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Distinctions between Various Types of Fuzzy-Extension HyperSoft Sets

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Abstract: We define the universes of discourses for all fuzzy and fuzzy-extension sets. Then present many types of Plithogenic Universes of discourse and their connections to HyperSoft Sets. Afterward, we make distinctions between various hybrid forms of HyperSoft Sets.

Keywords: HyperSoft Sets; Plithogenic; Fuzzy Sets.

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1. Introduction

We provide concrete examples for each type of fuzzy and fuzzy-extension HyperSoft Set and their many hybrid forms, including those with plithogenic sets. Gradually, we list the types of corresponding fuzzy and fuzzy-extensions universes of discourses in connection to the HyperSoft Sets.

2. Fuzzy and Fuzzy-extension Universes of Discourses

Let *U* be a classical (discrete or continuous, non-empty) **Universe** of discourse.

(i) **The fuzzy Universe** (*FU*) of discourse is defined as:

$FU = \{x(t), x \in U\}$, where t (that is the degree of truth-membership) of a generic element *x* from *FU*, is either a single number, an interval, or in general a subset of *[0, 1]*.

(ii) **The Intuitionistic Fuzzy Universe** (*IFU*) of discourse is:

$IFU = \{x(t, f), x \in U\}$, where *t* (that is the degree of truth-membership), and *f* (that is the degree of falsehood-nonmembership), of a generic element *x* from *IFU*, are either single numbers, intervals, or in general subsets from *[0, 1]*, with $\sup(t) + \sup(f) \le 1$.

(iii) **The Neutrosophic Universe** (*NU*) of discourse is:

$NU = \{x(t, i, f), x \in U\}$, where *t* (that is the degree of truth-membership), *i* (that is the degree of indeterminacy), *f* (that is the degree of falsehood-nonmembership), of a generic element *x* from *NU*, are either single numbers, or intervals, or in general subsets from *[0, 1]*, with $\sup(t) + \sup(i) + \sup(f) \le 3$.

(iv) In general, a **fuzzy-extension Universe** (*FEU*) of discourse, is:

$FEU = \{x(d), x \in U\}$,

where *d* is the fuzzy-extension degree of appurtenance of the generic element *x* to the universe *FEU*, and *d* should be included in *[0, 1]*.

{Exception to this restriction is for the fuzzy and fuzzy-extension **over-under-off-sets** [6], where the degrees are allowed to be outside of the interval *[0, 1]*}.

Such *fuzzy-extensions* may be:

Inconsistent Intuitionistic Fuzzy Set (Picture Fuzzy Set, Ternary Fuzzy Set), Pythagorean Fuzzy Set (Atanassov's Intuitionistic Fuzzy Set of second type), Fermatean Fuzzy Set, q-Rung Orthopair Fuzzy Set, Spherical Fuzzy Set, n-HyperSpherical Fuzzy Set, Refined Neutrosophic Set, MultiNeutrosophic Set, etc. [9].

3. Plithogenic Universes of Discourse

3.1 Informal Definition of Plithogenic Set [1]

A plithogenic set *P* is a set such that each element *x* is characterized by one or more attributes (parameters), and each attribute (parameter) may have many attribute values. With respect to each attribute-value *v,* a generic element *x* has a corresponding degree of appurtenance *d(x, v)* of the element *x* to the set *P*. These attributes (parameters) and their values may be independent, dependent, or partially independent and dependent - according to the applications to solve.

The degree of appurtenance *d(x, v)* may be fuzzy, intuitionistic fuzzy, neutrosophic, or any fuzzy-extension type.

(i) **Plithogenic Universe** (*PU*) of discourse is:

$$PU = \{x(a_1(d_1), a_2(d_2), ..., a_n(d_n)), x \in U\},$$

where $a_1, a_2, ..., a_n$ are attribute-values, for $n \geq 1$, with $a_1 \in A_1, a_2 \in A_2, ..., a_n \in A_n$, and

$A_1, A_2, ..., A_n$ are sets of attribute-values, with $A_i \cap A_j = \phi$, for $i \neq j$, and

$i, j \in \{1, 2, ..., n\}$.

While $d_1, d_2, …, d_n$ are the fuzzy or fuzzy-extension degrees of appurtenance of the generic element *x*, with respect to the attribute-values $a_1, a_2, …, a_n$ respectively, to the set *PU.*

(ii) **The Plithogenic Fuzzy Universe** (*PFU*) of discourse is

$$PFU = \{x(a_1(t_1), a_2(t_2), ..., a_n(t_n)), x \in U\}$$

It's a particular case of the Plithogenic Universe, where the degrees of appurtenances (truth-memberships) are fuzzy, with $t \in [0, 1]$.

(iii) **Plithogenic Intuitionistic Fuzzy Universe** (*PIFU*) of discourse is:

$$PIFU = \{x(a_1(t_1, f_1), a_2(t_2, f_2), ..., a_n(t_n, f_n)), x \in U\}$$

Again, it is a particular case of the Plithogenic Universe, where the degrees of appurtenances (truth-memberships, falsehood-non-memberships) are intuitionistic fuzzy, with $(t, f) \in [0, 1]^2$.

(iv) **The Plithogenic Neutrosophic Universe** (*PNU*) of discourse is

$$PNU = \{x(a_1(t_1, i_1, f_1), a_2(t_2, i_2, f_2), ..., a_n(t_n, i_n, f_n)), x \in U\}$$

Also, *PNU* is a particular case of the Plithogenic Universe, for the case when the degrees of appurtenance (truth-memberships, indeterminacy-membership, falsehood-nonmemberships) are neutrosophic, with $(t, i, f) \in [0, 1]^3$.

(v) In general, the **Plithogenic fuzzy-extension Universe** (*PFEU*) of discourse is:

$$PFEU = \{x(a_1(d_1), a_2(d_2), ..., a_n(d_n)), x \in U\}$$

Also, *PFEU* is a particular case of the Plithogenic Universe, for the case when the degrees of appurtenance *d* are *fuzzy extensions*.

4. Fuzzy and Fuzzy-extension HyperSoft Sets

(i) **HyperSoft Set** (founded by Smarandache [4], in 2018):

Let $\mathcal{U}$ be a universe of discourse, $(\mathcal{U})$ the power set of $\mathcal{U}$.

Let a_1 , a_2, …, a_n, for $n \geq 1$, be n distinct attributes, whose corresponding attribute-values are respectively the sets A_1 , A_2 , …, A_n, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i,j \in \{1, 2, …, n\}$.

Then, the pair $(F, A_1 \times A_2 \times … \times A_n)$, where: $F: A_1 \times A_2 \times … \times A_n \longrightarrow (\mathcal{U})$, is called a HyperSoft Set.

Concrete Example 1

Assume one has a set of houses $U = \{x_1, x_2, …, x_{10}\}$.

And their attributes are: *size, color, and position*, whose attribute values are respectively A_1, A_2, and A_3, where:

A_1 = *size* = *{big, medium, small}*

A_2 = *color* = *{green, red, white, yellow}*

A_3 = *position* = *{peripherical, central}*

Then $F : A_1 \times A_2 \times A_3 \to P(U)$ is a HyperSoft Set.

For example: $F(a_1, a_2, a_3) = \{x_1, x_2\}$, where $a_1 \in A_1, a_2 \in A_2, a_3 \in A_3$.

Let's take a_1 = *big*, a_2 = *white*, a_3 = *central* and F(*big, white, central*) = $\{x_1, x_2\} \in P(U)$, which means that the houses x_1 and x_2 are all big, white, and central.

(ii) Fuzzy HyperSoft Set

$$F : A_1 \times A_2 \times ... \times A_n \to P(FU)$$

The Cartesian product $A_1 \times A_2 \times ... \times A_n$ ensures the HyperSoft-ness of the set,

while the *FU* (Fuzzy Universe) ensures the fuzzy-ness degree of appurtenance of the elements *x* to the set *FU*.

Concrete Example 1 continued

For example, $F(a_1, a_2, a_3) = \{x_1(t_1), x_2(t_2)\}$.

F(big, white, central) = $\{x_1(0.7), x_2(0.8)\}$, which means that house x_1 is in a fuzzy degree of *70%* big and white and central, while the house x_2 is in a fuzzy degree of *80%* big, white and central.

(iii) Intuitionistic Fuzzy HyperSoft Set

$$F : A_1 \times A_2 \times ... \times A_n \to P(IFU)$$

Similarly, the Cartesian product $A_1 \times A_2 \times ... \times A_n$ ensures the HyperSoft-ness of the set,

while the *IFU* (Intuitionistic Fuzzy Universe) ensures the intuitionistic_fuzzy-ness degree of appurtenance of the elements *x* to the set *IFU*.

Concrete Example 1 continued

For example, $F(a_1, a_2, a_3) = \{x_1(t_1, f_1), x_2(t_2, f_2)\}$

$F(big, white, central) = \{x_1(0.7, 0.2), x_2(0.8, 0.1)\}$, which means that house x_1 is in an intuitionistic fuzzy degree of *70%* true and 20% false big and white and central, while the house x_2 is in an intuitionistic fuzzy degree of *80%* true and 10% false big and white and central.

(iv) Neutrosophic HyperSoft Set

$$F : A_1 \times A_2 \times ... \times A_n \to P(NU)$$

The same, the Cartesian product $A_1 \times A_2 \times ... \times A_n$ ensures the HyperSoft-ness of the set,

while the *NU* (Neutrosophic Universe) ensures the neutrosophic-ness degree of appurtenance of the elements *x* to the set *NU*.

Concrete Example 1 continued

For example, $F(a_1, a_2, a_3) = \{x_1(t_1, i_1, f_1), x_2(t_2, i_2, f_2)\}$.

$F(big, white, central) = \{x_1(0.7, 0.3, 0.2), x_2(0.8, 0.4, 0.1)\}$, which means that house x_1 is in a neutrosophic degree of *70%* true and 30% indeterminate and 20% false big and white and central, while the house x_2 is in a neutrosophic degree of *80%* true and 40% indeterminate and 10% false big and white and central.

(v) In general **fuzzy-extension HyperSoft Set**

$$F : A_1 \times A_2 \times ... \times A_n \to P(FEU)$$

Concrete Example 1 continued

For example, $F(a_1, a_2, a_3) = \{x_1(d_1), x_2(d_2)\}$, where *d* is the fuzzy-extension degree of appurtenance of the generic element *x* to the *FEU* set.

(vi) Plithogenic HyperSoft Set

$$F : A_1 \times A_2 \times ... \times A_n \to P(PU)$$

The cartesian product $A_1 \times A_2 \times ... \times A_n$ ensures, in the same way, the HyperSoft-ness, while the set *PU* ensures the plithogeny of the elements, i.e. each element *x* is characterized as in our real life by many attribute-values a_1, a_2, …, and the element x belongs to the set PU in a certain degree d_{ij} with respect to each individual attribute-value.

Concrete Example 1 continued

$F(a_1, a_2, a_3) = \{ x_1(a_1(d_{11}), a_2(d_{12}), a_3(d_{13})), x_2(a_1(d_{21}), a_2(d_{22}), a_3(d_{23})) \}$

For example, the element x_1 belongs to the set PU in a degree d_{11} with respect to its attribute-value a_1, in a degree d_{12} with respect to its attribute value a_2, and in a degree d_{13} with respect to its attribute-value a_3. Similarly for the element x_2.

The degrees of appurtenance d_{ij} of an element x to the set PU may be fuzzy, intuitionistic fuzzy, neutrosophic, or any other fuzzy-extension degrees.

(vii) Plithogenic Neutrosophic HyperSoft Set

$$F : A_1 \times A_2 \times ... \times A_n \rightarrow P(PNU)$$

When the degrees of appurtenance of an element to a plithogenic set are neutrosophic, we get a Plithogenic Neutrosophic HyperSoft Set.

Concrete Example 1 continued

$F(a_1, a_2, a_3) = \{ x_1(a_1(t_1, i_1, f_1), a_2(t_2, i_2, f_2), a_3(t_3, i_3, f_3)),$
$x_2(a_1(t_1, i_1, f_1), a_2(t_2, i_2, f_2), a_3(t_3, i_3, f_3)) \}$

F(big, white, central) =
= { x_1(big(0.7, 0.1, 0.6), white(0.4, 0.3, 0.1), central(0.9, 0.0, 0.1)),
= x_2(big(0.6, 0.0, 0.8), white(1.0, 0.1, 0.0), central(0.3, 0.4, 0.8)) }

Which means that, the element x_1 belongs to the set PU in a neutrosophic degree *of (0.7, 0.1. 0.6)* with respect to its attribute-value *big*, in a neutrosophic degree *(0.4, 0.3, 0.1)* with respect to its attribute-value *white*, and in a neutrosophic degree *(0.9, 0.0, 0.1)* with respect to its attribute-value *central.*

Similarly for the element x_2.

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The answer follows below, using a simple concrete example, according to the previous explanations.

(i) Neutrosophic HyperSoft Set:
F(big, white, central) = {x_1(0.7, 0.3, 0.2), x_2(0.8, 0.4, 0.1)},
which means that house x_1 is in a neutrosophic degree of 70% true 30% indeterminate and 20% false with respect to all three attribute-values together (big and white and central),

while the house x_2 is in a neutrosophic degree of 80% true and 40% indeterminate and 10% false also with respect to all three attribute-values together (big and white and central).

(ii) Plithogenic Neutrosophic HyperSoft Set:
(iii) F(big, white, central) =

= {x_1(big(0.7, 0.1, 0.6), white(0.4, 0.3, 0.1), central(0.9, 0.0, 0.1)),

= x_2(big(0.6, 0.0, 0.8), white(1.0, 0.1, 0.0), central(0.3, 0.4, 0.8)) },

which means that, the element x_1 belongs to the set PU in a neutrosophic degree with respect to each attribute-value independently;

in other words, in a neutrosophic degree (0.7, 0.1. 0.6) with respect to its individual attribute-value separately big, in a neutrosophic degree (0.4, 0.3, 0.1) with respect to its individual attribute-value white, and in a neutrosophic degree (0.9, 0.0, 0.1) with respect to its other individual attribute-value central.

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The authors declare that there is no conflict of interest in the research.

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