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Application of Trapezoidal Fuzzy Numbers in the Inventory Problem of Decision Science

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Abstract

The objectives of this research study focus on managing production networks efficiently through a systematic and integrated approach. The central idea is to combine inventory control and transportation scheduling to optimize the entire supply chain. The study introduces the concept of the overtime fuzzy inventory-transportation problem (FITP), specifically aiming to determine an optimal distribution plan that considers overtime to minimize total distribution costs. The use of trapezoidal fuzzy numbers (TrFN) is highlighted as a key methodology for addressing uncertainty and incomplete information in real-world problems. TrFNs are considered a generalization of crisp and fuzzy numbers, providing a flexible framework for modelling imprecision. The process of de-fuzzification is mentioned as a means to convert fuzzy numbers into crisp numbers for practical applications, thereby obtaining clear and precise solutions.

Keywords: Fuzzy Inventory-Transportation Problem, Trapezoidal Fuzzy Number, De-fuzzification, Minimum Row-Column Method.

1 | Introduction

Conventional wisdom holds that science should aim for certainty in all of its expressions; as a result, irregularity or uncertainty is viewed as unprofessional. But from a more advanced viewpoint, unavoidable uncertainty is seen as essential to research and of great benefit. Over the past 100 years, vulnerability and uncertainty have been a striking departure from the usual in science and mathematics. The cutting-edge idea of uncertainty came with the publication of a class paper by Zadeh [1] in 1965, who proposed a hypothesis whose components fuzzy set (FS) will be a set with unclear boundaries. Being a part of an FS from a more sophisticated viewpoint, uncertainty cannot be avoided but is considered crucial to research and greatly advantageous.

Fuzzy analysis of inventory management provides inventory control, which is crucial since practically every organization must maintain some inventory to operate smoothly and effectively. Continuous manufacturing

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is a challenge, so it is a matter of when and how much inventory to keep on hand. Harris [2] in 1915, first time created the inventory model. The conventional probabilistic models are unable to handle uncertainty effectively due to its existence of an unavoidable factor. The current research is based on how to specify inventory optimization tasks in such an uncertain environment and how to evaluate the best results. To overcome such unsettled problems in inventory modeling, Zadeh created the theory of FS. Later on in 1970, Bellman and Zadeh [3] introduced some extension principles of the FS for the solution in the area of decision-making problems in management sciences and operation research sciences. For more development on FS theory, see [4, 5].

According to Zadeh [6], FN rather than probabilistic methods with crisp numbers are preferable for new products and seasonal commodities. A fuzzy inventory model for decision-making in the presence of fuzzy variables was created by Jain [7] in 1976. In 1978, Dubois and Prade [8] defined some operations on FNs. Using fuzzy decisions, Kacpryzk and Staniewski [9] in 1982 developed an inventory model for long-term inventory policy planning. Zimmerman [10] tried to use fuzzy sets in operational research in 1983. In 1997, Gen et al. [11] demonstrated how FS theory may be applied to inventories.

When ordering cost and holding cost are represented by FN, inventory models in the fuzzy sense were established by Park [12] in 1987 and Vujosevic et al. [13] in 1996. Costs have been represented by Park as TrFNs, while ordering costs were represented by a triangular FN and holding costs by a TrFN by Vujosevic et al. The estimate of the fuzzy total cost was taken as the centroid. Another inventory model with fuzzy demand quantity and fuzzy production quantity was created by Yao and Lee [14, 15] in 1999. The inventory problem was created by Lin [16] in 2008 for a period review model with variable lead time, and the predicted demand deficit and backorder rate were fuzzed using the signed distance approach. Nagar and Surana [17] recently developed the associated fuzzy inventory model for fuzzy deteriorating items with fuzzy demand rate under complete backlogging. All inventory metrics, including the deterioration rate, are fuzzified into pentagonal FNs, and the average total inventory cost is not precisely calculated. A different recent model created by Maragatham and Lakshmidevi [18] in 2016 treats the holding cost, shortage cost, degradation cost, purchasing cost, and selling price as trapezoidal fuzzy values. The creation of a fuzzy inventory model with time-varying demand, deterioration, and salvage was examined in 2016 by Sahoo et al. [19]. A note on fuzziness in inventory management problems was discussed by Jayjayanti Ray [20] in 2017. The fuzzy model is observed to be providing a considerably better optimal solution when compared to the crisp model. For more development and applications of FNs in inventory models, see [21-39].

The primary goals of the study are to balance several approaches and compare the outcomes to find the best solutions for the fuzzy inventory-transportation problem (FITP) using an Excel solver. There are seven sections in this well-structured work. An overview of the current work along with some previous research is provided in the first section. The fundamental ideas of crisp and FS are covered in detail in the second part of the paper. Introduce de-fuzzification as a scoring function in the third part to transform TrFN into crisp values. The categorization and mathematical formulation of FITP are explained in the fourth section. We present the solutions in several tables along with a comparison, analysis, and conclusion in sections five, six, and seven. The final part of the article presents the findings and directions for the study project.

1.1 | Motivation of this Work

The motivation behind the work stems from the rapidly advancing economy and the increasing importance of efficient administration in contemporary enterprises. The production network in modern businesses plays a crucial role in this context. The authors are driven by the need to address real-world challenges characterized by uncertainty in today's scenarios. Specifically, the introduction of the overtime FITP is motivated by the goal of determining an optimal distribution plan from vendors to customers, considering overtime, to minimize the total distribution cost.

1.2 | Novelties of this Work

The work proposes an integrated concept that brings together inventory control and transportation scheduling as a holistic approach to optimizing the supply chain. This integration is presented as a novel contribution to the field. Also, the inclusion of the de-fuzzification process is highlighted as a novelty. This process involves converting fuzzy numbers into crisp numbers, enhancing the practical applicability of the proposed solutions.

1.3 | Solution Methods: Existing

The utilization of the existing method for solving the FITP is a distinctive aspect. The Excel solver showcases the versatility of the proposed methodology by employing the existing solution method.

1.4 | Real-life Numerical Example

The presentation of a real-life numerical example adds practical relevance and applicability to the work. The inclusion of a concrete scenario demonstrates the effectiveness of the proposed methodology in solving actual problems.

In summary, the motivation of the work lies in addressing contemporary challenges in production networks, while the novelties include an integrated approach, the introduction of the overtime FITP, the application of trapezoidal fuzzy numbers, the de-fuzzification process, and the use of various solution methods, all illustrated through a real-life numerical example.

For sensitivity the application of the proposed method in finding optimal distribution plans and minimizing distribution costs. The real-life numerical example serves to illustrate the practical implementation of the FITP concept. Overall, the research appears to contribute to the field of supply chain optimization, leveraging fuzzy logic and trapezoidal fuzzy numbers to handle uncertainty and improve decision-making processes.

2 | Preliminaries

Conventional wisdom holds that science should aim for certainty in all of its expressions; as a result, irregularity or uncertainty is viewed as unprofessional. But from a more advanced viewpoint, unavoidable uncertainty is seen as essential to research and of great benefit. Over the past 100 years, vulnerability and uncertainty have been a striking departure from the usual in science and mathematics. The cutting-edge idea of uncertainty came with the publication of a class paper by Zadeh [1] in 1965, who proposed a hypothesis whose components fuzzy set (FS) will be a set with unclear boundaries. Being a part of an FS from a more sophisticated viewpoint, uncertainty cannot be avoided but is considered crucial to research and greatly advantageous.

2.1 | Some Basic Definitions and Examples

Definition 2.1.1. A FS $\tilde{A}$ of a non-empty set X is defined as $\tilde{A}=\{(x,\mu_{\tilde{A}}(x)):x\in X\}$ where $\mu_{\tilde{A}}(x)$ is called the membership function such that $\mu_{\tilde{A}}(x):X\to[0,1]$.

Definition 2.1.2. A convex, normalized FS $\tilde{A}$ is called FN on the universal set of real numbers R, if the membership function $\mu_{\tilde{A}}$ of $\tilde{A}$ is continuous from X to [0,1]. Also $\mu_{\tilde{A}}(x)=0,$ for all $x\in(-\infty,a]\cup[d,\infty)$ and $\mu_{\tilde{A}}(x)$ is strictly increasing in [a, b] and strictly decreasing on [c, d]. For normalization $\mu_{\tilde{A}}(x)=1$ for all $x\in[b,\ c]$ where $a\le b\le c\le d$.

Definition 2.1.3. TrFN $\tilde{a}=\langle(a_1,a_2,a_3,a_4);\mu_{\tilde{a}}\rangle$ is a FN on the real line R, whose membership $\mu_{\tilde{a}}(x)$ is given as follows and shown in Figure 1.

$$\mu_{\tilde{a}}(x)=\begin{cases}\dfrac{(x-a_1)\mu_{\tilde{a}}}{a_2-a_1}, & \text{for } a_1\le x\le a_2,\\ \mu_{\tilde{a}}, & \text{for } a_2\le x\le a_3,\\ \dfrac{(a_4-x)\mu_{\tilde{a}}}{a_4-a_3}, & \text{for } a_3\le x\le a_4,\\ 0, & \text{for } x>a_4 \text{ and } x<a_1\end{cases}$$

where $\mu_{\tilde{a}}$ denotes the maximum membership degree, in [0,1] and $a_1,a_2,a_3,a_4\in\mathbf{R}$ such that $a_1\le a_2\le a_3\le a_4$. When $a_1>0$, $\tilde{a}=\langle(a_1,a_2,a_3,a_4);\mu_{\tilde{a}}\rangle$ is called positive TrFN, denoted by $\tilde{a}>0$, and if $a_4\le 0$, then $\tilde{a}=\langle(a_1,a_2,a_3,a_4);\mu_{\tilde{a}}\rangle$ becomes a negative TrFN denoted by $\tilde{a}<0$. If $0\le a_1\le a_2\le a_3\le a_4\le 1$, $\mu_{\tilde{a}}\in[0,1]$, then $\tilde{a}$ called a normalized TrFN. If $a_2=a_3$, then TrFN is reduces triangular FN, denoted as $\tilde{a}=\langle(a_1,a_3,a_4);\mu_{\tilde{a}}\rangle$.

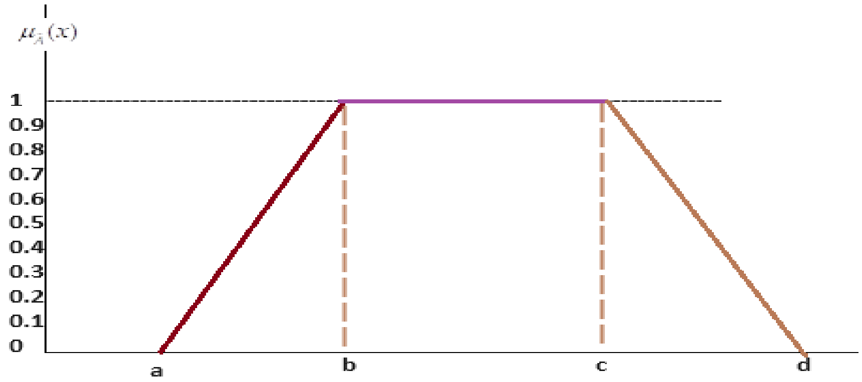

Figure 1. Trapezoidal fuzzy number.

If $b=c$ in TrFN $\tilde{A}=(a,b,c,d)$, then it becomes triangular FN $\tilde{A}=(a,b,d)$.

Example 2.1.1. Let X be a space with capability x_1, trustworthiness x_2 price x_3 and compatibility in x_4 [0, 1]. If expert wants "degree of good services", then a FN $\tilde{A}$ of X is defined as:

$$\tilde{A}=0.7/x_1+0.4/x_2+0.6/x_3+0.3/x_4.$$

Definition 2.1.4. A FN $\tilde{A}=\{\langle x,\mu_{\tilde{A}}(x):x\in X\}$ is called convex set on the real line; if the following conditions are satisfied $\forall\ x_1\in R$ and $\lambda\in[0,1]$ such that $\mu_{\tilde{A}}(\lambda x_1+(1-\lambda)x_2)\ge\min(\mu_{\tilde{A}}(x_1),\mu_{\tilde{A}}(x_2))$.

Definition 2.1.5. The parametric form $\tilde{A}$ of TrFN for some $0\le\alpha\le1$ is defined as $(\tilde{A})_\alpha=[\mu'_{\tilde{A}}(\alpha),\mu''_{\tilde{A}}(\alpha)]$, where $\mu'_{\tilde{A}}(\alpha)=p_1+\dfrac{\alpha}{\mu_{\tilde{A}}}(p_2-p_1)$, $\mu''_{\tilde{A}}(\alpha)=p_4-\dfrac{\alpha}{\mu_{\tilde{A}}}(p_4-p_3)$.

Example 2.1.2. Let us take $\tilde{A}=\langle(7,12,16,22);0.4\rangle$. The parametric representation are $\mu'_{0.4}(\alpha)=7+12.5\alpha$, and $\mu''_{0.4}(\alpha)=22-15\alpha$. For different values of α the degree of membership, shown in Table 1 and their graphical representation in Figure 2.

Table 1. Degree of Membership for different α.

α	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1.0**
$\mu'_{\tilde{A}}(\alpha)$	7.00	8.25	9.50	10.75	12.00	13.25	14.50	15.75	17.00	18.25	19.50
$\mu''_{\tilde{A}}(\alpha)$	22.00	20.50	19.5	17.50	16.00	14.50	13.00	11.50	10.00	8.50	7.00

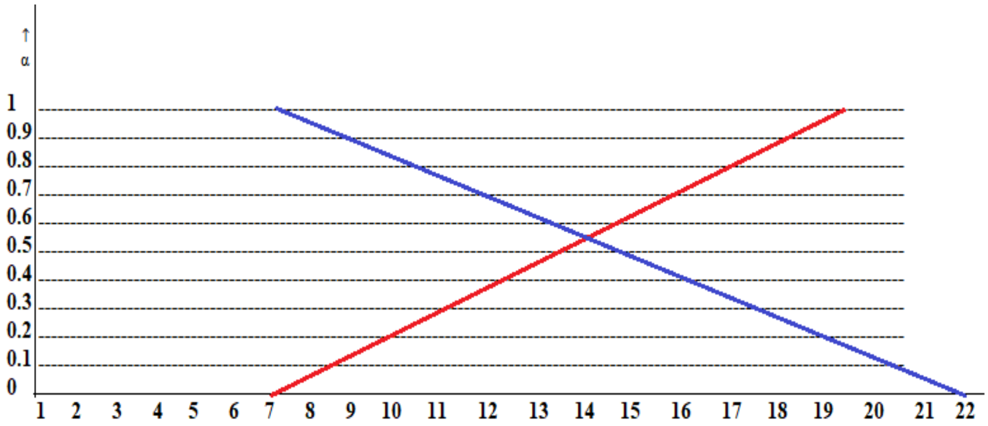

Figure 2. Degree of Membership for different α.

2.2 | Operational Laws on TrFN

Definition 2.2.1. If $\tilde{A}$ and $\tilde{B}$ are two TrFN with membership $\mu'_{\tilde{A}}(x)$, $\mu''_{\tilde{A}}(x)$ and a real numbers in [0.1], such as $\tilde{A} = \langle (p'_1, p'_2, p'_3, p'_4); \mu'_{\tilde{A}} \rangle$ and $\tilde{B} = \langle (p''_1, p''_2, p''_3, p''_4); \mu''_{\tilde{B}} \rangle$

- Addition of TrFN: $\tilde{C} = \tilde{A} + \tilde{B} = \langle (p'_1 + p''_1, p'_2 + p''_2, p'_3 + p''_3, p'_4 + p''_4); \mu'_{\tilde{A}} \vee \mu''_{\tilde{B}} \rangle$
- Negative of TrFN: If $\tilde{A} = \langle (p'_1, p'_2, p'_3, p'_4); \mu'_{\tilde{A}} \rangle$, then $-\tilde{A} = \langle (-p'_4, -p'_3, -p'_2, -p'_1); \mu'_{\tilde{A}} \rangle$
- Subtraction of TrFN: $\tilde{A} - \tilde{B} = \langle (p'_1 - p''_4, p'_2 - p''_3, p'_3 - p''_2, p'_4 - p''_1); \mu'_{\tilde{A}} \vee \mu''_{\tilde{B}} \rangle$
- Multiplication of TrFN: $$\tilde{A}^N \cdot \tilde{B}^N = \begin{cases} \langle (p'_1 \cdot p''_1, p'_2 \cdot p''_2, p'_3 \cdot p''_3, p'_4 \cdot p''_4); \mu'_{\tilde{A}} \vee \mu''_{\tilde{B}} \rangle, \ if \ p'_4 > 0, p''_4 > 0 \\ \langle (p'_1 \cdot p''_4, p'_2 \cdot p''_3, p'_3 \cdot p''_2, p'_4 \cdot p''_1); \mu'_{\tilde{A}} \vee \mu''_{\tilde{B}} \rangle, \ if \ p'_4 < 0, p''_4 > 0 \\ \langle (p'_4 \cdot p''_4, p'_3 \cdot p''_3, p'_2 \cdot p''_2, p'_1 \cdot p''_1); \mu'_{\tilde{A}} \vee \mu''_{\tilde{B}} \rangle, \ if \ p'_4 < 0, p''_4 < 0 \end{cases}$$
- Scalar multiplication of TrFN: $$k\tilde{A} = \begin{cases} \langle (kp'_1, kp'_2, kp'_3, kp'_4); \mu'_{\tilde{A}} \rangle, if \ k > 0, \\ \langle (kp'_4, kp'_3, kp'_2, kp'_1); \mu'_{\tilde{A}} \rangle, if \ k < 0. \end{cases}$$
- Inverse of TrFN: $$(\tilde{A})^{-1} = \frac{1}{\tilde{A}} = \begin{cases} \langle (\frac{1}{p'_4}, \frac{1}{p'_3}, \frac{1}{p'_2}, \frac{1}{p'_1}); \mu'_{\tilde{A}} \rangle, \ if \ p'_s > 0, \\ \langle (\frac{1}{p'_1}, \frac{1}{p'_2}, \frac{1}{p'_3}, \frac{1}{p'_4}); \mu'_{\tilde{A}} \rangle, \ if \ p'_s < 0. \end{cases}$$
- Division of TrFN: $$\frac{\tilde{A}}{\tilde{B}} = \begin{cases} \langle (\frac{p'_1}{p''_4}, \frac{p'_2}{p''_3}, \frac{p'_3}{p''_2}, \frac{p'_4}{p''_1}); \mu'_{\tilde{A}} \vee \mu''_{\tilde{B}} \rangle, \ if \ p'_4 > 0, p''_4 > 0 \\ \langle (\frac{p'_4}{p''_4}, \frac{p'_3}{p''_3}, \frac{p'_2}{p''_2}, \frac{p'_1}{p''_1}); \mu'_{\tilde{A}} \vee \mu''_{\tilde{B}} \rangle, \ if \ p'_4 < 0, p''_4 > 0; \\ \langle (\frac{p'_4}{p''_1}, \frac{p'_3}{p''_2}, \frac{p'_2}{p''_3}, \frac{p'_1}{p''_4}); \mu'_{\tilde{A}} \vee \mu''_{\tilde{B}} \rangle, \ if \ p'_4 < 0, p''_4 < 0. \end{cases}$$

Example 2.2.1. Let $\tilde{A} = \langle (8, 12, 18, 24), 0.6 \rangle$ and $\tilde{B} = \langle (7, 10, 15, 21), 0.5 \rangle$ be two TrFN, then

$\tilde{A} + \tilde{B} = \langle (15, 22, 33, 45), 0.6 \rangle$,

$\tilde{A}-\tilde{B}=\langle(-13,-3,8,17);0.6\rangle$

$\tilde{A}.\tilde{B}=\langle(56,120,270,504);0.6\rangle,$

$\tilde{A}/\tilde{B}=\langle(1.143,1.2,1.2,1.143);0.6\rangle,$

$3\tilde{A}=\langle(24,36,54,72);0.6\rangle$

3 | De-Fuzzification using Score Function

We use the score functions of a TrFN, defined by an expert [12] to compare any two TrFN. So that the score function for $\tilde{A}=\langle(p_1',p_2',p_3',p_4');\mu_{\tilde{A}}'\rangle$ is defined as $S(\tilde{A})=\left(\frac{p_1'+p_2'+p_3'+p_4'}{4}\right)\mu_{\tilde{A}}'$. For $\tilde{A}=\langle(8,12,18,24),0.6\rangle$, $S(\tilde{A})=9.3$.

Definition 3.1. (Comparison of TrFN). Let $\tilde{A}$ and $\tilde{B}$ are two TrFN, then one has the following:

- If $S(\tilde{A})<S(\tilde{B})\Rightarrow$ $\tilde{A}<\tilde{B}$
- If $S(\tilde{A})<S(\tilde{B})\Rightarrow$ $\tilde{A}<\tilde{B}$
- If $S(\tilde{A})=S(\tilde{B})$ then $\tilde{A}=\tilde{B}$

Example 3.2. Let $\tilde{A}=\langle(8,12,18,24),0.6\rangle$ and $\tilde{B}=\langle(7,10,15,21),0.5\rangle$ be two TrFN, then $S(\tilde{A})=9.3$, $S(\tilde{B})=6.625$ that is $S(\tilde{A})>S(\tilde{B})$ which implies $\tilde{A}>\tilde{B}$.

4 | The Mathematical Formulation of FITP

4.1 | FITP Classification

When at least one of the parameters in an inventory transportation problem (ITP), such as supply, demand, or cost, takes the form of FNs, the ITP is referred to as FITP. A FITP of type 1 is defined as having crisp cost but fuzzy demand and availability. FITP of type 2 refers to the FITP with crisp demand and crisp availability but fuzzy pricing. FITP is categorized as type 3 if all of the ITP criteria, including cost, demand, and availabilities, are combinations of sharp, triangular, or trapezoidal fuzzy numbers. It is referred to as overtime FITP or FITP of type 4 if all of the ITP's requirements must be expressed in fuzzy numbers.

4.2 | Mathematical Formulation of FITP

The ITP is very important for transporting goods from one source to another destination. In ITP if ambiguity occurs in cost, demand or supply then it is more difficult to solve it. To handle this type of impreciseness in cost to transferred product from i^{th} sources to j^{th} destination or uncertainty in supply and demand the decision maker introduce FITP of TrFN.

Here we consider two models in which the decision maker is unsettled about the specific values i.e. cost from i^{th} sources to j^{th} destination and also certainty or uncertainty in supply or demand of the product, so that a new type of TP is introduced namely FIP with parameters like cost, demand, and supply as TrFN. The FITP with assumptions and constraints is defined as the number of unites x_{ij} and the neutrosophic cost $\tilde{c}_{ij}$ are transported from i^{th} sources to j^{th} destination. For balance FIP $\sum_{i=0}^{m}a_i=\sum_{j=0}^{n}b_j$ i.e. total supply is equal to total demand.

For the formulation of FITP the following assumptions and constraints are required:

m Total number of source point

n	Total number of destination point
i	Table of source (for all m)
j	Table of destination (for all n)
$\tilde{x}_{ij}$	Number of transported fuzzy unites from i^{th} source to j^{th} destination
$\tilde{c}_{ij}$	Fuzzy cost of one unit transported from i^{th} source to j^{th} destination
$\tilde{a}_i$	Available fuzzy supply quantity from i^{th} source
$\tilde{b}_j$	Required fuzzy demand quantity to j^{th} destination
c_{ij}	Crisp cost of one unit quantity
x_{ij}	Number of transported crisps unites from i^{th} source to j^{th} destination
a_i	Available crisp supply quantity from i^{th} source
b_j	Required crisp demand quantity to j^{th} destination

The goal of FITP is to reduce the cost of product transportation from the point of origin to the final destination. The following is the mathematical expression of FIP with uncertain transported units, transportation costs, supply, and demand:

$$\text{Minimum} \quad \tilde{Z} = \sum_{i=0}^{m}\sum_{j=0}^{n} \tilde{x}_{ij}\tilde{c}_{ij}$$

$$\text{Subject to} \quad \sum_{j=0}^{n} \tilde{x}_{\boldsymbol{ij}} = \tilde{a}_i, \ \forall i(\text{sources}) = 1, 2, 3, \ldots, m,$$

$$\sum_{i=0}^{m} \tilde{x}_{ij} = \tilde{b}_j, \forall \ j(\text{destination}) = 1, 2, 3, \ldots, n,$$

$$\tilde{x}_{ij} \geq 0, \ \forall \ i = 1, 2, 3, \ldots, m, j = 1, 2, 3, \ldots, n.$$

4.3 | Solution Procedure of ITP and FITP

Since the overall inventory cost is independent of both distance and method of operation, we may formulate the problem in terms of either precise or imprecise manner. To solve the FITP, we first use the score function to turn all TrFNs into crisp values, which allows us to transform the FITP into a simple ITP. The following actions are necessary for the solution of FITP after balancing using the current method:

Step 1. Formulate the FITP form given uncertain data of company setup in new places.

Step 2. Convert the FITP into crisp ITP by using score function.

Step 3. Solve the crisp ITP by using Microsoft excel.

Step 4. Find the corresponding solution of FITP.

Step 5. Compare the crisp solution and fuzzy solution**.**

4.4 | Methodology for Solution of FIP

The methodology presented for solving the FITP and assessing the uncertainty of service networks is realistic and more appreciable in the current scenario. The methodology takes a holistic approach by combining various fuzzy-based techniques that address uncertainties in both demand and supply, recognizing the interconnected nature of these factors. The emphasis on thorough documentation and clear communication ensures that the methodology's findings are accessible and understandable to stakeholders. This transparency is crucial for gaining trust and facilitating effective decision-making. Thus, the methodology lies in its comprehensive and integrated approach, the involvement of subject matter specialists, the thoughtful use of fuzzy techniques, and its practical applicability through the development of a decision support system.

5 | Numerical Example

Keeping in mind of today's uncertainty in production, supply and demand, a producer of wedding suits, demand is low in the months before the wedding season but is expected to increase during those times. He can choose to manufacture after hours or to accumulate a stockpile throughout the earlier periods. Each period's normal fuzzy production capability is $\langle(30,40,50,60),0.7\rangle$ units, but during overtime he can generate up to $\langle(10,14,17,23),0.7\rangle$ units. While manufacturing during regular business hours fuzzy costs is Rs. $\langle(450,560,680,800),0.9\rangle$, producing during overtime would cost roughly Rs $\langle(950,1000,1150,1200),0.8\rangle$. The cost of maintaining inventory is Rs. $\langle(100,150,200,250),0.6\rangle$ per unit each period.

Also Period: P_1 P_2 P_3 P_4

Demand: $\langle(25,\ 35,\ 45,\ 55),0.6\rangle$ $\langle(45,\ 55,\ 65,\ 75),0.5\rangle$ $\langle(60,\ 70,\ 78\ 90),0.7\rangle$ $\langle(45,\ 55,\ 70,\ 75),0.6\rangle$

How many units should he produce in each period so as to minimize his cost? Also, determine the minimum cost so incurred. The problem can be formulated in tabular form as follows in Table 2.

Table 2. Tabular form of IVTP.

Periods $\rightarrow$ *Fabricate* $\downarrow$	P_1	P_2	P_3	P_4	**Supply**
M_1 $R_1 \rightarrow$	$\langle(600,800,1000,1200);0.8\rangle$	$\langle(620,840,1060,1280);0.8\rangle$	$\langle(640,920,1260,1360);0.8\rangle$	$\langle(660,960,1320,1440);0.8\rangle$	$\langle(55,70,80,95);0.8\rangle$
$O_1 \rightarrow$	$\langle(640,920,1260,1360);0.8\rangle$	$\langle(660,960,1320,1440);0.8\rangle$	$\langle(680,960,1240,1520);0.8\rangle$	$\langle(700,1000,1300,1600);0.8\rangle$	$\langle(14,17,20,24);0.8\rangle$
M_2 $R_2 \rightarrow$	M	$\langle(600,800,1000,1200);0.8\rangle$	$\langle(620,840,1060,1280);0.8\rangle$	$\langle(640,920,1260,1360);0.8\rangle$	$\langle(55,70,80,95);0.8\rangle$
$O_2 \rightarrow$	M	$\langle(640,920,1260,1360);0.8\rangle$	$\langle(660,960,1320,1440);0.8\rangle$	$\langle(680,960,1240,1520);0.8\rangle$	$\langle(14,17,20,24);0.8\rangle$
M_3 $R_3 \rightarrow$	M	M	$\langle(600,800,1000,1200);0.8\rangle$	$\langle(620,840,1060,1280);0.8\rangle$	$\langle(55,70,80,95);0.8\rangle$
$O_3 \rightarrow$	M	M	$\langle(640,920,1260,1360);0.8\rangle$	$\langle(660,960,1320,1440);0.8\rangle$	$\langle(14,17,20,24);0.8\rangle$
M_4 $R_4 \rightarrow$	M	M	M	$\langle(600,800,1000,1200);0.8\rangle$	$\langle(55,70,80,95);0.8\rangle$
$O_4 \rightarrow$	M	M	M	$\langle(640,920,1260,1360);0.8\rangle$	$\langle(14,17,20,24);0.8\rangle$
Demand	$\langle(35,45,55,65);0.7\rangle$	$\langle(55,65,85,95);0.8\rangle$	$\langle(65,75,90,110);0.8\rangle$	$\langle(61,79,94,126);0.6\rangle$	

Using score function, convert fuzzy values into crisp values as in Table 3, also find solution of balanced ITP by using excel solver as in Table 4.

Table 3. Formation of ITP after using score function

Periods → Fabricate ↓		P_1	P_2	P_3	P_4	Dummy	Supply
M_1	R_1 →	720	760	800	840	0	60
	O_1 →	800	840	880	920	0	15
M_2	R_2 →	0	720	760	800	0	60
	O_2 →	0	800	840	880	0	15
M_3	R_3 →	0	0	720	760	0	60
	O_3 →	0	0	800	840	0	15
M_4	R_4 →	0	0	0	720	0	60
	O_4 →	0	0	0	800	0	15
Demand		35	60	68	54	83	

Table 4. Solution of balanced ITP.

Periods → Fabricate ↓		P_1	P_2	P_3	P_4	**Dummy**	**Supply**	**Total Supply**
M_1	R_1 →	0	0	0	0	60	60	0
	O_1 →	0	0	0	0	15	15	0
M_2	R_2 →	28	0	0	32	0	60	25600
	O_2 →	7	0	0	0	8	15	0
M_3	R_3 →	0	45	0	15	0	60	11400
	O_3 →	0	15	0	0	0	15	0
M_4	R_4 →	0	0	53	7	0	60	5040
	O_4 →	0	0	15	0	0	15	0
Demand		35	60	68	54	83		
			Total Cost	**42040**				

According the crisp solution a small entry 7 in $M_2(O_2)_{21}$ gets introduce to remove degeneracy. The solution strategy for production in regular and over time as follows:

(i). Manufacturing unit M_1 produce nothing in any in regular or over time period.

(ii). Manufacturing unit M_2 produce 28 and 32 suits in regular time period P_1 and P_4 respectively and produce 7 suits in overtime period P_1.

(iii). Manufacturing unit M_3 produce 45 and 15 suits in regular time period P_2 and P_4 respectively and 15 suits in overtime period P_2.

(iv). Manufacturing unit M_4 produce 53 and 7 units in regular time period P_3 and P_4 respectively and 15 units in overtime period P_3.

The total cost for this allocation/production is Rs. 42040. The solution strategies for production in regular and over time shown in Figure 3 and Table 5.

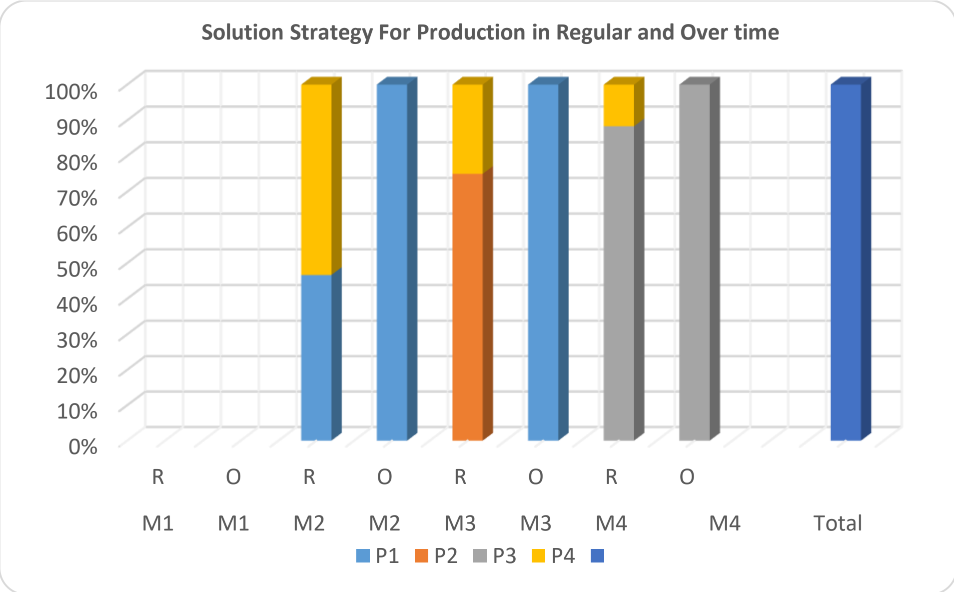

Figure 3. Solution strategies of FITP.

Table 5. Solution of corresponding balanced FITP.

Periods → *Fabricate* ↓	P_1	P_2	P_3	P_4
M_1 $R_1 \rightarrow$	-	-	-	-
$O_1 \rightarrow$	-	-	-	-
M_2 $R_2 \rightarrow$	M	-	-	$\left\langle \begin{pmatrix} 25,35, \\ 45,55 \end{pmatrix};0.8 \right\rangle$
$O_2 \rightarrow$	$\left\langle \begin{pmatrix} 3,7, \\ 11,14 \end{pmatrix};0.8 \right\rangle$	-	-	-
M_3 $R_3 \rightarrow$	M	M	-	$\left\langle \begin{pmatrix} 14,17, \\ 20,24 \end{pmatrix};0.8 \right\rangle$
$O_3 \rightarrow$	M	M	-	-
M_4 $R_4 \rightarrow$	M	M	M	$\left\langle \begin{pmatrix} 3,7, \\ 11,14 \end{pmatrix};0.8 \right\rangle$
$O_4 \rightarrow$	M	M	M	-

According the fuzzy solution a small entry $\left\langle \begin{pmatrix} 3,7, \\ 11,14 \end{pmatrix};0.8 \right\rangle$ in $M_2(O_2)_{21}$ gets introduce to remove degeneracy.

The solution strategy for production in regular and over time as follows:

- Manufacturing unit M_1 produce nothing in any in regular or over time period.
- Manufacturing unit M_2 produce $\left\langle \begin{pmatrix} 24,32, \\ 40,44 \end{pmatrix};0.8 \right\rangle$ and $\left\langle \begin{pmatrix} 25,35, \\ 45,55 \end{pmatrix};0.8 \right\rangle$ suits in regular time period P_1 and P_4 respectively and produce $\left\langle \begin{pmatrix} 3,7, \\ 11,14 \end{pmatrix};0.8 \right\rangle$ suits in overtime period P_1.

- Manufacturing unit M_3 produce $\left\langle \begin{pmatrix} 41,54, \\ 63,67 \end{pmatrix}; 0.8 \right\rangle$ and $\left\langle \begin{pmatrix} 14,17, \\ 20,24 \end{pmatrix}; 0.8 \right\rangle$ suits in regular time period P_2 and $\left\langle \begin{pmatrix} 15,18, \\ 19,23 \end{pmatrix}; 0.8 \right\rangle$ P_4 respectively and 15 suits in overtime period P_2.

- Manufacturing unit M_4 produce $\left\langle \begin{pmatrix} 48,59, \\ 73,85 \end{pmatrix}; 0.8 \right\rangle$ and $\left\langle \begin{pmatrix} 3,7, \\ 11,14 \end{pmatrix}; 0.8 \right\rangle$ units in regular time period P_3 and P_4 respectively and $\left\langle \begin{pmatrix} 14,17, \\ 20,24 \end{pmatrix}; 0.8 \right\rangle$ units in overtime period P_3.

- The minimum fuzzy cost of FITP and its corresponding crisp cost minimum $\tilde{Z}$ is:

$$\text{Minimum } \tilde{Z} = \left\langle \begin{pmatrix} 640,920, \\ 1260,1360 \end{pmatrix}; 0.8 \right\rangle . \left\langle \begin{pmatrix} 25,35, \\ 45,55 \end{pmatrix}; 0.8 \right\rangle + \left\langle \begin{pmatrix} 620,840, \\ 1060,1280 \end{pmatrix}; 0.8 \right\rangle . \left\langle \begin{pmatrix} 14,17, \\ 20,24 \end{pmatrix}; 0.8 \right\rangle + \left\langle \begin{pmatrix} 600,800 \\ 1000,1200 \end{pmatrix}; 0.8 \right\rangle . \left\langle \begin{pmatrix} 3,7, \\ 11,14 \end{pmatrix}; 0.8 \right\rangle$$

$$= \left\langle \begin{pmatrix} 16000,32200, \\ 56700,174800 \end{pmatrix}; 0.8 \right\rangle + \left\langle \begin{pmatrix} 8680,14280, \\ 21200,30720 \end{pmatrix}; 0.8 \right\rangle + \left\langle \begin{pmatrix} 1800,5600 \\ 11000,16800 \end{pmatrix}; 0.8 \right\rangle = \left\langle \begin{pmatrix} 26480,52080 \\ 88900,122320 \end{pmatrix}; 0.8 \right\rangle \approx 57956$$

6 | Results and Discussion

In this present study the optimal solution in crisp form of inventory problem is Rs. 42040, which is minimum while in fuzzy form the solution is $\left\langle \begin{pmatrix} 26480,52080 \\ 88900,122320 \end{pmatrix}; 0.8 \right\rangle$, which is equivalent to Rs. 57956 for level of truthfulness. The difference indicates the vague or uncertainty in decision. The degree of membership or acceptance, is defined as $\mu_{\tilde{A}}(x) \times 100$, where x denotes the total cost as presented in Table 6 and Figure 4.

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{0.8(x-26480)}{25600}, & \text{for } 26480 \le x \le 52080, \\ 0.8, & \text{for } 52080 \le x \le 88900, \\ \dfrac{0.8(122320-x)}{33420}, & \text{for } 88900 \le x \le 122320, \\ 0, & \text{for otherwise.} \end{cases}$$

With the help of degree of membership, we can conclude the total fuzzy cost from the range of 20000 to 130000 for favorable to schedule the initial inventory budget allocation.

Table 6. Total cost.

$x \rightarrow$ Degree $\downarrow$	**20000**	**40000**	**70000**	**90000**	**110000**	**130000**
$\mu_{\tilde{A}}(x) \times 100$	0	42.25	80.00	77.36	29.49	0

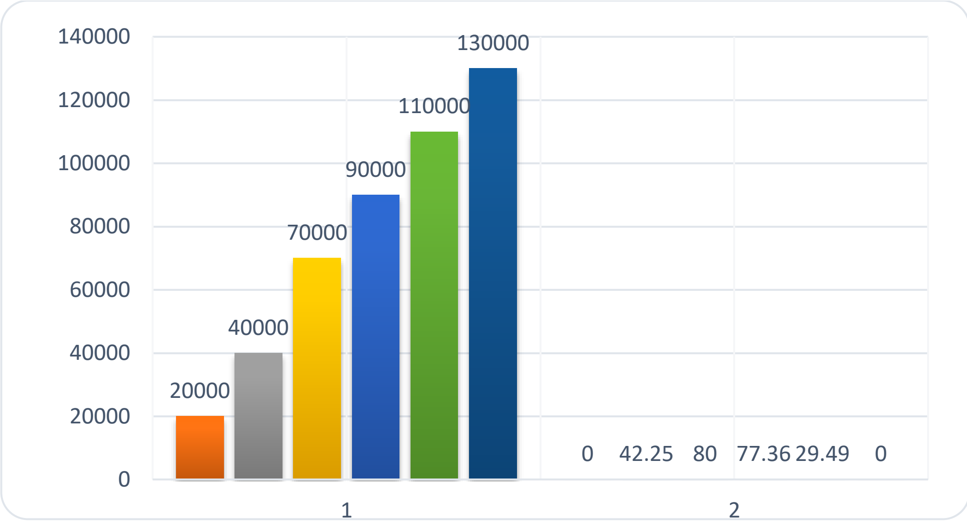

Figure 4. Total cost.

7 | Conclusions

The excerpt outlines a research paper that focuses on applied inventory modeling in a contemporary, uncertain environment. The central theme involves leveraging the concept of fuzzy values or numbers to address uncertainty in decision science. The proposed method aims to offer a more practical structure, taking into account various characteristics of inventory problems in today's uncertain environment. In summary, the research paper aims to contribute to the field of inventory modeling by leveraging fuzzy logic and introducing a futuristic approach to the FITP.

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Author Contributaion

All authors contributed equally to this work.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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