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A Short Note for Hypersoft Rough Graphs

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Abstract

Graph theory, a branch of mathematics, explores relationships among entities using vertices and edges. To address the uncertainties inherent in real-world networks, Uncertain Graph Theory has emerged as a significant extension within this field. Hypersoft Graphs advance traditional graphs further by incorporating multi-attribute nodes, allowing each node to represent multiple attribute values, thus capturing more complex, multi-dimensional relationships. The Soft Rough Set and Soft Rough Graph combine the principles of Soft Sets and Rough Sets to manage uncertainty in a more refined manner. In this paper, we introduce the Hypersoft Rough Graph and analyze its relationships with other graph classes.

Keywords: Hypersoft Rough Graph; Rough Graph; Soft Graph; Hypersoft Graph; Soft Set.

1 | Introduction

1.1 | Soft Graph and Rough Graph Classes

Graph theory, a fundamental branch of mathematics, models relationships within networks using vertices (nodes) and edges to represent connections and interactions among entities such as individuals, locations, or organizations. Since its inception in the 1700s, graph theory has evolved significantly, yielding profound theoretical advancements and a wide range of applications [31, 32, 47, 153, 162, 236]. In recent years, the concept of graphs has also found broad application in artificial intelligence, particularly in areas like Graph Neural Networks, further demonstrating its relevance [22, 35, 49, 83, 134, 135, 182, 216, 218, 237].

To address real-world uncertainty, mathematical frameworks such as Fuzzy Sets [224 229, 231], Soft Sets [141], Rough Sets [158], and Neutrosophic Sets [188-191, 203, 205] provide essential tools for analyzing ambiguous or imprecise information. This paper examines several models of uncertain graphs, which hold promise in a variety of real-world applications, inspiring diverse graph classes [55-59, 61-71, 73, 73-82].

A Soft Set [15, 16, 137, 219], or Soft Graph [7, 9, 110], offers a flexible framework for dealing with uncertainty by linking elements to specific parameters, thereby supporting adaptable decision-making processes. Extensive research has been conducted on soft graphs, soft sets, and their various extensions [8, 25, 26, 28, 34, 41, 48, 93, 110, 114, 123, 126, 140, 150, 184-186, 213].

Similarly, Rough Sets [101, 157-160] and Rough Graphs [37, 43, 111, 150, 212] manage uncertainty by using lower and upper boundaries to approximate imprecise data.

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Hypersoft Graphs build on traditional graphs by incorporating multi-attribute nodes, enabling each node to represent multiple attribute values and thereby capture more complex, multi-dimensional relationships [171, 175-178]. Conceptually, this development aligns with Hypersoft Sets [1, 40, 138, 144, 145, 172, 181, 192, 223].

The Soft Rough Set and Soft Rough Graph [5, 6, 13, 18, 42, 52, 53, 125, 128, 151, 183, 214, 222, 235] combine principles from Soft Sets and Rough Sets, providing nuanced approaches to uncertainty. Similarly, Hypersoft Rough Sets integrates concepts from Hypersoft Sets and Rough Sets, extending their applicability to uncertain data analysis [124, 169, 208, 209].

1.2 | Contributions

As previously mentioned, while research on Soft Expert Graphs and related fields is advancing, studies on Hypersoft Expert Graphs remain scarce. In this paper, we define the Hypersoft Rough Graph and explore its relationships with other graph classes. In the Future Tasks section, we discuss concepts related to Hypersets, Superhypersets, and N-soft sets. These efforts are anticipated to further the study of Soft Graph Theory and Rough Graph Theory.

2 | Preliminaries and Definitions

This section offers an overview of the fundamental definitions and notations used throughout the paper. Additionally, some foundational concepts from set theory are applied in parts of this work. For further details, please consult relevant references as needed [54, 109, 113, 116, 127].

2.1 | Basic Graph Concepts

This section provides a concise overview of essential concepts in graph theory. For a more detailed study and additional notational conventions, see [45-47, 102, 215].

Definition 1 (Graph). [47] A graph G is a mathematical structure used to model pairwise relations between objects. It is composed of a set of vertices $V(G)$ and a set of edges $E(G)$, where each edge represents a connection between two vertices. Formally, a graph is denoted by $G = (V, E)$, with V as the set of vertices and E as the set of edges.

Definition 2 (Vertex Degree). [47] For a graph $G = (V, E)$, the degree of a vertex $v \in V$, denoted $\deg(v)$, is defined as the number of edges incident to v. For an undirected graph, this is expressed as:

$$\deg(v) = |\{e \in E \mid v \in e\}|$$

This measure reflects the connectivity of v within the graph.

2.2 | Soft Graph

A soft set over a universe U associates subsets of U to parameters from a parameter set A, capturing flexible relationships in uncertain data. A soft graph is a soft set over the vertices V of a graph G where each subset forms a connected subgraph, capturing parameter-based connections within G. The definitions are provided as follows.

Definition 3. [141] Let U be a non-empty finite set, called the universe of discourse, and let E be a non-empty set of parameters. A soft set over U is defined as follows:

$$F = (F, A) \text{ over } U \text{ is an ordered pair, where } A \subseteq E \text{ and } F: A \to P(U)$$

where $F(a) \subseteq U$ for each $a \in A$ and $P(U)$ denotes the power set of U. The set of all soft sets over U is denoted by $S(U)$.

1. Soft Subset: Let $F = (F, A)$ and $G = (G, B)$ be two soft sets over the common universe U. We say that F is a soft subset of G, denoted $F \subseteq G$, if:

- $A \subseteq B$
- $F(a) \subseteq G(a)$ for all $a \in A$.

2. Union of Soft Sets: The union of two soft sets $F = (F, A)$ and $G = (G, B)$ over U is defined as $H = (H, C)$ where $C = A \cup B$ and

$$H(e) = \begin{cases} F(e), & e \in A - B \\ G(e), & e \in B - A \\ F(e) \cup G(e), & e \in A \cap B \end{cases}$$

3. The intersection of Soft Sets: The intersection of two soft sets $F = (F, A)$ and $G = (G, B)$ with disjoint parameter sets $A \cap B = \emptyset$ is defined as $H = (H, C)$, where $C = A \cap B$ and

$$H(e) = F(e) \cap G(e), \forall e \in C$$

Definition 4. [7, 110] Let $G = (V, E)$ be a simple graph, where V is the set of vertices and E is the set of edges. Let A be a non-empty set of parameters and $R \subseteq A \times V$ be a relation between elements of A and elements of V. Define a set-valued function $F: A \to P(V)$ by

$$F(x) = \{y \in V \mid xRy\}$$

The pair (F, A) is a soft set over V.

A Soft Graph of G is defined as follows:

A soft set (F, A) over V is said to be a soft graph of G if the subgraph $F(x)$ is a connected subgraph of G for all $x \in A$. The set of all soft graphs of G is denoted by $SG(G)$.

2.3 | Hypersoft Graph

A HyperSoft Graph represents multi-attribute nodes where each node can hold distinct attribute values, enabling complex, multi-dimensional relationships. The definition of a Hypersoft Graph is provided as follows [171, 175-178].

Definition 5 (Hypersoft Set). [192] Let X be a non-empty finite universe, and let $T_1, T_2, \ldots, T_n$ be n-distinct attributes with corresponding disjoint sets $J_1, J_2, \ldots, J_n$. A pair (F, J) is called a hypersoft set over the universal set X, where F is a mapping defined by

$$F: J \to \mathcal{P}(X)$$

with $J = J_1 \times J_2 \times \cdots \times J_n$.

Definition 6 (Hypersoft Graph). Let $G = (V, E)$ be a simple connected graph, where V is the set of vertices and E is the set of edges. Consider $J = J_1 \times J_2 \times \cdots \times J_n$, where each $J_i \subseteq V$ and $J_i \cap J_j = \emptyset$ for $i \neq j$. A Hypersoft Graph (HS-Graph) of G is defined as a hypersoft set (F, J) over V such that for each $x \in J, F(x)$ induces a connected subgraph of G. The set of all HS-Graphs of G is denoted by $\mathrm{HsG}(G)$.

Proposition 7. Any Hypersoft Graph can be transformed into a Soft Graph and subsequently into a Classical Graph.

Proof. Let $G = (V, E)$ be a Hypersoft Graph defined with $J = J_1 \times J_2 \times \cdots \times J_n$ and a hypersoft set $F: J \to \mathcal{P}(V)$. For each attribute tuple $x = (j_1, j_2, \ldots, j_n) \in J$, the mapping $F(x) \subseteq V$ defines subsets of V that induce connected subgraphs of G. To transform this into a Soft Graph, we redefine the parameter set $A = J_1 \cup J_2 \cup \cdots \cup J_n$ by flattening the multi-attribute parameters into a single layer of parameters. For each $a \in A$, define $F'(a)$ as the union of all $F(x)$ where $x \in J$ includes$a$ as one of its components. This mapping $F': A \to \mathcal{P}(V)$ now defines a Soft Graph, where each subset $F'(a)$ still induces connected subgraphs of G

based on parameter a. Thus, the Hypersoft Graph (F, J) has been transformed into a Soft Graph (F', A) over G.

And let $G = (V, E)$ be the Soft Graph obtained from the previous transformation with parameter set A and mapping. $F': A \to \mathcal{P}(V)$. To reduce this to a Classical Graph, we ignore the parameterization by A and consider only the underlying vertex set V and edge set E. Define a Classical Graph $G' = (V, E')$, where $E' \subseteq E$ consists of edges connecting vertices within any subset $F'(a), a \in A$, such that each $F'(a)$ maintains connectivity among its vertices. Since $F'(a)$ induces connected subgraphs in the Soft Graph, the resultant graph G' will be a connected graph without parameterization, representing the underlying structure of the Soft Graph.

By this two-step process, a Hypersoft Graph can be systematically reduced to a Soft Graph, which can then be further reduced to a Classical Graph by eliminating parameters.

2.4 | Rough Set and Rough Graph

Rough Set provides a mathematical tool to handle uncertain information by approximating the set of elements. Rough Graph extends Rough Set Theory to graphs, where uncertainty in relationships (edges) is represented through lower and upper approximations. Rough Graphs are especially useful in knowledge mining, social networks, and systems where connections among entities are partially known or inherently uncertain.

Definition 8 (Partition of Sets). Let U be a non-empty set. A partition of U is a collection of non-empty, pairwise disjoint subsets. $\{P_i\}_{i \in I}$ Such that:

- $P_i \subseteq U$ for each $i \in I$,
- $P_i \cap P_j = \emptyset$ for all $i \neq j$,
- $\bigcup_{i \in I} P_i = U$.

Each subset P_i is called a block or cell of the partition, and the elements within each block are equivalence-related under some equivalence relation R.

Definition 9 (Rough Set). [101, 157-160] Let U be a universe of discourse and R a relation on U that induces a partition of U into equivalence classes. For a subset $X \subseteq U$, the lower approximation of Xconcerning R (denoted $R(X)$) is the set of elements that are certainly in X given the information provided by R :

$$R(X) = \{x \in U \mid [x]_R \subseteq X\}$$

The upper approximation of X (denoted (X)) is the set of elements that possibly belong to X :

$$R(X) = \{x \in U \mid [x]_R \cap X \neq \emptyset\}.$$

The pair $(R(X), R(X))$ is called the Rough Set approximation of X with respect to R. **Definition 10** (Rough Graph). [37, 43, 111, 150, 212] Let $G = (V, E)$ be a graph where V is the set of vertices and E is the set of edges. Let R be an attribute set on E, inducing an equivalence relation on the edges. For any edge set $X \subseteq E$, the lower approximation of X with respect to R (denoted $R(X)$) is defined as:

$$R(X) = \{e \in E \mid [e]_R \subseteq X\}$$

where $[e]_R$ Denotes the equivalence class of e under R. The upper approximation of $X($ denoted $R(X))$ is defined as:

$$R(X) = \{e \in E \mid [e]_R \cap X \neq \emptyset\}.$$

A graph $G = (V, E)$ is called an R-rough graph if X is not exactly definable under R, and it is characterized by the pair $(R(X), R(X))$, where $R(X)$ is the lower approximation graph and $R(X)$ is the upper approximation graph.

2.5 | Soft Rough Set and Soft Rough Graph

A soft rough set extends soft sets by using lower and upper approximations over a universe U, capturing uncertainty within the set [53]. A soft rough graph combines soft rough set concepts over a graph's vertices and edges, forming subgraphs to represent uncertain or approximate connections. The definitions are provided as follows.

Definition 11. [53] Consider a soft approximation space $S = (U, F)$, where:

- U represents the universe of discourse.
- $F: A \to P(U)$ is a set-valued mapping from a set $A \subseteq P$, where P is the set of all parameters.

A soft rough set over a subset $X \subseteq U$ has both lower and upper approximations defined as follows:

$$F_*(X) = \{u \in U \mid \exists a \in A \text{ such that } u \in F(a) \subseteq X\}$$
$$F^*(X) = \{u \in U \mid \exists a \in A \text{ such that } u \in F(a) \text{ and } F(a) \cap X \neq \emptyset\}$$

If $F_*(X) = F^*(X)$, then X is said to be softly definable; otherwise, X is a soft rough set, denoted by $(F_*(X), F^*(X), A)$.

Definition 12. [151] Let $G = (V, E)$ be a simple graph where V is the set of vertices and E is the set of edges. Suppose $S = (U, F)$ is a soft approximation space, and $A \subseteq P$ is a non-empty set of parameters. Define $(F_*(X), F^*(X), A)$ as a soft rough set over V, and $(K_*(X), K^*(X), A)$ as a soft rough set over E, with:

$$K_*(X) = \{e \in E \mid \exists a \in A \text{ such that } e \in K(a) \subseteq X\}$$
$$K^*(X) = \{e \in E \mid \exists a \in A \text{ such that } e \in K(a) \text{ and } K(a) \cap X \neq \emptyset\}$$

Here, $K: A \to P(E)$ maps each parameter $a \in A$ to subsets of E based on adjacency relations. A graph $\tilde{G} = (G, F_*, K_*, F^*, K^*, A, X)$ is called a soft rough graph if it satisfies:

1. $G = (V, E)$ is a simple graph.
2. A is a non-empty set of parameters.
3. $X \subseteq U$ is a non-empty subset.
4. $(F_*(X), F^*(X), A)$ is a soft rough set over V.
5. $(K_*(X), K^*(X), A)$ is a soft rough set over E.
6. $H_*(X) = (F_*(X), K_*(X))$ and $H^*(X) = (F^*(X), K^*(X))$ are subgraphs of G.

A soft rough graph can be represented by

$$\tilde{G} = \langle F_*, K_*, F^*, K^*, A, X \rangle = \{H_*(X), H^*(X)\}$$

The collection of all soft rough graphs of G is denoted by $\mathrm{SRG(G)}$.

Proposition 13. Any Soft Rough Graph can be transformed into a Rough Graph and a Soft Graph. **Proof.** To prove this theorem, we demonstrate the transformations of a Soft Rough Graph into a Rough Graph and then into a Soft Graph.

To transform a Soft Rough Graph $\tilde{G} = \langle F_*, K_*, F^*, K^*, A, X \rangle$ Into a Rough Graph, we ignore the parameter set A while retaining the approximation sets $F_*(X)$ and $F^*(X)$ over vertices, and $K_*(X)$ and $K^*(X)$ over edges. The rough graph thus formed retains the inherent approximations in vertex and edge connectivity without explicit parameter dependence. Specifically, the lower approximation graph $H_*(X) = (F_*(X), K_*(X))$ and the upper approximation graph $H^*(X) = (F^*(X), K^*(X))$ Serve as the representations of uncertainty through rough approximations, and this transformed graph is a Rough Graph over G.

Next, we consider transforming the Soft Rough Graph into a Soft Graph. We redefine the parameterization by considering only one approximation level at a time. For instance, we treat either the lower or the upper approximation set $F_*(X)$ or $F^*(X)$ over vertices, and $K_*(X)$ or $K^*(X)$ over edges, as separate soft sets $F: A \to P(V)$ and $K: A \to P(E)$, while associating each approximation level with a distinct parameter. This enables the conversion of the uncertainty in connectivity into a parameterized soft graph structure. By this construction, the approximations of vertices and edges in the Soft Rough Graph are reinterpreted within a flexible parameterized structure, thus forming a Soft Graph over G without explicitly relying on rough approximations.

Consequently, we conclude that any Soft Rough Graph $\tilde{G}$ Can be effectively transformed into both a Rough Graph and a Soft Graph by reinterpreting its approximation components and parameter dependencies.

2.6 | Hypersoft Rough Set

A hypersoft rough set uses lower and upper approximations over an approximation space to represent multi-attribute uncertainty in sets. Note that an approximation space is a mathematical structure that models uncertainty, consisting of a universe of objects and an equivalence relation, enabling the definition of lower and upper approximations for subsets.

Definition 14 (Pawlak Approximation Space). [220, 221] Let U be a non-empty set, known as the universe of discourse, and let R be an equivalence relation on U. The pair (U, R) is called a Pawlak approximation space. The relation R induces a partition of U into equivalence classes, where each element in U is indiscernible from others within its equivalence class.

Given a subset $X \subseteq U$, the lower approximation $R(X)$ and the upper approximation $\bar{R}(X)$ of X with respect to R are defined as follows:

- The lower approximation of X, denoted $R(X)$, is the set of all elements $u \in U$ such that the equivalence class $[u]_R \subseteq X$:

$$R(X) = \{u \in U \mid [u]_R \subseteq X\}$$

  This set contains elements that are certainly in X based on the information provided by R.

- The upper approximation of X, denoted $\bar{R}(X)$, is the set of all elements $u \in U$ such that the intersection $[u]_R \cap X \neq \emptyset$:

$$\bar{R}(X) = \{u \in U \mid [u]_R \cap X \neq \emptyset\}$$

  This set contains elements that possibly belong to X given the information provided by R.

The pair $(R(X), \bar{R}(X))$ represents the rough set approximation of X within the Pawlak approximation space (U, R).

Definition 15 (Hypersoft Rough Set). [124] Let (X, R) be a Pawlak approximation space, where R is an equivalence relation on X. Given a Hypersoft Set (F, J) over X, the Hypersoft Lower Approximation $F_*(\mathrm{j})$ and Hypersoft Upper Approximation $F^*(\mathrm{j})$ of F with respect to R are defined for each $\mathrm{j} \in J$ as:

$$F_*(\mathrm{j}) = \{x \in X \mid [x]_R \subseteq F(\mathrm{j})\},$$
$$F^*(\mathrm{j}) = \{x \in X \mid [x]_R \cap F(\mathrm{j}) \neq \emptyset\}$$

where $[x]_R$ Denotes the equivalence class of x under R.

The Hypersoft Rough Set is then the pair. (F_*, F^*, J).

3 | Results

The results of this paper are described below.

3.1 | Hypersoft Rough Graph

A hypersoft rough graph extends rough graph theory by using multi-attribute approximations over vertices and edges to capture complex relationships.

Definition 16 (Hypersoft Rough Graph). Let $G = (V, E)$ be a simple graph, and let R be an equivalence relation on V. Consider n distinct attributes with disjoint parameter sets $J_1, J_2, \dots, J_n$, and let $J = J_1 \times J_2 \times \cdots \times J_n$.

Define a mapping $F: J \to \mathcal{P}(V)$, assigning to each parameter tuple $\mathrm{j} \in J$ a subset of vertices. The Hypersoft Lower Approximation $F_*(\mathrm{j})$ and Hypersoft Upper Approximation $F^*(\mathrm{j})$ of F with respect to R are:

$$F_*(\mathrm{j}) = \{v \in V \mid [v]_R \subseteq F(\mathrm{j})\}$$
$$F^*(\mathrm{j}) = \{v \in V \mid [v]_R \cap F(\mathrm{j}) \neq \emptyset\}$$

Similarly, define a mapping $K: J \to \mathcal{P}(E)$ for edges, and define the edge approximations. $K_*(\mathrm{j})$ and $K^*(\mathrm{j})$.

A Hypersoft Rough Graph $\tilde{G} = (G, F_*, K_*, F^*, K^*, J)$ is defined, where for each $\mathrm{j} \in J$:

- $H_*(\mathrm{j}) = (F_*(\mathrm{j}), K_*(\mathrm{j}))$ is a subgraph of G.
- $H^*(\mathrm{j}) = (F^*(\mathrm{j}), K^*(\mathrm{j}))$ is a subgraph of G.

Theorem 17. Every Hypersoft Rough Graph can be transformed into a Soft Rough Graph, a Rough Graph, and a Soft Graph.

Proof. We will demonstrate a systematic transformation from a Hypersoft Rough Graph to a Soft Rough Graph, then to a Rough Graph, and finally to a Soft Graph.

Let $\tilde{G} = (G, F_*, K_*, F^*, K^*, J)$ be a Hypersoft Rough Graph, where $J = J_1 \times J_2 \times \cdots \times J_n$.

Select one parameter set, say $J_1 = A$, and fix the other parameters. Define a mapping $F': A \to \mathcal{P}(V)$ by:

$$F'(a) = \bigcup_{\substack{\mathrm{j} \in J \\ j_1 = a}} F(\mathrm{j}), \ \forall a \in A$$

Define the lower and upper approximations:

$$F'_*(a) = \bigcup_{\substack{\mathrm{j} \in J \\ j_1 = a}} F_*(\mathrm{j}), \ F'^*(a) = \bigcup_{\substack{\mathrm{j} \in J \\ j_1 = a}} F^*(\mathrm{j})$$

Similarly, define $K': A \to \mathcal{P}(E)$ and $K'_*(a), K'^*(a)$.

Then $\tilde{G}' = (G, F'_*, K'_*, F'^*, K'^*, A)$ is a Soft Rough Graph. Justification:

- The mapping F' is from A to $\mathcal{P}(V)$, satisfying the definition of a soft set over V.
- The lower and upper approximations $F'_*(a)$ and $F'^*(a)$ are well-defined and correspond to those in Soft Rough Sets.
- The subgraphs $H'_*(a) = (F'_*(a), K'_*(a))$ and $H'^*(a) = (F'^*(a), K'^*(a))$ are subgraphs of G.

Next, consider the Soft Rough Graph $\tilde{G}' = (G, F'_*, K'_*, F'^*, K'^*, A)$.

Define:

$$V_R = \bigcup_{a \in A} F'^*(a), \; E_R = \bigcup_{a \in A} K'^*(a)$$

Then $G_R = (V_R, E_R)$ is a Rough Graph.

Justification:

- The vertex set V_R and edge set E_R are formed by the upper approximations over all parameters in A.
- This represents the rough approximation of G with respect to the equivalence relation R.

Step 3: Transforming a Rough Graph into a Soft Graph

Define a mapping $F'': A \to \mathcal{P}(V)$ by:

$$F''(a) = F'^*(a), \; \forall a \in A$$

Then (F'', A) is a Soft Graph over G.

Justification:

- The mapping F'' assigns to each parameter $a \in A$ a subset of V, forming a soft set over V.
- If the subgraphs induced by $F''(a)$ are connected, (F'', A) Satisfies the definition of a Soft Graph.

Through these transformations, we have shown that any Hypersoft Rough Graph can be systematically transformed into a Soft Rough Graph, a Rough Graph, and a Soft Graph.

4 | Future Tasks and Discussion of this Research

This section outlines future directions for this research.

4.1 | Discussion: N-Soft Graphs, N-hypersoft Graphs and N-bipolar hypersoft Graphs

The Soft Set, due to its intriguing mathematical structure and ease of application, has led to the development of various derivative concepts, such as the N -Soft Set [3, 4, 14, 50, 51], N-HyperSoft Set [148], bipolar hypersoft set [11, 27, 142, 146, 147], and N-bipolar hypersoft set [143, 149]. Visualizing these sets as graphs could provide valuable insights and further facilitate their application in future studies. **Definition 18** (N-Soft Set). [14] Let U be a universe of objects under consideration, and let E denote a set of attributes with $A \subseteq E$. Let $R = \{0,1,2,\dots,N-1\}$ represent an ordered set of grades, where $N \in \{2,3,\dots\}$.

An N-soft set on U is defined as a triple (F, A, N), where F is a mapping from A to $\mathcal{P}(U \times R)$ such that for each $a \in A$ and $u \in U$, there exists a specific pair $(u, r) \in U \times R$ with $(u, r) \in F(a)$.

The interpretation of the pair $(u, r) \in F(a)$ is that the element u belongs to the a-approximation of U with grade r. The tabular representation of an N-soft set is given as follows:

(F, A, N)	a_1	a_2	…	a_q
u_1	$\{r_{11}\}$	$\{r_{12}\}$	…	$\{r_{1q}\}$
u_2	$\{r_{21}\}$	$\{r_{22}\}$	…	$\{r_{2q}\}$
⋮	⋮	⋮	⋱	⋮
u_p	$\{r_{p1}\}$	$\{r_{p2}\}$	…	$\{r_{pq}\}$

Definition 19 (N-Soft Graph). Let $G=(V,E)$ be a simple graph, where V is the set of vertices and E is the set of edges. Let A be a non-empty set of parameters, and let $R=\{0,1,2,\ldots,N-1\}$ be an ordered set of grades, with $N \geq 2$.

An N-Soft Graph over G is defined as a triple (F,A,N), where:

- $F: A \rightarrow \mathcal{P}(V \times R)$ is a mapping such that for each $a \in A$ and each $v \in V$, there exists a unique grade $r \in R$ with $(v,r) \in F(a)$ (equivalently, $F(a)(v)=r$).

  The interpretation of $(v,r) \in F(a)$ is that the vertex v has a grade r with respect to the parameter a. The N -Soft Graph (F,A,N) represents graded relationships between parameters and vertices in G.

Theorem 20. Every N-Soft Graph can be transformed into a Soft Graph.

Proof. Let $G=(V,E)$ be a simple graph, A be a non-empty set of parameters, and (F,A,N) be an N-Soft Graph over G, where $F: A \rightarrow \mathcal{P}(V \times R)$ with $R=\{0,1,\ldots,N-1\}$.

We define a Soft $\text{Graph}(F',A)$ over G as follows:

For each parameter $a \in A$, define the mapping $F': A \rightarrow \mathcal{P}(V)$ by:

$$F'(a)=\{v \in V \mid F(a)(v) \geq T\}$$

Where T is a fixed threshold in R.

Explanation:

- For each parameter $a, F(a)(v)$ is the grade assigned to vertex v.
- We select a threshold $T \in R$ to determine which vertices are included in $F'(a)$.
- The set $F'(a)$ includes all vertices v such that $F(a)(v)$ meets or exceeds the threshold T.

Verification:

- F' maps A to subsets of V, i.e., $F': A \rightarrow \mathcal{P}(V)$, satisfying the definition of a Soft Graph.
- Since F is defined for all $a \in A$ and $v \in V$, so is F'.

  By applying the threshold T, any N-Soft $\text{Graph}(F,A,N)$ can be transformed into a $\text{SoftGraph}(F',A)$.

Definition 21 (N-Hypersoft Set). [148] Let Ω be a universe of objects, and let E denote a set of parameters, partitioned into disjoint subsets $E_1,E_2,\ldots,E_n$. Each E_i represents a distinct category of parameters. Define $R=\{0,1,2,\ldots,N-1\}$ as an ordered set of grades, where $N \geq 2$. An N-hypersoft set over Ω is a triple (∇,J,N), where:

- $J=E_1 \times E_2 \times \cdots \times E_n$ is the set of all tuples formed by selecting one parameter from each subset E_i,
- ∇ is a mapping $\nabla: J \rightarrow \mathcal{P}(\Omega \times R)$, with the property that for each tuple $\mathrm{j} \in J$ and each object $\omega \in \Omega$, there exists a unique grade $r_{\mathrm{j}} \in R$ such that $(\omega, r_{\mathrm{j}}) \in \nabla(\mathrm{j})$.

The interpretation of $(\omega, r_{\mathrm{j}}) \in \nabla(\mathrm{j})$ is that the object ω has a grade r_{j} under the parameter tuple j . The N-hypersoft set can be represented in tabular form, where each row corresponds to an object in Ω and each column corresponds to a parameter tuple in J.

Thus, the N-hypersoft set (∇, J, N) can be expressed as:

$$(\nabla, J, N) = \{(\mathrm{j}, \{(\omega, \nabla(\mathrm{j})(\omega)) \mid \omega \in \Omega\}) \mid \mathrm{j} \in J\}$$

Definition 22 (N-Hypersoft Graph). Let $G = (V, E)$ be a simple graph. Consider n distinct attributes with corresponding disjoint parameter sets $E_1, E_2, \dots, E_n$, and let $J = E_1 \times E_2 \times \cdots \times E_n$. Let $R = \{0,1,2, \dots, N - 1\}$ be an ordered set of grades, where $N \geq 2$.

An N-Hypersoft Graph over G is defined as a triple (∇, J, N), where:

- $\nabla: J \to \mathcal{P}(V \times R)$ is a mapping such that for each parameter tuple $\mathrm{j} \in J$ and each $v \in V$, there exists a unique grade $r_{\mathrm{j}} \in R$ with $(v, r_{\mathrm{j}}) \in \nabla(\mathrm{j})$ (equivalently, $\nabla(\mathrm{j})(v) = r_{\mathrm{j}}$). The interpretation of $(v, r_{\mathrm{j}}) \in \nabla(\mathrm{j})$ is that the vertex v has a grade r_{j} under the parameter tuple j . The N -Hypersoft Graph captures multi-parameter graded relationships within G.

Theorem 23. The following holds.

1. Every N-Hypersoft Graph can be transformed into an N-Soft Graph.
2. Every N-Hypersoft Graph can be transformed into a Hypersoft Graph.
3. Every N-Hypersoft Graph can be transformed into a Soft Graph.

Proof. Let $G = (V, E)$ be a simple graph, and (∇, J, N) be an N-Hypersoft Graph over G, where $J = E_1 \times E_2 \times \cdots \times E_n$, and $\nabla: J \to \mathcal{P}(V \times R)$ with $R = \{0,1, \dots, N - 1\}$.

Part 1: Transformation into an N-Soft Graph

We can transform the N-Hypersoft Graph into an N-Soft Graph (F, A, N) over G as follows:

- Let $A = J$ (i.e., the set of parameter tuples in).
- Define $F: A \to \mathcal{P}(V \times R)$ by $F(a) = \nabla(a)$ for all $a \in A$.

Explanation:

- Since ∇ maps parameter tuples to graded subsets of V, we can view each tuple $a \in A$ as a single parameter in the N-Soft Graph.
- The mapping F retains the grading information from ∇.

  Thus, (∇, J, N) can be viewed as an N-Soft Graph (F, A, N).

  Part 2: Transformation into a Hypersoft Graph We can transform the N-Hypersoft Graph into a Hypersoft Graph (∇', J) Over G by selecting a threshold $T \in R$ and defining:

$$\nabla'(\mathrm{j}) = \{v \in V \mid \nabla(\mathrm{j})(v) \geq T\}$$

for all $\mathrm{j} \in J$.

The mapping ∇' now assigns to each parameter tuple j a subset of V, where the grade meets or exceeds the threshold T. Thus, (∇, J, N) can be transformed into a Hypersoft Graph. (∇', J).

Part 3: Transformation into a Soft Graph To further transform the Hypersoft Graph into a Soft Graph, we can:

Fix one parameter set E_i and select a particular value $e_i \in E_i$.

Consider the reduced parameter set $J' = E_1 \times \cdots \times E_{i-1} \times E_{i+1} \times \cdots \times E_n$.

Define $F'': J' \to \mathcal{P}(V)$ by:

$$F''(a) = \nabla'(a, e_i)$$

for all $a \in J'$.

Therefore, (∇, J, N) can be transformed into a Soft Graph (F'', J').

Definition 24 (Bipolar Hypersoft Set (BHS Set)). [142] Let Ω be a universe of objects, E a set of attributes, and $Q_1 \subseteq E$ a subset of attributes. A bipolar hypersoft set over Ω is defined as a triple (Δ, r, Q_1), where:

- $\Delta: Q_1 \rightarrow \mathcal{P}(\Omega)$ is a mapping assigning each parameter $q \in Q_1$ a subset of Ω.
- $r: \neg Q_1 \rightarrow \mathcal{P}(\Omega)$ is a mapping assigning each parameter $\neg q \in \neg Q_1$ (where $\neg q$ is the complement or opposite of q) a subset of Ω.
- For each $q \in Q_1, \Delta(q) \cap r(\neg q) = \emptyset$.

The BHS set (Δ, r, Q_1) can be represented as:

$$(\Delta, r, Q_1) = \{(q, \Delta(q), r(\neg q)) \mid q \in Q_1, \Delta(q), r(\neg q) \subseteq \Omega\}$$

Definition 25 (Bipolar Hypersoft Graph). Let $G = (V, E)$ be a simple graph, and let $Q_1 \subseteq E$ be a subset of parameters. The Bipolar Hypersoft Graph over G is defined as a triple (Δ, r, Q_1), where:

- $\Delta: Q_1 \rightarrow \mathcal{P}(V)$ assigns to each parameter $q \in Q_1$ a subset $\Delta(q) \subseteq V$.
- $r: \neg Q_1 \rightarrow \mathcal{P}(V)$ assigns to each opposite parameter $\neg q \in \neg Q_1$ a subset $r(\neg q) \subseteq V$.
- For each $q \in Q_1$, it holds that $\Delta(q) \cap r(\neg q) = \emptyset$.

The Bipolar Hypersoft Graph (Δ, r, Q_1) captures both positive and negative relationships between parameters and vertices in G.

Theorem 26. The following holds.

1. Every Bipolar Hypersoft Graph can be transformed into a Hypersoft Graph.
2. Every Bipolar Hypersoft Graph can be transformed into a Soft Graph.

Proof. Let $G = (V, E)$ be a simple graph, and (Δ, r, Q_1) be a Bipolar Hypersoft Graph over G. Part 1: Transformation into a Hypersoft Graph

We can construct a Hypersoft Graph (∇, J) over G as follows:

- Let $J = Q_1$.
- Define $\nabla(q) = \Delta(q)$ for all $q \in Q_1$.

Thus, the Bipolar Hypersoft Graph (Δ, r, Q_1) can be transformed into a Hypersoft Graph (∇, J). Part 2: Transformation into a Soft Graph

To further transform the Hypersoft Graph into a Soft Graph:

- If Q_1 consists of single parameters (not tuples), then (∇, Q_1) is a Soft Graph over G.

Therefore, the Bipolar Hypersoft Graph can be transformed into a Soft Graph.

Definition 27 (N-Bipolar Hypersoft Set (N-BHS Set)). [143] Let Ω be a universe of objects, E a set of attributes, and $Q_1 \subseteq E$ a subset of attributes. Consider $R = \{0,1, \ldots, N-1\}$ as an ordered set of grades, where $N \geq 2$. An N-bipolar hypersoft set over Ω is defined as a quadruple (Δ, r, Q_1, N), where:

- $\Delta: Q_1 \rightarrow \mathcal{P}(\Omega \times R)$ maps each parameter $q \in Q_1$ to a subset of $\Omega \times R$.
- $r: \neg Q_1 \rightarrow \mathcal{P}(\Omega \times R)$ maps each parameter $\neg q \in \neg Q_1$ (opposite of q) to a subset of $\Omega \times R$.

- For each $q \in Q_1$ and $\omega \in \Omega$, there exist unique values $r_q, r_{\neg q} \in R$ such that $(\omega, r_q) \in \Delta(q)$ and $(\omega, r_{\neg q}) \in r(\neg q)$ with the condition $r_q + r_{\neg q} \leq N - 1$.

The N-BHS set (Δ, r, Q_1, N) can be written as:

$$(\Delta, r, Q_1, N) = \{(q, \{(\omega, \Delta(q)(\omega), r(\neg q)(\omega))\}) \mid q \in Q_1, \omega \in \Omega, \Delta(q)(\omega), r(\neg q)(\omega) \in R\}$$

Definition 28 (N-Bipolar Hypersoft Graph). Let $G = (V, E)$ be a simple graph, $Q_1 \subseteq E$ a subset of parameters, and $R = \{0,1,2, \ldots, N - 1\}$ an ordered set of grades, with $N \geq 2$.

An N-Bipolar Hypersoft Graph over G is defined as a quadruple (Δ, r, Q_1, N), where:

- $\Delta: Q_1 \to \mathcal{P}(V \times R)$ assigns to each parameter $q \in Q_1$ a set of pairs (v, r_q), where $v \in V$ and $r_q \in R$.
- $r: \neg Q_1 \to \mathcal{P}(V \times R)$ assigns to each opposite parameter $\neg q \in \neg Q_1$ a set of pairs $(v, r_{\neg q})$, where $v \in V$ and $r_{\neg q} \in R$.
- For each $q \in Q_1$ and $v \in V$, there exist unique grades $r_q, r_{\neg q} \in R$ such that:
    1. $(\Delta(q))(v) = r_q$ and $(r(\neg q))(v) = r_{\neg q}$.
    2. The grades satisfy $r_q + r_{\neg q} \leq N - 1$.

The N-Bipolar Hypersoft Graph (Δ, r, Q_1, N) captures graded positive and negative relationships between parameters and vertices in G, with the grading condition ensuring the compatibility between positive and negative assessments.

Theorem 29. The following holds.

1. Every N-Bipolar Hypersoft Graph can be transformed into a Bipolar Hypersoft Graph.
2. Every N-Bipolar Hypersoft Graph can be transformed into a Hypersoft Graph.
3. Every N-Bipolar Hypersoft Graph can be transformed into an N-Soft Graph.
4. Every N-Bipolar Hypersoft Graph can be transformed into an N-Hypersoft Graph.

Proof. Let $G = (V, E)$ be a simple graph, and (Δ, r, Q_1, N) be an N-Bipolar Hypersoft Graph over G. Part 1: Transformation into a Bipolar Hypersoft Graph

Define a Bipolar Hypersoft Graph (Δ', r', Q_1) over G as follows:

$$\Delta'(q) = \{v \in V \mid \Delta(q)(v) \geq T\}$$
$$r'(\neg q) = \{v \in V \mid r(\neg q)(v) \geq T'\}$$

where $T, T' \in R$ are thresholds.

Thus, (Δ, r, Q_1, N) can be transformed into a Bipolar Hypersoft Graph (Δ', r', Q_1).

Part 2: Transformation into a Hypersoft Graph

By ignoring the negative mapping r', we define a Hypersoft $\text{Graph}(\nabla, J)$:

Let $J = Q_1$.

Define $\nabla(q) = \Delta'(q)$ for all $q \in Q_1$.

(Δ, r, Q_1, N) can be transformed into a Hypersoft $\text{Graph}(\nabla, J)$.

Part 3: Transformation into an N-Soft Graph

Define an N-Soft Graph (F, A, N) as follows:

$$A = Q_1,\ F(a) = \Delta(a) \text{ for all } a \in A$$

(Δ, r, Q_1, N) can be transformed into an N-Soft $\mathrm{Graph}(F, A, N)$.

Part 4: Transformation into an N-Hypersoft Graph

If Q_1 is a Cartesian product, i.e., $Q_1 = E_1 \times E_2 \times \cdots \times E_n$, then we define an N-Hypersoft Graph (∇, J, N) as follows:

$$J = Q_1,\ \nabla(\mathrm{j}) = \Delta(\mathrm{j}) \text{ for all } \mathrm{j} \in J$$

(Δ, r, Q_1, N) can be transformed into an N-Hypersoft $\mathrm{Graph}(\nabla, J, N)$. .

4.2 | Future tasks: Hypersoft Hyperset and SuperHypersoft Superhyperset

This paper outlines future directions for exploration. In the realm of soft set theory, related concepts such as hypersoft sets and super hypersoft sets [96, 139, 197, 201] are recognized for their potential to broaden the structure and applications of generalized sets. In graph theory, analogous concepts like hypergraphs [33 98-100, 161] and super hypergraphs [72, 94, 104-106, 193-196, 200] are well-known, as construct such as hyperalgebras [115, 87, 204] serve to generalize traditional graph and algebraic concepts. Additionally, in set theory, we find concepts such as hypersets [30, 97, 152] and supersets, which we propose to integrate into a unified concept termed the Superhyperset. We aim to investigate the mathematical structures and applications of sets, hypersets, and super hypersets, further enriched by the theoretical foundations of soft, hypersoft, and superhypersoft frameworks.

Although still in the conceptual stage, we outline below the definitions of Superhypersets and related concepts.

Definition 30 (Hyperset). [97] A Hyperset is a set that can contain itself as a member, either directly or indirectly. This is permitted in Non-Well-Founded Set Theory, where the Axiom of Foundation is replaced by the AntiFoundation Axiom (AFA). In this context, sets are allowed to have membership structures that include cycles or infinite descending chains.

Example 31 (Hyperset). Let A be defined as:

$$A = \{A\}$$

Here, A contains itself as its sole member, which is permissible in Hyperset Theory under the Anti-Foundation Axiom.

Definition 32 (Superset). In set theory, a Superset is a set whose elements are themselves sets. Formally, a set S is a Superset if and only if for every element $x \in S, x$ is a set. That is,

$$\forall x \in S,\ x \text{ is a set.}$$

Example 33 (Superset). Let $B = \{1,2,3\}$ and $C = \{1,2\}$. Since $C \subseteq B$, we have:

$$B \supseteq C.$$

Every element of C is contained in B, so B is a Superset of C.

Definition 34 (Superhyperset). Let $\mathcal{H}$ be the class of all Hypersets. A Superhyperset S is a set such that:

$$S \subseteq \mathcal{P}(\mathcal{H})$$

Where $\mathcal{P}(\mathcal{H})$ denotes the power set of $\mathcal{H}$. Elements of S are subsets of Hypersets, and S may include itself as an element, directly or indirectly, thus allowing for non-well-founded and higher-order membership structures.

Example 35 (Superhyperset). Let $A = \{A\}$ (a Hyperset) and $H = \{A\}$. Define:

$$S = \{\{A\}, \{A, H\}\}$$

Here, $S \subseteq \mathcal{P}(\mathcal{H})$ with $\mathcal{H} = \{A, H\}$, and S is a Superhyperset since its elements are subsets of Hypersets.

Theorem 36. Every Superhyperset can be transformed into a Hyperset and a Superset.

Proof. Let S be a Superhyperset, so $S \subseteq \mathcal{P}(\mathcal{H})$, where $\mathcal{H}$ is the class of all Hypersets. Transformation into a Hyperset:

Define a function $f: S \to \mathcal{H}$ by selecting an element from each subset $s \in S$. Since each $s \subseteq \mathcal{H}$, we have $f(s) \in \mathcal{H}$.

Consider the set:

$$H = \{f(s) \mid s \in S\}$$

Since $f(s) \in \mathcal{H}$ for all $s \in S, H$ is a set of Hypersets. If H contains itself as an element (directly or indirectly), then H is a Hyperset.

Transformation into a Superset:

Let:

$$U = \bigcup_{s \in S} s$$

Then U is a set containing all elements of the subsets in S. For any set A such that $A \in s$ for some $s \in S$, we have $A \in U$, so U is a Superset of A.

Thus, the Superhyperset S can be associated with a Hyperset H and a Superset U, demonstrating that a Superhyperset can be transformed into both a Hyperset and a Superset.

Additionally, We would like to extend the concept of the SuperHyperSoft Graph to Rough Graphs, Expert Graphs, Fuzzy Graphs, and Neutrosophic Graphs, and to explore their mathematical structures as well as applications in decision-making and related areas.

Definition 37 (SuperHyperSoft Set). [197] Let U be a universe of discourse, and let $P(U)$ denote the power set of U. Let $a_1, a_2, \ldots, a_n$ be n distinct attributes, where $n \geq 1$. Each attribute a_i has a corresponding set of attribute values A_i, with the property that $A_i \cap A_j = \emptyset$ for all $i \neq j$.

Let $P(A_i)$ denote the power set of A_i for each $i = 1,2, \ldots, n$.

Then, the pair $\left(F, P(A_1) \times P(A_2) \times \cdots \times P(A_n)\right)$, where

$$F: P(A_1) \times P(A_2) \times \cdots \times P(A_n) \to P(U)$$

is called a SuperHyperSoft Set over U.

Definition 38 (SuperHyperSoft Graph). [60] Let $G = (V, E)$ be a graph, where V is the set of vertices and E is the set of edges. Let $U = V \cup E$, and let $P(U)$ denote the power set of U.

Let $a_1, a_2, \ldots, a_n$ be n distinct attributes, each with a corresponding set of attribute values A_i, such that $A_i \cap A_j = \emptyset$ for all $i \neq j$.

Let $P(A_i)$ denote the power set of A_i for each $i = 1,2, \ldots, n$.

Define two functions:

1. Vertex Function:

$$F_V: P(A_1) \times P(A_2) \times \cdots \times P(A_n) \to P(V)$$

Which maps combinations of attribute value subsets to subsets of vertices.

2. Edge Function:

$$F_E: P(A_1) \times P(A_2) \times \cdots \times P(A_n) \to P(E)$$

Which maps combinations of attribute value subsets to subsets of edges.

Then, the pair (F_V, F_E) is called a SuperHyperSoft Graph over G.

Due to the versatility and ease of application of soft sets, a variety of soft set extensions, beyond the above-mentioned Superhypersoft, have been developed. The concepts of IndetermSoft Sets [198, 199], IndetermHyperSoft Sets [198, 199], Binary soft sets [2, 155, 173], Generalized soft rough sets [17, 18, 23] and TreeSoft Sets [44, 154, 198, 199, 202] are recognized as extensions of the Soft Set. In the future, additional research into the mathematical structures and applications of these sets in the contexts of Expert Sets, Rough Sets, Expert Graphs, and Rough Graphs is anticipated.

4.3 | Discussion: Hypersoft Digraphs

We intend to explore the mathematical structures of Hypersoft Digraphs, extending the concepts of Soft Directed Graphs as established by [89, 117-120]. A Directed Graph consists of vertices connected by directed edges (arcs), where each edge has a direction from one vertex to another. The definitions, along with relevant related concepts, will be outlined below.

Definition 39. A directed graph (or digraph) is a pair $D = (V, A)$ where:

- V is a non-empty finite set of elements called vertices or nodes.
- $A \subseteq V \times V$ is a set of ordered pairs of distinct vertices called arcs or directed edges.

Definition 40. [117] Let $D^* = (V, A)$ be a directed graph and P be a non-empty set of parameters. A soft-directed graph over D^* is a quadruple $D = (D^*, J, L, P)$, where:

- $D^* = (V, A)$ is a directed graph.
- P is a non-empty set of parameters.
- $J: P \to \mathcal{P}(V)$ is a mapping that assigns to each parameter $p \in P$ a subset $J(p) \subseteq V$.
- $L: P \to \mathcal{P}(A)$ is a mapping that assigns to each parameter $p \in P$ a subset $L(p) \subseteq A$.

These mappings satisfy the condition that for each $p \in P$, the pair $M(p) = (J(p), L(p))$ is a subdigraph of D^*, i.e., $L(p) \subseteq J(p) \times J(p)$.

The collection $D = \{M(p) \mid p \in P\}$ represents the soft directed graph over D^*.

Definition 41. Let $D^* = (V, A)$ be a directed graph. Consider n distinct attributes with corresponding disjoint parameter sets $E_1, E_2, \dots, E_n$, and let $J = E_1 \times E_2 \times \cdots \times E_n$.

A hypersoft directed graph over D^* is a triple $D = (D^*, \nabla, J)$, where:

- $D^* = (V, A)$ is a directed graph.
- $J = E_1 \times E_2 \times \cdots \times E_n$ is the set of all parameter tuples formed by selecting one parameter from each E_i.
- $\nabla: J \to \mathcal{P}(V)$ is a mapping that assigns to each parameter tuple $\mathrm{j} \in J$ a subset $\nabla(\mathrm{j}) \subseteq V$.

The mapping ∇ must satisfy the condition that for each $\mathrm{j} \in J$, the pair $M(\mathrm{j}) = (\nabla(\mathrm{j}), L(\mathrm{j}))$ is a subdigraph of D^*, where:

$$L(\mathrm{j}) = \{(u, v) \in A \mid u, v \in \nabla(\mathrm{j})\}$$

The collection $D = \{M(\mathrm{j}) \mid \mathrm{j} \in J\}$ represents the hypersoft directed graph over D^*.

Theorem 42. Every hypersoft-directed graph can be transformed into a soft-directed graph and a directed graph.

Proof. Let $D = (D^*, \nabla, J)$ be a hypersoft directed graph over the directed graph $D^* = (V, A)$, where $J = E_1 \times E_2 \times \cdots \times E_n$ and $\nabla: J \rightarrow \mathcal{P}(V)$.

We can construct a soft-directed graph $D' = (D^*, J', L', P)$ as follows:

- Let $P = J$ (i.e., the set of all parameter tuples).
- Define the mapping $J': P \rightarrow \mathcal{P}(V)$ by:

$$J'(\mathrm{j}) = \nabla(\mathrm{j}),\ \forall \mathrm{j} \in P$$

- Define the mapping $L': P \rightarrow \mathcal{P}(A)$ by:

$$L'(\mathrm{j}) = \{(u, v) \in A \mid u, v \in \nabla(\mathrm{j})\},\ \forall \mathrm{j} \in P$$

Since $M(\mathrm{j}) = (J'(\mathrm{j}), L'(\mathrm{j}))$ is a subdigraph of D^* for each $\mathrm{j} \in P$, the quadruple $D' = (D^*, J', L', P)$ Satisfies the definition of a soft-directed graph.

To transform the hypersoft directed graph D into a directed graph D'', we can consider the union of all subdigraphs $M(\mathrm{j})$:

$$V'' = \bigcup_{\mathrm{j} \in J} \nabla(\mathrm{j}) \subseteq V,$$

$$A'' = \bigcup_{\mathrm{j} \in J} L(\mathrm{j}) \subseteq A.$$

Then, $D'' = (V'', A'')$ is a subdigraph of D^*, and thus a directed graph.

By constructing D' and D'' as shown, we have demonstrated that a hypersoft-directed graph D can be transformed into a soft-directed graph D' and a directed graph D''.

4.4 | Discussion: Hypersoft Semingraphs

We aim to explore the mathematical structures of Hypersoft Semigraphs, building on the concepts of Soft Semigraphs as developed in works such as [84-88, 90, 91]. A Semigraph generalizes graphs by allowing edges to connect multiple vertices with specified order but with constraints on shared vertices between edges. Below, we will provide the definitions and discuss relevant related concepts. We also plan to extend these ideas by incorporating Directed Graphs, Soft Expert Graphs, and Rough Graphs to further analyze their mathematical structures and potential applications.

Definition 43. [180] A semigraph is a pair $G^* = (V, E)$ where:

- V is a non-empty finite set whose elements are called vertices.
- E is a set of finite sequences (tuples) of distinct vertices called edges, such that:
  1. Any two edges have at most one vertex in common.
  2. Two edges $(v_1, v_2, \ldots, v_n)$ and $(u_1, u_2, \ldots, u_m)$ are considered equal if and only if:
     - $n = m$, and
     - Either $v_i = u_i$ for all $1 \leq i \leq n$, or $v_i = u_{n-i+1}$ for all $1 \leq i \leq n$ (i.e., they are reverses of each other).

An edge $e = (v_1, v_2, \ldots, v_n)$ is an ordered sequence where:

- v_1 and v_n are called the end vertices of e.

- The vertices $v_2, v_3, \dots, v_{n-1}$ are called the middle vertices or m-vertices of e.

Definition 44. [92] Let $G^* = (V, E)$ be a semigraph, and let A be a non-empty set of parameters. A soft semigraph over G^* is a quadruple $G = (G^*, Q, W, A)$, where:

- $G^* = (V, E)$ is a semigraph.
- A is a non-empty set of parameters.
- $Q: A \to \mathcal{P}(V)$ is a mapping that assigns to each parameter $a \in A$ a subset $Q(a) \subseteq V$.
- $W: A \to \mathcal{P}(E_p)$ is a mapping that assigns to each parameter $a \in A$ a set $W(a)$ of maximal partial edges induced by $Q(a)$, where E_p is the set of all partial edges of G^*.

The mappings Q and W must satisfy the condition that for each $a \in A$, the pair $H(a) = (Q(a), W(a))$ is a partial semigraph of G^*.

- Partial Edge: A partial edge of an edge $e = (v_1, v_2, \dots, v_n)$ is a subsequence $e' = (v_{i_1}, v_{i_2}, \dots, v_{i_k})$, where $1 \leq i_1 < i_2 < \cdots < i_k \leq n$ or $n \geq i_k > i_{k-1} > \cdots > i_1 \geq 1$.
- Maximal Partial Edge: A partial edge that is not a subsequence of any other partial edge induced by the same vertex set.

Definition 45. Let $G^* = (V, E)$ be a semigraph. Consider n distinct attributes with corresponding disjoint parameter sets $E_1, E_2, \dots, E_n$, and let $J = E_1 \times E_2 \times \cdots \times E_n$. A hypersoft semigraph over G^* is a triple $G = (G^*, F, J)$, where:

- $G^* = (V, E)$ is a semigraph.
- $J = E_1 \times E_2 \times \cdots \times E_n$ is the set of all parameter tuples formed by selecting one parameter from each E_i.
- $F: J \to \mathcal{P}(V)$ is a mapping that assigns to each parameter tuple $\mathrm{j} \in J$ a subset $F(\mathrm{j}) \subseteq V$.

For each $\mathrm{j} \in J$, define $W(\mathrm{j})$ as the set of maximal partial edges induced by $F(\mathrm{j})$. The pair $H(\mathrm{j}) = (F(\mathrm{j}), W(\mathrm{j}))$ must be a partial semigraph of G^*.

The collection $G = \{H(\mathrm{j}) \mid \mathrm{j} \in J\}$ represents the hypersoft semigraph over G^*.

Theorem 46. Every hypersoft semigraph can be transformed into a semigraph and a soft semigraph. Proof. Let $G = (G^*, F, J)$ be a hypersoft semigraph over the semigraph $G^* = (V, E)$, where $J = E_1 \times E_2 \times \cdots \times E_n$ and $F: J \to \mathcal{P}(V)$.

To transform the hypersoft semigraph G into a semigraph $G' = (V', E')$, proceed as follows:

Let $V' = \bigcup_{\mathrm{j} \in J} F(\mathrm{j}) \subseteq V$.

Let $E' = \bigcup_{\mathrm{j} \in J} W(\mathrm{j}) \subseteq E_p$, where $W(\mathrm{j})$ is the set of maximal partial edges induced by $F(\mathrm{j})$.

Since each $H(\mathrm{j}) = (F(\mathrm{j}), W(\mathrm{j}))$ is a partial semigraph of G^*, their union $G' = (V', E')$ forms a semigraph, as it satisfies the conditions of a semigraph:

Any two edges in E' have at most one vertex in common because this property holds in G^*.

The edges in E' are sequences of distinct vertices, satisfying the edge definition in a semigraph.

We can construct a soft semigraph $G'' = (G^*, Q, W', A)$ as follows:

Let $A = J$ (i.e., the set of all parameter tuples).

Define the mapping $Q: A \to \mathcal{P}(V)$ by:

$$Q(\mathrm{j}) = F(\mathrm{j}),\ \forall \mathrm{j} \in A$$

Define the mapping $W': A \to \mathcal{P}(E_p)$ by:

$$W'(\mathrm{j}) = W(\mathrm{j}),\ \forall \mathrm{j} \in A,$$

where $W(\mathrm{j})$ is the set of maximal partial edges induced by $F(\mathrm{j})$.

Since $H(\mathrm{j}) = (Q(\mathrm{j}), W'(\mathrm{j}))$ is a partial semigraph of G^* for each $\mathrm{j} \in A$, the quadruple $G'' = (G^*, Q, W', A)$ satisfies the definition of a soft semigraph.

By constructing G' and G'' as shown, we have demonstrated that a hypersoft semigraph G can be transformed into a semigraph G' and a soft semigraph G''.

4.5 | Future Tasks: Extension of Near Set and Decision-theoretic Rough Set

Future research directions include various extensions of existing concepts. For example, We aim to define new sets, such as Soft Near Sets, Soft Expert Near Sets, Hypersoft Near Sets, Soft Decision-Theoretic Rough Sets, and Hypersoft Decision-Theoretic Rough Sets by extending Near Sets [108, 112, 165-167] and Decision Theoretic Rough Sets [129-133, 170] through Soft Sets, Soft Expert Sets(cf., 10, 12, 20, 21, 168, 179]), and Hypersoft Sets. Near sets are collections that are either spatially or descriptively close, often sharing characteristics or elements. Decision-Theoretic Rough Sets, meanwhile, utilize rough set approximations for classification, leveraging Bayesian decision theory to minimize risk based on observed evidence. Alongside these definitions, I intend to explore their graphical representations, mathematical properties, potential applications, and construction algorithms.

4.6 | Future Tasks: Various Hypersoft Set

In the future, we intend to explore the mathematical structures of Z-hypersoft sets and Z-hypersoft fuzzy rough sets as extensions of Z-soft sets and Z-soft fuzzy rough sets [232-234, 238]. This exploration includes related concepts, such as Z-numbers [19, 38, 156, 230]. Furthermore, We anticipate progress in future research on Z-hypersoft graphs and Z-hypersoft fuzzy rough graphs, which graphically represent these concepts.

Other types of sets, such as Positive Sets [107, 163], Boffa Sets [29, 36 136], Meta Sets [122, 206, 207], Hyperfuzzy Sets [95, 121], and Naive Sets [103, 211], are also recognized. We aim to consider generalized concepts of Soft Sets, Hypersoft Sets, Soft Graphs, Hypersoft Graphs, and Rough Graphs by incorporating these ideas. For overviews of well-known types of sets, please refer to survey papers as needed [79].

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Author Contribution

All authors contributed equally to this work.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

References

[1] Mujahid Abbas, Ghulam Murtaza, and Florentin Smarandache. Basic operations on hypersoft sets and hypersoft point. Neutrosophic Sets and Systems, 35:407-421, 2020.

[2] Ahu Acikgoz and Nihal Tas. Binary soft set theory. European Journal of Pure and Applied Mathematics, 9(4):452-463, 2016.

[3] Muhammad Akram, Arooj Adeel, and José Carlos Rodriguez Alcantud. Group decision-making methods based on hesitant n-soft sets. Expert Syst. Appl., 115:95-105, 2019.

[4] Muhammad Akram, Arooj Adeel, and José Carlos Rodriguez Alcantud. Hesitant fuzzy n-soft sets: A new model with applications in decision-making. J. Intell. Fuzzy Syst., 36:6113-6127, 2019.

[5] Muhammad Akram, Ghous Ali, and José Carlos R Alcantud. New decision-making hybrid model: intuitionistic fuzzy n-soft rough sets. Soft Computing, 23:9853-9868, 2019.

[6] Muhammad Akram, Ghous Ali, and Noura Omair Alshehri. A new multi-attribute decision-making method based on m-polar fuzzy soft rough sets. Symmetry, 9:271, 2017.

[7] Muhammad Akram and Saira Nawaz. Operations on soft graphs. Fuzzy Information and Engineering, 7:423 - 449, 2015.

[8] Muhammad Akram and Sundas Shahzadi. Neutrosophic soft graphs with application. J. Intell. Fuzzy Syst., 32:841-858, 2017.

[9] Muhammad Akram, Fariha Zafar, Muhammad Akram, and Fariha Zafar. Fuzzy soft graphs. Hybrid Soft Computing Models Applied to Graph Theory, pages 211-259, 2020.

[10] Yousef Al-Qudah and Nasruddin Hassan. Bipolar fuzzy soft expert set and its application in decision making. Int. J. Appl. Decis. Sci, 10:175-191, 2017.

[11] Ashraf Al-Quran, Faisal Al-Sharqi, Kifayat Ullah, Mamika Ujianita Romdhini, Marwa Balti, and Mohammed Alomair. Bipolar fuzzy hypersoft set and its application in decision making. Infinite Study, 2023.

[12] Ashraf Al-Quran and Nasruddin Hassan. Neutrosophic vague soft expert set theory. J. Intell. Fuzzy Syst., 30:3691-3702, 2016.

[13] Ashraf Al-Quran, Nasruddin Hassan, and Emad A. Marei. A novel approach to neutrosophic soft rough set under uncertainty. Symmetry, 11:384, 2019.

[14] José Carlos R Alcantud, Feng Feng, and Ronald R Yager. An n-soft set approach to rough sets. IEEE Transactions on Fuzzy Systems, 28(11):2996-3007, 2019.

[15] José Carlos R Alcantud, Azadeh Zahedi Khameneh, Gustavo Santos-García, and Muhammad Akram. A systematic literature review of soft set theory. Neural Computing and Applications, 36(16):8951-8975, 2024.

[16] M Irfan Ali, Feng Feng, Xiaoyan Liu, Won Keun Min, and Muhammad Shabir. On some new operations in soft set theory. Computers & Mathematics with Applications, 57(9):1547-1553, 2009.

[17] Muhammad Irfan Ali, Mostafa K El-Bably, and El-Sayed A Abo-Tabl. Correction to: Topological approach to generalized soft rough sets via near concepts. Soft Comput., 26(6):3127, 2022.

[18] Muhammad Irfan Ali, Mostafa K El-Bably, and El-Sayed A Abo-Tabl. Topological approach to generalized soft rough sets via near concepts. Soft Computing, 26:499-509, 2022.

[19] Rafik Aziz Aliev, Akif Alizadeh, Rashad Rafig Aliyev, and Oleg H Huseynov. Arithmetic of Z-numbers, the: theory and applications. World Scientific, 2015.

[20] Shawkat Alkhazaleh and Abdul Razak Salleh. Soft expert sets. Adv. Decis. Sci., 2011:757868:1-757868:12, 2011.

[21] Shawkat Alkhazaleh and Abdul Razak Salleh. Fuzzy soft expert set and its application. Applied Mathematics-a Journal of Chinese Universities Series B, 5:1349-1368, 2014.

[22] Uri Alon and Eran Yahav. On the bottleneck of graph neural networks and its practical implications. arXiv preprint arXiv:2006.05205, 2020.

[23] Abd El-Latif AM. Generalized soft rough sets and generated soft ideal rough topological spaces. Journal of Intelligent & Fuzzy Systems, 34(1):517-524, 2018.

[24] Reza Ameri and Tahereh Nozari. Fuzzy hyperalgebras. Comput. Math. Appl., 61:149-154, 2011.

[25] Umair Amin, Aliya Fahmi, Yaqoob Naveed, Aqsa Farid, and Muhammad Arshad Shehzad Hassan. Domination in bipolar fuzzy soft graphs. J. Intell. Fuzzy Syst., 46:6369-6382, 2024.

[26] Shabana Anwar, Muhammad Azeem, and Muhammad Kamran Jamil. Topological numbers of fuzzy soft graphs and their applications in globalizing the world by mutual trade. Appl. Soft Comput., 159:111642, 2024.

[27] Baravan A Asaad, Sagvan Y Musa, and Zanyar A Ameen. Fuzzy bipolar hypersoft sets: A novel approach for decisionmaking applications. Mathematical and Computational Applications, 29(4):50, 2024.
[28] Muhammad Azeem, Shabana Anwar, Muhammad Kamran Jamil, Muhammad Haris Saeed, and Muhammet Deveci. Topological numbers of fuzzy soft graphs and their application. Inf. Sci., 667:120468, 2024.
[29] David Ballard and Karel Hrbácek. Standard foundations for nonstandard analysis. The Journal of Symbolic Logic, 57(2):741-748, 1992.
[30] Jon Barwise and Larry Moss. Hypersets. The Mathematical Intelligencer, 13:31-41, 1991.
[31] John Adrian Bondy, Uppaluri Siva Ramachandra Murty, et al. Graph theory with applications, volume 290. Macmillan London, 1976.
[32] Andreas Brandstädt, Van Bang Le, and Jeremy P Spinrad. Graph classes: a survey. SIAM, 1999.
[33] Alain Bretto. Hypergraph theory. An introduction. Mathematical Engineering. Cham: Springer, 1, 2013.
[34] Said Broumi, Assia Bakali, Mohamed Talea, Florentin Smarandache, and Faruk Karaaslan. Interval valued neutrosophic soft graphs. viXra, 2018.
[35] Derun Cai, Moxian Song, Chenxi Sun, Baofeng Zhang, Shenda Hong, and Hongyan Li. Hypergraph structure learning for hypergraph neural networks. In IJCAI, pages 1923-1929, 2022.
[36] Ross P Cameron. Explanation and plenitude in non-well-founded set theories. Philosophia Mathematica, 32(3):275-306, 2024.
[37] S Robinson Chellathurai and L Jesmalar. Core in rough graph and weighted rough graph. International Journal of Contemporary Mathematical Sciences, 11(6):251-265, 2016.
[38] Huakun Chen, Jingping Shi, Yongxi Lyu, and Qianlei Jia. A decision-making model with cloud model, z-numbers, and interval-valued linguistic neutrosophic sets. Entropy, 2024.
[39] Edward Cline, Brian J. Parshall, and Leonard L. Scott. Cohomology, hyperalgebras, and representations. Journal of Algebra, 63:98-123, 1980.
[40] Somen Debnath. Fuzzy hypersoft sets and its weightage operator for decision making. 2021.
[41] Irfan Deli and Serkan Karatas. Interval valued intuitionistic fuzzy parameterized soft set theory and its decision making. J. Intell. Fuzzy Syst., 30:2073-2082, 2016.
[42] NAIME DEMIRTAS, SABIR HUSSAIN, and ORHAN DALKILIC. New approaches of inverse soft rough sets and their applications in a decision making problem. Journal of applied mathematics & informatics, 38(3_4):335-349, 2020.
[43] R Aruna Devi and K Anitha. Construction of rough graph to handle uncertain pattern from an information system. arXiv preprint arXiv:2205.10127, 2022.
[44] G. Dhanalakshmi, S. Sandhiya, and Florentin Smarandache. Selection of the best process for desalination under a treesoft set environment using the multi-criteria decision-making method. International Journal of Neutrosophic Science, 2024.
[45] Reinhard Diestel. Graduate texts in mathematics: Graph theory.
[46] Reinhard Diestel. Graph theory 3rd ed. Graduate texts in mathematics, 173(33):12, 2005.
[47] Reinhard Diestel. Graph theory. Springer (print edition); Reinhard Diestel (eBooks), 2024.
[48] Azadeh Zahedi Khameneh etc. Multi-attribute decision-making based on soft set theory: a systematic review. Soft Computing, 23:6899-6920, 2018.
[49] Wenqi Fan, Yao Ma, Qing Li, Yuan He, Eric Zhao, Jiliang Tang, and Dawei Yin. Graph neural networks for social recommendation. In The world wide web conference, pages 417-426, 2019.
[50] Fatia Fatimah and José Carlos Rodriguez Alcantud. The multi-fuzzy n-soft set and its applications to decision-making. Neural Computing and Applications, 33:11437 - 11446, 2021.
[51] Fatia Fatimah, Fatia Fatimah, Dedi Rosadi, R. B. Fajriya Hakim, and José Carlos Rodriguez Alcantud. N-soft sets and their decision making algorithms. Soft Computing, 22:3829 - 3842, 2017.
[52] Feng Feng. Soft rough sets applied to multicriteria group decision making. 2022.
[53] Feng Feng, Xiaoyan Liu, Violeta Leoreanu-Fotea, and Young Bae Jun. Soft sets and soft rough sets. Information Sciences, 181(6):1125-1137, 2011.
[54] Ronald C. Freiwald. An introduction to set theory and topology. 2014.
[55] Takaaki Fujita. Antipodal turiyam graphs.
[56] Takaaki Fujita. Neutrosophic circular-arc graphs and proper circular-arc graphs.
[57] Takaaki Fujita. Bipolar turiyam graph and interval-valued turiyam graph, June 2024.
[58] Takaaki Fujita. Bunch graph in fuzzy, neutrosophic, and plithogenic graphs, June 2024. Preprint, File available.
[59] Takaaki Fujita. Claw-free graph and at-free graph in fuzzy, neutrosophic, and plithogenic graphs, June 2024. License: CC BY 4.0.
[60] Takaaki Fujita. A comprehensive discussion on fuzzy hypersoft expert, superhypersoft, and indetermsoft graphs. 2024.
[61] Takaaki Fujita. Contact graphs in fuzzy and neutrosophic graphs. Preprint, May 2024, File available, 2024.
[62] Takaaki Fujita. Domination number and chromatic number in turiyam and plithogenic graphs, July 2024. License: CC BY 4.0.
[63] Takaaki Fujita. Extension of bipolar, interval-valued, and complex neutrosophic soft expert graphs to general plithogenic soft rough expert mixed graphs. July 2024.
[64] Takaaki Fujita. General plithogenic soft rough graphs and some related graph classes, June 2024. License: CC BY 4.0.
[65] Takaaki Fujita. Interval graphs and proper interval graphs in fuzzy and neutrosophic graphs. Preprint, July 2024, File available, 2024.

[66] Takaaki Fujita. Mixed graph in fuzzy, neutrosophic, and plithogenic graphs. June 2024.

[67] Takaaki Fujita. Permutation graphs in fuzzy and neutrosophic graphs. Preprint, July 2024. File available.

[68] Takaaki Fujita. Plithogenic line graph, star graph, and regular graph. Journal of Graph Structures and Applications, 2024.

[69] Takaaki Fujita. Pythagorean, fermatean, and complex turiyam graphs: Relations with general plithogenic graphs, June 2024. License: CC BY 4.0.

[70] Takaaki Fujita. A review of the hierarchy of plithogenic, neutrosophic, and fuzzy graphs: Survey and applications. ResearchGate(Preprint), 2024.

[71] Takaaki Fujita. Short note of even-hole-graph for uncertain graph. Uncertain Graph Theory Journal, 2024.

[72] Takaaki Fujita. Short note of supertree-width and n-superhypertree-width. Neutrosophic Sets and Systems, 77:54-78, 2024.

[73] Takaaki Fujita. Some graph structures in fuzzy, neutrosophic, and plithogenic graphs. 2024.

[74] Takaaki Fujita. Study for general plithogenic soft expert graphs. June 2024. License CC BY 4.0.

[75] Takaaki Fujita. Survey of intersection graphs, fuzzy graphs and neutrosophic graphs. ResearchGate, July 2024.

[76] Takaaki Fujita. Survey of planar and outerplanar graphs in fuzzy and neutrosophic graphs. ResearchGate, July 2024.

[77] Takaaki Fujita. Survey of trees, forests, and paths in fuzzy and neutrosophic graphs. July 2024.

[78] Takaaki Fujita. Uncertain automata and uncertain graph grammar, June 2024. License CC BY 4.0.

[79] Takaaki Fujita. Uncertain labeling graphs and uncertain graph classes (with survey for various uncertain sets). July 2024.

[80] Takaaki Fujita. Unit disk graphs in fuzzy and neutrosophic graphs. June 2024. Available under CC BY 4.0 license.

[81] Takaaki Fujita. Vague soft expert graph and complex fuzzy soft expert graph. ResearchGate, June 2024. License CC BY 4.0.

[82] Takaaki Fujita. Various properties of various ultrafilters, various graph width parameters, and various connectivity systems. arXiv preprint arXiv:2408.02299, 2024.

[83] Fernando Gama, Joan Bruna, and Alejandro Ribeiro. Stability properties of graph neural networks. IEEE Transactions on Signal Processing, 68:5680-5695, 2020.

[84] Bobin George, S.P. George, Rajesh K. Thumbakara, and Jinta Jose. Delving into combined graphs and matrices in soft semigraphs. International Journal of Apllied Mathematics, 2024.

[85] Bobin George, Jinta Jose, and Rajesh K. Thumbakara. Cartesian product and composition of soft semigraphs. New Mathematics and Natural Computation, 2023.

[86] Bobin George, Jinta Jose, and Rajesh K. Thumbakara. Connectedness in soft semigraphs. New Math. Nat. Comput., 20:157-182, 2023.

[87] Bobin George, Jinta Jose, and Rajesh K. Thumbakara. Investigating the traits of soft semigraph-associated degrees. New Math. Nat. Comput., 20:647-663, 2023.

[88] Bobin George, Jinta Jose, and Rajesh K. Thumbakara. Eulerian and hamiltonian soft semigraphs. International Journal of Foundations of Computer Science, 2024.

[89] Bobin George, Jinta Jose, and Rajesh K Thumbakara. Modular product of soft directed graphs. TWMS Journal of Applied and Engineering Mathematics, 2024.

[90] Bobin George, Jinta Jose, and Rajesh K. Thumbakara. Soft semigraph isomorphisms: classification and characteristics. New Mathematics and Natural Computation, 2024.

[91] Bobin George, Rajesh K. Thumbakara, and Jinta Jose. Soft semigraphs and some of their operations. New Math. Nat. Comput., 19:369-385, 2022.

[92] Bobin George, Rajesh K Thumbakara, and Jinta Jose. Soft semigraphs and some of their operations. New Mathematics and Natural Computation, 19(02):369-385, 2023.

[93] Dimitis N. Georgiou and Athanasios C. Megaritis. Soft set theory and topology. Applied general topology, 15:93-109, 2014.

[94] Masoud Ghods, Zahra Rostami, and Florentin Smarandache. Introduction to neutrosophic restricted superhypergraphs and neutrosophic restricted superhypertrees and several of their properties. Neutrosophic Sets and Systems, 50:480-487, 2022.

[95] Jayanta Ghosh and Tapas Kumar Samanta. Hyperfuzzy sets and hyperfuzzy group. Int. J. Adv. Sci. Technol, 41:27-37, 2012.

[96] Daniela Gifu. Soft sets extensions: Innovating healthcare claims analysis. Applied Sciences, 14(19):8799, 2024.

[97] Ben Goertzel. Hyperset models of self, will and reflective consciousness. International Journal of Machine Consciousness, 3(01):19-53, 2011.

[98] Georg Gottlob, Nicola Leone, and Francesco Scarcello. Hypertree decompositions and tractable queries. In Proceedings of the eighteenth ACM SIGMOD-SIGACT-SIGART symposium on Principles of database systems, pages 21-32, 1999.

[99] Georg Gottlob, Nicola Leone, and Francesco Scarcello. Hypertree decompositions: A survey. In Mathematical Foundations of Computer Science 2001: 26th International Symposium, MFCS 2001 Mariánské Lázne, Czech Republic, August 27-31, 2001 Proceedings 26, pages 37-57. Springer, 2001.

[100] Georg Gottlob and Reinhard Pichler. Hypergraphs in model checking: Acyclicity and hypertree-width versus cliquewidth. SIAM Journal on Computing, 33(2):351-378, 2004.

[101] Salvatore Greco, Benedetto Matarazzo, and Roman Slowinski. Rough sets theory for multicriteria decision analysis. European journal of operational research, 129(1):1-47, 2001.

[102] Jonathan L Gross, Jay Yellen, and Mark Anderson. Graph theory and its applications. Chapman and Hall/CRC, 2018.

[103] Paul Richard Halmos. Naive set theory. van Nostrand, 1960.

[104] Mohammad Hamidi, Florentin Smarandache, and Elham Davneshvar. Spectrum of superhypergraphs via flows. Journal of Mathematics, 2022(1):9158912, 2022.

[105] Mohammad Hamidi, Florentin Smarandache, and Mohadeseh Taghinezhad. Decision Making Based on Valued Fuzzy Superhypergraphs. Infinite Study, 2023.
[106] Mohammad Hamidi and Mohadeseh Taghinezhad. Application of Superhypergraphs-Based Domination Number in Real World. Infinite Study, 2023.
[107] Wolfhard Hansen and Victor Klee. Intersection theorems for positive sets. Proceedings of the American Mathematical Society, 22(2):450-457, 1969.
[108] Aboul Ella Hassanien, Ajith Abraham, James F Peters, Gerald Schaefer, and Christopher Henry. Rough sets and near sets in medical imaging: A review. IEEE Transactions on Information Technology in Biomedicine, 13(6):955-968, 2009.
[109] Felix Hausdorff. Set theory, volume 119. American Mathematical Soc., 2021.
[110] Khizar Hayat, Muhammad Irfan Ali, Bing yuan Cao, and Xiaopeng Yang. A new type-2 soft set: Type-2 soft graphs and their applications. Adv. Fuzzy Syst., 2017:6162753:1-6162753:17, 2017.
[111] Tong He, Yong Chen, and Kaiquan Shi. Weighted rough graph and its application. In Sixth International Conference on Intelligent Systems Design and Applications, volume 1, pages 486-491. IEEE, 2006.
[112] Christopher James Henry. Near sets: theory and applications. University of Manitoba (Canada), 2011.
[113] Karel Hrbacek and Thomas Jech. Introduction to set theory, revised and expanded. 2017.
[114] R. Jahir Hussain and M. S. Afya Farhana. Fuzzy chromatic number of fuzzy soft cycle and complete fuzzy soft graphs. AIP Conference Proceedings, 2023.
[115] Sirus Jahanpanah and Roohallah Daneshpayeh. An outspread on valued logic superhyperalgebras. Facta Universitatis, Series: Mathematics and Informatics, 2024.
[116] Thomas Jech. Set theory: The third millennium edition, revised and expanded. Springer, 2003.
[117] Jinta Jose, Bobin George, and Rajesh K Thumbakara. Soft directed graphs, their vertex degrees, associated matrices and some product operations. New Mathematics and Natural Computation, 19(03):651-686, 2023.
[118] Jinta Jose, Bobin George, and Rajesh K Thumbakara. Corona product of soft directed graphs. New Mathematics and Natural Computation, pages 1-18, 2024.
[119] Jinta Jose, Bobin George, and Rajesh K Thumbakara. Soft graphs: A comprehensive survey. New Mathematics and Natural Computation, pages 1-52, 2024.
[120] Jinta Jose, Bobin George, Rajesh K Thumbakara, and Sijo P George. Understanding normal and restricted normal products in soft directed graphs. Journal of the Nigerian Society of Physical Sciences, pages 2156-2156, 2024.
[121] Young Bae Jun, Kul Hur, and Kyoung Ja Lee. Hyperfuzzy subalgebras of bck/bci-algebras. Annals of Fuzzy Mathematics and Informatics, 2017.
[122] Magdalena Kacprzak, Bartłomiej Starosta, and Katarzyna Węgrzyn-Wolska. Metasets and opinion mining in new decision support system. In Artificial Intelligence and Soft Computing: 14th International Conference, ICAISC 2015, Zakopane, Poland, June 14-18, 2015, Proceedings, Part II 14, pages 625-636. Springer, 2015.
[123] Hüseyin Kamac and Subramanian Petchimuthu. Bipolar n-soft set theory with applications. Soft Computing, 24:16727 - 16743, 2020.
[124] Hüseyin Kamacı. On hybrid structures of hypersoft sets and rough sets. International Journal of Modern Science and Technology, 6(4):69-82, 2021.
[125] Faruk Karaaslan and Naim Çağman. Bipolar soft rough sets and their applications in decision making. Afrika Matematika, 29:823-839, 2018.
[126] L Subha Lakshmi and NR Santhi Maheswari. A study on constant and regular hesitancy fuzzy soft graphs. International J. Math. Combin. Vol, 4:33-43, 2023.
[127] Azriel Levy. Basic set theory. Courier Corporation, 2012.
[128] Zhaowen Li and Tusheng Xie. The relationship among soft sets, soft rough sets and topologies. Soft Computing, 18(4):717-728, 2014.
[129] Decui Liang and Dun Liu. Systematic studies on three-way decisions with interval-valued decision-theoretic rough sets. Inf. Sci., 276:186-203, 2014.
[130] Decui Liang and Dun Liu. Deriving three-way decisions from intuitionistic fuzzy decision-theoretic rough sets. Inf. Sci., 300:28-48, 2015.
[131] Decui Liang and Dun Liu. A novel risk decision making based on decision-theoretic rough sets under hesitant fuzzy information. IEEE Transactions on Fuzzy Systems, 23:237-247, 2015.
[132] Dun Liu, Tianrui Li, and Da Ruan. Probabilistic model criteria with decision-theoretic rough sets. Inf. Sci., 181:3709-3722, 2011.
[133] Dun Liu, Yiyu Yao, and Tianrui Li. Three-way investment decisions with decision-theoretic rough sets. Int. J. Comput. Intell. Syst., 4:66-74, 2011.
[134] Qi Liu, Maximilian Nickel, and Douwe Kiela. Hyperbolic graph neural networks. Advances in neural information processing systems, 32, 2019.
[135] Xiaorui Liu, Wei Jin, Yao Ma, Yaxin Li, Hua Liu, Yiqi Wang, Ming Yan, and Jiliang Tang. Elastic graph neural networks. In International Conference on Machine Learning, pages 6837-6849. PMLR, 2021.
[136] Nuno Maia and Matteo Nizzardo. Identity and extensionality in boffa set theory. Philosophia Mathematica, 32(1):115-123, 2024.
[137] Pradip Kumar Maji, Ranjit Biswas, and A Ranjan Roy. Soft set theory. Computers & mathematics with applications, 45(4-5):555-562, 2003.

[138] Nivetha Martin and Florentin Smarandache. Introduction to combined plithogenic hypersoft sets. Neutrosophic Sets and Systems, 35:503-510, 2020.

[139] Mona Mohamed, Alaa Elmor, Florentin Smarandache, and Ahmed A Metwaly. An efficient superhypersoft framework for evaluating llms-based secure blockchain platforms. Neutrosophic Sets and Systems, 72:1-21, 2024.

[140] Dmitri A. Molodtsov. Soft set theory-first results. Computers & Mathematics With Applications, 37:19-31, 1999.

[141] Dmitriy Molodtsov. Soft set theory-first results. Computers & mathematics with applications, 37(4-5):19-31, 1999.

[142] Sagvan Y Musa. N-bipolar hypersoft sets: Enhancing decision-making algorithms. Plos one, 19(1):e0296396, 2024.

[143] Sagvan Y Musa. N-bipolar hypersoft sets: Enhancing decision-making algorithms. Plos one, 19(1):e0296396, 2024.

[144] Sagvan Y. Musa and Baravan A. Asaad. Bipolar hypersoft sets. Mathematics, 2021.

[145] Sagvan Y. Musa and Baravan A. Asaad. Topological structures via bipolar hypersoft sets. Journal of Mathematics, 2022.

[146] Sagvan Y Musa and Baravan A Asaad. Topological structures via bipolar hypersoft sets. Journal of Mathematics, 2022(1):2896053, 2022.

[147] Sagvan Y Musa and Baravan A Asaad. A novel approach towards parameter reduction based on bipolar hypersoft set and its application to decision-making. Neutrosophic Sets and Systems, 55(1):33, 2023.

[148] Sagvan Y Musa, Ramadhan A Mohammed, and Baravan A Asaad. N-hypersoft sets: An innovative extension of hypersoft sets and their applications. Symmetry, 15(9):1795, 2023.

[149] SY Musa and BA Asaad. Bipolar m-parametrized n-soft sets: a gateway to informed decision-making. Journal of Mathematics and Computer Science, 36(1):121-141, 2025.

[150] R Noor, I Irshad, and I Javaid. Soft rough graphs. arXiv preprint arXiv:1707.05837, 2017.

[151] Rehana Noor, Iqra Irshad, and Imran Javaid. Soft rough graphs. arXiv: General Mathematics, 2017.

[152] Eugenio G Omodeo and Alberto Policriti. Solvable set/hyperset contexts: I. some decision procedures for the pure, finite case. Communications on Pure and Applied Mathematics, 48(9):1123-1155, 1995.

[153] János Pach. Geometric graph theory. Handbook of Discrete and Computational Geometry, 2nd Ed., pages 219-238, 2004.

[154] Paolo Paolo. A study using treesoft set and neutrosophic sets on possible soil organic transformations in urban agriculture systems. International Journal of Neutrosophic Science.

[155] CR Parvathy and A Sofia. A study on new classes of binary soft sets in topological rough approximation space. Bulletin of the Karaganda University. Mathematics series, 112(4):79-94, 2023.

[156] Purvag Patel, Shahram Rahimi, and Elham Khorasani. Applied z-numbers. In 2015 Annual Conference of the North American Fuzzy Information Processing Society (NAFIPS) held jointly with 2015 5th World Conference on Soft Computing (WConSC), pages 1-6. IEEE, 2015.

[157] Vasile Patrascu. Rough sets on four-valued fuzzy approximation space. In 2007 IEEE International Fuzzy Systems Conference, pages 1-5. IEEE, 2007.

[158] Zdzislaw Pawlak. Rough set theory and its applications to data analysis. Cybernetics & Systems, 29(7):661-688, 1998.

[159] Zdzislaw Pawlak, Lech Polkowski, and Andrzej Skowron. Rough set theory. KI, 15(3):38-39, 2001.

[160] Zdzislaw Pawlak and Andrzej Skowron. Rudiments of rough sets. Information sciences, 177(1):3-27, 2007.

[161] Nicole Pearcy, Jonathan J Crofts, and Nadia Chuzhanova. Hypergraph models of metabolism. International Journal of Biological, Veterinary, Agricultural and Food Engineering, 8(8):752-756, 2014.

[162] Sriram Pemmaraju and Steven Skiena. Computational discrete mathematics: Combinatorics and graph theory with mathematica. Cambridge university press, 2003.

[163] Jean-Paul Penot. Positive sets, conservative sets and dissipative sets. Journal of Convex Analysis, 16(3-4):973-986, 2009.

[164] J. M. Pérez-Izquierdo. Algebras, hyperalgebras, nonassociative bialgebras and loops. Advances in Mathematics, 208:834-876, 2007.

[165] James F Peters. Near sets. special theory about nearness of objects. Fundamenta Informaticae, 75(1-4):407-433, 2007.

[166] ames F Peters and Piotr Wasilewski. Foundations of near sets. Information Sciences, 179(18):3091-3109, 2009.

[167] Jim Peters and Som Naimpally. Applications of near sets. Amer. Math. Soc. Notices, 59(4), 2012.

[168] Surapati Pramanik, Partha Pratim Dey, and Bibhas Chandra Giri. Topsis for single valued neutrosophic soft expert set based multi-attribute decision making problems. 2015.

[169] Rukchart Prasertpong and Aiyared Iampan. Approximation approaches for rough hypersoft sets based on hesitant bipolar-valued fuzzy hypersoft relations on semigroups. Journal of Mathematics and Computer Science, 2022.

[170] Yuhua Qian, Xinyan Liang, Guoping Lin, Qian Guo, and Jiye Liang. Local multigranulation decision-theoretic rough sets. Int. J. Approx. Reason., 82:119-137, 2017.

[171] Atiqe Ur Rahman, Muhammad Haris Saeed, Ebenezer Bonyah, and Muhammad Arshad. Graphical exploration of generalized picture fuzzy hypersoft information with application in human resource management multiattribute decisionmaking. Mathematical Problems in Engineering, 2022.

[172] Atiqe Ur Rahman, Muhammad Haris Saeed, and Florentin Smarandache. Convex and concave hypersoft sets with some properties. 2020.

[173] PB Remya and A Francina Shalini. Vague binary soft sets and their properties. International Journal of Engineering, Science and Mathematics, 7(11):56-73, 2018.

[174] Ivo G. Rosenberg. An algebraic approach to hyperalgebras. Proceedings of 26th IEEE International Symposium on Multiple-Valued Logic (ISMVL'96), pages 203-207, 1996.

[175] Muhammad Saeed, Atiqe Ur Rahman, and Muhammad Arshad. A study on some operations and products of neutrosophic hypersoft graphs. Journal of Applied Mathematics and Computing, 68(4):2187-2214, 2022.
[176] Muhammad Saeed, Muhammad Khubab Siddique, Muhammad Ahsan, Muhammad Rayees Ahmad, and Atiqe Ur Rahman. A novel approach to the rudiments of hypersoft graphs. Theory and Application of Hypersoft Set, Pons Publication House, Brussel, pages 203-214, 2021.
[177] Muhammad Haris Saeed, Atiqe Ur Rahman, and Muhammad Arshad. A novel approach to neutrosophic hypersoft graphs with properties. 2021.
[178] Muhammad Haris Saeed, Atiqe Ur Rahman, and Muhammad Arshad. A novel approach to neutrosophic hypersoft graphs with properties. 2021.
[179] Mehmet Sahin, Shawkat Alkhazaleh, and Vakkas Ulucay. Neutrosophic soft expert sets. Applied Mathematics-a Journal of Chinese Universities Series B, 06:116-127, 2015.
[180] E Sampathkumar, CM Deshpande, BY Bam, L Pushpalatha, and V Swaminathan. Semigraphs and their applications. Report on the DST Project, 2000.
[181] Muhammad Saqlain, Naveed Jafar, Sana Moin, Muhammad Farhan Saeed, and Said Broumi. Single and multi-valued neutrosophic hypersoft set and tangent similarity measure of single valued neutrosophic hypersoft sets. Neutrosophic Sets and Systems, 32:20, 2020.
[182] Franco Scarselli, Marco Gori, Ah Chung Tsoi, Markus Hagenbuchner, and Gabriele Monfardini. The graph neural network model. IEEE transactions on neural networks, 20(1):61-80, 2008.
[183] Muhammad Shabir, Muhammad Irfan Ali, and Tanzeela Shaheen. Another approach to soft rough sets. KnowledgeBased Systems, 40:72-80, 2013.
[184] Nasir Shah and Asim Hussain. Neutrosophic soft graphs. viXra, 2016.
[185] Sundas Shahzadi, Musavarah Sarwar, and Muhammad Akram. Decision-making approach with fuzzy type-2 soft graphs. Journal of Mathematics, 2020(1):8872446, 2020.
[186] R. Sivasamy and M. Mohammed Jabarulla. Products on interval valued pythagorean fuzzy soft graphs. Journal of Interdisciplinary Mathematics, 2024.
[187] F. Smarandache. Introduction to superhyperalgebra and neutrosophic superhyperalgebra. Journal of Algebraic Hyperstructures and Logical Algebras, 2022.
[188] Florentin Smarandache. A unifying field in logics: Neutrosophic logic. In Philosophy, pages 1-141. American Research Press, 1999.
[189] Florentin Smarandache. Neutrosophic set-a generalization of the intuitionistic fuzzy set. International journal of pure and applied mathematics, 24(3):287, 2005.
[190] Florentin Smarandache. Neutrosophic physics: More problems, more solutions. 2010.
[191] Florentin Smarandache. Neutrosophic Overset, Neutrosophic Underset, and Neutrosophic Offset. Similarly for Neutrosophic Over-/Under-/Off-Logic, Probability, and Statistics. Infinite Study, 2016.
[192] Florentin Smarandache. Extension of soft set to hypersoft set, and then to plithogenic hypersoft set. Neutrosophic Sets and Systems, 22:168-170, 2018.
[193] Florentin Smarandache. n-superhypergraph and plithogenic n-superhypergraph. Nidus Idearum, 7:107-113, 2019.
[194] Florentin Smarandache. Extension of HyperGraph to n-SuperHyperGraph and to Plithogenic n-SuperHyperGraph, and Extension of HyperAlgebra to n-ary (Classical-/Neutro-/Anti-) HyperAlgebra. Infinite Study, 2020.
[195] Florentin Smarandache. Introduction to the n-SuperHyperGraph-the most general form of graph today. Infinite Study, 2022.
[196] Florentin Smarandache. Decision making based on valued fuzzy superhypergraphs. 2023.
[197] Florentin Smarandache. Foundation of the superhypersoft set and the fuzzy extension superhypersoft set: A new vision. Neutrosophic Systems with Applications, 11:48-51, 2023.
[198] Florentin Smarandache. New types of soft sets: Hypersoft set, indetermsoft set, indetermhypersoft set, and treesoft set. International Journal of Neutrosophic Science, 2023.
[199] Florentin Smarandache. New types of soft sets" hypersoft set, indetermsoft set, indetermhypersoft set, and treesoft set": An improved version. Neutrosophic Systems with Applications, 2023.
[200] Florentin Smarandache. SuperHyperFunction, SuperHyperStructure, Neutrosophic SuperHyperFunction and Neutrosophic SuperHyperStructure: Current understanding and future directions. Infinite Study, 2023.
[201] Florentin Smarandache. Foundation of superhyperstructure & neutrosophic superhyperstructure. Neutrosophic Sets and Systems, 63(1):21, 2024.
[202] Florentin Smarandache. Treesoft set vs. hypersoft set and fuzzy-extensions of treesoft sets. HyperSoft Set Methods in Engineering, 2024.
[203] Florentin Smarandache and Said Broumi. Neutrosophic graph theory and algorithms. IGI Global, 2019.
[204] Florentin Smarandache, Memet Şahin, Derya Bakbak, Vakkas Uluçay, and Abdullah Kargın. Neutrosophic SuperHyperAlgebra and New Types of Topologies. Infinite Study, 2023.
[205] Florentin Smarandache and AA Salama. Neutrosophic crisp set theory. 2015.
[206] Bartlomiej Starosta and Witold Kosiński. Meta sets-another approach to fuzziness. In Views on Fuzzy Sets and Systems from Different Perspectives: Philosophy and Logic, Criticisms and Applications, pages 509-532. Springer, 2009.
[207] Bartlomiej Starosta and Witold Kosiński. Metasets, intuitionistic fuzzy sets and uncertainty. In Artificial Intelligence and Soft Computing: 12th International Conference, ICAISC 2013, Zakopane, Poland, June 9-13, 2013, Proceedings, Part I 12, pages 388-399. Springer, 2013.

[208] V. S. Subha and R. Selvakumar. An innovative electric vehicle selection with multi-criteria decision-making approach in indian brands on a neutrosophic hypersoft rough set by using reduct and core. 2024 Fourth International Conference on Advances in Electrical, Computing, Communication and Sustainable Technologies (ICAECT), pages 1-7, 2024.

[209] V. S. Subha and R. Selvakumar. A novel approach to plithogenic neutrosophic hypersoft rough set and its application to decision making problem with reference to bakery industry. Communications in Mathematics and Applications, 2024.

[210] Mitsuhiro Takeuchi and Mitsuhiro Takeuchi. Tangent coalgebras and hyperalgebras i. Japanese journal of mathematics :transactions and abstracts, 42:1-143, 1974.

[211] Kazushige Terui. Light affine set theory: A naive set theory of polynomial time. Studia Logica, 77:9-40, 2004.

[212] He Tong, Xue Peijun, and Shi Kaiquan. Application of rough graph in relationship mining. Journal of Systems Engineering and Electronics, 19(4):742-747, 2008.

[213] Vakkas Ulucay. Q-neutrosophic soft graphs in operations management and communication network. Soft Computing, 25:8441-8459, 2021.

[214] Yinyu Wang, Azmat Hussain, Tahir Mahmood, Muhammad Irfan Ali, Hecheng Wu, and Yun Jin. Decision-making based on q-rung orthopair fuzzy soft rough sets. Mathematical Problems in Engineering, 2020.

[215] Douglas Brent West et al. Introduction to graph theory, volume 2. Prentice hall Upper Saddle River, 2001.

[216] Lingfei Wu, Yu Chen, Kai Shen, Xiaojie Guo, Hanning Gao, Shucheng Li, Jian Pei, Bo Long, et al. Graph neural networks for natural language processing: A survey. Foundations and Trends ^® in Machine Learning, 16(2):119-328, 2023.

[217] Zonghan Wu, Shirui Pan, Fengwen Chen, Guodong Long, Chengqi Zhang, and S Yu Philip. A comprehensive survey on graph neural networks. IEEE transactions on neural networks and learning systems, 32(1):4-24, 2020.

[218] Keyulu Xu, Weihua Hu, Jure Leskovec, and Stefanie Jegelka. How powerful are graph neural networks? arXiv preprint arXiv:1810.00826, 2018.

[219] Xibei Yang, Dongjun Yu, Jingyu Yang, and Chen Wu. Generalization of soft set theory: from crisp to fuzzy case. In Fuzzy Information and Engineering: Proceedings of the Second International Conference of Fuzzy Information and Engineering (ICFIE), pages 345-354. Springer, 2007.

[220] Yiyu Yao and Jilin Yang. Granular rough sets and granular shadowed sets: three-way approximations in pawlak approximation spaces. International Journal of Approximate Reasoning, 2021.

[221] Yiyu Yao and Jilin Yang. Granular rough sets and granular shadowed sets: Three-way approximations in pawlak approximation spaces. Int. J. Approx. Reason., 142:231-247, 2022.

[222] Adem Yolcu, Aysun Benek, and Taha Yasin Öztürk. A new approach to neutrosophic soft rough sets. Knowledge and Information Systems, 65:2043-2060, 2023.

[223] Adem Yolcu, Florentin Smarandache, and Taha Yasin Ozturk. Intuitionistic fuzzy hypersoft sets. 2021.

[224] Lotfi A Zadeh. Fuzzy sets. Information and control, 8(3):338-353, 1965.

[225] Lotfi A Zadeh. A fuzzy-set-theoretic interpretation of linguistic hedges. 1972.

[226] Lotfi A Zadeh. Fuzzy sets and their application to pattern classification and clustering analysis. In Classification and clustering, pages 251-299. Elsevier, 1977.

[227] Lotfi A Zadeh. Fuzzy sets versus probability. Proceedings of the IEEE, 68(3):421-421, 1980.

[228] Lotfi A Zadeh. Fuzzy logic, neural networks, and soft computing. In Fuzzy sets, fuzzy logic, and fuzzy systems: selected papers by Lotfi A Zadeh, pages 775-782. World Scientific, 1996.

[229] Lotfi A Zadeh. Fuzzy sets and information granularity. In Fuzzy sets, fuzzy logic, and fuzzy systems: selected papers by Lotfi A Zadeh, pages 433-448. World Scientific, 1996.

[230] Lotfi A. Zadeh. A note on z-numbers. Inf. Sci., 181:2923-2932, 2011.

[231] Lotfi Asker Zadeh. Fuzzy sets as a basis for a theory of possibility. Fuzzy sets and systems, 1(1):3-28, 1978.

[232] Jianming Zhan, Muhammad Irfan Ali, and Nayyar Mehmood. On a novel uncertain soft set model: Z-soft fuzzy rough set model and corresponding decision making methods. Appl. Soft Comput., 56:446-457, 2017.

[233] Jianming Zhan and Kuanyun Zhu. A novel soft rough fuzzy set: Z-soft rough fuzzy ideals of hemirings and corresponding decision making. Soft Computing, 21:1923-1936, 2016.

[234] Jianming Zhan, Kuanyun Zhu, and Muhammad Irfan Ali. Study on z-soft fuzzy rough sets in hemirings. J. Multiple Valued Log. Soft Comput., 30:359-377, 2018.

[235] Di Zhang, Pi-Yu Li, and Shuang An. N-soft rough sets and its applications. Journal of Intelligent & Fuzzy Systems, 40(1):565-573, 2021.

[236] Ping Zhang and Gary Chartrand. Introduction to graph theory. Tata McGraw-Hill, 2:2-1, 2006.

[237] Jie Zhou, Ganqu Cui, Shengding Hu, Zhengyan Zhang, Cheng Yang, Zhiyuan Liu, Lifeng Wang, Changcheng Li, and Maosong Sun. Graph neural networks: A review of methods and applications. AI open, 1:57-81, 2020.

[238] Kuanyun Zhu, Jingru Wang, and Yongwei Yang. A study on z -soft fuzzy rough sets in bc i -algebras. 2020.