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Futher Operations on Neutrosophic Hypersoft Matrices and Application in Decision Making

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Abstract

The main objective of this paper is to extend the concept of Neutrosophic Hypersoft Matrix theory. Neutrosophic soft matrix parametrically evaluates the attributes chosen whereas Neutrosophic hypersoft matrix can parametrically evaluate the sub-attributes of the attributes chosen. Some notions and operations related to Neutrosophic Hypersoft matrices (NHSMs) such as Row-NHSM, Column-NHSM, Diagonal-NHSM, Proper-NHS submatrix, Disjoint NHSM, Extended union (NHSM), Extended intersection (NHSM), addition, subtraction, AND-product and OR-product on NHSM have been introduced with examples. Further a new NHSM-algorithm has been developed based on value matrix, grace matrix and mean matrix to solve Neutrosophic Hypersoft Set based decision-making problems.

Keywords: Soft Set; Neutrosophic Soft Set; Neutrosophic Hypersoft Set; Neutrosophic Hypersoft Matrix; NHSM.

1 | Introduction

To deal with uncertainty, Lotfi A. Zadeh [1] in 1965 introduced the concept of Fuzzy logic and Fuzzy sets. In Fuzzy logic, it represents the degree of truth as an extension of valuation. To deal with imprecise and vague information K. Atanassov [2] in 1986 introduced the concept of Intuitionistic fuzzy sets and Intuitionistic fuzzy logic. Similarly, Pythagorean fuzzy sets, Pythagorean fuzzy numbers, and several other concepts and their applications in MCDM, MADM, and MAGDM were proposed in [3-11]. Thomason [12] expanded the idea of Fuzzy sets to Fuzzy matrices (FM) and talked about the convergence of powers of Fuzzy matrices. Fuzzy matrices only take into account membership values while solving the Decision-making problems. To deal with both membership and non-membership values, Pal et al. [13] transformed the well-known Fuzzy matrix into the Intuitionistic fuzzy matrix (IFM), whose constituent parts come from the unit interval [0, 1]. The parametrization of the attributes is not discussed in any of the aforementioned studies. Molodtsov [14] 1999 generalized the concept of fuzzy set theory to soft set theory which helps to deal with uncertainty. Some basic properties of soft set theory were proposed by P. K. Maji et al. [15]. Later on several interesting results based on Soft set theory were obtained by embedding the idea of Fuzzy set, Intuitionistic fuzzy set, Vague set, Rough set, Interval-valued intuitionistic fuzzy set and so on. Also, various applications of the above-mentioned sets in decision-making problems were developed in [16-22].

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Naim Cagman et al. [23] introduced the notion of Soft matrix (SM), which are representations of Soft sets and also defined products of such matrices. They also have offered a soft max-min decision-making algorithm to solve some problems with uncertainties. However, because of the different product order, this method does not satisfy the commutative law, as it could lead to two different outcomes when used to solve identical decision-making problems. Further, this approach will be wholly invalid if a decision-making problem requires the perspectives of at least three observers. To overcome such limitations Yong et al. [24] introduced Fuzzy soft matrix (FSM). Further, to deal with both membership and non-membership values in a parametric manner, Rajarajeswari et al. [25], proposed the Intuitionistic fuzzy soft matrix (IFSM). This idea handles the uncertain object more accurately with their parametrization and ensures that the sum of membership degrees and non-membership degrees does not exceed 1. Abhishek et al. [26] proposed the idea of the Pythagorean fuzzy soft matrix (PFSM) that altered the condition MV + NMV ≤ 1 to MV 2 + NMV 2 ≤ 1.

F. Samarandache [27] 1995 introduced the concept of Neutrosophic sets and Neutrosophic logic with indeterminate data. Similarly, in [41] F. Smarandache gave some definitions based on neutrosophic sets. Neutrosophic soft sets were introduced by Maji in [28]. He also gave an application on Neutrosophic soft sets in decision-making problems. The concept of Generalized Neutrosophic soft set theory was proposed by Said Broumi [29]. Similarly, several concepts based on the Neutrosophic Soft set theory have emerged in recent days. Later on, in 2015 Irfan Deli et al. [30] introduced the concept of Neutrosophic soft matrix (NSM) and their operators which are more functional to make theoretical studies in the Neutrosophic soft set theory. Also, Tanushree Mitra Basa et al. [31], in 2015 developed the Neutrosophic soft matrix theory by defining various operations on them. In 2017 Tuhin Bera et al. [32] further extended the concept of Neutrosophic soft matrix theory and presented an application in decision making. Following this, Sujit Das et al. [33] and Faruk karaaslan [42] gave applications in group decision making. Similarly, Jayasudha et al. [34] gave an application of neutrosophic soft matrices in decision-making.

Smarandache [35] introduced Hypersoft sets which deal with multi-attribute functions. Further, Muhammad Saqlain et al. [36] developed a new concept called the Neutrosophic Hypersoft set and also studied some operations on it. Rana Muhammad Zulqarnain et al. [37] developed the generalized version of aggregate operators on Neutrosophic Hypersoft sets. In 2021 Abdul Samad et al. [38] devised a method that is an extension of the TOPSIS technique using Neutrosophic hypersoft sets based on correlation coefficient to determine the effectiveness of hand sanitizer to reduce COVID-19 effects. To reduce the complicated framework of Neutrosophic Hyper-soft sets, Rana Muhammad Zulqarnain et al. [39] developed the concept of Neutrosophic Hypersoft Matrix and provided certain basic operators and operations on them. Further, certain new notions, operations, and properties of Neutrosophic Hypersoft Matrices have been explored by Naveed Jafar et al. [40]. Similarly, Jayasudha et al. [43] developed the NHSM theory by defining basic notions of classical matrix theory in NHSM has been discussed with examples.

The present study aims to extend the concept of NHSM theory by developing some basic notions and operations along with examples. The organization of our manuscript is as follows: In Section 2 we recall some fundamental definitions that would be further helpful to extend the NHSM theory. In section 3, the so mentioned notions and operations include Row-NHSM, ColumnNHSM, Diagonal-NHSM, Proper-NHS submatrix, Disjoint NHSM, Extended union (NHSM), Extended intersection (NHSM), subtraction, addition, OR-product, AND-product which have been examined with examples and some properties. In section 4, to solve the decision-making problem an NHSM-algorithm has been devised and it is used in the selection of data entry clerks by the manager of a cooperative bank.

1.1 | Preliminaries

In this section, we recall some fundamentals such as Soft set, Neutrosophic set, Neutrosophic soft set, Hypersoft set, Neutrosophic hypersoft set, Neutrosophic hypersoft matrix, etc., which would further help to extend the Neutrosophic Hypersoft Matrix theory. Throughout this paper, a Neutrosophic Hypersoft Matrix is represented by NHSM.

Definition 1.1.1. [14] Let U be an initial universal set and E be a set of parameters. Let P(U) denote the power set of U. Consider a nonempty set B⊂E. A pair (G, B) is called a soft set of U, where G is a mapping given by G: B →P(U). A soft set over U is a parameterized family of subsets of the universe U. For ε ∈ B, G(ε) may be considered as the set of ε-approximate elements of the soft set (G, B).

Definition 1.1.2 [41] A neutrosophic set B on the universal set Y is attributed to three individualistic degrees namely, truth-membership degree (η), indeterminacy-membership degree (υ), and falsity-membership degree (φ), which is defined as;

$$B\{< y, \eta_B(y), \upsilon_B(y), \varphi_B(y) >: y \in Y\}$$

where η_B, υ_B, φ_B: $Y \rightarrow]^-0,1^+[$ and $^-0 \leq \eta_B(y) + \upsilon_B(y) + \varphi_B(y) \leq 3^+$.

Definition 1.1.3. [42] A neutrosophic soft set g over Y is a neutrosophic set valued function from E to $N(Y)$. It can be written as $g = \{(e, \eta_{g(e)}(y), \upsilon_{g(e)}(y), \varphi_{g(e)}(y) >: y \in Y\}): e \in E\}$ where, $N(Y)$Denotes all neutrosophic sets over Y.

Definition 1.1.4. [36] Let ξ be the universal set and $P(\xi)$.be the power set of ξ. Consider $l^1, l^2, l^3, \ldots, l^n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attribute values are respectively the set L^1, L^2, $L^3, \ldots, L^n$ with $L^i \cap L^j = \emptyset$, for $i \neq j$ and $i, j \in \{1, 2, 3, \ldots, n\}$, then the pair $(G, L^1 \times L^2 \times L^3 \ldots L^n)$ is said to be Hypersoft set over ξ where

$$G: L^1 \times L^2 \times L^3 \ldots L^n \rightarrow P(\xi)$$

Definition 1.1.5. [36] Let ξ be the universal set and $P(\xi)$.be the power set of ξ. Consider $l^1, l^2, l^3, \ldots, l^n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attribute values are respectively the set L^1, L^2, $L^3, \ldots, L^n$ with $L^i \cap L^j = \emptyset$, for $i \neq j$ and $i, j \in \{1, 2, 3, \ldots, n\}$ and their relation $L^1 \times L^2 \times L^3 \ldots L^n = S$, then the pair (G, S) is said to be Neutrosophic Hypersoft set over ξ where $G: L^1 \times L^2 \times L^3 \ldots L^n \rightarrow P(\xi)$ and $G(L^1 \times L^2 \times L^3 \ldots L^n) = \{< x, \eta(G(S)), \upsilon(G(S)), \varphi(G(S)) >, x \in \xi\}$ where η is the *truth-membership value*, υ *is the indeterminacy-membership value* and φ is the *falsity-membership value* such that $\eta, \upsilon, \varphi : \xi \rightarrow [0, 1]$ also $0 \leq \eta(G(S)) + \upsilon(G(S)) + \varphi(G(S)) \leq 3$.

Definition 1.1.6. [37] Let $U = \{u^1, u^2, \ldots, u^\alpha\}$ and (U) be the universal set and power set of universal set, respectively, and also consider $L_1, L_2, \ldots, L_\beta$ for $\beta \geq 1, \beta$ well-defined attributes, whose corresponding attribute values are, respectively, the set $L_1^a, L_2^b, \ldots, L_\beta^z$ and their relation $L_1^a \times L_2^b \times \ldots \times L_\beta^z$, where $a, b, c, \ldots, z = 1, 2, \ldots, \beta$, then the pair $(G, L_1^a \times L_2^b \times \ldots \times L_\beta^z)$is said to be neutrosophic hypersoft set over U, where $G: L_1^a \times L_2^b \times \ldots \times L_\beta^z \rightarrow P(U)$, and it is defined as

$$G(L_1^a \times L_2^b \times \ldots \times L_\beta^z) = \{u, \eta_i(u), \upsilon_i(u), \varphi_i(u): u \in U, i \in L_1^a \times L_2^b \times \ldots \times L_\beta^z\}.$$

Let $R_i = L_1^a \times L_2^b \times \ldots \times L_\beta^z$ be the relation, and its characteristic function is

$$X_{R_i} = L_1^a \times L_2^b \times \ldots \times L_\beta^z \rightarrow P(U);$$

it is defined as $X_{R_i} = \{u, \eta_i(u), \upsilon_i(u), \varphi_i(u): u \in U, i \in L_1^a \times L_2^b \times \ldots \times L_\beta^z\}$ and can be a representation of R_i as given in Table 1.

Table 1. Tabular representation of the characteristic function.

U	L_1^a	L_2^b	…	L_β^z
u^1	$X_{R_i}(u^1, L_1^a)$	$X_{R_i}(u^1, L_2^b)$	…	$X_{R_i}(u^1, L_\beta^z)$
u^2	$X_{R_i}(u^2, L_1^a)$	$X_{R_i}(u^2, L_2^b)$	…	$X_{R_i}(u^2, L_\beta^z)$
⋮	⋮	⋮	⋮	⋮
u^α	$X_{R_i}(u^\alpha, L_1^a)$	$X_{R_i}(u^\alpha, L_2^b)$	…	$X_{R_i}(u^\alpha, L_\beta^z)$

If $M_{ij} = X_{R_i}(u^i, L_j^k)$, where $i = 1,2,3,\dots,\alpha, j = 1,2,3,\dots,\beta, k = a,b,c,\dots,z,$ then the matrix is defined as

$$[M_{ij}]_{\alpha\times\beta} = \begin{pmatrix} M_{11} & M_{12}\cdots & M_{1\beta} \\ M_{21} & M_{22}\cdots & M_{2\beta} \\ \vdots & \vdots\ \vdots & \vdots \\ M_{\alpha 1} & M_{\alpha 2}\cdots & M_{\alpha\beta} \end{pmatrix}$$

Where $M_{ij} = \left(\eta_{L_j^k}(u_i), \upsilon_{L_j^k}(u_i), \varphi_{L_j^k}(u_i), u_i \in U, L_j^k \in L_1^a \times L_2^b \times \dots \times L_\beta^z\right) = (\eta_{ijk}^M, \upsilon_{ijk}^M, \varphi_{ijk}^M)$.

Thus, we can represent any neutrosophic hypersoft set in terms of the neutrosophic hypersoft matrix (NHSM), and it means that they are interchangeable.

Definition 1.1.7. [37] Let $O = [O_{ij}]$ be the square NHSM of order $\varsigma \times \varsigma$, where $O_{ij} = (\eta_{ijk}^O, \upsilon_{ijk}^O, \varphi_{ijk}^O)$, then O^t is said to be the transpose of square NHSM if rows and columns of O are interchanged. It is denoted as

$$O^t = [O_{ij}]^t = (\eta_{ijk}^O, \upsilon_{ijk}^O, \varphi_{ijk}^O)^t = (\eta_{jki}^O, \upsilon_{jki}^O, \varphi_{jki}^O) = [O_{ji}].$$

Definition 1.1.8. [37] Let $O = [O_{ij}]$ and $M = [m_{ij}]$ Be two NHSMs of order $\varsigma \times \nu$, where $O_{ij} = (\eta_{ijk}^O, \upsilon_{ijk}^O, \varphi_{ijk}^O)$and $M_{ij} = (\eta_{ijk}^M, \upsilon_{ijk}^M, \varphi_{ijk}^M)$. Then, their union is defined as follows:

$$O \cup M = D, where\ \eta_{ijk}^D = \max(\eta_{ijk}^O, \eta_{ijk}^M), \upsilon_{ijk}^D = \frac{(\upsilon_{ijk}^O + \upsilon_{ijk}^M)}{2}, \varphi_{ijk}^O = \min(\varphi_{ijk}^O, \varphi_{ijk}^M).$$

Definition 1.1.9. [37] Let $O = [O_{ij}]$ and $M = [m_{ij}]$ be two NHSMs of order $\varsigma \times \nu$, where $O_{ij} = (\eta_{ijk}^O, \upsilon_{ijk}^O, \varphi_{ijk}^O)$and $M_{ij} = (\eta_{ijk}^M, \upsilon_{ijk}^M, \varphi_{ijk}^M)$. Then, their intersection is defined as follows:

$$O \cap M = D, where\ \eta_{ijk}^D = \min(\eta_{ijk}^O, \eta_{ijk}^M), \upsilon_{ijk}^D = \frac{(\upsilon_{ijk}^O + \upsilon_{ijk}^M)}{2}, \varphi_{ijk}^O = \max(\varphi_{ijk}^O, \varphi_{ijk}^M).$$

Definition 1.1.10. [37] Let $O = [O_{ij}]$ be the NHSM of order $p \times \mathrm{T}$, where $O_{ij} = (\eta_{ijk}^O, \upsilon_{ijk}^O, \varphi_{ijk}^O)$, then O is said to be square NHSM if $p = \mathrm{T}$. It means that if an NHSM has the same number of rows and columns, then it is square NHSM.

Definition 1.1.11. [37] Let $O = [O_{ij}]$ be the square NHSM of order $p \times \mathrm{T}$, where $O_{ij} = (\eta_{ijk}^O, \upsilon_{ijk}^O, \varphi_{ijk}^O)$, then O is said to be transpose of square NHSM if rows and columns of O are interchanged. It is denoted as $O^t = [O_{ij}]^t = (\eta_{ijk}^O, \upsilon_{ijk}^O, \varphi_{ijk}^O)^t = (\eta_{jki}^O, \upsilon_{jki}^O, \varphi_{jki}^O) = [O_{ji}]$.

Definition 1.1.12. [37] Let $K = [k_{ij}]_{p\times\mathrm{T}}$ be a Neutrosophic hypersoft matrix where $k_{ij} = (\eta_{ijk}^k, \upsilon_{ijk}^k, \varphi_{ijk}^k)$.

If indeterminacy membership degree lies in favor of falsity-membership degree then the value matrix of the matrix K which is symbolized by $V(K)$ and is defined as $V(K) = [v_{ij}^k]_{p\times\mathrm{T}}$, where $v_{ij}^k = [\eta_{ijk}^k$- $(\upsilon_{ijk}^k + \varphi_{ijk}^k)]$, $\forall i, j, k$.

1.2 | Neutrosophic Hypersoft Matrix

In this section, we introduce some basic notions based on NHSM and study their properties.

Example 1.2.1. Let U be the collection of TV's appeared within the TV showroom:

$$U = \{T_1 = Panasonic, T_2 = Samsung, T_3 = LG, T_4 = SonyBravia\}$$

The decision-maker offers his conclusion around the choice procedure of the alternatives such as; $G_1 = USB\ Port, G_2 = Inches, G_3 = Resolution, G_4 = Speakers$. Moreover, the above-mentioned attributes have advanced sub-attributes and can be classified as follows: $G_1^a = \{1, 2, 3\}$, $G_2^b = \{43, 49, 55\}$, $G_3^c = \{1920 \times 1080P, 3840 \times 2160P\}$, $G_4^d = \{20W, 40W\}$.

Let the function be $R = G_1^a \times G_2^b \times G_3^c \times G_4^d \to P(U)$. The neutrosophic hypersoft set is defined as; $R: (G_1^a \times G_2^b \times G_3^c \times G_4^d) \to P(U)$.

Assume that $R(G_1^a \times G_2^b \times G_3^c \times G_4^d) = R(1, 49, 1920 \times 1080P, 40W) = \{T_1, T_2\}$. Then, the neutrosophic hypersoft set of above expected connection is;

$$R(G_1^a \times G_2^b \times G_3^c \times G_4^d) = R(1, 49, 1920 \times 1080P, 40W)$$

$$= \begin{Bmatrix} (T_1, \{0.7, 0.3, 0.5\}, \{0.6, 0.4, 0.2\}, \{0.4, 0.8, 0.1\}, \{0.5, 0.2, 0.7\}) \\ (T_2, \{0.7, 0.3, 0.5\}, \{0.7,0.2,0.3\}, \{0.3,0.8,0.1\}, \{0.4,0.8,0.5\}) \end{Bmatrix}$$

Further, the matrix representation of the above Neutrosophic hypersoft set is:

$$[R]_{2\times4} = \begin{pmatrix} (0.7, 0.3, 0.5) & (0.6, 0.4, 0.2) & (0.4, 0.8, 0.1) & (0.5, 0.2, 0.7) \\ (0.3, 0.8, 0.1) & (0.7, 0.2, 0.3) & (0.3, 0.8, 0.1) & (0.4, 0.8, 0.5) \end{pmatrix}$$

Definition 1.2.2. An NHSM with order $1 \times \tau$, i.e., with a single row is called a row-NHSM. Formally, a row-NHSM compares to a Neutrosophic hypersoft set whose universal set contains only one alternative.

Example 1.2.3. Consider a NHSM R related to the Neutrosophic hypersoft set $R: (G_1^a \times G_2^b \times G_3^c \times G_4^d) \to P(U)$ Over the same universe and attributes as in Example 1.2.1.

$$R(G_1^a \times G_2^b \times G_3^c \times G_4^d) = R(1, 49, 1920 \times 1080P, 40W)$$

$$= \{(T_1, \{0.7, 0.3, 0.5\}, \{0.6, 0.4, 0.2\}, \{0.4, 0.8, 0.1\}, \{0.5, 0.2, 0.7\})\}$$

The NSHM $[R]$ is given by,

$[R]_{1\times4} = ((0.7, 0.3, 0.5)(0.6, 0.4, 0.2)(0.4, 0.8, 0.1)(0.5, 0.2, 0.7))$ Which is a row NHSM.

Definition 1.2.4. A NHSM with order $p \times 1$, i.e., with a single column is called a column-NHSM. Formally, a column-NHSM compares to a Neutrosphic hypersoft set whose universal set contains only one attribute.

Example 1.2.5. From the same universe and attributes as of Example 1.2.1., there exists a NHSM $[S]$ related to the Neutrosophic hypersoft set $S: (G_1^a \times G_2^b \times G_3^c \times G_4^d) \to P(U)$

$$S(G_1^a \times G_2^b \times G_3^c \times G_4^d) = S(1) = \begin{Bmatrix} (T_1, \{0.7, 0.3, 0.5\}) \\ (T_2, \{0.6, 0.4, 0.2\}) \\ (T_3, \{0.4, 0.5, 0.6\}) \\ (T_4, \{0.5, 0.2, 0.7\}) \end{Bmatrix}$$

Hence the NHSM $[S]$ is written by,

$$S_{4\times 1} = \begin{pmatrix} (0.7, 0.3, 0.5) \\ (0.6, 0.4, 0.2) \\ (0.4, 0.5, 0.6) \\ (0.5, 0.2, 0.7) \end{pmatrix}$$

Which is a column NHSM.

Definition 1.2.6. A square of a NHSM with order $p \times \tau$ is called a diagonal NHSM if all of its non-diagonal elements are $(0,0,1)$.

Example 1.2.7. Consider the same universe and attributes as of Example 1.2.1. Then a diagonal NHSM $[R]$ is given by;

$$R_{4\times 4} = \begin{pmatrix} (0.7, 0.2, 0.4) & (0.0, 0.0, 1.0) & (0.0, 0.0, 1.0) & (0.0, 0.0, 1.0) \\ (0.0, 0.0, 1.0) & (0.5, 0.3, 0.9) & (0.0, 0.0, 1.0) & (0.0, 0.0, 1.0) \\ (0.0, 0.0, 1.0) & (0.0, 0.0, 1.0) & (0.9, 0.1, 0.4) & (0.0, 0.0, 1.0) \\ (0.0, 0.0, 1.0) & (0.0, 0.0, 1.0) & (0.0, 0.0, 1.0) & (0.6, 0.4\ 0.7) \end{pmatrix}$$

Definition 1.2.8. NHSM $R = [r_{ij}]$ is a proper neutrosophic hypersoft submatrix of $S = [s_{ij}] \in$ NHSM, denoted by $[r_{ij}] \subset [s_{ij}]$, if $t^r_{ijk} < t^s_{ijk}, i^r_{ijk} < i^s_{ijk}, f^r_{ijk} > f^s_{ijk}$.

Example 1.2.9. Consider the NHSM $[R]_{2\times 4}$ of Example 1.2.1.

$$[R]_{2\times 4} = \begin{pmatrix} (0.7, 0.3, 0.5)(0.6, 0.4, 0.2)(0.4, 0.8, 0.1) & (0.5, 0.2, 0.7) \\ (0.3, 0.8, 0.1)(0.7, 0.2, 0.3)(0.3, 0.8, 0.1) & (0.4, 0.8, 0.5) \end{pmatrix}$$

Presently consider another NHSM. $[S]$ Related with the Neutrosphic hypersoft set $S: (G_1^a \times G_2^b \times G_3^c \times G_4^d) \rightarrow P(U)$ Over the same universe and attributes as in Example 1.2.1.

$$S(G_1^a \times G_2^b \times G_3^c \times G_4^d) = S(1, 49, 1920 \times 1080P, 40W)$$

$$= \begin{Bmatrix} (T_1, \{0.8, 0.5, 0.3\}, \{0.7, 0.6, 0.1\}, \{0.6, 0.8, 0.5\}, \{0.6, 0.4, 0.5\}) \\ (T_2, \{0.5, 0.8, 0.1\}, \{0.8,0.3,0.2\}, \{0.5,0.8,0.1\}, \{0.8,0.8,0.4\}) \end{Bmatrix}$$

Hence the NHSM $[S]$ is written by,

$$[S]_{2\times 4} = \begin{pmatrix} (0.8, 0.5, 0.3)(0.7, 0.6, 0.1)(0.6, 0.8, 0.5) & (0.6, 0.4, 0.5) \\ (0.5, 0.8, 0.1)(0.8, 0.3, 0.2)(0.5, 0.8, 0.1) & (0.8, 0.8, 0.4) \end{pmatrix}$$

Hence, we can observe that the membership value of $T_1$for $USB\ Port$ in both sets is $(0.7, 0.3, 0.5)$ and $(0.8, 0.5, 0.3)$ which satisfies the definition of Proper Neutrosophic Hypersoft Sub-matrix as $0.7 < 0.8, 0.3 < 0.5, 0.5 > 0.3$. This shows that $(0.7, 0.3, 0.5) \subset (0.8, 0.5, 0.3)$ and the same was the case with the rest of the attributes of NHSM $[R]$ and NHSM $[S]$.

Definiton 1.2.10. Let $R = [r_{ij}]$ and $S = [s_{ij}]$ Be two NHSMs with order $p \times \tau$, where $r_{ij} = (t^r_{ijk}, i^r_{ijk}, f^r_{ijk})$ and $s_{ij} = (t^s_{ijk}, i^s_{ijk}, f^s_{ijk})$. Then $[r_{ij}]$ and $[s_{ij}]$ are said to be disjoint, if $[r_{ij}] \cap [s_{ij}] = [0]$ for all i, j, k.

$[0]$ and $[1]$ denotes the Null-neutrosophic hypersoft matrix, $(t_{ijk}, i_{ijk}, f_{ijk}) = (0, 1, 1)$ and Universal neutrosophic hypersoft matrix, $(t_{ijk}, i_{ijk}, f_{ijk}) = (1, 0, 0)$.

Proposition 1.2.11. Let $R = [r_{ij}] \in$ NHSM with order $p \times \tau$, where $r_{ij} = (t^r_{ijk}, i^r_{ijk}, f^r_{ijk})$. Then,

1) $\left([r_{ij}]^c\right)^c = [r_{ij}]$
2) $[0]^c \neq [1]$

Proof: Since $R = [r_{ij}] \in$ NHSM, we have;

$$[r_{ij}] = [(t^r_{ijk}, i^r_{ijk}, f^r_{ijk})]$$

$$([r_{ij}]^c) = [(t^r_{ijk}, i^r_{ijk}, f^r_{ijk})]^c$$

$$= [(f^r_{ijk}, i^r_{ijk}, t^r_{ijk})]$$

$$([r_{ij}]^c)^c = [(f^r_{ijk}, i^r_{ijk}, t^r_{ijk})]^c$$

$$= [(t^r_{ijk}, i^r_{ijk}, f^r_{ijk})]$$

$$([r_{ij}]^c)^c = [r_{ij}]$$

Proposition 1.2.12. Let $R = [r_{ij}], S = [s_{ij}], L = [l_{ij}] \in$ NHSMs with order $p \times \tau$, where $r_{ij} = (t^r_{ijk}, i^r_{ijk}, f^r_{ijk}), s_{ij} = (t^s_{ijk}, i^s_{ijk}, f^s_{ijk}), l_{ij} = (t^l_{ijk}, i^l_{ijk}, f^l_{ijk})$. Then,

1) $[r_{ij}] \nsubseteq [1]$
2) $[0] \nsubseteq [r_{ij}]$
3) $[r_{ij}] \subseteq [r_{ij}]$
4) $[r_{ij}] \subseteq [s_{ij}]$ and $[s_{ij}] \subseteq [l_{ij}] \Rightarrow [r_{ij}] \subseteq [l_{ij}]$

Proof: The proof is straight forward.

Proposition 1.2.13. Let $R = [r_{ij}], S = [s_{ij}], L = [l_{ij}] \in$ NHSMs with order $p \times \tau$, where $r_{ij} = (t^r_{ijk}, i^r_{ijk}, f^r_{ijk}), s_{ij} = (t^s_{ijk}, i^s_{ijk}, f^s_{ijk}), l_{ij} = (t^l_{ijk}, i^l_{ijk}, f^l_{ijk})$. Then,

1) $[r_{ij}] = [s_{ij}]$ and $[s_{ij}] = [l_{ij}] \Leftrightarrow [r_{ij}] = [l_{ij}]$
2) $[r_{ij}] \subseteq [s_{ij}]$ and $[s_{ij}] \subseteq [r_{ij}] \Leftrightarrow [r_{ij}] = [s_{ij}]$

Proof: The proof is straight forward.

Definition 1.2.14. Extended Union of two NHSMs: Let $R = [r_{ij}]\ and\ S = [s_{ij}]$ be two NHSMs, where $r_{ij} = (t^r_{ijk}, i^r_{ijk}, f^r_{ijk}), s_{ij} = (t^s_{ijk}, i^s_{ijk}, f^s_{ijk})$,then their extended union is;

$$t([R \cup S]) = t([r_{ij}] \cup [s_{ij}]) = \begin{cases} t^r_{ijk} & if\ x \in R \\ t^s_{ijk} & if\ x \in S \\ \max\ (t^r_{ijk}, t^s_{ijk}) & if\ x \in R \cup S \end{cases}$$

$$i([R \cup S]) = i([r_{ij}] \cup [s_{ij}]) = \begin{cases} i^r_{ijk} & if\ x \in R \\ i^s_{ijk} & if\ x \in S \\ \dfrac{i^r_{ijk} + i^s_{ijk}}{2} & if\ x \in R \cup S \end{cases}$$

$$f([R \cup S]) = f([r_{ij}] \cup [s_{ij}]) = \begin{cases} f^r_{ijk} & if\ x \in R \\ f^s_{ijk} & if\ x \in S \\ \min(f^r_{ijk}, f^s_{ijk}) & if\ x \in R \cup S \end{cases}$$

Definition 1.2.15. Extended Intersection of two NHSMs: Let $R = [r_{ij}]\ and\ S = [s_{ij}]$ be two NHSMs, where $r_{ij} = (t^r_{ijk}, i^r_{ijk}, f^r_{ijk}),\ s_{ij} = (t^s_{ijk}, i^s_{ijk}, f^s_{ijk})$,then their extended intersection is;

$$t([R \cap S]) = t([r_{ij}] \cap [s_{ij}]) = \begin{cases} t^r_{ijk} & if \ \ x \ \in R \\ t^s_{ijk} & if \ \ x \ \in S \\ \min(t^r_{ijk}, t^s_{ijk}) & if \ \ x \ \in R \cap S \end{cases}$$

$$i([R \cap S]) = i([r_{ij}] \cap [s_{ij}]) = \begin{cases} i^r_{ijk} & if \ \ x \ \in R \\ i^s_{ijk} & if \ \ x \ \in S \\ \dfrac{i^r_{ijk} + i^s_{ijk}}{2} & if \ \ x \ \in R \cap S \end{cases}$$

$$f([R \cap S]) = f([r_{ij}] \cap [s_{ij}]) = \begin{cases} f^r_{ijk} & if \ \ x \ \in R \\ f^s_{ijk} & if \ \ x \ \in S \\ \max(f^r_{ijk}, f^s_{ijk}) & if \ \ x \ \in R \cap S \end{cases}$$

Proposition 1.2.16. Let $R = [r_{ij}]$ and $S = [s_{ij}]$be twoNHSMs with order $p \times \tau$, where $r_{ij} = (t^s_{ijk}, i^r_{ijk}, f^r_{ijk})$, $s_{ij} = (t^s_{ijk}, i^s_{ijk}, f^s_{ijk})$. Then,

1) $[r_{ij}] \cup [s_{ij}] = [r_{ij}]$
2) $[r_{ij}] \cup [0] \neq [r_{ij}]$
3) $[r_{ij}] \cup [1] \neq [1]$

Proof: Proof is straight forward by the definition 1.2.14.

Proposition 1.2.17. Let $R = [r_{ij}]$ and $S = [s_{ij}]$be twoNHSMs with order $p \times \tau$, where $r_{ij} = (t^r_{ijk}, i^r_{ijk}, f^r_{ijk})$, $s_{ij} = (t^s_{ijk}, i^s_{ijk}, f^s_{ijk})$. Then,

1) $[r_{ij}] \cap [r_{ij}] = [r_{ij}]$
2) $[r_{ij}] \cap [0] \neq [0]$
3) $[r_{ij}] \cap [1] \neq [r_{ij}]$

Proof: Proof is straight forward from the above definition.

1.3 | Operations on NHSM

This section presents some operations on NHSM such as addition, subtraction, OR-product and AND-product along with their properties.

Definition 1.3.18. Two NHSMs, $R = [r_{ij}]$, $S = [s_{ij}]$with order $p \times \tau$, where $r_{ij} = (t^r_{ijk}, i^r_{ijk}, f^r_{ijk})$ and $s_{ij} = (t^s_{ijk}, i^s_{ijk}, f^s_{ijk})$ are said to be conformable for addition, in case they have the same order. The addition of two NHSMs $[r_{ij}]$ and $[s_{ij}]$ is given by $[l_{ij}] = [r_{ij}] \oplus [s_{ij}]$, where $[l_{ij}]$ is also the NHSM of order $p \times \tau$ and

$$l_{ij} \ = (max(t^r_{ijk}, t^s_{ijk}), avg(i^r_{ijk}, i^s_{ijk}), \min(f^r_{ijk}, f^s_{ijk}))$$

Definition 1.3.19. Two NHSMs, $R = [r_{ij}]$, $S = [s_{ij}]$ with order $p \times \tau$, where $r_{ij} = (t^r_{ijk}, i^r_{ijk}, f^r_{ijk})$ and $s_{ij} = (t^s_{ijk}, i^s_{ijk}, f^s_{ijk})$ are said to be conformable for subtraction, in case they have the same order. The subtraction of two NHSMs $[r_{ij}]$ and $[s_{ij}]$ is given by $[l_{ij}] = [r_{ij}] \ominus [s_{ij}]$, where $[l_{ij}]$ is also the NHSM of order $p \times \tau$ and

$$[l_{ij} = (min(t^r_{ijk}, t^{s^c}_{ijk}), avg(i^r_{ijk}, i^{s^c}_{ijk}), \max(f^r_{ijk}, f^{s^c}_{ijk}))$$

where $t^{s^c}_{ijk}, i^{s^c}_{ijk}$ and $f^{s^c}_{ijk}$ denotes the complement of t^s_{ijk}, i^s_{ijk} and f^s_{ijk} respectively.

Example 1.3.20. Consider two NHSMs $[r_{ij}]$ and $[s_{ij}]$ which are given by;

$$[r_{ij}]_{4\times 4} = \begin{pmatrix} (0.6, 0.5, 0.3)(0.2, 0.5, 0.1)(0.8, 0.1\ 0.7) \\ (0.2, 0.8, 0.7)(0.4, 0.8, 0.6)(0.7, 0.3, 0.1) \\ (0.7, 0.4, 0.2)(0.3, 0.4, 0.6)(0.5, 0.2, 0.8) \end{pmatrix}$$

$$[s_{ij}]_{4\times 4} = \begin{pmatrix} (0.7, 0.3, 0.1)(0.8, 0.2, 0.6)(0.5, 0.3, 0.6) \\ (0.8, 0.5, 0.3)(0.5, 0.2, 0.7)(0.6, 0.1, 0.5) \\ (0.5, 0.2, 0.8)(0.2, 0.8, 0.1)(0.3, 0.4, 0.7) \end{pmatrix}$$

Then,

$$[r_{ij}] \oplus [s_{ij}] = \begin{pmatrix} (0.7, 0.4, 0.1)(0.8, 0.35, 0.1)(0.8, 0.2, 0.6) \\ (0.8, 0.65, 0.3)(0.5, 0.5, 0.6)(0.7, 0.2, 0.1) \\ (0.7, 0.3, 0.2)(0.3, 0.6, 0.1)(0.5, 0.3, 0.7) \end{pmatrix}$$

$$[s_{ij}]^c = \begin{pmatrix} (0.1, 0.3, 0.7)(0.6, 0.2, 0.8)(0.6, 0.3, 0.5) \\ (0.3, 0.5, 0.8)(0.7, 0.2, 0.5)(0.5, 0.1, 0.6) \\ (0.8, 0.2, 0.5)(0.1, 0.8, 0.2)(0.7, 0.4, 0.3) \end{pmatrix}$$

$$[r_{ij}] \ominus [s_{ij}] = \begin{pmatrix} (0.1, 0.4, 0.7)(0.2, 0.35, 0.8)(0.6, 0.2, 0.7) \\ (0.2, 0.65, 0.8)(0.4, 0.5, 0.6)(0.5, 0.2, 0.6) \\ (0.7, 0.3, 0.5)(0.1, 0.6, 0.6)(0.5, 0.3, 0.8) \end{pmatrix}$$

Proposition 1.3.21. Let R, S and L bethree NHSMs with order $p \times \tau$. Then,

1) $R \oplus S = S \oplus R$
2) $(R \oplus S) \oplus L \neq R \oplus (S \oplus L)$
3) $R \ominus S \neq S \ominus R$
4) $(R \ominus S) \ominus L \neq R \ominus (S \ominus L)$
5) $R \ominus R \neq \emptyset$

Proof: The proof is straight forward.

Theorem 1.3.22. If $R$is asquare NHSM with order $p \times p$, then $(R^t)^t = R$.

Proof: Since $R \in NHSM_{p\times p}$, then R^t and $(R^t)^t$ also $\in NHSM_{p\times p}$. Now,

$$R^t = \left[\left(t^r_{ijk}, i^r_{ijk}, f^r_{ijk}\right)\right]^t = \left[\left(t^r_{jki}, i^r_{jki}, f^r_{jki}\right)\right]$$

$$(R^t)^t = \left[\left(t^r_{jki}, i^r_{jki}, f^r_{jki}\right)\right]^t = \left[\left(t^r_{ijk}, i^r_{ijk}, f^r_{ijk}\right)\right] = R$$

Hence $(R^t)^t = R$.

Theorem 1.3.23. If R and S are two square NHSMs with order $p \times p$, then $(R \oplus S)^t = R^t \oplus S^t$.

Proof: Let $R = [r_{ij}],\ S = [s_{ij}]$ be two NHSMs of order $p \times p$. Then,

$$L.H.S = (R \oplus S)^t = \left([r_{ij}] \oplus [s_{ij}]\right)^t = Q, \text{ where } Q = [q_{ij}]$$

According to Definition 1.2.18 we have,

$$[r_{ij}] \oplus [s_{ij}] = \left[\left(max(t^r_{jki}, t^s_{jki}), avg\left(i^r_{jki}, i^s_{jki}\right), \min\left(f^r_{jki}, f^s_{jki}\right)\right)\right]$$

$$R.H.S = R^t \oplus S^t = [r_{ij}]^t \oplus [s_{ij}]^t = [r_{ji}] \oplus [s_{ji}]$$

According to Definiton 1.3.18 we have,

$[r_{ji}] \oplus [s_{ji}] = [(max(t^r_{jki}, t^s_{jki}), avg(i^r_{jki}, i^s_{jki}), \min(f^r_{jki}, f^s_{jki}))]$

Hence $(R \oplus S)^t = R^t \oplus S^t$.

Theorem 1.3.24. If R is a square NHSM with order $p \times p$, then $(R \oplus R^t)$ is symmetric.

Proof: Let $R = [r_{ij}]$. Then, $R^t = [r_{ij}]^t = [r_{ji}] = [(t^r_{jki}, i^r_{jki}, f^r_{jki})]$.

Now, $(R \oplus R^t) = [r_{ij}] \oplus [r_{ji}] = [l_{ij}]$.

where, $[l_{ij}] = [(\max(t^r_{ijk}, t^r_{jki}), avg(i^r_{ijk}, i^r_{jki}), \min(f^r_{ijk}, f^r_{jki}))]$.

Now, $[l_{ji}] = [l_{ij}]^t = [(\max(t^r_{jki}, t^r_{ijk}), avg(i^r_{jki}, i^r_{ijk}), \min(f^r_{jki}, f^r_{ijk}))] = [l_{ij}]$.

Thus $(R \oplus R^t)$ is symmetric.

Theorem 1.2.25. If R and S are two square NHSMs with order $p \times p$ and if R and S are symmetric, then $R \oplus S$ is symmetric.

Proof: Since R and S are symmetric,

$R^t = R$ and $S^t = S$. Therefore $R^t \oplus S^t = R \oplus S$.

By theorem 1.3.23 we have, $(R \oplus S)^t = R^t \oplus S^t = R \oplus S$.

Hence $R \oplus S$ is symmetric.

Definition 1.3.26. Let $R = [r_{ij}], N = [n_{ij}]$ be two NHSMs with order $p \times \tau$, where $[r_{ij]} = (t^r_{ijk}, i^r_{ijk}, f^r_{ijk})$, $[n_{il]} = (t^n_{ilk}, i^n_{ilk}, f^n_{ilk})$. Then AND-product of $[r_{ij}]$ and $[n_{ij}]$ is defined by;

$$\wedge: NHSM_{p\times\tau} \times NHSM_{p\times\tau} \to NHSM_{p\times\tau^2}$$

$[r_{ij}]\wedge[n_{il}] = [g_{ip}]$, where $[g_{ip}] = (t^g_{ipk}, i^g_{ipk}, f^g_{ipk})$

where, $t^g_{ipk} = t(t^r_{ijk}, t^n_{ilk})$, $i^g_{ipk} = s(i^r_{ijk}, i^n_{ilk})$, $f^g_{ipk} = s(f^r_{ijk}, f^n_{ilk})$, such that $p = \beta(j-1) + l$.

Definition 1.3.27. Let $R = [r_{ij}], N = [n_{ij}]$ be two NHSMs with order $p \times \tau$, where $[r_{ij}] = (t^r_{ijk}, i^r_{ijk}, f^r_{ijk})$, $[n_{il}] = (t^n_{ilk}, i^n_{ilk}, f^n_{ilk})$. Then OR-product of $[r_{ij}]$ and $[n_{ij}]$ is defined by;

$$\vee: NHSM_{p\times\tau} \times NHSM_{p\times\tau} \to NHSM_{p\times\tau^2}$$

$[r_{ij}]\vee[n_{il}] = [g_{ip}]$, where $[g_{ip}] = (t^g_{ipk}, i^g_{ipk}, f^g_{ipk})$

where, $t^g_{ipk} = s(t^r_{ijk}, t^n_{ilk})$, $i^g_{ipk} = t(i^r_{ijk}, i^n_{ilk})$, $f^g_{ipk} = t(f^r_{ijk}, f^n_{ilk})$, such that $p = \beta(j-1) + l$.

Example 1.3.28. Two NHSMs $[r_{ij}]$ and $[n_{ij}]$ are given by;

$$[r_{ij}]_{2\times2} = \begin{pmatrix} (0.6, 0.5, 0.3) & (0.2, 0.5, 0.1) \\ (0.2, 0.8, 0.7) & (0.4, 0.8, 0.6) \end{pmatrix}$$

$$[n_{ij}]_{2\times2} = \begin{pmatrix} (0.7, 0.3, 0.1) & (0.8, 0.2, 0.6) \\ (0.8, 0.5, 0.3) & (0.5, 0.2, 0.7) \end{pmatrix}$$

Then,

$$[r_{ij}]\vee[n_{ij}]_{2\times4} = \begin{pmatrix} (0.7, 0.3, 0.1) & (0.8, 0.2, 0.3) & (0.7, 0.3, 0.1) & (0.8, 0.2, 0.1) \\ (0.8, 0.5, 0.3) & (0.5, 0.2, 0.7) & (0.8, 0.5, 0.3) & (0.5, 0.2, 0.6) \end{pmatrix}$$

$$[r_{ij}]\wedge[n_{ij}]_{2\times4} = \begin{pmatrix} (0.6, 0.5\ 0.3) & (0.6, 0.5, 0.6) & (0.2, 0.5, 0.1) & (0.2, 0.5, 0.6) \\ (0.2, 0.8, 0.7) & (0.2, 0.8, 0.7) & (0.4, 0.8, 0.6) & (0.4, 0.8, 0.7) \end{pmatrix}$$

Proposition 1.3.29. Let $R = [r_{ij}], N = [n_{ij}]$ be two NHSMs with order $p \times \text{т}$, where $[r_{ij}] = (t^r_{ijk}, i^r_{ijk}, f^r_{ijk})$, $[n_{il}] = (t^n_{ilk}, i^n_{ilk}, f^n_{ilk})$, then the De-morgan's law holds;

1) $([r_{ij}] \wedge [n_{il}])^c = [r_{ij}]^c \vee [n_{il}]^c$
2) $([r_{ij}] \vee [n_{il}])^c = [r_{ij}]^c \wedge [n_{il}]^c$

Proof: 1) From Definition 1.3.26 we have,

$$[r_{ij}] \wedge [n_{il}] = \left[\left(t(t^r_{ijk}, t^n_{ilk}), s(i^r_{ijk}, i^n_{ilk}), s(f^r_{ijk}, f^n_{ilk})\right)\right]$$

$$([r_{ij}] \wedge [n_{il}])^c = \left[\left(t(t^r_{ijk}, t^n_{ilk}), s(i^r_{ijk}, i^n_{ilk}), s(f^r_{ijk}, f^n_{ilk})\right)\right]^c$$

$$= \left[\left(s(f^r_{ijk}, f^n_{ilk}), t(i^r_{ijk}, i^n_{ilk}), t(t^r_{ijk}, t^n_{ilk})\right)\right]$$

$$= [(f^r_{ijk}, i^r_{ijk}, t^r_{ijk}) \vee (f^n_{ilk}, i^n_{ilk}, t^n_{ilk})]$$

$$([r_{ij}] \wedge [n_{il}])^c = [r_{ij}]^c \vee [n_{il}]^c$$

2) From Definition 1.3.27 we have,

$$[r_{ij}] \vee [n_{il}] = \left[\left(s(t^r_{ijk}, t^n_{ilk}), t(i^r_{ijk}, i^n_{ilk}), t(f^r_{ijk}, f^n_{ilk})\right)\right]$$

$$([r_{ij}] \vee [n_{il}])^c = \left[\left(s(t^r_{ijk}, t^n_{ilk}), t(i^r_{ijk}, i^n_{ilk}), t(f^r_{ijk}, f^n_{ilk})\right)\right]^c$$

$$= \left[\left(t(f^r_{ijk}, f^n_{ilk}), s(i^r_{ijk}, i^n_{ilk}), s(t^r_{ijk}, t^n_{ilk})\right)\right]$$

$$= [(f^r_{ijk}, i^r_{ijk}, t^r_{ijk}) \wedge (f^n_{ilk}, i^n_{ilk}, t^n_{ilk})]$$

$$([r_{ij}] \vee [n_{il}])^c = [r_{ij}]^c \wedge [n_{il}]^c$$

1.4 | An Application of NHSM in Decision-Making

Based on some of these matrix operations an efficient methodology named NHSM-algorithm can be developed to solve Neutrosophic hypersoft-based decision-making problems.

Definition 1.4.1 Let $K = [k_{ij}]_{p \times \text{т}}$ be a Neutrosophic hypersoft matrix where $k_{ij} = (\eta^k_{ijk}, \upsilon^k_{ijk}, \varphi^k_{ijk})$.

1) If the indeterminacy membership degree lies in favor of the truth-membership degree then the Grace matrix of the matrix K which is symbolized by $G(K)$ and is defined as $G(K) = [g^k_{ij}]_{p \times \text{т}}$, where $g^k_{ij} = [\eta^k_{ijk} + \upsilon^k_{ijk} - \varphi^k_{ijk}], \forall i, j, k$.
2) The Mean matrix $M(K)$ for Value matrix $V(K)$ and Grace matrix $G(K)$ is defined as; $M(K) = \frac{V(K)+G(K)}{2}$.
3) (3) In the case of multi-observer, the Score matrix of two Mean matrices $M(K)$ and $M(L)$ is given by $S(O) = S(K, L) = [s_{ij}]_{p \times \text{т}}$, where $s_{ij} = m^k_{ij} + m^l_{ij}$, such that $S(K, L) = M(K) + M(L)$. Hence m^k_{ij} and m^l_{ij} Are entries in the Mean matrices $M(K)$ and $M(L)$ Respectively.
4) Hence, the Total score for each alternative in U is described as $\sum_{j=1}^{n} s_{ij}$, where s_{ij} Are entries in the Score matrix.

1.4.1 | Properties of Mean Function

The Grace matrix and Value matrix comply with all properties of the genuine matrix. Similarly, the Mean function also satisfies all properties of the genuine matrix, since it is obtained from the Grace matrix and Value matrix.

1.4.2 | Methodology

The decision-making needs to select the suitable alternative form given p number of alternatives. For this selection of appropriate alternatives, the attributes (τ) are selected by the decision-makers. In case, any one of the attributes has encouraged sub-attributes that frame a connection like NHSM, at that point decision-makers give their opinion to each alternative conjuring to the sub-attributes of the selected attributes within the structure of NHSMs. Hence, an NHSM with order $p \times \tau$ is obtained. From this NHSM we compute the Value matrix and Grace matrix. Then the Mean matrix, Score matrix, and finally the Total score of each object is calculated.

The proposed Algorithm for the above approach is ;

NHSM-algorithm

Input the Neutrosophic hypersoft set from the given situation based on the attributes selected.

Construct the NHSM based on 1.

From 2, calculate the Grace matrix and Value matrix.

Evaluate the Mean matrix.

From the Mean matrices, calculate the Score matrix.

Determine the Total score matrix from the result of 5.

The alternative with the maximum score value will be the optimal solution.

Suppose more than one alternative processes the maximum score value, then any one alternative can be chosen according to the decision maker.

Statement of the problem

A Cooperative bank wants to select an appropriate person for the position of Data entry clerk. Ten applications were received from the suitable candidates. Based, on the sub-attributes relation four candidates are shortlisted, and now the decision-making team interviews the 4 candidates and shares their opinion for each alternative. After this, the NHSM algorithm is applied to select the appropriate candidate for the post.

1.4.3 | Application in banking Sector

Example 1.4.2. Let U be the set of applicants who applied for the position of Data entry clerk in the Cooperative bank:

$$U = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}\}$$

The bank manager assists a team of decision-makers $\{K, L\}$ in the selection of the most appropriate candidate. Additionally, the Manager instructs the decision makers about the selection procedure of the alternative such as;

$$G = \{G_1 = Grade, G_2 = Typing\ speed, G_3 = Qualification, G_4 = Computer\ Knowledge\}$$

Moreover, the above-given attributes have advanced sub-attributes and can be classified as follows:

$$G_1^a = \{Junior, Senior, Steno\ typist\}$$

$$G_2^b = \{65\ WPM, 70\ WPM, 90\ WPM, 100\ WPM\}$$

$$G_3^c = \{12th, Diploma, Under - graduate, B.E\}$$

$$G_4^d = \{MS - Office, COA, C - language, Tally\}$$

Let the function be $R = G_1^a \times G_2^b \times G_3^c \times G_4^d \to P(U)$. The neutrosophic hypersoft set is defined as; $R: (G_1^a \times G_2^b \times G_3^c \times G_4^d) \to P(U)$.

The relation $R(G_1^a \times G_2^b \times G_3^c \times G_4^d) = R(Senior, 70\ WPM, Under - graduate, COA)$ is the actual requirement of the bank for appropriate candidate selection. On this basis, four candidates P_1, P_3, P_7 and P_9 are shortlisted. The decision-makers team $\{K, L\}$ interviews the shortlisted candidates and shares their opinion in the form of a Neutrosophic hypersoft set for each alternative as follows:

The Neutrosophic hypersoft set based on a decision of the group K is given by,

$$K = R(Senior, 70\ WPM, Under\ graduate, COA)$$

$$= \begin{Bmatrix} (P_1, \{0.6, 0.3, 0.5\}, \{0.5, 0.7, 0.8\}, \{0.8, 0.2, 0.1\}, \{0.4, 0.6, 0.7\}) \\ (P_3, \{0.3, 0.6, 0.2\}, \{0.2, 0.7, 0.6\}, \{0.1, 0.9, 0.3\}, \{0.7, 0.2, 0.3\}) \\ (P_7, \{0.7, 0.5, 0.3\}, \{0.8, 0.1, 0.3\}, \{0.5, 0.8, 0.2\}, \{0.6, 0.2, 0.3\}) \\ (P_9, \{0.5, 0.7, 0.2\}, \{0.3, 0.8, 0.7\}, \{0.7, 0.2, 0.8\}, \{0.8, 0.6, 0.1\}) \end{Bmatrix}$$

The Neutrosophic hypersoft matrix derived from the above Neutrosophic hypersoft set is,

$$K = \begin{pmatrix} (0.6, 0.3, 0.5) & (0.5, 0.7, 0.8) & (0.8, 0.2, 0.1) & (0.4, 0.6, 0.7) \\ (0.3, 0.6, 0.2) & (0.2, 0.7, 0.6) & (0.1, 0.9, 0.3) & (0.7, 0.2, 0.3) \\ (0.7, 0.5, 0.3) & (0.8, 0.1, 0.3) & (0.5, 0.8, 0.2) & (0.6, 0.2, 0.3) \\ (0.5, 0.7, 0.2) & (0.3, 0.8, 0.7) & (0.7, 0.2, 0.8) & (0.8, 0.6, 0.1) \end{pmatrix}$$

The Value matrix $V(K)$ is;

$$V(K) = \begin{pmatrix} -0.2 & -1.0 & 00.5 & -0.9 \\ -0.5 & -1.1 & -1.1 & 00.2 \\ -0.1 & 00.4 & -0.5 & 00.1 \\ -0.4 & -1.2 & -0.3 & 00.1 \end{pmatrix}$$

The Grace matrix $G(K)$ is;

$$G(K) = \begin{pmatrix} 0.4 & 0.4 & 0.9 & 0.3 \\ 0.7 & 0.3 & 0.7 & 0.6 \\ 0.9 & 0.6 & 1.1 & 0.5 \\ 1.0 & 0.4 & 0.1 & 1.3 \end{pmatrix}$$

The Mean matrix $M(K)$ is;

$$M(K) = \begin{pmatrix} 0.1 & -0.3 & 00.7 & -0.3 \\ 0.1 & -0.4 & -0.2 & 00.4 \\ 0.4 & 00.5 & 00.3 & 00.3 \\ 0.3 & -0.4 & -0.1 & 00.7 \end{pmatrix}$$

The Neutrosophic hypersoft set based on the decision of the group K is given by,

$$L = R(Senior, 70\ WPM, Under\ graduate, COA)$$

$$= \begin{Bmatrix} (P_1, \{0.8, 0.6, 0.3\}, \{0.8, 0.2, 0.5\}, \{0.4, 0.4, 0.7\}, \{0.3, 0.8, 0.2\}) \\ (P_3, \{0.4, 0.6, 0.8\}, \{0.6, 0.7, 0.3\}, \{0.8, 0.2, 0.1\}, \{0.3, 0.4, 0.6\}) \\ (P_7, \{0.8, 0.2, 0.7\}, \{0.4, 0.6, 0.9\}, \{0.9, 0.1, 0.2\}, \{0.3, 0.6, 0.1\}) \\ (P_9, \{0.6, 0.1, 0.4\}, \{0.6, 0.8, 0.2\}, \{0.7, 0.3, 0.1\}, \{0.1, 0.7, 0.6\}) \end{Bmatrix}$$

The Neutrosophic hypersoft matrix derived from the above Neutrosophic hypersoft set is,

$$L = \begin{pmatrix} (0.8,0.6,0.3) & (0.8,0.2,0.5) & (0.4,0.4,0.7) & (0.3,0.8,0.2) \\ (0.4,0.6,0.8) & (0.6,0.7,0.3) & (0.8,0.2,0.1) & (0.3,0.4,0.6) \\ (0.8,0.2,0.7) & (0.4,0.6,0.9) & (0.9,0.1,0.2) & (0.3,0.6,0.1) \\ (0.6,0.1,0.4) & (0.6,0.8,0.2) & (0.7,0.3,0.1) & (0.1,0.7,0.6) \end{pmatrix}$$

The Value matrix $V(L)$ is;

$$V(L) = \begin{pmatrix} -0.1 & 00.1 & -0.7 & -0.7 \\ -1.0 & -0.4 & 00.5 & -0.7 \\ -0.1 & -1.1 & 00.6 & -0.4 \\ 00.1 & -0.4 & 00.3 & -1.2 \end{pmatrix}$$

The Grace matrix $G(L)$ is;

$$G(L) = \begin{pmatrix} 1.1 & 0.5 & 0.1 & 0.9 \\ 0.2 & 1.0 & 0.9 & 0.1 \\ 0.3 & 0.1 & 0.8 & 0.8 \\ 0.3 & 1.2 & 0.9 & 0.2 \end{pmatrix}$$

The Mean matrix $M(L)$ is;

$$M(L) = \begin{pmatrix} 00.1 & 00.3 & -0.3 & 00.1 \\ -0.4 & 00.3 & 00.7 & -0.3 \\ 00.1 & -0.5 & 00.7 & 00.2 \\ 00.2 & 00.4 & 00.6 & -0.5 \end{pmatrix}$$

The Score matrix $S(O) = S(K, L) = [s_{ij}]_{p\times\mathrm{T}} = M(K) + M(L)$ is,

$$S(O) = \begin{pmatrix} 00.6 & 00.0 & 00.4 & -0.2 \\ -0.3 & -0.1 & 00.5 & 00.1 \\ 00.5 & 00.0 & 01.0 & 00.5 \\ 00.5 & 00.0 & 00.5 & 00.2 \end{pmatrix}$$

The Total score matrix $T(O) = \sum_{j=1}^{n} s_{ij}$ is,

$$T(O) = \begin{pmatrix} 0.8 \\ 0.2 \\ 2.0 \\ 1.2 \end{pmatrix}$$

From the above Total score matrix, the maximum score value is, $\max_{1\le i\le 4}\{s_{ij}\} = P_7 = 2.$ Therefore, according to the NHSM algorithm, the candidate P_7 will be selected for the position of Data entry clerk by the manager of a cooperative bank.

1.4.4 | Comparative Analysis

This section will compare the currently developed Neutrosophic Hypersoft matrix theory with the existing theories. Further, the comparative analysis will examine the efficiency, clarity, and tractability of the currently developed Neutrosophic hyper-soft matrix theory along with its advantages. Table 2 introduces the comparison between the Neutrosophic Hypersoft matrix and some existing methods. Here, P denotes parameterization, and A denotes Attributes.

Table 2. Comparison between the Neutrosophic Hypersoft matrix and some existing methods.

Author	Matrix	Truth	Indeterminacy	Falsity	P	A	Sub-Attribute
Thomason et al. [12]	FM	×	×	×	×	Y	×
Pal et al. [13]	IFM	Y	×	Y	×	Y	×
Naim Cagman et al.[23]	SM	Y	×	×	Y	Y	×
Yong Yang et al. [24]	FSM	Y	×	×	Y	Y	×
Rajarajeswari et al. [25]	IFSM	Y	×	Y	Y	Y	×
Abshishek Guleria et al. [26]	PFSM	Y	×	Y	Y	Y	×
Irfan Deli et al. [30]	NSM	Y	Y	Y	Y	Y	×
Currently developed method	NHSM	Y	Y	Y	Y	Y	Y

1.4.5 | Discussion

It may be inferred from the current investigation and comparisons that the findings obtained by the suggested methodology are more flexible when compared to the available approaches. The fundamental advantage of the suggested method is that it includes more information and addresses data uncertainty by taking into account the membership, non-membership, and indeterminacy of sub-attributes. It is also a helpful tool for the decision-making process when dealing with faulty and imprecise data. In all the existing matrix theories except the Neutrosophic hypersoft matrix the motivation for the score value assigned to one parameter will not impact the other values. This results in more information loss. On the contrary, the suggested technique does not result in any significant information loss. The advantage of the proposed approach over existing methods is that it not only detects the level of discrimination but also the level of similarity between observations, preventing choices from being made for unfavorable reasons. As a result, it is also an appropriate technique for drawing the right conclusions in Decision-making problems even though the information is uncertain.

1.4.6 | Limitation

The present methodology cannot deal with the situation when the decisions are provided in interval form. For which Interval-valued neutrosophic hypersoft set and Interval-valued neutrosophic hypersoft expert set can be developed to deal with such situations.

2 | Conclusion

In this paper, some notions based on NHSM have been presented and a few operations on them such as addition, subtraction, OR-product, and AND-product have been examined along with examples. Furthermore, an application of NHSM in a multi-attributed decision-making issue has been proposed. Decision-making problems based on NHSM in the field of medicine, the economy of a country, etc., can be solved using the proposed algorithm. Further, this study can be extended to the digital field.

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Author Contributions

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Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Data Availability

There was no data used in the inquiry that was as stated in the article.

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