., Casanova, R. S., & Smarandache, F. (2024). ANÁLISIS NEUTROSÓFICO DE LAS ACTITUDES HACIA LA MÁQUINA DE EXPERIENCIA DE NOZICK. Investigación Operacional, 45(4).

[14] Smarandache, F. (1999). A unifying field in logics: Neutrosophic logic. En Philosophy (pp. 1-141). American Research Press.

[15] Wang, H., Smarandache, F., Zhang, Y., & Sunderraman, R. (2012). Single valued neutrosophic sets. Rev. Air Force Acad.

[16] Mandal, K. (2020). On Deneutrosophication. Neutrosophic Sets and Systems, 38, 409-423.

[17] Ragin, Charles C. and Sean Davey. 2022. Fuzzy-Set/Qualitative Comparative Analysis 4.0. Irvine, California: Department of Sociology, University of California.

[18] Ragin, Charles C. 2018. User's Guide to Fuzzy-Set/Qualitative Comparative Analysis 3.0. Irvine, California: Department of Sociology, University of California.
[19] Ragin, C. C. (2006). Set relations in social research: Evaluating their consistency and coverage. Political analysis, 14(3), 291-310.