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The Order Involving the Neutrosophic Hyperstructures, the Construction and Setting up of a Typical Neutrosophic Group

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Abstract

The concepts involving the n^{th}-Power Set of a Set, SuperHyperOperation, SuperHyperAxiom, SuperHyperAlgebra, and their corresponding Neutrosophic SuperHyperOperation, Neutrosophic SuperHyper-Axiom and Neutrosophic SuperHyperAlgebra have been considered and treated by Florentin Smarandache and it was actually ascertained that in general, in any field of knowledge, one actually encounters SuperHyperStructures (or more accurately (m; n)-SuperHyperStructures). In this paper, efforts are intensified as much as possible to explicitly and clearly formulate the cardinality of the nth degree of the power set of a given order. We also made some constructions and setting up of typical neutrosophic groups.

Keywords: Soft Set; HyperSoft Set; IndetermSoft Set; IndetermHyperSoft Set; IndetermSoft Operators; IndetermSoft Algebra; Indeterminacy; Neutrosophic Groups.

1 | Introduction

The notion of Neutrosophy as a new branch of philosophy throughout the realm of the mathematical sciences has been believed to have its basic and foundational introduction from Florentin Smarandache. Neutrosophy is the base of neutrosophic logic which is an extension of the fuzzy logic in which indeterminacy is included. In the neutrosophic logic, each proposition is estimated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F. Since the world is full of indeterminacy, several real-world problems involving indeterminacy arising from law, medicine, sociology, psychology, politics, engineering, industry, economics, management and decision making, finance, stocks and share, meteorology, artificial intelligence, IT, communication etc can be solved by neutrosophic logic. This has over time extended to several mathematically viable and applicable levels and areas. These areas include but are not limited to SuperHyperAlgebra and Neutrosophic SuperHyperAlgebra. Our hope and expectations are that these new fields of SuperHyperAlgebra, its outcomes as well as its extensions will inspire researchers to study several interesting particular cases, such as the SuperHyperGroupoid, SuperHyperMonoid, SuperHyperSemigroup, SuperHyperGroup, SuperHyperRing, SuperHyperVectorSpace, and other relevant concepts.

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2 | Orders and Cardinalities Involving the SuperHyperAgebra and Neutrosophic SuperHyperAlgebra

The concepts of the SuperHyperAgebra and Neutrosophic SuperHyperAlgebra were introduced and developed by Smarandache [3-5].

Definition 1 [1]: Let U be a universe of discourse and H a non-empty set, H ⊂ U. A classical Binary HyperOperation α_2^* can be defined as α_2^* : $H^2 \rightarrow P_*(H)$, where H is a discrete or continuous set, and $P_*(H)$ is the power set of H without the empty set ∅, or $P_*(H) = P(H) \setminus \{\emptyset\}$. A classical m-ary HyperOperation o_m^* is defined as $o_m^* : H^m \rightarrow P_*(H)$, for integer m ≥ 1. For m = 1, one gets a Unary HyperOperation. The classical HyperStructures are structures endowed with classical HyperOperations. It was Marty F. who introduced the classical HyperOperations and classical HyperStructures in 1934 [6].

Definition 2: (The n^{th}-Power Set of a Set)

In [3-5] The n^{th}-Powerset of a Set was introduced. Hence, $P^n(H)$, is considered as the n^{th}-Powerset of the Set H, for integer n ≥ 1. It is then recursively defined as $P^2(H) = P(P(H))$, $P^3(H) = P(P^2(H)) = P(P(P(H)))$, · · · ,$P^n(H) = P(P^{(n-1)}(H))$, where $P^o(H) = H$, and $P^1(H) = P(H)$.

The n^{th}-Powerset of a Set better reflects our complex reality, since a set H (that may represent a group, a society, a country, a continent, etc.) of elements (such as people, objects, and in general any items) is organized onto subsets P(H), and these subsets are again organized onto subsets of subsets P(P(H)), and so on. That's our world. In the classical HyperOperation and classical HyperStructures, the empty set ∅ does not belong to the power set, or $P_*(H) = P(H) \setminus \{\emptyset\}$. However, in the real world, we encounter many situations when a HyperOperation ∘ is indeterminate, for example, a ∘ b = ∅ (unknown, or undefined), or partially indeterminate, for example, c ∘ d = {[0:2; 0:3]; ∅}. In our everyday lives, many more operations and laws have some degree of indeterminacy (vagueness, unclearness, unknowingness, contradiction, etc.), than those that are determinate. Hence, Florentin in 2016 extended the classical HyperOperation to the Neutrosophic Hyper-Operation, by taking the whole power P(H) (that includes the empty-set ∅ as well), instead of $P_*(H)$ (that does not include the empty-set ∅).

Let $P^n{}_*(H)$ be the n^{th}-Powerset of the set H such that none of P(H), $P^2(H)$, …, $P^n(H)$ contain the empty set ∅.

Our main focus here is to determine the cardinality and order involving some of the neutrosophic hyperstructures.

Theorem 1 [2]: Let A be a set such that |A| = n. If ℘(A) represents the power set of A (i.e. the family of all the subsets of A), then $|\wp(A)| = 2^n$.

Proof:

Consider the table below:

Number of elements in a subset	0	1	2	3	4	5	. . .	n -1	n
Number of subsets	${}^nc_o$	${}^nc_1$	${}^nc_2$	${}^nc_3$	${}^nc_4$	${}^nc_5$	. . .	${}^nc_{n-1}$	${}^nc_n$

From the table, te number of subsets of A is given by: ${}^nc_o + {}^nc_1 + {}^nc_2 + \ldots + {}^nc_{n-1} + {}^nc_n = \sum_{k=o}^{n} {}^nC_k$.

Now, observe that: $(a + b)^n = {}^nc_oa^n + {}^nc_1a^{n-1}b + {}^nc_2a^{n-2}b^2 + {}^nc_3a^{n-3}b^3 + \ldots + {}^nc_ka^{n-k}b^k + \ldots + {}^nc_{n-1}ab^{n-1} + b^n$.

Now, set a = b = 1, we have that:

$(1 + 1)^n = {}^nc_o + {}^nc_1 + {}^nc_2 + {}^nc_3 + \ldots + {}^nc_k + \ldots + {}^nc_{n-1} + {}^nc_n. = \sum_{k=o}^{n} {}^nC_k$.

Therefore, $\sum_{k=o}^{n} {}^{n}C_k = {}^{n}C_o + {}^{n}C_1 + {}^{n}C_2 + {}^{n}C_3 + \ldots + {}^{n}C_k + \ldots + {}^{n}C_{n-1} + {}^{n}C_n. = 2^n = |\wp(A)|$

Let $A_o = \{\} = \Phi$ {the null set}. Then, it is very clear that the power set of A_o denoted by $\wp(A_o) = \{\Phi\}$

Now, set $A_1 = \wp(A_o) = \{\Phi\}$, such that $|A_1| = |\wp(A_o)| = 1$.

The same way, $A_2 = \wp(\wp(A_o)) = \wp^2(\wp(A_o))$ such that

$|A_2| = 2^1 = 2$.

We have that $A_n = \wp^n(\wp(A_o))$

Proposition 1: If $A_n = \wp^n(\wp(A_o))$, then $|A_n| = 2^{|An-1|} - (n-2)$

$= 2^{|A\,n-1|} + 2 - n$ for $n \geq 2$

Proof: By proper application of the theorem given above, we are going to have that

$|A_n| = {}^{|An-1|}C_o + ({}^{n}C_1 - 1) + ({}^{n}C_2 - 1) + ({}^{n}C_3 - 1) + \ldots + ({}^{n}C_k - 1) + \ldots + ({}^{n}C_{n-2} - 1) + {}^{n}C_n v = \sum_{k=o}^{n} |A_{n-1}|_{C_k}$ $- 1(n-2)$

$= 2 - n + \sum_{k=o}^{n} |A_{n-1}|_{C_k}$.

Proposition 2: If $A_n = \wp(A_{n-1})$ and $|A_n| = n \geq 1$, then $|A_n| = 2^{|An|} - {}^{|An|}C_o + (2^{|An-2|} - |A_{n-2}|$.

Proof:

$|A_n| = 2^{|An-1|} - {}^{|An|}C_o - \sum_{k=2}^{|A_{n-2}|} |A_{n-2}|_{C_k}$

$= 2^{|An|} - {}^{|An|}C_o + (2^{|An-2|} - |A_{n-2}|$.

Now, we are about to deal with the case without the empty (null set)

Proposition 3: Set $A_o = \{a\}$, then, $\wp *_{(A_n)} = 1$

Proof:

Since $A_o = \{a\}$, $|A_o| = 1$. Now, since the subsets are without the null set, $\wp *_{(A_o)} = \{A_o\} = A_1$ say. Implying that $|A_1| = 1$. We also have that

$\wp *_{(A_1)} = \{A_1\} = A_2$ say, such that $|A_2| = 1$. And that $\wp *_{(A_2)} = \{A_2\} = A_3$ say, such that $|A_3| = 1$. The process continues the same way, and we finally have that: $\wp *_{(A_n)} = \{A_n\} = A_{n+1}$ say, such that $|A_{n+1}| = \wp *_{(A_n)} = 1$

Proposition 4: In the foregoing proposition, suppose that $|A_o| \geq 2$, and let $A_{n+1} = \wp *_{(A_n)}$, then $|A_n| = 2^{|An-1|} - 1 - \sum_{k=2}^{|A_{n-2}|} |A_{n-2}|_{C_k}$.

Proof:

$|A_n| = 2^{|An-1|} - \sum_{k=2}^{|A_{n-2}|} |A_{n-2}|_{C_k} = 2^{|An-1|} - \sum_{k=2}^{|A_{n-2}|} |A_{n-2}|_{C_k} - 1$

$= 2^{|An-1|} - 1 - \sum_{k=2}^{|A_{n-2}|} |A_{n-2}|_{C_k}$.

3 | The Construction and Setting up of A Typical Neutrosophic Group

Ever since the concepts and general idea of Neutrosophy were introduced and established by Florentin Smarandache in 1998, other developments and several advancements have emerged [7, 9-11]. Also, Agboola was able to define and establish some basic properties for the neutrosophic groups [8]. Notwithstanding, not much has been said or done about the creation of typical neutrosophic groups. This aspect of the paper is thus designed to fill the obvious vacuum and the conspicuous gap by diligently considering how various typical neutrosophic groups could be successfully constructed. Examples and various relevant illustrations have been put in place to establish our definitions, proofs, and clarifications.

Definition 3. A neutrosophic number is of the form $\boldsymbol{a+bI}$, where $\boldsymbol{a}, \boldsymbol{b} \in \mathbb{R}$ $\boldsymbol{or}$ $\mathbb{C}$ and $\boldsymbol{I}$ is referred to as the neutrosophic indeterminacy.

3.1 | Operations on Neutrosophic Numbers and Elements

3.1.1 | Neutrosophic Numbers

A classical Neutrosophic Number has the standard form: $a+bI$, where a, and b are real or complex coefficients, and I = indeterminacy, such $0{\cdot}I = 0$ and I. I = $I^2 = I$. It results that I.I.I. . . .I (n times) = $I^n = I$ and I + I = 2I , and by extension, I +I + I + I+ I +I . . . I (n copies) = nI for all positive integers n [3]. Also, it should be noted that: Now, given any two neutrosophic numbers, x = a + bI, and y = c + dI. Addition and subtraction operations are done componentwise. Thus, x $\pm$ y = (a $\pm$ c) + (b $\pm$ d)I. The multiplication operation is done termwise. Thus, (a + bI). (c + dI) = ac + adI + bcI + bdI = ac + (ad + bc + bd)I.

In general, every binary operation defined on the neutrosophic set of numbers must follow the definitions and properties stipulated by the binary operations.

Definition 4. Let S be any set, of neutrosophic numbers or elements such that a binary operation is defined on S. Then, S is called a neutrosophic groupoid. This is denoted by (S, *). A neutrosophic groupoid that is associative is called a neutrosophic semigroup. A neutrosophic semigroup with identity element {e} is called a neutrosophic monoid. And finally, a neutrosophic monoid in which every element has an inverse is called a neutrosophic group.

Definition 5. Let G be any group of sets of elements as defined above, and I, be an indeterminacy, then, there exists a group G∪I which is the union of the group and the indeterminacy. By representation and notation, G(I) = G∪I. This is known as the neutrosophic group.

Definition 6. Suppose there exists a neutrosophic group H(I) = H∪I such that H(I) ⊂ G(I). Then, H(I) is called the neutrosophic subgroup of the neutrosophic group G(I). A neutrosophic group G(I) is said to be cyclic if there exists an element x = a + bI such that G(I) is generated by x, and we write G(I) = $< x > = < a + bI >$.

Definition 7 (Idempotent element). An element (a + bI) = T ∈ G(I) is called an idempotent element in G(I) if $T^2 = T$ i.e. $T^2 = (a + bI)^2 = a + bI \in G(I)$.

In this case, $a^2 + (2ab + b^2)I = a + bI$. If we equate components on both sides, we are going to have that $a^2 = a$, and $2ab + b^2 = b$. $a^2 = a \Rightarrow a(a - 1) = 0 \Rightarrow$ either a = 0 or a = 1.

Also, $2ab + b^2 = b \Rightarrow 2ab + b^2 - b = 0 \Rightarrow b^2 + 2ab - b = 0 \Rightarrow b(b + 2a - 1) = 0$

Case 1. If a = 0, then, $b(b - 1) = 0 \Rightarrow b = 0$ or b = 1.

Case 2. If a = 1 then $b(b + 1) = 0 \Rightarrow b = 0$ or b = -1

Possibilities for the values of T: a + bI

a = 0, b = 0 ⇒ T = 0 + oI = 0 (a trivial case)

a = 0, b = 1 ⇒ T = 0 + I = I

a = 1, b = 0 ⇒ T = 1 + 0I = 1 (also, a trivial case)

a = 1, b = - 1 ⇒ T = 1 – I.

Hence, we have the following theorem 2:

Theorem 2. Let G(I) be a neutrosophic group. Then, the idempotent non–trivial elements of order two are given by: $\mathbb{I}_2$ = {I, I – 1} | $\mathbb{I}_2$ | = 2

Proof:

i). By our convention of the indeterminacy I, $I^2 = I$

ii). $(1 – I)^2 = (1 – I)(1 – I) = 1 – 2I + I = 1 – I$

3.2 | The Neutrosophic Group of A Given Order

Let G(I) = x = $< \boldsymbol{a+bI} >$ i.e. G is generated by x = $\{\boldsymbol{a+bI}\}$. Since G is a group, x in G must possess an inverse element. Hence, we seek an element y such that xy = yx = 1 ∈ G (the identity element of G). By this, both x, y ∈ G. Thus, let y = c + dI and we have that (a + bI) (c + dI) = 1.

By expansion, (a + bI) (c + dI) = = ac + [ad + bc + bd] I = 1. ⇒ ac = 1 or c = $\frac{\boldsymbol{1}}{\boldsymbol{a}}$, in the first part.

In the second part, ad + bc + bd = 0. By substituting for c, we have ad + b$\frac{\boldsymbol{1}}{\boldsymbol{a}}$ + bd = 0. ⇒ d = $\frac{\boldsymbol{-b}}{\boldsymbol{a(b+a)}}$.

We must have that G(I) = { 1 , a +bI , $\frac{\boldsymbol{1}}{\boldsymbol{a}}$ - $\frac{\boldsymbol{b}}{\boldsymbol{a(b+a)}}$ I | a ≠ - b , a ≠ 0}.

3.2.1 | The Neutrosophic Group Whose Each Element is of Order Two

Suppose that we are to construct a neutrosophic group having every element of order two. Then we must have that (a + bI)2 = 1 ⇒ a2 + [2ab + b2] I = 1 ⇒ a2 = 1, and b2 + 2ab = 0. We have that a = ±1, and b(b + 2a) = 0. And since b ≠ 0 for a non–trivial case, we must have that b = - 2a.

⇒ G(I) = { 1, a -2aI , $\frac{\boldsymbol{1}}{\boldsymbol{a}}$ - $\frac{\boldsymbol{2}}{\boldsymbol{a}}$ I | a = ±1, a ≠ 0 }= < 1 -2I, - 1 + 2I > = {1, - 1, 1 -2I, - 1 + 2I}.

Here, G(I) = { 1, - 1, 1 -2I, - 1 + 2I }, G(I) ≅ $\mathbb{Z}_2 \boldsymbol{x} \mathbb{Z}_2$ and |G(I)|= | $\mathbb{Z}_2 \boldsymbol{x} \mathbb{Z}_2$ |= 4.

3.2.2 | Idempotent Elements of Order N

Let (a + bI) = x ∈ G(I). Then (a + bI)n =

$$\text{an} + \left[\!\left[\binom{n}{2}a^{n-1}b \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \cdots + \binom{n}{k}a^{n-k}b^k + \cdots + \binom{n}{n-1}ab^{n-1} + b^n\right]\!\right] I = 1$$

⇒ an = 1 ⇒ an - 1 = 0 ⇒ (a – 1)(an – 1 + an – 2 + an – 3 + an – 4 +…+ a + 1)

- **Case 1.** Set n = 1 ⇒ a + bI = a + bI on both sides (the trivial case)
- **Case 2.** Set n = 2 ⇒ (a + bI)2 = a2 + [2ab + b2] I = a + bI ⇒ a2 = a , and b = 2ab

  This case was treated earlier, and we have that: $\mathbb{J}_2$ = {I, I – 1}.
- **Case 3.** Set n = 3 ⇒ (a + bI)3 = a3 + [3a2b + 3ab2 + b3] I = a + bI ⇒ a3 = a, ⇒ a(a2 – 1) = 0

  ⇒ a = { 0 , 1 , - 1 } and b = 3a2b + 3ab2 + b3 ⇒ b(1 - 3a2 + 3ab + b2) = 0

  b = 0 gives a trivial case , hence , we must have that : b2 + 3ab + 3a2 - 1 = 0.

  If a = 0, b2 – 1 = 0, b = 1 or b = - 1

  a = 1 ⇒ b2 + 3b + 2 = 0 ⇒ b = {1.2}

  $\mathbb{I}_3$ = {I, - I, 1 - I, 1 - 2I, - 1 + I, - 1 + 2I}, | $\mathbb{I}_3$ |= 6.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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