HyperSoft Set Methods in Engineering

Journal Homepage: sciencesforce.com/hsse

HyperSoft Set Meth. Eng. Vol. 1 (2024) 95–108

Paper Type: Original Article

Fully Fuzzy Distribution Planning Problem by Using Interval-Valued Bipolar Trapezoidal Fuzzy Number

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Received: 25 Nov 2023 **Revised:** 10 Feb 2024 **Accepted:** 11 Mar 2024 **Published:** 16 Mar 2024

Abstract

Distribution planning (DP) is a process in which we study the way to get materials and distribute the product from the delivery point to the consuming point after production planning in the supply chain. The limits of possible creation in a model are stock holding, deferred buying, and transportation costs while thinking about the time value of money. Since uncertainty is an undeniable issue in any evident creation framework, fuzzy sets (FS) have been applied in the proposed mathematical modeling. During the COVID-19 pandemic, to maintain physical distance among, humans, used & unused equipment, and daily needs, the researchers kept interval-valued fuzzy numbers (IVFNs) in place of crisp numbers that are much more effective to address uncertainty & hesitation in real-world situations. The cost, consumption, and delivery in distribution planning problems (DPP) are not as effective as crisp numbers in compression of fuzzy numbers (FNs). A realistic numerical model in the form of fully fuzzy DPP (FFDPP) has been introduced to show the practical application of the model. The solution procedure and results show the feasibility and validity of the mathematical model. Here we propose the concept of interval-valued bipolar trapezoidal fuzzy number (IVBTrFN) and its operations in the FFDPP, where fuzzy variables are required to be equal to either 0 or 1. The use of IVBTrFN in place of crisp numbers is more suitable for distributing the necessary equipment, medicines, food products, and other relevant items from one place to another in situations like COVID-19. The solution with the conclusion of FFDPP is introduced to better understand and execute our proposed methodology and results with IVBTrFNs.

Keywords: Interval-Valued Fuzzy Numbers, Interval-Valued Bipolar Trapezoidal Fuzzy Number, Fully Fuzzy Distribution Planning Problem.

1 | Introduction

Nowadays, manufacturing organizations have been constrained to find methods to plan and work powerful inventory chains to boost the advantage of the uncertain business environment and to fulfill client needs with globalization and the advancement of the business community in recent times [7], [27]. To optimize

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https://doi.org/10.61356/j.hsse.2024.17750

production and distribution planning problems simultaneously, extraordinary efforts are needed. In different manufacturing environments, the advantages of integrated production-distribution planning have been adopted by researchers [1], [22], [23]. The latest research on the maximum integrated production-distribution models is deterministic in the area of integrated production and distribution planning, although one of the real factors of current creation frameworks is that the objectives and other significant data sources, market demand, and production rates are not deterministic. In such a circumstance, fuzzy and stochastic mathematical modeling can be applied to adapt to the decision-making under the uncertainties of manufacturing environments [2], [20], [24]. Most recently, fuzzy programming, as one of the methods that is able to take into account the uncertainty of manufacturing systems has been applied in integrated production-distribution planning problems. For more study in this field see [3], [16], [17], [18], [21], [28]. Due to some vague information, inexact perception, and environmental factors, the parameters of integer linear programming problems are essential. To handle such type of uncertainty, Zadeh [29] 1965 introduced the concept of FS, by which the researchers can check the uncertainty in engineering, industrial, distribution, and management problems [11-15], [30].

Lee in 2000 introduced the concept of bipolar FS, which was the extension of FS [19]. According to Bosc and Pivert, bipolarity represents the tendency of the human mind to reason and make decisions based on pessimistic and optimistic outcomes [5]. Optimistic information reflects, what is permitted, desirable, satisfactory or acceptable, while a pessimistic statement reflects, what is impossible, non-reachable, revertible or forbidden. The values or objects that are to be rejected or not satisfy the constraints correspond to negative preference, while positive preference corresponds to user wishes, which are more acceptable than others. For more rapid developments in bipolar FS and its operation see ([6], [8], [9] and references therein).

In this FFTP, all parameters such as capital budgeting, fixed cost, distribution system, and product market share are in the form of IVBTrFNs by using 0-1 variables to maintain indeterminacy. For penetrating the quality solution of FFTP, there exist truth, indeterminacy, and falsity membership functions. This study aims to design a fully fuzzy mathematical model of the distribution system that determines only the economical and best site selection that provides minimum transportation cost for shipping the products to the issuing nodes in the unsettled domain of the supply chain.

This study aims to develop a production-distribution mathematical model that not only determines the production planning of the company but also selects the best location for setting up a new manufacturing plant from where the products can be distributed to the canters or retailers efficiently. Moreover, since some data from real-world manufacturing environments are unobtainable or imprecise, to provide a more realistic mathematical model, FS theory has been applied in this study.

2 | Problem Description

This study assumes that a hardware manufacturing company produces different kinds of hardware items in a fixed city. The main setup of the company is at city C from where items are dispatched to several distribution centers located at D_1, D_2 and D_3, to satisfy imprecise demand. On the other hand, the company has more demand for products in different parts of the country, so that the production group of the company plans to increase the capacity by setting up new branches of the company in new cities say C_1, C_2, C_3 and C_4. The company capacity, demand, and price are imprecise and fuzzy due to uncertain or incomplete available records. An FFDPP mathematical model not only determines the optimal production but also to set up new plants in different cities to enhance the economic condition that provides the minimum transportation cost of the new distribution centers in an uncertain environment.

3 | Preliminaries

To handle some uncertainties in FS [1], the extensions of FS, bipolar fuzzy sets [10], and interval-valued bipolar fuzzy sets with application are introduced.

Definition 3.1. [30]: A FS $\tilde{A}$ of a non-empty set X is defined as $\tilde{A}=\{\langle x,\mu_{\tilde{A}}(x)\rangle / x\in X\}$ where $\mu_{\tilde{A}}(x): X\to[0,1]$ is the membership function.

Definition 3.2. A FN (Figure 1)on the universal set $\boldsymbol{R}$ is a convex, normalized fuzzy set $\tilde{A}$, where the membership function $\mu_{\tilde{A}}(x): X\to[0,1]$ is continuous, strictly increasing on [a, b] and strictly decreasing on [c, d] , $\mu_{\tilde{A}}(x)=1,$ for all $x\in[b,c]$, where $a\le b\le c\le d$ and $\mu_{\tilde{A}}(x)=0,$ for all $x\in(-\infty,a]\cup[d,\infty)$.

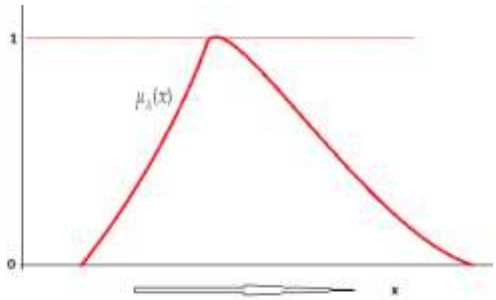

Figure 1. Fuzzy number.

Definition 3.3. [26]: A trapezoidal fuzzy number [TrFN] denoted as $\tilde{A}=(a,b,c,d)$, with its membership function $\mu_{\tilde{A}}(x)$ on $\boldsymbol{R}$, is given by

$$\mu_{\tilde{A}}(x)=\begin{cases}(x-a)/(b-a), & \text{for } a\le x<b\\ 1, & \text{for } b\le x<c\\ (d-x)/(d-c), & \text{for } c<x\le d\\ 0, & \text{otherwise}\end{cases}$$

If $b=c$ in TrFN $\tilde{A}=(a,b,c,d)$, then it becomes triangular FN $\tilde{A}=(a,b,c)$.

Definition 3.4. [10]: Let X be a non-empty set. Then, a bipolar valued fuzzy set, denoted by $\tilde{A}_{bi}$ is defined as;

$$\tilde{A}_{bi}(x)=\{\langle x,\mu^{+}_{bi}(x),\mu^{-}_{bi}(x)\rangle : x\in X\}$$

Where: $\mu^{+}_{bi}(x): X\to[0,1]$ and $\mu^{-}_{bi}(x): X\to[0,1]$. The positive membership degree $\mu^{+}_{bi}(x)$ denotes the satisfaction degree of an element x to the property corresponding to $\tilde{A}_{bi}$ and the negative membership degree $\mu^{-}_{bi}(x)$ denotes the satisfaction degree of x to some implicit counter property of $\tilde{A}_{bi}$.

4 | Interval Valued Bipolar Fuzzy Set

Interval valued bipolar fuzzy set (IVBFS) and its operations are as follows:

Definition 4.1. An IVBFS is denoted as $\tilde{A}^{IV}_{bi}$ in X and defined as $\tilde{A}^{IV}_{bi}=\left[\left\{x,\left\langle T^{+}_{\tilde{A}^{IV}_{bi}}(x),T^{-}_{\tilde{A}^{IV}_{bi}}(x)\right\rangle\right\}: x\in X\right]$, where $T^{+}_{\tilde{A}^{IV}_{bi}}(x)=\left[\mu^{L^+}_{\tilde{A}^{IV}_{bi}}(x),\mu^{R^+}_{\tilde{A}^{IV}_{bi}}(x)\right]$, $T^{-}_{\tilde{A}^{IV}_{bi}}(x)=\left[\mu^{L^-}_{\tilde{A}^{IV}_{bi}}(x),\mu^{R^-}_{\tilde{A}^{IV}_{bi}}(x)\right]$, $\mu^{L^+}_{\tilde{A}^{IV}_{bi}},\mu^{R^+}_{\tilde{A}^{IV}_{bi}}: X\to[0,1]$ and $\mu^{L^-}_{\tilde{A}^{IV}_{bi}},\mu^{R^-}_{\tilde{A}^{IV}_{bi}}: X\to[0,1]$. If X has only one element then IVBFS becomes interval-valued bipolar fuzzy number [IVBFN] and denoted as

$$\tilde{A}^{IV}_{bi}=\left\{x,\left\langle\left[\mu^{L^+}_{\tilde{A}^{IV}_{bi}}(x),\mu^{R^+}_{\tilde{A}^{IV}_{bi}}(x)\right],\left[\mu^{L^-}_{\tilde{A}^{IV}_{bi}}(x),\mu^{R^-}_{\tilde{A}^{IV}_{bi}}(x)\right]\right\rangle\right\}$$

Definition 4.2. An interval-valued bipolar trapezoidal fuzzy number (IVBTrFN) is a special FS on the set of real numbers R as shown in Figure 2, defined as:

$$\tilde{A}_{bi}^{IV}=\left\{(a,b,c,d),\left\langle\left[\mu_{\tilde{A}_{bi}^{IV}}^{L^{+}}(x),\mu_{\tilde{A}_{bi}^{IV}}^{R^{+}}(x)\right],\left[\mu_{\tilde{A}_{bi}^{IV}}^{L^{-}}(x),\mu_{\tilde{A}_{bi}^{IV}}^{R^{-}}(x)\right]\right\rangle\right\}$$

where the left membership function is $\left[\mu_{\tilde{A}_{bi}^{IV}}^{L^{-}}(x),\mu_{\tilde{A}_{bi}^{IV}}^{R^{-}}(x)\right]$, the right membership $\left[\mu_{\tilde{A}_{bi}^{IV}}^{L^{+}}(x),\mu_{\tilde{A}_{bi}^{IV}}^{R^{+}}(x)\right]$, are respectively as follows:

$$T_{\tilde{A}_{bi}^{IV}}^{-}(x)=\begin{cases}(x-a)\mu_{\tilde{A}_{bi}^{IV}}^{-}/(b-a); & a\le x\le b\\ \mu_{\tilde{A}_{bi}^{IV}}^{-}; & b\le x\le c\\ (d-x)\mu_{\tilde{A}_{bi}^{IV}}^{-}/(d-c); & c\le x\le d\\ 0; & \text{otherwise}\end{cases}\qquad T_{\tilde{A}_{bi}^{IV}}^{+}(x)=\begin{cases}(x-a)\mu_{\tilde{A}_{bi}^{IV}}^{+}/(b-a); & a\le x\le b\\ \mu_{\tilde{A}_{bi}^{IV}}^{+}; & b\le x\le c\\ (d-x)\mu_{\tilde{A}_{bi}^{IV}}^{+}/(d-c); & c\le x\le d\\ 0; & \text{otherwise}\end{cases}$$

If $a\ge 0$ and at least one $d>0,$ then $\tilde{A}_{bi}^{IV}$ called positive IVBTrFN and denoted as $\tilde{A}_{bi}^{IV}>0.$ similarly if $d\le 0,$ and atleast $a<0$, then IVBTrFN called negative i.e. $\tilde{A}_{bi}^{IV}<0.$

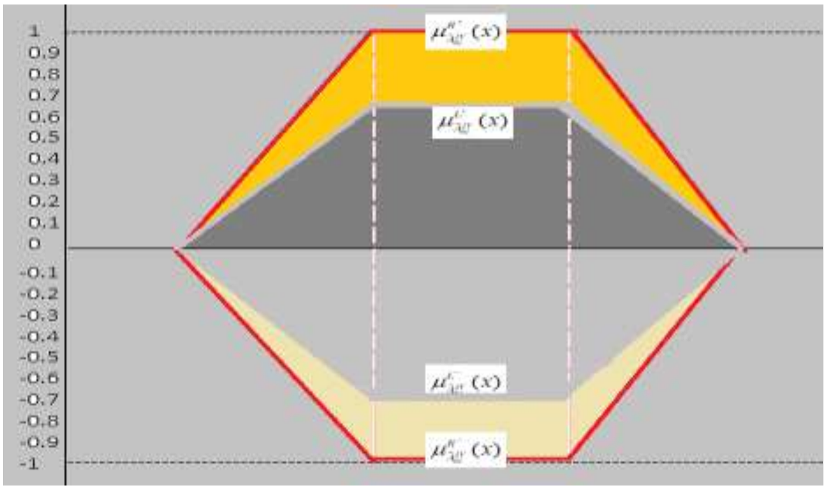

Figure 2. IVBSVTrFN.

Some important max-min norm operations on IVBTrFN as follows:

Let $\tilde{A}_{bi}^{IV}=\left\langle(a_1,b_1,c_1,d_1);\left\{\left[\mu_{\tilde{A}_{bi}^{IV}}^{L^{-}}(x),\mu_{\tilde{A}_{bi}^{IV}}^{R^{-}}(x)\right],\left[\mu_{\tilde{A}_{bi}^{IV}}^{L^{+}}(x),\mu_{\tilde{A}_{bi}^{IV}}^{R^{+}}(x)\right]\right\}\right\rangle$

$\tilde{B}_{bi}^{IV}=\left\langle(a_2,b_2,c_2,d_2);\left\{\left[\mu_{\tilde{B}_{bi}^{IV}}^{L^{-}}(x),\mu_{\tilde{B}_{bi}^{IV}}^{R^{-}}(x)\right],\left[\mu_{\tilde{B}_{bi}^{IV}}^{L^{+}}(x),\mu_{\tilde{B}_{bi}^{IV}}^{R^{+}}(x)\right]\right\}\right\rangle$, then

I. $$\tilde{A}_{bi}^{IV}+\tilde{B}_{bi}^{IV}=\left\langle(a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2);\begin{bmatrix}\left\{\max\left(\mu_{\tilde{A}_{bi}^{IV}}^{L^{+}},\mu_{\tilde{B}_{bi}^{IV}}^{L^{+}}\right),\max\left(\mu_{\tilde{A}_{bi}^{IV}}^{R^{+}},\mu_{\tilde{B}_{bi}^{IV}}^{R^{+}}\right)\right\},\\ \left\{\min\left(\mu_{\tilde{A}_{bi}^{IV}}^{L^{-}},\mu_{\tilde{B}_{bi}^{IV}}^{L^{-}}\right),\min\left(\mu_{\tilde{A}_{bi}^{IV}}^{R^{-}},\mu_{\tilde{B}_{bi}^{IV}}^{R^{-}}\right)\right\}\end{bmatrix}\right\rangle$$

II. $$\tilde{A}_{bi}^{IV}-\tilde{B}_{bi}^{IV}=\left\langle(a_1-d_2,b_1-c_2,c_1-b_2,d_1-a_2);\begin{bmatrix}\left\{\min\left(\mu_{\tilde{A}_{bi}^{IV}}^{L^{-}},\mu_{\tilde{B}_{bi}^{IV}}^{L^{-}}\right),\min\left(\mu_{\tilde{A}_{bi}^{IV}}^{R^{-}},\mu_{\tilde{B}_{bi}^{IV}}^{R^{-}}\right)\right\},\\ \left\{\max\left(\mu_{\tilde{A}_{bi}^{IV}}^{L^{+}},\mu_{\tilde{B}_{bi}^{IV}}^{L^{+}}\right),\max\left(\mu_{\tilde{A}_{bi}^{IV}}^{R^{+}},\mu_{\tilde{B}_{bi}^{IV}}^{R^{+}}\right)\right\}\end{bmatrix}\right\rangle$$

III. $$\tilde{A}_{bi}^{IV}.\tilde{B}_{bi}^{IV}=\left\langle(a_1.a_2,b_1.b_2,c_1.c_2,d_1.d_2);\left[\left\{\mu_{\tilde{A}_{bi}^{IV}}^{L^{+}}.\mu_{\tilde{B}_{bi}^{IV}}^{L^{+}},\mu_{\tilde{A}_{bi}^{IV}}^{R^{+}}.\mu_{\tilde{B}_{bi}^{IV}}^{R^{+}}\right\},\left\{\mu_{\tilde{A}_{bi}^{IV}}^{L^{-}}.\mu_{\tilde{B}_{bi}^{IV}}^{L^{-}},\mu_{\tilde{A}_{bi}^{IV}}^{R^{-}}.\mu_{\tilde{B}_{bi}^{IV}}^{R^{-}}\right\}\right],(d_1>0,d_2>0)\right\rangle$$

IV. $$\lambda \tilde{A}_{bi}^{IV} = \begin{cases} \left\langle (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1); \left[\left(\mu_{\tilde{A}_{bi}^{IV}}^{L^-}, \mu_{\tilde{A}_{bi}^{IV}}^{R^-} \right), \left(\mu_{\tilde{A}_{bi}^{IV}}^{L^+} \mu_{\tilde{A}_{bi}^{IV}}^{R^+} \right) \right] \right\rangle; \lambda > 0 \\ \left\langle (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1); \left[\left(\mu_{\tilde{A}_{bi}^{IV}}^{L^-}, \mu_{\tilde{A}_{bi}^{IV}}^{R^-} \right), \left(\mu_{\tilde{A}_{bi}^{IV}}^{L^+} \mu_{\tilde{A}_{bi}^{IV}}^{R^+} \right) \right] \right\rangle; \lambda < 0 \end{cases}$$

V. $$\tilde{A}_{bi}^{IV} / \tilde{B}_{bi}^{IV} = \left\langle (a_1/a_2, b_1/b_2, c_1/c_2, d_1/d_2); \begin{bmatrix} \left\{ \mu_{\tilde{A}_{bi}^{IV}}^{L^+} / \mu_{\tilde{B}_{bi}^{IV}}^{L^+}, \mu_{\tilde{A}_{bi}^{IV}}^{R^+} / \mu_{\tilde{B}_{bi}^{IV}}^{R^+} \right\}, \\ \left\{ \mu_{\tilde{A}_{bi}^{IV}}^{L^-} / \mu_{\tilde{B}_{bi}^{IV}}^{L^-}, \mu_{\tilde{A}_{bi}^{IV}}^{L^-} / \mu_{\tilde{B}_{bi}^{IV}}^{L^-} \right\} \end{bmatrix}; (d_1 > 0, d_2 > 0) \right\rangle$$

VI. $$\left(\tilde{A}_{bi}^{IV}\right)^{\lambda} = \begin{cases} \left\langle (a_1^{\lambda}, b_1^{\lambda}, c_1^{\lambda}, d_1^{\lambda}); \left[\left(\mu_{\tilde{A}_{bi}^{IV}}^{L^-}, \mu_{\tilde{A}_{bi}^{IV}}^{R^-} \right), \left(\mu_{\tilde{A}_{bi}^{IV}}^{L^+} \mu_{\tilde{A}_{bi}^{IV}}^{R^+} \right) \right] \right\rangle; \lambda > 0 \\ \left\langle ((d_1^{\lambda}, c_1^{\lambda}, b_1^{\lambda}, a_1^{\lambda}); \left[\left(\mu_{\tilde{A}_{bi}^{IV}}^{L^-}, \mu_{\tilde{A}_{bi}^{IV}}^{R^-} \right), \left(\mu_{\tilde{A}_{bi}^{IV}}^{L^+} \mu_{\tilde{A}_{bi}^{IV}}^{R^+} \right) \right] \right\rangle; \lambda < 0 \end{cases}$$

VII. $$\left(\tilde{A}_{bi}^{IV}\right)^{-1} = \left\langle (a_1^{-1}, b_1^{-1}, c_1^{-1}, d_1^{-1}); \left\{ \left[\mu_{\tilde{A}_{bi}^{IV}}^{L^-}(x), \mu_{\tilde{A}_{bi}^{IV}}^{R^-}(x) \right], \left[\mu_{\tilde{A}_{bi}^{IV}}^{L^+}(x), \mu_{\tilde{A}_{bi}^{IV}}^{R^+}(x) \right] \right\} \right\rangle$$

Definition 4.3. Let $\tilde{A}_{bi}^{IV} = \left\langle (a_1, b_1, c_1, d_1); \left\{ \left[\mu_{\tilde{A}_{bi}^{IV}}^{L^-}(x), \mu_{\tilde{A}_{bi}^{IV}}^{R^-}(x) \right], \left[\mu_{\tilde{A}_{bi}^{IV}}^{L^+}(x), \mu_{\tilde{A}_{bi}^{IV}}^{R^+}(x) \right] \right\} \right\rangle$ be a IVBTrFN. The primary application of score function is to drag the judgment of conversion of IVBTrFN into crisp number. The mean of IVBTrFN components is $\left(\frac{a_1 + b_1 + c_1 + d_1}{4} \right)$ and the score value of the membership portion is $\left\{ \frac{\mu_{\tilde{A}_{bi}^{IV}}^{L^-} + \mu_{\tilde{A}_{bi}^{IV}}^{R^-} + \mu_{\tilde{A}_{bi}^{IV}}^{L^+} + \mu_{\tilde{A}_{bi}^{IV}}^{R^+}}{4} \right\}$ then the score function $s\left(\tilde{A}_{bi}^{IV}\right)$ of a IVBTrFN are defined as follows:

$$s\left(\tilde{A}_{bi}^{IV}\right) = \frac{(a_1 + b_1 + c_1 + d_1) \times \left(\mu_{\tilde{A}_{bi}^{IV}}^{L^-} + \mu_{\tilde{A}_{bi}^{IV}}^{R^-} + \mu_{\tilde{A}_{bi}^{IV}}^{L^+} + \mu_{\tilde{A}_{bi}^{IV}}^{R^+} \right)}{16}$$

Definition 4.4. Let $\tilde{A}_{bi}^{IV} = \left\langle (a_1, b_1, c_1, d_1); \left\{ \left[\mu_{\tilde{A}_{bi}^{IV}}^{L^-}(x), \mu_{\tilde{A}_{bi}^{IV}}^{R^-}(x) \right], \left[\mu_{\tilde{A}_{bi}^{IV}}^{L^+}(x), \mu_{\tilde{A}_{bi}^{IV}}^{R^+}(x) \right] \right\} \right\rangle$

$\tilde{B}_{bi}^{IV} = \left\langle (a_2, b_2, c_2, d_2); \left\{ \left[\mu_{\tilde{B}_{bi}^{IV}}^{L^-}(x), \mu_{\tilde{B}_{bi}^{IV}}^{R^-}(x) \right], \left[\mu_{\tilde{B}_{bi}^{IV}}^{L^+}(x), \mu_{\tilde{B}_{bi}^{IV}}^{R^+}(x) \right] \right\} \right\rangle$, are two IVBTrFNs on the set of real numbers, then if

(i). $s\left(\tilde{A}_{bi}^{IV}\right) < s\left(\tilde{B}_{bi}^{IV}\right) \Rightarrow \tilde{A}_{bi}^{IV} < \tilde{B}_{bi}^{IV}$

(ii). $s\left(\tilde{A}_{bi}^{IV}\right) > s\left(\tilde{B}_{bi}^{IV}\right) \Rightarrow \tilde{A}_{bi}^{IV} > \tilde{B}_{bi}^{IV}$

(iii). $s\left(\tilde{A}_{bi}^{IV}\right) = s\left(\tilde{B}_{bi}^{IV}\right) \Rightarrow \tilde{A}_{bi}^{IV} = \tilde{B}_{bi}^{IV}$

Example 4.1. Let $X = \{x_1, x_2, x_3\}$. The two IVBTrFN in X are

$\tilde{A}_{bi}^{IV} = \langle (2,5,7,9); [0.4, 0.6], [-0.2, -0.1] \rangle$ and $\tilde{B}_{bi}^{IV} = \langle (1,3,6,8); [0.5, 0.6], [-0.5, -0.2] \rangle$ then $s\left(\tilde{A}_{bi}^{IV}\right) = 1.00625$, $s\left(\tilde{B}_{bi}^{IV}\right) = 0.45$. Here $s\left(\tilde{A}_{bi}^{IV}\right) > s\left(\tilde{B}_{bi}^{IV}\right)$ implies that $\tilde{A}_{bi}^{IV} > \tilde{B}_{bi}^{IV}$.

5 | Mathematical Formulation

The mathematical formulation of FFDPP, where delivered units, cost, demands and supplies are in the form of IVBTrFN. The FFDPP defined as follows:

(FFDPP) Min $\tilde{Z}_{bi}^{IV} = \sum_{i=0}^{m} \sum_{j=0}^{n} \left(\tilde{x}_{bi}^{IV}\right)_{ij} \left(\tilde{c}_{bi}^{IV}\right)_{ij}$

Subject to $\sum_{j=0}^{n}\left(\tilde{x}_{bi}^{IV}\right)_{ij} \approx \left(\tilde{a}_{bi}^{IV}\right)_{i}$, $i = 1,2,3,\ldots,m$(shipping sources),

$\sum_{i=0}^{m}\left(\tilde{x}_{bi}^{IV}\right)_{ij} \approx \left(\tilde{b}_{bi}^{IV}\right)_{j}$, $j = 1,2,3,\ldots,n$(destination), $\left(\tilde{x}_{bi}^{IV}\right)_{ij} \geq \tilde{0},\ \forall\ i = 1,2,3,\ldots,m,\ j = 1,2,3,\ldots,n.$

where $\left(\tilde{x}_{bi}^{IV}\right)_{12}$,........$\left(\tilde{x}_{bi}^{IV}\right)_{53}$, are according to

$$\left(\tilde{x}_{bi}^{IV}\right)_{ij} = \left\langle (a_{\tilde{x}_{bi}^{IV}}, b_{\tilde{x}_{bi}^{IV}}, c_{\tilde{x}_{bi}^{IV}}, d_{\tilde{x}_{bi}^{IV}}), \left[\left(\mu^{L^-}_{\tilde{x}_{bi}^{IV}}, \mu^{R^-}_{\tilde{x}_{bi}^{IV}}\right), \left(\mu^{L^+}_{\tilde{x}_{bi}^{IV}}, \mu^{R^+}_{\tilde{x}_{bi}^{IV}}\right)\right]\right\rangle;\ \ i = 1,2,3,4,5;\ j = 1,2,3.$$

$\left(\tilde{y}_{bi}^{IV}\right)_{1}$,.......$\left(\tilde{y}_{bi}^{IV}\right)_{4}$ are according to $\left(\tilde{y}_{bi}^{IV}\right)_{k} = \left\langle (a_{\tilde{y}_{bi}^{IV}}, b_{\tilde{y}_{bi}^{IV}}, c_{\tilde{y}_{bi}^{IV}}, d_{\tilde{y}_{bi}^{IV}}), \left[\left(\mu^{L^-}_{\tilde{y}_{bi}^{IV}}, \mu^{R^-}_{\tilde{y}_{bi}^{IV}}\right), \left(\mu^{L^+}_{\tilde{y}_{bi}^{IV}}, \mu^{R^+}_{\tilde{y}_{bi}^{IV}}\right)\right]\right\rangle; k = 1,2,3,4.$

and $\left(\tilde{x}_{bi}^{IV}\right)_{ij} \geq 0; i = 1,2,3,4,5;\ j = 1,2,3.$

where $\tilde{x}_{bi}^{IV}$ = the number of delivered fuzzy units shipped (in thousands) from plant *i* to distribution center *j*, for each *i=1,2,3,4,5 and j=1,2,3*.

$\tilde{c}_{bi}^{IV}$ = the shipping fuzzy cost data of one unit transported from i^{th} source to j^{th} destination.

$\tilde{a}_{bi}^{IV}$ = available fuzzy supply quantity from i^{th} plant

$\tilde{b}_{bi}^{IV}$ = required fuzzy demand quantity from j^{th} distribution centre.

Also

$$\left(\tilde{c}_{bi}^{IV}\right)_{ij} = \left\langle (a_{\tilde{c}_{bi}^{IV}}, b_{\tilde{c}_{bi}^{IV}}, c_{\tilde{c}_{bi}^{IV}}, d_{\tilde{c}_{bi}^{IV}}), \left[\left(\mu^{L^-}_{\tilde{c}_{bi}^{IV}}, \mu^{R^-}_{\tilde{c}_{bi}^{IV}}\right), \left(\mu^{L^+}_{\tilde{c}_{bi}^{IV}}, \mu^{R^+}_{\tilde{c}_{bi}^{IV}}\right)\right]\right\rangle,$$

$$\left(\tilde{x}_{bi}^{IV}\right)_{ij} = \left\langle (a_{\tilde{x}_{bi}^{IV}}, b_{\tilde{x}_{bi}^{IV}}, c_{\tilde{x}_{bi}^{IV}}, d_{\tilde{x}_{bi}^{IV}}), \left[\left(\mu^{L^-}_{\tilde{x}_{bi}^{IV}}, \mu^{R^-}_{\tilde{x}_{bi}^{IV}}\right), \left(\mu^{L^+}_{\tilde{x}_{bi}^{IV}}, \mu^{R^+}_{\tilde{x}_{bi}^{IV}}\right)\right]\right\rangle,$$

$$\left(\tilde{a}_{bi}^{IV}\right)_{ij} = \left\langle (a_{\tilde{a}_{bi}^{IV}}, b_{\tilde{a}_{bi}^{IV}}, c_{\tilde{a}_{bi}^{IV}}, d_{\tilde{a}_{bi}^{IV}}), \left[\left(\mu^{L^-}_{\tilde{a}_{bi}^{IV}}, \mu^{R^-}_{\tilde{a}_{bi}^{IV}}\right), \left(\mu^{L^+}_{\tilde{a}_{bi}^{IV}}, \mu^{R^+}_{\tilde{a}_{bi}^{IV}}\right)\right]\right\rangle,$$

$$\left(\tilde{b}_{bi}^{IV}\right)_{ij} = \left\langle (a_{\tilde{b}_{bi}^{IV}}, b_{\tilde{b}_{bi}^{IV}}, c_{\tilde{b}_{bi}^{IV}}, d_{\tilde{b}_{bi}^{IV}}), \left[\left(\mu^{L^-}_{\tilde{b}_{bi}^{IV}}, \mu^{R^-}_{\tilde{b}_{bi}^{IV}}\right), \left(\mu^{L^+}_{\tilde{b}_{bi}^{IV}}, \mu^{R^+}_{\tilde{b}_{bi}^{IV}}\right)\right]\right\rangle,$$

The above FFDPP may be written as:

$$\text{Min } \tilde{Z}_{bi}^{IV} = \sum_{i=0}^{m}\sum_{j=0}^{n}\begin{pmatrix} \left\langle (a_{\tilde{c}_{bi}^{IV}}, b_{\tilde{c}_{bi}^{IV}}, c_{\tilde{c}_{bi}^{IV}}, d_{\tilde{c}_{bi}^{IV}}), \left[\left(\mu^{L^-}_{\tilde{c}_{bi}^{IV}}, \mu^{R^-}_{\tilde{c}_{bi}^{IV}}\right), \left(\mu^{L^+}_{\tilde{c}_{bi}^{IV}}, \mu^{R^+}_{\tilde{c}_{bi}^{IV}}\right)\right]\right\rangle. \\ \left\langle (a_{\tilde{x}_{bi}^{IV}}, b_{\tilde{x}_{bi}^{IV}}, c_{\tilde{x}_{bi}^{IV}}, d_{\tilde{x}_{bi}^{IV}}), \left[\left(\mu^{L^-}_{\tilde{x}_{bi}^{IV}}, \mu^{R^-}_{\tilde{x}_{bi}^{IV}}\right), \left(\mu^{L^+}_{\tilde{x}_{bi}^{IV}}, \mu^{R^+}_{\tilde{x}_{bi}^{IV}}\right)\right]\right\rangle \end{pmatrix}$$

Subject to $\sum_{j=0}^{n}\left\langle (a_{\tilde{x}_{bi}^{IV}}, b_{\tilde{x}_{bi}^{IV}}, c_{\tilde{x}_{bi}^{IV}}, d_{\tilde{x}_{bi}^{IV}}), \left[\left(\mu^{L^-}_{\tilde{x}_{bi}^{IV}}, \mu^{R^-}_{\tilde{x}_{bi}^{IV}}\right), \left(\mu^{L^+}_{\tilde{x}_{bi}^{IV}}, \mu^{R^+}_{\tilde{x}_{bi}^{IV}}\right)\right]\right\rangle \approx \left\langle (a_{\tilde{a}_{bi}^{IV}}, b_{\tilde{a}_{bi}^{IV}}, c_{\tilde{a}_{bi}^{IV}}, d_{\tilde{a}_{bi}^{IV}}), \left[\left(\mu^{L^-}_{\tilde{a}_{bi}^{IV}}, \mu^{R^-}_{\tilde{a}_{bi}^{IV}}\right), \left(\mu^{L^+}_{\tilde{a}_{bi}^{IV}}, \mu^{R^+}_{\tilde{a}_{bi}^{IV}}\right)\right]\right\rangle$

$i = 1,2,3,\ldots,m$(shipping sources),

$\sum_{j=0}^{n}\left\langle (a_{\tilde{x}_{bi}^{IV}}, b_{\tilde{x}_{bi}^{IV}}, c_{\tilde{x}_{bi}^{IV}}, d_{\tilde{x}_{bi}^{IV}}), \left[\left(\mu^{L^-}_{\tilde{x}_{bi}^{IV}}, \mu^{R^-}_{\tilde{x}_{bi}^{IV}}\right), \left(\mu^{L^+}_{\tilde{x}_{bi}^{IV}}, \mu^{R^+}_{\tilde{x}_{bi}^{IV}}\right)\right]\right\rangle \approx \left\langle (a_{\tilde{b}_{bi}^{IV}}, b_{\tilde{b}_{bi}^{IV}}, c_{\tilde{b}_{bi}^{IV}}, d_{\tilde{b}_{bi}^{IV}}), \left[\left(\mu^{L^-}_{\tilde{b}_{bi}^{IV}}, \mu^{R^-}_{\tilde{b}_{bi}^{IV}}\right), \left(\mu^{L^+}_{\tilde{b}_{bi}^{IV}}, \mu^{R^+}_{\tilde{b}_{bi}^{IV}}\right)\right]\right\rangle$

$j = 1,2,3,\ldots,n$(destination),

and $\left\langle (a_{\tilde{x}_{bi}^{IV}}, b_{\tilde{x}_{bi}^{IV}}, c_{\tilde{x}_{bi}^{IV}}, d_{\tilde{x}_{bi}^{IV}}), \left[\left(\mu^{L^-}_{\tilde{x}_{bi}^{IV}}, \mu^{R^-}_{\tilde{x}_{bi}^{IV}}\right), \left(\mu^{L^+}_{\tilde{x}_{bi}^{IV}}, \mu^{R^+}_{\tilde{x}_{bi}^{IV}}\right)\right]\right\rangle \geq \tilde{0},\ \forall\ i = 1,2,3,\ldots,m,\ j = 1,2,3,\ldots,n.$

5.1 | Solution Procedure

The total distribution cost does not depends on the mode of distribution and distance, also the framework of the problem will be denoted by either crisp or fuzzy. For solution of FFDPP, first we convert all IVBTrFNs into crisp values by using score function and so the FFDPP converted into simple DPP. After balancing by existing method, the following steps are required for solution of FFDPP:

Step 5.1.1. Formulate the FFDPP form given uncertain data of company setup in new places.

Step 5.1.2. Convert the FFDPP into crisp TP by using score function.

Step 5.1.3. Solve the crisp DPP by using Microsoft excel.

Step 5.1.4. Find the corresponding solution of FFDPP.

Step 5.1.5. Compare the crisp solution and fuzzy solution.

5.2 | Steps for Balancing of FFTP

For solution of FFTP, first we convert all cost, availability and requirement, which are in the form IVBTrFNs into crisp values by using score function. If $\sum_{i=0}^{m}(\tilde{a}_{bi}^{IV})_i < \sum_{j=0}^{n}(\tilde{b}_{bi}^{IV})_j$ or $\sum_{i=0}^{m}(\tilde{a}_{bi}^{IV})_i > \sum_{j=0}^{n}(\tilde{b}_{bi}^{IV})_j$ for all *i, j*, then for balance, make sure as $\sum_{i=0}^{m}(\tilde{a}_{bi}^{IV})_i = \sum_{j=0}^{n}(\tilde{b}_{bi}^{IV})_j$, for all *i, j* by adding a row or column with zero IVBTrFNs cost entries in cost matrix. The proposed approach is applied in the following example where the author considered an examples on FFDPP.

5.3 | Steps for Balancing of FFTP by Minimum Row-Column Method (MRCM)

The MRCM for balancing the DPP introduced by Saini [26] as follows:

Step 5.3.1. Convert IVBTrFN cost $\tilde{c}_{ij}^{IV}$ IVBTrFN delivery $\tilde{a}_i^{IV}$ and IVBTrFN requirement $\tilde{b}_j^{IV}$ of FFDPP in cost matrix to crisp values by using score function $S(\tilde{a}_{bi}^{IV})$.

Step 5.3.2. If FFDPP is unbalance i.e. $\sum_{i=0}^{m}\tilde{a}_i^{IV} < \text{or} > \sum_{j=0}^{n}\tilde{b}_j^{IV}, \forall i, j$ than we find

$$\tilde{a}_{i(m+1)}^{IV} = \sum_{i=0}^{m}\tilde{a}_i^{IV} \text{ and } \tilde{b}_{j(n+1)}^{IV} = \sum_{j=0}^{n}\tilde{b}_j^{IV} \oplus \text{excess availability, or}$$

$$\tilde{b}_{j(n+1)}^{IV} = \sum_{j=0}^{n}\tilde{b}_j^{IV} \text{ and } \tilde{a}_{i(m+1)}^{IV} = \sum_{i=1}^{m}\tilde{a}_i^{IV} \oplus \text{excess requirement.}$$

The unit distribution costs are taken as follows:

$$\tilde{c}_{i(n+1)}^{IV} = \min_{1\le j\le n} \tilde{c}_{ij}^{IV}, 1\le i\le m, \quad \tilde{c}_{(m+1)j}^{IV} = \min_{1\le i\le m} \tilde{c}_{ij}^{IV}, 1\le j\le n,$$

$$\tilde{c}_{ij}^{IV} = \tilde{c}_{ji}^{IV}, 1\le i\le m, 1\le j\le n, \text{ and } \tilde{c}_{(m+1)(n+1)}^{IV} = 0.$$

Step 5.3.3. Obtain optimal solution of FFDPP by excel solver. Let the fuzzy optimal solution obtained as $\tilde{x}_{ij}^{IV}, 1\le i\le m+1, 1\le j\le n+1.$

Step 5.3.4. By assuming $\tilde{\omega}_{m+1}^{'IV} = 0$ and using the relation $\tilde{\omega}_i^{'IV} \oplus \tilde{v}_j^{'IV} = \tilde{\sigma}_{ij}^{'IV}$ for basic variables, find the values of all the dual variables $\tilde{\omega}_i^{'IV}, 1\le i\le m$ and $\tilde{v}_j^{'IV}, 1\le j\le n+1,$

Step 5.3.5. According to MRCM, $\tilde{\omega}_i^{'IV} = \tilde{\omega}_i^{IV}$ and $\tilde{v}_j^{'IV} = \tilde{v}_j^{IV}$ for $1\le i\le m, 1\le j\le n$, obtain only central rank zero duals.

Table 1. Estimated requirement.

Distribution centre	**D1**	$\langle(15,25,40,45);[0.5,0.6],[-0.4,-0.1]\rangle$
	D2	$\langle(05,15,30,45);[0.5,0.7],[-0.3,-0.2]\rangle$
	D3	$\langle(10,20,30,40);[0.5,0.6],[-0.3,-0.2]\rangle$

6 | Numerical Problem

A hardware manufacturing company has a setup in a city (C) with annual fuzzy capacities of approximately $\langle(17,28,39,48);[0.4,0.6],[-0.4,-0.2]\rangle$ units. The company shipped these products to the distribution centers located at D_1, D_2 and D_3 with annual fuzzy demand are as follows:

Suppose the company has more requirements for products in different parts of the country, so that the production group of the company plans to increase the capacity by setting up new branches of the company in new cities say C_1, C_2, C_3 and C_4. Since the market fluctuates due to uncertainty, so let's take the estimated fixed fuzzy price and the annual fuzzy capacities in four cities as in Table 2:

Table 2. Annual fuzzy capacities.

Branch at	Estimated fixed value in IVBTrFN	Estimated Expense in IVBTrFN
C_1	$\langle(105,127,191,288);[0.5,0.8],[-0.4,-0.2]\rangle$	$\langle(07,09,12,14);[0.5,0.7],[-0.6,-0.2]\rangle$
C_2	$\langle(205,275,345,395);[0.4,0.7],[-0.4,-0.3]\rangle$	$\langle(7.5,15,25,40);[0.4,0.6],[-0.5,-0.1]\rangle$
C_3	$\langle(215,335,435,525);[0.4,0.8],[-0.4,-0.3]\rangle$	$\langle(10,25,40,50);[0.5,0.7],[-0.4,-0.1]\rangle$
C_4	$\langle(355,455,555,655);[0.6,0.7],[-0.7,-0.31]\rangle$	$\langle(18,38,49,58);[0.6,0.7],[-0.4,-0.3]\rangle$

If the company plan to setup the branch at the city C_1, then $\left(\tilde{y}_{bi}^{IV}\right)_1=\langle(1,1,1,1);[1,0],[0,-1]\rangle$ and the total cost shipped from city C_1 to the three cities i.e. at D_1, D_2 and D_3 must be less than or equal to $\langle(07,09,12,14);[0.5,0.7],[-0.6,-0.2]\rangle$ units, otherwise, it will be similar $\left(\tilde{y}_{bi}^{IV}\right)_2$, $\left(\tilde{y}_{bi}^{IV}\right)_3$ and $\left(\tilde{y}_{bi}^{IV}\right)_4$ are equal to $\langle(1,1,1,1);[1,0],[0,-1]\rangle$, if the company plans to set up the branch at city C_2, city C_3 or city C_4 respectively, $\left(\tilde{y}_{bi}^{IV}\right)_2$, $\left(\tilde{y}_{bi}^{IV}\right)_3$ and $\left(\tilde{y}_{bi}^{IV}\right)_4$ are equal to $\langle(0,0,0,0);[1,0],[0,-1]\rangle$. To set up new branches of the company, the annual fuzzy price is as follows:

$$\langle(105,127,191,288);[0.5,0.8],[-0.4,-0.2]\rangle.\left(\tilde{y}_{bi}^{IV}\right)_1+\langle(207,275,345,395);[0.4,0.7],[-0.4,-0.3]\rangle.\left(\tilde{y}_{bi}^{IV}\right)_2+$$
$$+\langle(215,335,435,525);[0.4,0.8],[-0.4,-0.3]\rangle.\left(\tilde{y}_{bi}^{IV}\right)_3+\langle(355,455,555,655);[0.6,0.7],[-0.7,-0.31]\rangle.\left(\tilde{y}_{bi}^{IV}\right)_4$$

The above problem is formulated as follows:

$$\begin{aligned}\text{Min } \tilde{Z}_{bi}^{IV}=&\left(\tilde{c}_{bi}^{IV}\right)_{11}\left(\tilde{x}_{bi}^{IV}\right)_{11}+\left(\tilde{c}_{bi}^{IV}\right)_{12}\left(\tilde{x}_{bi}^{IV}\right)_{12}+\left(\tilde{c}_{bi}^{IV}\right)_{13}\left(\tilde{x}_{bi}^{IV}\right)_{13}+\left(\tilde{c}_{bi}^{IV}\right)_{21}\left(\tilde{x}_{bi}^{IV}\right)_{21}+\left(\tilde{c}_{bi}^{IV}\right)_{22}\left(\tilde{x}_{bi}^{IV}\right)_{22}+\\&\left(\tilde{c}_{bi}^{IV}\right)_{23}\left(\tilde{x}_{bi}^{IV}\right)_{23}+\left(\tilde{c}_{bi}^{IV}\right)_{31}\left(\tilde{x}_{bi}^{IV}\right)_{31}+\left(\tilde{c}_{bi}^{IV}\right)_{32}\left(\tilde{x}_{bi}^{IV}\right)_{32}+\left(\tilde{c}_{bi}^{IV}\right)_{33}\left(\tilde{x}_{bi}^{IV}\right)_{33}+\left(\tilde{c}_{bi}^{IV}\right)_{41}\left(\tilde{x}_{bi}^{IV}\right)_{41}+\\&\left(\tilde{c}_{bi}^{IV}\right)_{42}\left(\tilde{x}_{bi}^{IV}\right)_{42}+\left(\tilde{c}_{bi}^{IV}\right)_{43}\left(\tilde{x}_{bi}^{IV}\right)_{43}+\left(\tilde{c}_{bi}^{IV}\right)_{51}\left(\tilde{x}_{bi}^{IV}\right)_{51}+\left(\tilde{c}_{bi}^{IV}\right)_{52}\left(\tilde{x}_{bi}^{IV}\right)_{52}+\left(\tilde{c}_{bi}^{IV}\right)_{53}\left(\tilde{x}_{bi}^{IV}\right)_{53}+\\&\left(\tilde{C}_{bi}^{IV}\right)_1\left(\tilde{y}_{bi}^{IV}\right)_1+\left(\tilde{C}_{bi}^{IV}\right)_2\left(\tilde{y}_{bi}^{IV}\right)_2+\left(\tilde{C}_{bi}^{IV}\right)_3\left(\tilde{y}_{bi}^{IV}\right)_3+\left(\tilde{C}_{bi}^{IV}\right)_4\left(\tilde{y}_{bi}^{IV}\right)_4\end{aligned}$$

where

$\left(\tilde{c}_{bi}^{IV}\right)_{11}=\langle(2,4,7,9);[0.5,0.7],[-0.4,-0.3]\rangle$, $\left(\tilde{c}_{bi}^{IV}\right)_{12}=\langle(1,2,3,4);[0.4,0.6],[-0.5,-0.3]\rangle$

$\left(\tilde{c}_{bi}^{IV}\right)_{13} = \left\langle (2.5, 3, 3.5, 5); [0.5, 0.6], [-0.4, -0.1] \right\rangle$, $\left(\tilde{c}_{bi}^{IV}\right)_{21} = \left\langle (3, 4, 5, 6); [0.4, 0.5], [-0.5, -0.3] \right\rangle$

$\left(\tilde{c}_{bi}^{IV}\right)_{22} = \left\langle (2.5, 3, 4, 4.5); [0.3, 0.5], [-0.4, -0.2] \right\rangle$, $\left(\tilde{c}_{bi}^{IV}\right)_{23} = \left\langle (3.1, 4.2, 5.1, 6.6); [0.2, 0.5], [-0.4, -0.3] \right\rangle$

$\left(\tilde{c}_{bi}^{IV}\right)_{31} = \left\langle (5.5, 7.5, 10, 14); [0.5, 0.7], [-0.4, -0.2] \right\rangle$, $\left(\tilde{c}_{bi}^{IV}\right)_{32} = \left\langle (4.5, 6, 9, 11.5); [0.5, 0.6], [-0.3, -0.1] \right\rangle$

$\left(\tilde{c}_{bi}^{IV}\right)_{33} = \left\langle (2.5, 4.5, 6.5, 8.0); [0.6, 0.7], [-0.3, -0.2] \right\rangle$, $\left(\tilde{c}_{bi}^{IV}\right)_{41} = \left\langle (6.5, 9, 12.5, 15); [0.6, 0.7], [-0.4, -0.3] \right\rangle$

$\left(\tilde{c}_{bi}^{IV}\right)_{42} = \left\langle (3, 4, 5, 6); [0.6, 0.7], [-0.4, -0.2] \right\rangle$, $\left(\tilde{c}_{bi}^{IV}\right)_{43} = \left\langle (2, 3, 4, 7); [0.6, 0.7], [-0.5, -0.3] \right\rangle$

$\left(\tilde{c}_{bi}^{IV}\right)_{51} = \left\langle (5, 7, 9, 11); [0.6, 0.7], [-0.3, -0.2] \right\rangle$, $\left(\tilde{c}_{bi}^{IV}\right)_{52} = \left\langle (2.7, 4.3, 6.5, 8.5); [0.5, 0.6], [-0.5, -0.4] \right\rangle$

$\left(\tilde{c}_{bi}^{IV}\right)_{53} = \left\langle (2.5, 3.5, 5.5, 6.5); [0.5, 0.7], [-0.4, -0.2] \right\rangle$,

Subject to: $\left(\tilde{x}_{bi}^{IV}\right)_{11} + \left(\tilde{x}_{bi}^{IV}\right)_{12} + \left(\tilde{x}_{bi}^{IV}\right)_{13} \leq \left\langle (07, 09, 12, 14); [0.5, 0.7], [-0.6, -0.2] \right\rangle \left(\tilde{y}_{bi}^{IV}\right)_{1}$

$\left(\tilde{x}_{bi}^{IV}\right)_{21} + \left(\tilde{x}_{bi}^{IV}\right)_{22} + \left(\tilde{x}_{bi}^{IV}\right)_{23} \leq \left\langle (7.5, 15, 25, 40); [0.4, 0.6], [-0.5, -0.1] \right\rangle \left(\tilde{y}_{bi}^{IV}\right)_{2}$

$\left(\tilde{x}_{bi}^{IV}\right)_{31} + \left(\tilde{x}_{bi}^{IV}\right)_{32} + \left(\tilde{x}_{bi}^{IV}\right)_{33} \leq \left\langle (10, 25, 40, 50); [0.5, 0.7], [-0.4, -0.1] \right\rangle \left(\tilde{y}_{bi}^{IV}\right)_{3}$

$\left(\tilde{x}_{bi}^{IV}\right)_{41} + \left(\tilde{x}_{bi}^{IV}\right)_{42} + \left(\tilde{x}_{bi}^{IV}\right)_{43} \leq \left\langle (18, 38, 49, 58); [0.6, 0.7], [-0.4, -0.1] \right\rangle \left(\tilde{y}_{bi}^{IV}\right)_{4}$

while for city C it is written as $\left(\tilde{x}_{bi}^{IV}\right)_{51} + \left(\tilde{x}_{bi}^{IV}\right)_{52} + \left(\tilde{x}_{bi}^{IV}\right)_{53} \leq \left\langle (17, 28, 39, 48); [0.3, 0.6], [-0.5, -0.2] \right\rangle$.

In the same manner the requirement (demand) at cities D_1, D_2 and D_3 respectively is as follows:

$\left(\tilde{x}_{bi}^{IV}\right)_{11} + \left(\tilde{x}_{bi}^{IV}\right)_{21} + \left(\tilde{x}_{bi}^{IV}\right)_{31} + \left(\tilde{x}_{bi}^{IV}\right)_{41} + \left(\tilde{x}_{bi}^{IV}\right)_{51} = \left\langle (15, 25, 40, 45); [0.5, 0.6], [-0.4, -0.1] \right\rangle$

$\left(\tilde{x}_{bi}^{IV}\right)_{12} + \left(\tilde{x}_{bi}^{IV}\right)_{22} + \left(\tilde{x}_{bi}^{IV}\right)_{32} + \left(\tilde{x}_{bi}^{IV}\right)_{42} + \left(\tilde{x}_{bi}^{IV}\right)_{52} = \left\langle (05, 15, 30, 45); [0.2, 0.4], [-0.5, -0.2] \right\rangle$

$\left(\tilde{x}_{bi}^{IV}\right)_{13} + \left(\tilde{x}_{bi}^{IV}\right)_{23} + \left(\tilde{x}_{bi}^{IV}\right)_{33} + \left(\tilde{x}_{bi}^{IV}\right)_{43} + \left(\tilde{x}_{bi}^{IV}\right)_{53} = \left\langle (10, 20, 30, 40); [0.3, 0.5], [-0.5, -0.2] \right\rangle$

and $\left(\tilde{x}_{bi}^{IV}\right)_{ij} \geq 0; i = 1, 2, 3, 4, 5; j = 1, 2, 3.$

Distribution of FFDPP shown in Figure 3 as follows:

Using score function the, fuzzy cost, fuzzy requirement and fuzzy distribution in form of IVTrBFN converted into crisp number as shown in the follows distribution matrix Table 3:

Table 3. Distribution matrix of DPP.

	D_1	D_2	D_3	**Requirement**
C_1	0.6875	0.125	0.525	**1.05**
C_2	0.1125	0.1875	0	**2.1875**
C_3	1.3875	1.35625	1.075	**5.46875**
C_4	1.6125	0.7875	0.5	**6.113**
C	1.6	0.275	0.675	**3.3**
Availability	**4.6875**	**4.156**	**3.75**	

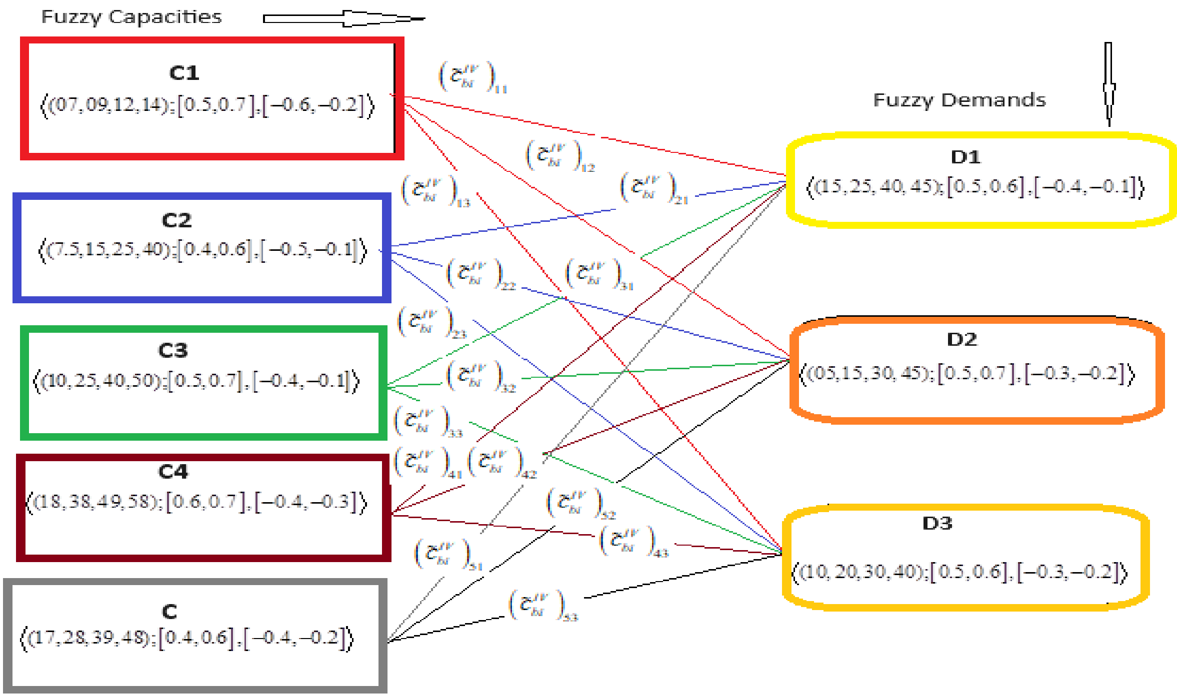

Figure 3. FFDPP.

Table 4. Balanced DPP.

	D_1	D_2	D_3	**Dummy**	**Requirement**
C_1	0.6875	0.125	0.525	0	**1.05**
C_2	0.1125	0.1875	0	0	**2.1875**
C_3	1.3875	1.35625	1.075	0	**5.46875**
C_4	1.6125	0.7875	0.5	0	**6.113**
C	1.6	0.275	0.675	0	**3.3**
Availability	**4.6875**	**4.156**	**3.75**	**5.52575**	

With help of Excel Solver, the solution of DPP is shown in Tables 4 and 5 as follows:

Table 5. Solution of DPP.

	D_1	D_2	D_3	**Dummy**	**Requirement**
C_1	0	0	0	1.05	**1.05**
C_2	0	0	1.01175	1.17575	**2.1875**
C_3	0	2.7305	2.73825	0	**5.46875**
C_4	4.6875	1.4255	0	0	**6.113**
C	0	0	0	3.3	**3.3**
Availability	**4.6875**	**4.156**	**3.75**	**5.52575**	

Here $\left(\tilde{x}_{bi}^{IV}\right)_{32} = 2.7305,$, $\left(\tilde{x}_{bi}^{IV}\right)_{33} = 2.73825,$ $\left(\tilde{x}_{bi}^{IV}\right)_{41} = 4.6875,$ $\left(\tilde{x}_{bi}^{IV}\right)_{42} = 1.4255$, and hence $\text{Min } \tilde{Z}_{bi}^{IV} = 15.32803$

The corresponding fuzzy values are $\left(\tilde{x}_{bi}^{IV}\right)_{32} = \langle(-38,-9,32,72);[0.5,0.6],[-0.3,-0.1]\rangle$

$\left(\tilde{x}_{bi}^{IV}\right)_{33} = \langle(-62,-7,49,88;[0.5,0.6],[-0.3,-0.1]\rangle$

$$\left(\tilde{x}_{bi}^{IV}\right)_{41} = \left\langle (15, 25, 40, 45); [0.5, 0.6], [-0.4, -0.1] \right\rangle$$

$$\left(\tilde{x}_{bi}^{IV}\right)_{42} = \left\langle (-27, -2, 24, 43); [0.5, 0.6], [-0.4, -0.1] \right\rangle$$

And $\text{Min } \tilde{Z}_{bi}^{IV} = \left\langle (-309.5, 131.5, 1226.5, 2465); [0.6, 0.7], [-0.1, -0.1] \right\rangle$.

In conclusion the company should be enhance their product by setup new plants at city C_3 and from where $\left\langle (-38, -9, 32, 72); [0.5, 0.6], [-0.3, -0.1] \right\rangle$ units shipped to city D_2 and $\left\langle (-62, -7, 49, 88; [0.5, 0.6], [-0.3, -0.1] \right\rangle$ units shipped to city D_3 respectively. Also company setup plants at city C_4 from where $\left\langle (15, 25, 40, 45); [0.5, 0.6], [-0.4, -0.1] \right\rangle$ units shipped to city D_1 and $\left\langle (-27, -2, 24, 43); [0.5, 0.6], [-0.4, -0.1] \right\rangle$ units shipped to city D_2 respectively.

The above unbalanced distribution problem in Table 1 can also be balanced by using MRCM [26] as follows:

Table 6. Distribution Matrix of DPP after balanced by MRCM.

	D_1	D_2	D_3	**dummy**	**Requirement**
C_1	0.6875	0.125	0.525	0.125	**1.05**
C_2	0.1125	0.1875	0	0	**2.1875**
C_3	1.3875	1.35625	1.075	1.075	**5.46875**
C_4	1.6125	0.7875	0.5	0.5	**6.113**
C	1.6	0.275	0.675	0.275	**3.3**
dummy	0.1125	0.125	0	0	**7.06775**
Availability	**4.6875**	**4.156**	**3.75**	**12.5935**	

With the help of excel solver, the solution of DPP is shown in Table 6 as follows:

Table 7. Solution of CTP.

	D_1	D_2	D_3	**dummy**	**Requirement**
C_1	0	0	0	1.05	**1.05**
C_2	0	0	0	2.1875	**2.1875**
C_3	0.659475	0.304233	0.304233	4.200809	**5.46875**
C_4	1.51289	1.433656	1.298323	1.868131	**6.113**
C	0.763558	0.745767	0.610433	1.180242	**3.3**
dummy	1.751577	1.672344	1.537011	2.106819	**7.06775**
Availability	**4.6875**	**4.156**	**3.75**	**12.5935**	

Here $\text{Min } \tilde{Z}_{bi}^{IV} = 14.02306$, which shows that MRCM for balancing unbalance DPP is more suitable method. For the comparison of objective fuzzy values and crisp values, we find the following Figure 4 and Figure 5 :

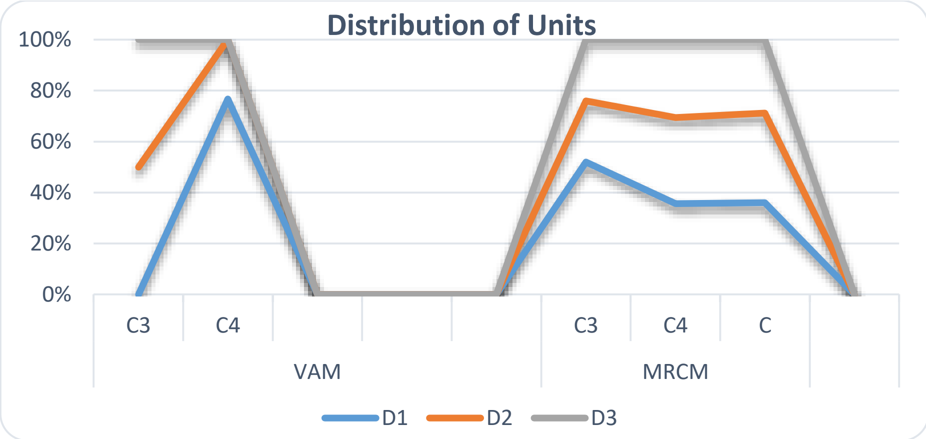

Figure 4. Comparative values of FFDPP.

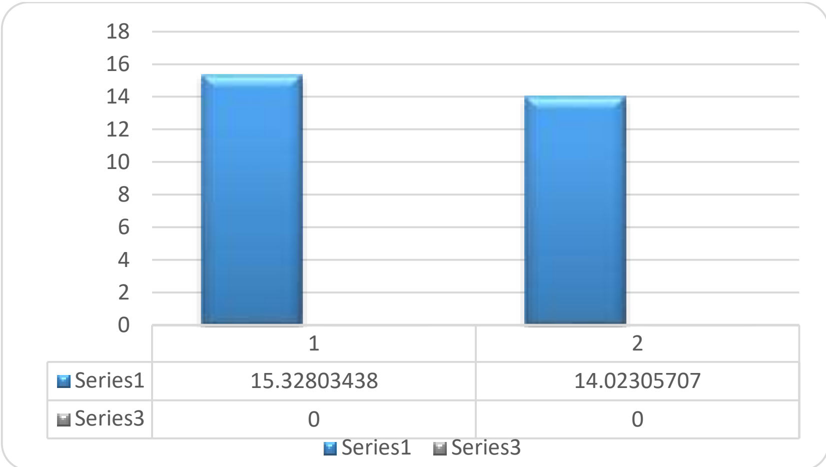

Figure 5. Comparative values of DPP.

7 | Conclusions

In today's competitive and uncertain business environment, design and location selection are crucial for every company to meet customer demands and maximize profit. The above research article introduces a new concept to design a fully fuzzy mathematical model of the distribution system that determines the economical and best site selection that provides the minimum distribution cost for shipping the products to the issuing nodes in the unsettled domain of the supply chain. We use here IVBTrFN in place of crisp numbers that can handle the uncertain information more flexibly in the optimization. For example, the crisp distribution cost is 15.32803438, when we balance the DPP by the existing method, while when we balance the same unbalanced DPP by MRCM, the value is 14.02305707, which shows that the different unusual things and materials can affect the final results. This type of study may be very useful in different scenarios to maintain the physical distance between humans and used & unused equipment during pandemics like COVID-19.

Acknowledgments

The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Author Contributaion

Chhavi Jain and R K Saini: conceptualized and designed the study, the mathematical formulation of the case study, conducted data analysis, and provided supervision throughout the research process.

Atul Sangal and Manisha: contributed to data collection, conduction of data analysis, and provided some writing assistance at various stages.

Funding

This research has no funding source.

Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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