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TreeSoft Set vs. HyperSoft Set and Fuzzy-Extensions of TreeSoft Sets

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Abstract

The TreeSoft Set is more general than the HyperSoft Set. The TreeSoft Set is a union of HyperSoft Sets.
To distinguish between the levels of attributes, we use the comma notation i.e. an attribute of level k has k-1 commas in between its indexes.

Keywords: TreeSoft; HyperSoft; Fuzzy Extensions.

1 | Definitions and Examples

Let $\mathcal{U}$ be a universe of discourse, and $\mathcal{H}$ a non-empty subset of $\mathcal{U}$, with $\mathcal{P}(\mathcal{H})$ being the power set of $\mathcal{H}$.

Let A be a set of attributes $A_1, A_2, \dots, A_n$, $n \geq 2$,

$A = \{A_1, A_2, \dots, A_n\}$,

where each attribute A_i, $1 \leq i \leq n$, is a class of sub-attributes $A_i = \{A_{i,1}, A_{i,2}, \dots, A_{i,n_i}\}$.

1.1 | HyperSoft Set

Then: $f: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \dots \times \mathcal{P}(A_n) \to \mathcal{P}(\mathcal{H})$ is called a HyperSoft Set over $\mathcal{U}$.

1.2 | Practical Example of HyperSoft Set

Let $\mathcal{U}$ be a universal set of houses, and $\mathcal{H} = \{h_1, h_2, \dots, h_{10}\}$ of subset of $\mathcal{U}$, from a certain geographical zone (for example, the south-west side of United States). And $\mathcal{P}(\mathcal{H})$ is the powerset of $\mathcal{H}$.

Let's consider the set of attributes A be

$A = \{A_1, A_2\}$,

where $A_1 = \text{size} = \{\text{Small}, \text{Big}\}$, and $A_2 = \text{Arizonian cities} = \{\text{Phoenix}, \text{Tucson}\}$.

$F: A_1 \times A_2 \to \mathcal{P}(\mathcal{H})$

$F(\text{Big}, \text{Phoenix}) = \{h_1, h_3\}$.

$F(\text{Big}, \text{Tucson}) = \{h_5, h_7\}$.

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2 | Definition of TreeSoft Set

Let $\mathcal{U}$ be a universe of discourse, and $\mathcal{H}$ a non-empty subset of $\mathcal{U}$, with $\mathcal{P}(\mathcal{H})$ the powerset of $\mathcal{H}$.

Let A be a set of attributes (parameters, factors, etc.),

A= $\{A_1, A_2, \ldots, A_n\}$, for integer $n \geq 1$, where $A_1, A_2, \ldots, A_n$ are considered attributes of first level (since they have one-digit indexes).

Each attribute $A_i, 1 \leq i \leq n$, is formed by sub-attributes:

$A_1 = \{A_{1,1}, A_{1,2}, \ldots\}$

$A_2 = \{A_{2,1}, A_{2,2}, \ldots\}$

....

$A_n = \{A_{n,1}, A_{n,2}, \ldots\}$

where the above $A_{i,j}$ are sub-attributes (or attributes of second level) (since they have two-digit indexes).

Again, each sub-attribute $A_{i,j}$ is formed by sub-sub-attributes (or attributes of third level):

$A_{i,j,k}$

and so on, as much refinement as needed into each application, up to sub-sub-…-sub-attributes (or attributes of m-level (or having m digits into the indexes):

$A_{i1,i2,\ldots,im}$

Therefore, a graph-tree is formed, that we denote as Tree(A), whose root is A (considered of level zero), then nodes of level 1, level 2, up to level m.

We call leaves of the graph-tree, all terminal nodes (nodes that have no descendants).

Then the TreeSoft Set is defined as: F: $\mathcal{P}(\text{Tree}(A)) \rightarrow \mathcal{P}(\mathcal{H})$

Tree(A) is the set of all nodes and leaves (from level 1 to level m) of the graph-tree, and $\mathcal{P}(\text{Tree}(A))$ is the powerset of the Tree(A).

2.1 | Remark

We use comma (,) in between indexes to clearly distinguish between their levels.

For example, A_{12} is an attribute of the first level (has no comma between indexes) being part of the attributes $A_1, A_2, \ldots, A_{10}, A_{11}, A_{12}, A_{13}, \ldots$;

while $A_{1,2}$ is an attribute of second-level (or sub-attribute) [has one comma between indexes], being part of the sub-attributes:

$A_{1,1}, A_{1,2}, A_{1,3}, \ldots$.

Therefore $A_{i,j} \neq A_{ij}$ and similarly for all k-levels of attributes.

2.2 | Property

An attribute of level k has k-1 commas in between indexes.

Example: $A_{2,11,4}$ is an attribute of level 3 (has 2 commas in between indexes).

3 | Fuzzy Extensions of the TreeSoft Set

Let's denote by $d°(h)$ the fuzzy (or fuzzy extension, such as: Intuitionistic Fuzzy Set, Inconsistent Intuitionistic Fuzzy Set (Picture Fuzzy Set, Ternary Fuzzy Set), Pythagorean Fuzzy Set (Atanassov's Intuitionistic Fuzzy Set of second type), q-Rung Orthopair Fuzzy Set, Spherical Fuzzy Set, n-HyperSpherical Fuzzy Set, etc.) degree of appurtenance of the element *h* with respect to the set it belongs to.

$F: \mathcal{P}(\text{Tree}(A)) \rightarrow \mathcal{P}(\mathcal{H}_{d°(h)})$.

For any subset $\alpha \in \text{Tree}(A)$, one has $F(\alpha) = (h_{i(d°(h_i))}, h_i \in \mathcal{H})$.

From the previous Example 2.2.,

Neutrosophic HyperSoft Set

$F(\text{Big, Phoenix}) = \{h_1(0.7, 0.4, 0.3), h_3(0.6, 0.2, 0.3)\}$

$F(\text{Big, Tucson}) = \{h_5(0.1, 0.8, 0.3), h_7(0.9, 0.0, 0.5)\}$

Neutrosophic TreeSoft Set:

F(Big, Arizona) = F(Big, Phoenix) U F(Big, Tucson) =

$= \{h_1(0.7, 0.4, 0.3, h_3(0.6, 0.2, 0.3), h_5(0.1, 0.8, 0.3), h_7(0.9, 0.0, 0.5)\}$.

Fuzzy HyperSoft Set

$F(\text{Big, Phoenix}) = \{h_1(0.8), h_3(0.5)\}$,

$F(\text{Big, Tucson}) = \{h_5(0.2), h_7(0.6)\}$,

Fuzzy TreeSoft Set:

F(Big, Arizona) = F(Big, Phoenix) U F(Big, Tucson)=$\{h_1(0.8), h_3(0.5), h_5(0.2), h_7(0.6)\}$.

The first set is formed by the nodes of level 1, second set by the nodes of level 2, third set by the nodes of level 3, and so on, the last set is formed by the nodes of level *m*.

4 | Practical Example of TreeSoft Set of Level 3 [3]

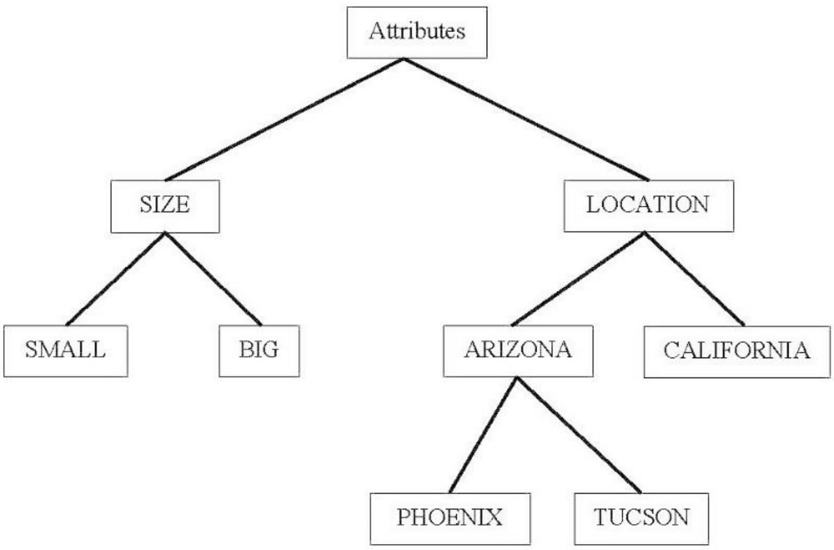

Figure 1. A TreeSoft Set of Level 3.

Assume a town has many houses.

This is a classical tree, whose:

Level 0 (the root) is the node: Attributes;

Level 1 is formed by the nodes: Size, Location;

Level 2 is formed by the nodes: Small, Big, Arizona, and California;

Level 3 is formed by the nodes: Phoenix, Tucson.

Let's consider $H = \{h_1, h_2, ..., h_{10}\}$ be a set of houses, and $P(H)$ the powerset of H.

And the set of first-level Attributes: $A = \{A_1, A_2\}$, where A_1 = Size, A_2 = Location.

Then $A_1 = \{A_{1,1}, A_{1,2}\}$ = {Small, Big} second-level attributes, and

$A_2 = \{A_{2,1}, A_{2,2}\}$ = {Arizona, California} as American states, also second-level attributes.

Further on, $A_{2,1} = \{A_{2,1,1}, A_{2,1,2}\}$ = {Phoenix, Tucson} as Arizonian cities, as third-level attributes.

Let's assume that the function F gets the following values:

F(Big, Arizona, Phoenix} = { h_1, h_3 }

F(Big, Arizona, Tucson) = { h_5, h_7}

F(Big, Arizona) = all big houses from both cities, Phoenix and Tucson =

= F(Big, Arizona, Phoenix) ∪ F(Big, Arizona, Tucson) =

={ h_1, h_3, h_5, h_7 }.

5 | Theorem

In general, the TreeSoft Set is a union of HyperSoft Sets.

Let $\mathcal{U}$ be a universe of discourse, $\mathcal{H}$ a non-empty subset of $\mathcal{U}$, and $\mathcal{P}(\mathcal{H})$ the powerset of $\mathcal{H}$.

Let $A = \{A_1, A_2, \dots, A_n\}$, $n \geq 2$, be a set of attributes of first-level.

Then, $A_i = \{A_{i,1}, A_{i,2}, ..., A_{i,n_i}\}$, for $1 \leq i \leq n$, is a set of attributes of second-level.

Similarly, $A_{i,j} = \{A_{i,j,1}, A_{i,j,2}, ..., A_{i,j,n_{ij}}\}$, for all i, j, a set of attributes of third-level.

$A_{i,j,k} = \{A_{i,j,k,1}, A_{i,j,k,2}, ..., A_{i,j,k,n_{ijk}}\}$ attributes of the third-level (sub-sub-attributes).

Up to attributes of level m:

$$A_{i_1,i_2,...,i_m} = \{A_{i_1,i_2,...,i_m,1}, A_{i_1,i_2,...,i_m,2}, ..., A_{i_1,i_2,...,i_m,n_{i_1i_2,...,i_m}}\}$$

where $A, A_i, A_{i,j}, A_{i,j,k}, \dots, A_{i_1,i_2,...,i_m}$ generate a Tree of m levels, denoted as Tree(A), and $\mathcal{P}(\text{Tree}(A))$ is the powerset of the Tree(A).

All as in Definition 3.

$F: \mathcal{P}(\text{Tree}(A)) \to \mathcal{P}(\mathcal{H})$ is the TreeSoft Set.

The powerset of Tree(A) is formed by elements that are nodes or leaves of the Tree(A).

The leaves are okay, as in HyperSoft Set, while a node is replaced by the union of all leaves that descend from it.

In the previous Example 5, if one takes the node Arizona, that is equivalent with (and replaced by) Phoenix and Tucson:

ARIZONA = {Phoenix, Tucson} or Phoenix ∪ Tucson.

If the node is LOCATION, then it is equivalent to, and replaced by {Phoenix, Tucson, California}.

The HyperSoft Set works with the leaves only.

But, because each node, no matter its level, is equivalent to the union of leaves that descend from it, one gets that the TreeSoft Set is equal to a union of HyperSoft Sets.

Therefore, the Treesoft Set is more general than the HyperSoft Set, and it is a union of HyperSoft Sets.

Coming back to the previous example, we have:

HyperSoft Sets

f(Big, Phoenix) = $\{h_1, h_3\}$, meaning big houses in Phoenix.

f(Big, Tucson) = $\{h_5, h_7\}$, meaning big houses in Tucson.

TreeSoft Set

f(Big, Arizona) = *f*(Big, {Phoenix, Tucson}) =

= *f*(Big, Phoenix) ∪ *f*(Big, Tucson) = $\{h_1, h_3\} \cup \{h_5, h_7\} = \{h_1, h_3, h_5, h_7\}$.

6 | Conclusion

The TreeSoft Set is equivalent to a union of HyperSoft Sets.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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