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# A Novel Multi-criteria Decision Making Approach Based on Bipolar Neutrosophic Set for Evaluating Financial Markets in Egypt

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## Abstract

In the evolving landscape of financial markets, assessing company performance with precision and reliability is crucial for investors and analysts. This paper introduces an innovative framework for evaluating the financial performance of companies listed on the Egyptian Exchange (EGX) by integrating Bipolar Neutrosophic Sets (BNS) with Multi-Criteria Decision-Making (MCDM) methods. The proposed framework utilizes BNS to incorporate the inherent uncertainty and imprecision of financial data, and integrates MCDM methods to provide a comprehensive assessment of multiple performance indicators. Specifically, the evaluation focuses on seven critical criteria: Stock Returns, Volatility, Liquidity, Market Capitalization, Earnings Growth, Revenue Growth, and Analyst Ratings. By utilizing Bipolar Neutrosophic Weighted Average (BNWA) operators and similarity measures, the study effectively ranks companies through a nuanced analysis of these criteria. The results highlight the effectiveness of the proposed approach in providing a robust and accurate financial performance evaluation framework tailored to the unique dynamics of the Egyptian market. The study demonstrates that the hybrid model not only accommodates variations in criteria weights but also offers valuable insights for investment decisions in emerging markets. The approach exhibits resilience and adaptability, providing a more informed and reliable basis for evaluating financial performance. This research contributes to the advancement of financial performance assessment methodologies by showcasing the potential of combining BNS with MCDM techniques. The research also emphasizes the framework's practical applications and suggests future research directions for refining the model and exploring its use in diverse financial contexts.

**Keywords:** BNS, MCDM, Weighted Average, Similarity Measures, Financial Market, Financial Performance, Egyptian Exchange.

# 1 | Introduction

Financial markets play a vital role in supporting economic growth and development by facilitating the flow of capital, managing risk, and generating returns on investments [1]. The stock exchange, as a cornerstone of financial markets, significantly influences economic growth and stability. Stock exchanges play a critical role in the functioning of global financial markets as they provide companies with access to capital, help investors grow their wealth, and serve as an indicator of economic health [2]. Financial markets rely heavily on

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informational transparency to ensure that the markets set prices that are efficient and appropriate [3]. Evaluating financial markets involves assessing financial performance and its impact on economic growth. A financial performance assessment encompasses a comprehensive analysis of a company's financial health, operational efficiency, and overall profitability, which is essential for stakeholders—including investors, managers, and analysts—to make informed decisions [4]. Financial performance assessment is crucial in evaluating a company's overall financial health, which involves analyzing various financial metrics to determine how well a company is generating revenues, managing its assets and liabilities, and safeguarding the financial interests of its stakeholders and stockholders [5]. Financial performance assessment is vital in evaluating a company's stock performance. So, financial performance assessment of financial markets using stock returns is a crucial aspect of evaluating a company's overall financial health [6]. Company financial performance is a fundamental factor in influencing changes in stock returns obtained by investors [7]. Evaluating financial performance involves analyzing how effectively a firm or financial market uses its assets to generate revenues and maintain overall financial health [8]. Given this, the evaluation criteria and methods need to account for both objective financial metrics and subjective assessments. In the context of stock returns, financial performance assessment is crucial in evaluating the performance of a company's stock. Stock returns are influenced by a company's financial performance, and investors use financial performance metrics to make informed investment decisions [9]. Assessing the financial performance of financial markets, particularly focusing on stock returns, involves analyzing various factors and metrics to understand market performance and gauge the expected returns for investors [10]. A multi-faceted approach is required to assess the financial performance of financial markets, especially stock returns. This includes analyzing historical data, market conditions, financial ratios, and investment strategies [11]. By combining quantitative analysis with contextual understanding, investors and analysts can make informed decisions and develop strategies to maximize returns while managing risk.

The importance of measuring financial performance, particularly in the financial sector, has indeed surged with the increasing complexity of financial markets and the imperative for transparency [12]. An accurate assessment of financial performance is crucial for companies to make informed decisions and maintain investor confidence [13]. Multi-criteria decision-making (MCDM) methods provide a robust framework for evaluating the financial performance of financial markets, particularly focusing on stock returns [14]. MCDM involves a structured approach that accounts for multiple factors and criteria influencing stock performance, allowing stakeholders to gain a nuanced understanding of stock performance and make more informed investment decisions [15]. Financial Performance Assessment of Financial Markets using Stock Returns with MCDM BNS is a novel approach to evaluating the financial performance of companies. This method combines the strengths of MCDM and BNS to handle complex, inconsistent, and ambiguous data [16]. BNS is designed to effectively handle bipolar, inconsistent, and ambiguous data. They are characterized by positive and negative membership degrees, which can represent both positive and negative preferences in decision-making processes [17]. In the context of financial performance assessment, BNS can be used to evaluate the stock returns of companies based on multiple criteria, such as Volatility, Liquidity, Market Capitalization, and Revenue Growth. Integrating BNS with MCDM methods offers a powerful approach to assessing financial performance in the financial markets, including those in Egypt. The Egyptian financial market faces unique challenges and opportunities. This approach accommodates the complexity and ambiguity of financial data, providing stakeholders with a more accurate and comprehensive understanding of stock performance. By considering multiple criteria and their respective weights, stakeholders can make more informed investment decisions, optimizing their portfolios and managing risks effectively. Figure 1 provides a visual representation of the top 10 companies in the Egyptian financial market in the last half of the current year, based on their daily average value in EGP, allowing stakeholders to quickly identify the leading performers and make informed decisions [18]. By integrating BNS with MCDM methods, stakeholders can further analyze these companies based on multiple criteria to gain a more comprehensive understanding of their stock performance.

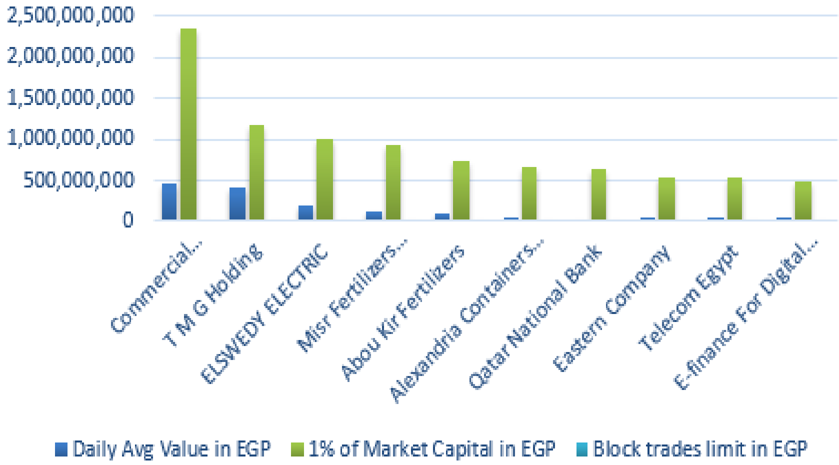


**Figure 1.** Top 10 Companies in the Egyptian Financial Market.

This framework also leverages a group of neutrosophic analytical network processes to resolve challenging decision-making issues [19]. Essentially, BNS extends the notion of fuzzy sets and neutrosophic sets by considering both positive and negative membership degrees, in a more comprehensive and nuanced manner [16]. The application of BNS is particularly useful in MCDM problems, where the assessment scores for alternatives on criteria take the form of bipolar neutrosophic numbers (BNN) [17]. This enables the selection of the most preferred alternatives determined by the accuracy, certainty, and score functions. The application of BNS in financial markets is particularly useful for addressing MCDM problems. Investors and analysts often face complex decision-making scenarios where they need to evaluate multiple investment alternatives based on various criteria. BNS also provides a robust framework to handle the bipolar nature of financial information, allowing for a more comprehensive evaluation of investments. The accuracy, certainty, and score functions can be employed for comparison and rank the alternatives based on their bipolar neutrosophic (BN) evaluation values. The Bipolar Neutrosophic Weighted Average Operator (BNWA) or bipolar neutrosophic weighted geometric operator (BNWG) can be used to aggregate the BN facts and select the most desirable stocks. Similarity measures in BNS can be used to compare different investment alternatives by evaluating the degree of similarity between their BN evaluation values.

The proposed approach for evaluating financial performance using MCDM and BNS aims to:

- Handle Complexity and Ambiguity: Address the complexities and ambiguities inherent in financial data by modeling both positive and negative information using BNS.
- Provide Comprehensive Evaluation: Offer a robust framework that accommodates multiple criteria and factors affecting stock performance, ensuring a thorough evaluation of investment alternatives.
- Optimize Investment Decisions: Assist stakeholders in making more informed and strategic investment decisions by providing a nuanced understanding of stock performance and associated risks.

The integration of BNS with MCDM methods contributes significantly to the field of financial performance assessment in several ways:

- BNS allows for the modeling of both positive and negative aspects of financial information, leading to a more balanced and accurate assessment of stock performance, and aiding investors in developing strategies to maximize returns while managing risks.
- Combining BNS with MCDM methods provides a comprehensive evaluation framework that enables the consideration of multiple criteria simultaneously, such as volatility, liquidity, market capitalization, and revenue growth.

- The application of BNWA and geometric operators facilitates the aggregation of information, helping to rank and select the most desirable stocks.
- The application of similarity measures helps in comparing and ranking investment alternatives based on their bipolar neutrosophic evaluation values, further refining the decision-making process.
- The approach is particularly valuable in emerging markets like Egypt, where financial markets face unique challenges and opportunities. It adapts to the local context by addressing specific market dynamics and regulatory environments.

The remaining parts are organized as follows: Section 2: Literature Review of BNS and Financial Performance Assessment using MCDM. Section 3: Fundamentals of BNS. Section 4: Bipolar Neutrosophic Sets in MCDM. Section 5: Case Study: Financial Performance Assessment of Financial Markets with MCDM and BNS. Section 6: Discussion and Comparative Analysis. Section 7: Conclusions.

## 2 | Literature Review

The literature review highlights some reviews of BNS and financial performance assessments using MCDM.

MCDM methods are a set of techniques used to evaluate and compare alternatives based on multiple criteria [20]. These methods have been widely applied in various fields, including finance, engineering, and management [21]. In the context of financial performance assessment, MCDM methods can be used to evaluate the performance of companies based on multiple criteria, such as financial ratios, risk metrics, and growth indicators [22]. MCDM methods have proven to be quite successful in determining financial performance, especially when dealing with multiple ratios or criteria [23]. Several studies have used MCDM methods in financial performance evaluation. Nese Yalcin et al. [24] applied fuzzy MCDM methods for the financial performance assessment of Turkish manufacturing industries. Abdolhamid et al. [25] also applied Fuzzy MCDM for financial performance evaluation but for IRANIAN Companies. Nida Turegun [15] used the VIKOR method for financial performance evaluation. Mahmut Baydaş et al. [26] provided a comprehensive MCDM assessment for economic data using CODAS, and fuzzy approaches. Nazanin Ghaemi-Zadeh and Maryam Eghbali-Zarch evaluate the financial performance of corporations and investors' behavior using D-CRITIC and fuzzy MULTI-MOORA techniques [27]. Abdulrahman T. Alsanousi et al. [28] applied hybrid BWM and the TOPSIS methods to evaluate Saudi Stocks based on Financial Performance. Ahmet Kaya et al. [13] used MCDM with simulation to decide the financial performance of firms in the Borsa. Mohamed et al. [29] applied a combined plithogenic MCDM technique to assess manufacturing industries' financial performance. Sanjay Gupta et al. [30] applied the MCDM methodology for assessing the public sector Banks' financial performance in India. These studies primarily aim to rank the alternatives according to financial performance criteria and identify the companies with the highest performance.

BNS was introduced by Deli et al. [16] as a generalization of neutrosophic sets, which allows for the representation of bipolar information, i.e., information that has both positive and negative aspects. This concept has been applied in various studies, particularly in the context of MCDM. Several studies have applied the integration of BNS and MCDM methods. Abdel-Basset et al. [19] utilized BNS and TOPSIS for professional selection. Similarly, Pamučar et al. [31] applied BNS and MABAC for Sustainable Energy Selection. Abdullah, Lazim and Rahim, Siti Nuraini applied DEMATEL based on bipolar neutrosophic for urban sustainable development [32]. Muhammad Akram et al. [33] applied BNS-TOPSIS-ELECTRE-1 for decision-making. Irvanizam Irvanizam et al. [34] provided an Improved EDAS method based on BNS. Deli et al. [16] suggested the AW and GW operators collect the BN information; these operators enable the combination of bipolar neutrosophic sets in a meaningful way. STANUJKIC et al. [35] developed a new strategy for assessing the reliability of BNS and its implementation in MCDM. Vakkas Uluc et al. [36] provided similarity measures of BNS and their application to MCDM. Mehmet Şahin et al. [37] developed a Jaccard Vector Similarity Measure of BNS In light of MCDM. These studies demonstrate the potential of BNS in

MCDM and highlight the importance of developing new aggregation operators and similarity measures for BNS.

## 3 | Preliminaries

BNS extends the notion of neutrosophic sets by considering both positive and negative membership degrees [16]. According to Deli et al. [16] in a BNS, each element has a positive and a negative membership degree, both of which are defined by truth $(\mathrm{Tru})$, indeterminacy $(\mathrm{In})$, and falsity $(\mathrm{Fa})$ components. This allows BNS to handle more complex and uncertain information than traditional neutrosophic sets. The BNS Scale symbolizes the degrees of $\mathrm{Tru}$, $\mathrm{In}$, and $\mathrm{Fa}$ in both positive and negative membership functions. Each component is typically represented by a value within the interval [0, 1]. According to Deli et al. [16] the BNS Scale is shown as:

$\mathrm{B_i} = \{\langle \mathrm{z}, \mathrm{Tru^{Po}(z)}, \mathrm{In^{po}(z)}, \mathrm{Fa^{po}(z)}, \mathrm{Tru^{ne}(z)}, \mathrm{In^{ne}(z)}, \mathrm{Fa^{ne}(z)} \rangle : \mathrm{z} \in \mathrm{Z}\}$,

Where $\mathrm{Tru^{Po}}, \mathrm{In^{po}}, \mathrm{Fa^{po}}$ : Z → [1, 0] and$\mathrm{Tru^{ne}}, \mathrm{In^{ne}}, \mathrm{Fa^{ne}}$: Z → [-1, 0]

Overview of the theory of BNS and a few of its functions:

**Definition 3.1.** A BNS $\mathrm{B_i}$ in Z is described as an item of the type:

$\mathrm{B_i} = \{\langle \mathrm{z}, \mathrm{Tru^{Po}(z)}, \mathrm{In^{po}(z)}, \mathrm{Fa^{po}(z)}, \mathrm{Tru^{ne}(z)}, \mathrm{In^{ne}(z)}, \mathrm{Fa^{ne}(z)} \rangle : \mathrm{z} \in \mathrm{Z}\}$,

Where $\mathrm{Tru^{Po}}, \mathrm{In^{po}}, \mathrm{Fa^{po}}$ : Z → [1, 0] and$\mathrm{Tru^{ne}}, \mathrm{In^{ne}}, \mathrm{Fa^{ne}}$: Z → [-1, 0]

The positive membership $\mathrm{Tru^{Po}(z)}, \mathrm{In^{po}(z)}, \mathrm{Fa^{po}(z)}$ indicate the $\mathrm{Tru}$, In and $\mathrm{Fa}$ membership of an element $\in \mathrm{Z}$ which corresponds to a BNS $\mathrm{B_i}$. Negative membership is also indicated as $\mathrm{Tru^{ne}(z)}, \mathrm{In^{ne}(z)}, \mathrm{Fa^{ne}(z)}$.

**Definition 3.2.** Assume : $\mathrm{B_{i1}} = \{\langle \mathrm{z}, \mathrm{Tru_1^{po}(z)}, \mathrm{In_1^{po}(z)}, \mathrm{Fa_1^{po}(z)}, \mathrm{Tru_1^{ne}(z)}, \mathrm{In_1^{ne}(z)}, \mathrm{Fa_1^{ne}(z)} \rangle\}$ and $\mathrm{B_{i2}} = \{\langle \mathrm{z}, \mathrm{Tru_2^{po}(z)}, \mathrm{In_2^{po}(z)}, \mathrm{Fa_2^{po}(z)}, \mathrm{Tru_2^{ne}(z)}, \mathrm{In_2^{ne}(z)}, \mathrm{Fa_2^{ne}(z)} \rangle\}$ Be two BNS. Then $\mathrm{B_{i1}} \subseteq \mathrm{B_{i2}}$ if and only if $\mathrm{Tru_1^{po}(z)} \le \mathrm{Tru_2^{po}(z)}, \mathrm{In_1^{po}(z)} \le \mathrm{In_2^{po}(z)}, \mathrm{Fa_1^{po}(z)} \le \mathrm{Fa_2^{po}(z)}$ and if $\mathrm{Tru_1^{ne}(z)} \ge \mathrm{Tru_2^{ne}(z)}, \mathrm{In_1^{ne}(z)} \ge \mathrm{In_2^{ne}(z)}, \mathrm{Fa_1^{ne}(z)} \ge \mathrm{Fa_2^{ne}(z)}$ for all $\mathrm{z} \in \mathrm{Z}$.

**Definition 3.3.** Let $\mathrm{B_{i1}} = \{\langle \mathrm{z}, \mathrm{Tru_1^{po}(z)}, \mathrm{In_1^{po}(z)}, \mathrm{Fa_1^{po}(z)}, \mathrm{Tru_1^{ne}(z)}, \mathrm{In_1^{ne}(z)}, \mathrm{Fa_1^{ne}(z)} \rangle\}$ and

$\mathrm{B_{i2}} = \{\langle \mathrm{z}, \mathrm{Tru_2^{po}(z)}, \mathrm{In_2^{po}(z)}, \mathrm{Fa_2^{po}(z)}, \mathrm{Tru_2^{ne}(z)}, \mathrm{In_2^{ne}(z)}, \mathrm{Fa_2^{ne}(z)} \rangle\}$ Be two BNS. Then $\mathrm{B_{i1}} = \mathrm{B_{i2}}$ if $\mathrm{Tru_1^{po}(z)} = \mathrm{Tru_2^{po}(z)}, \mathrm{In_1^{po}(z)} = \mathrm{In_2^{po}(z)}, \mathrm{Fa_1^{po}(z)} = \mathrm{Fa_2^{po}(z)}$ and if $\mathrm{Tru_1^{ne}(z)} = \mathrm{Tru_2^{ne}(z)}, \mathrm{In_1^{ne}(z)} = \mathrm{In_2^{ne}(z)}, \mathrm{Fa_1^{ne}(z)} = \mathrm{Fa_2^{ne}(z)}$ for all $\mathrm{z} \in \mathrm{Z}$.

**Definition 3.4.** Assume $\mathrm{B_{i1}} = \{\langle \mathrm{z}, \mathrm{Tru_1^{po}(z)}, \mathrm{In_1^{po}(z)}, \mathrm{Fa_1^{po}(z)}, \mathrm{Tru_1^{ne}(z)}, \mathrm{In_1^{ne}(z)}, \mathrm{Fa_1^{ne}(z)} \rangle\}$ and

$\mathrm{B_{i2}} = \{\langle \mathrm{z}, \mathrm{Tru_2^{po}(z)}, \mathrm{In_2^{po}(z)}, \mathrm{Fa_2^{po}(z)}, \mathrm{Tru_2^{ne}(z)}, \mathrm{In_2^{ne}(z)}, \mathrm{Fa_2^{ne}(z)} \rangle\}$ Be two BNS. The definition of their union is:

$$(\mathrm{B_{i1}} \cup \mathrm{B_{i2}})(\mathrm{z}) = \begin{bmatrix} \max\left(\mathrm{Tru_1^{po}(z)}, \mathrm{Tru_2^{po}(z)}\right), \frac{\mathrm{In_1^{po}(z)} + \mathrm{In_2^{po}(z)}}{2}, \min\left(\mathrm{Fa_1^{po}(z)}, \mathrm{Fa_2^{po}(z)}\right), \\ \min\left(\mathrm{Tru_1^{ne}(z)}, \mathrm{Tru_2^{ne}(z)}\right), \frac{\mathrm{In_1^{ne}(z)} + \mathrm{In_2^{ne}(z)}}{2}, \max(\mathrm{Fa_1^{ne}(z)}, \mathrm{Fa_2^{ne}(z)}) \end{bmatrix} \text{For all } \mathrm{z} \in \mathrm{Z}.$$

**Definition 3.5.** Suppose $\mathrm{B_{i1}} = \{\langle \mathrm{z}, \mathrm{Tru_1^{po}(z)}, \mathrm{In_1^{po}(z)}, \mathrm{Fa_1^{po}(z)}, \mathrm{Tru_1^{ne}(z)}, \mathrm{In_1^{ne}(z)}, \mathrm{Fa_1^{ne}(z)} \rangle\}$ and

$\mathrm{B_{i2}} = \{\langle \mathrm{z}, \mathrm{Tru_2^{po}(z)}, \mathrm{In_2^{po}(z)}, \mathrm{Fa_2^{po}(z)}, \mathrm{Tru_2^{ne}(z)}, \mathrm{In_2^{ne}(z)}, \mathrm{Fa_2^{ne}(z)} \rangle\}$ Be two BNS. The definition of their intersection is

$$(B_{i1} \cap B_{i2})(z) = \begin{bmatrix} \min\left(Tru_1^{po}(z), Tru_2^{po}(z)\right), \dfrac{In_1^{po}(z) + In_2^{po}(z)}{2}, \max\left(Fa_1^{po}(z), Fa_2^{po}(z)\right), \\ \max\left(Tru_1^{ne}(z), Tru_2^{ne}(z)\right), \dfrac{In_1^{ne}(z) +, In_2^{ne}(z)}{2}, \min(Fa_1^{ne}(z), Fa_2^{ne}(z)) \end{bmatrix} \text{for all } z \in Z.$$

**Definition 3.6.** Suppose $B_i = \{\langle z, Tru^{Po}(z), In^{po}(z), Fa^{po}(z), Tru^{ne}(z), In^{ne}(z), Fa^{ne}(z) \rangle : z \in Z\}$, be a BNS in Z . Then the complement of $B_i$ is denoted by ${B_i}^c$ and is defined by

$$Tru_{A^c}^{po}(z) = \{1^+\} - Tru_A^{po}(z), In_{A^c}^{po}(z) = \{1^+\} - In_A^{po}(z),\ Fa_{A^c}^{po}(z) = \{1^+\} - Fa_A^{po}(z)$$

and

$$Tru_{A^c}^{ne}(z) = \{1^-\} - Tru_A^{ne}(z), In_{A^c}^{ne}(z) = \{1^-\} - In_A^{ne}(z),\ Fa_{A^c}^{ne}(z) = \{1^-\} - Fa_A^{ne}(z)$$

**Definition 3.7.** Suppose $B_i = \{\langle z, Tru^{Po}(z), In^{po}(z), Fa^{po}(z), Tru^{ne}(z), In^{ne}(z), Fa^{ne}(z) \rangle : z \in Z\}$, be a BNS in Z . Then, the score function $S(B_i)$, accuracy function $a(B_i)$ and certainty function $C(B_i)$ of a BN are defined as follows:

$$S(B_i) = \left[\frac{1}{6}\right]\left(Tru^{Po} + 1 - In^{po} + 1 - Fa^{po} + 1 + Tru^{ne} - In^{ne} - Fa^{ne}\right) \quad (1)$$

$$a(B_i) = Tru^{po} - Fa^{po} + Tru^{ne} - Fa^{ne} \quad (2)$$

$$C(B_i) = Tru^{po} - Fa^{ne} \quad (3)$$

**Definition 3.8.** Suppose $B_{i1} = \{\langle z, Tru_1^{po}(z), In_1^{po}(z), Fa_1^{po}(z), Tru_1^{ne}(z), In_1^{ne}(z), Fa_1^{ne}(z) \rangle\}$ and $B_{i2} = \{\langle z, Tru_2^{po}(z), In_2^{po}(z), Fa_2^{po}(z), Tru_2^{ne}(z), In_2^{ne}(z), Fa_2^{ne}(z)\rangle\}$ Be two BNS. The definition of a comparison method is as:

$S(B_{i1}) > S(B_{i2})$, then $B_{i1}$ is higher than $B_{i2}$ , that is $B_{i1}$ superior to $B_{i2}$ denoted by $B_{i1} > B_{i2}$

$S(B_{i1}) = S(B_{i2})$ and $a(B_{i1}) > a(B_{i2})$ , then $B_{i1}$ is higher than $B_{i2}$ , that is $B_{i1}$ superior to $B_{i2}$ denoted by $B_{i1} > B_{i2}$

$S(B_{i1}) = S(B_{i2})$ and $a(B_{i1}) = a(B_{i2})$ and $c(B_{i1}) > c(B_{i2})$ then $B_{i1}$ is greater than $B_{i2}$, that is $B_{i1}$ superior to $B_{i2}$ denoted by $B_{i1} > B_{i2}$

$S(B_{i1}) = S(B_{i2})$ and $a(B_{i1}) = a(B_{i2})$ and $c(B_{i1}) = c(B_{i2})$ then $B_{i1}$ is equal to $B_{i2}$ , that is $B_{i1}$ is indifferent to $B_{i2}$ denoted by $B_{i1} = B_{i2}$

Considering the research presented in [16] the following WA operators connected to BNS are presented:

**Definition 3.9.** Let $B_{ij} = \left( Tru_j^{po}, In_j^{po}, Fa_j^{po}, Tru_j^{ne}, In_j^{ne}, Fa_j^{ne} \right)$ $j = (1, 2, \dots, n)$ be a family of BNS. A mapping $A_w: Q_n \to Q$ is called BNWA operator if it satisfies: $A_w(B_{i1}, B_{i2}, \dots, B_{in}) =$

$$\sum_{j=1}^{n} w_j B_{ij} = (1 - \prod_{j=1}^{n}(1 - Tru_j^{po})^{w_j}, \prod_{j=1}^{n} {In_j^{po}}^{w_j}, \prod_{j=1}^{n} {Fa_j^{po}}^{w_j}, - \prod_{j=1}^{n}(-Tru_j^{ne})^{w_j}, -(1 - \prod_{j=1}^{n}(1 - (-In_j^{ne}))^{w_j}), -(1 - \prod_{j=1}^{n}(1 - (-Fa_j^{ne}))^{w_j})) \quad (4)$$

As $w_j$ is the weight of $B_{ij}$ $(j = 1, 2, \dots, n)$ , $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$

**Definition 3.10.** Let $B_{ij} = \left( Tru_j^{po}, In_j^{po}, Fa_j^{po}, Tru_j^{ne}, In_j^{ne}, Fa_j^{ne} \right)$ $j = (1, 2, \dots, n)$ be a family of BNS. A mapping $G_w: Q_n \to Q$ is BNWG operator if it satisfies

$$G_w(B_{i1}, B_{i2}, \dots, B_{in}) = \prod_{j=1}^{n} {B_{ij}}^{w_j} = \prod_{j=1}^{n} {Tru_j^{po}}^{w_j}, 1 - \prod_{j=1}^{n}(1 - In_j^{po})^{w_j}, 1 - \prod_{j=1}^{n}(1 - Fa_j^{Po})^{w_j}, -(1 - \prod_{j=1}^{n}(1 - (-Tru_j^{ne}))^{w_j}), - \prod_{j=1}^{n}(-In_j^{ne})^{w_j}, - \prod_{j=1}^{n}(-Fa_j^{ne})^{w_j} \quad (5)$$

As $w_j$ is the weight of $B_{ij}$ $(j = 1, 2, \dots, n)$ , $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$

In light of the findings reported in [36], the following similarity measures between BNS are provided.

**Definition 3.11**: Suppose $B_{i1} = \{\langle z, Tru_1^{po}(z), In_1^{po}(z), Fa_1^{po}(z), Tru_1^{ne}(z), In_1^{ne}(z), Fa_1^{ne}(z) \rangle\}$ and $B_{i2} = \{\langle z, Tru_2^{po}(z), In_2^{po}(z), Fa_2^{po}(z), Tru_2^{ne}(z), In_2^{ne}(z), Fa_2^{ne}(z)\rangle\}$ be two BNS;

Then, Dice similarity measure between BNS $B_{i1}$ and $B_{i2}$, indicated $D(B_{i1}, B_{i2})$, is described as $D(B_{i1}, B_{i2}) =$

$$\frac{1}{n}\sum_{i=1}^{n}\left(\frac{Tru_1^{po}(z).Tru_2^{po}(z)+In_1^{po}(z).In_2^{po}(z)+Fa_1^{pa}(z).Fa_2^{po}(z)-Tru_1^{ne}(z).Tru_2^{ne}(z)+In_1^{ne}(z).In_2^{ne}(z)+Fa_1^{ne}(z).Fa_2^{ne}(z)}{(Tru_1^{po}(z))^2+(In_1^{po}(z))^2+(Fa_1^{po}(z))^2+(Tru_2^{po}(z))^2+(In_2^{po}(z))^2+(Fa_2^{po}(z))^2-(Tru_1^{ne}(z))^2+(In_1^{ne}(z))^2+(Fa_1^{ne}(z))^2-(Tru_2^{ne}(z))^2+(In_2^{ne}(z))^2+(Fa_2^{ne}(z))^2}\right. \quad (6)$$

**Definition 3.12.** Suppose $B_{i1} = \{\langle z, Tru_1^{po}(z), In_1^{po}(z), Fa_1^{po}(z), Tru_1^{ne}(z), In_1^{ne}(z), Fa_1^{ne}(z)\rangle\}$ and $B_{i2} = \{\langle z, Tru_2^{po}(z), In_2^{po}(z), Fa_2^{po}(z), Tru_2^{ne}(z), In_2^{ne}(z), Fa_2^{ne}(z)\rangle\}$ be two BNS and $w_i \in [0,1]$ be each element's weight x_i for i =1,2,...,n as $\sum_{i=1}^{n} w_i = 1$.

Then, the weighted Dice similarity measure between BNS $B_{i1}$ and $B_{i2}$, indicated $D_w(B_{i1}, B_{i2})$ is described as

$$D_w(B_{i1}, B_{i2}) = \sum_{i=1}^{n} w_i\left(\frac{Tru_1^{po}(z).Tru_2^{po}(z) + In_1^{po}(z).In_2^{po}(z) + Fa_1^{po}(z).Fa_2^{po}(z) - Tru_1^{ne}(z).Tru_2^{ne}(z) + In_1^{ne}(z).In_2^{ne}(z) + Fa_1^{ne}(z).Fa_2^{ne}(z)}{(Tru_1^{po}(z))^2 + (In_1^{po}(z))^2 + (Fa_1^{po}(z))^2 + (Tru_2^{po}(z))^2 + (In_2^{po}(z))^2 + (Fa_2^{po}(z))^2 - (Tru_1^{ne}(z))^2 + (In_1^{ne}(z))^2 + (Fa_1^{ne}(z))^2 - (Tru_2^{ne}(z))^2 + (In_2^{ne}(z))^2 + (Fa_2^{ne}(z))^2}\right. \quad (7)$$

**Definition 3.13.** Suppose $B_{i1} = \{\langle z, Tru_1^{po}(z), In_1^{po}(z), Fa_1^{po}(z), Tru_1^{ne}(z), In_1^{ne}(z), Fa_1^{ne}(z) \rangle\}$ and $B_{i2} = \{\langle z, Tru_2^{po}(z), In_2^{po}(z), Fa_2^{po}(z), Tru_2^{ne}(z), In_2^{ne}(z), Fa_2^{ne}(z)\rangle\}$ be two BNS Then, a hybrid vector similarity measure between BNS $B_{i1}$ and $B_{i2}$ indicated $HybV(B_{i1}, B_{i2})$, is described as

$$HybV(B_{i1}, B_{i2}) = \lambda . \frac{1}{n} . \sum_{i=1}^{n}\left(\frac{Tru_1^{po}(z).Tru_2^{po}(z) + In_1^{po}(z).In_2^{po}(z) + Fa_1^{po}(z).Fa_2^{po}(z) - Tru_1^{ne}(z).Tru_2^{ne}(z) + In_1^{ne}(z).In_2^{ne}(z) + Fa_1^{ne}(z).Fa_2^{ne}(z)}{2\left((Tru_1^{po}(z))^2 + (In_1^{po}(z))^2 + (Fa_1^{po}(z))^2 + (Tru_2^{po}(z))^2 + (In_2^{po}(z))^2 + (Fa_2^{po}(z))^2 - (Tru_1^{ne}(z))^2 + (In_1^{ne}(z))^2 + (Fa_1^{ne}(z))^2 - (Tru_2^{ne}(z))^2 + (In_2^{ne}(z))^2 + (Fa_2^{ne}(z))^2\right.}\right.$$

$$+(1-\lambda)\frac{1}{n} . \sum_{i=1}^{n}\left(\frac{Tru_1^{po}(z).Tru_2^{po}(z) + In_1^{po}(z).In_2^{po}(z) + Fa_1^{po}(z).Fa_2^{po}(z) - Tru_1^{ne}(z).Tru_2^{ne}(z) + In_1^{ne}(z).In_2^{ne}(z) + Fa_1^{ne}(z).Fa_2^{ne}(z)}{2\left(\sqrt{(Tru_1^{po}(z))^2 + (In_1^{po}(z))^2 + (Fa_1^{po}(z))^2} . \sqrt{(Tru_2^{po}(z))^2 + (In_2^{po}(z))^2 + (Fa_2^{po}(z))^2} - \sqrt{(Tru_1^{ne}(z))^2 + (In_1^{ne}(z))^2 + (Fa_1^{ne}(z))^2} . \sqrt{(Tru_2^{ne}(z))^2 + (In_2^{ne}(z))^2 + (Fa_2^{ne}(z))^2}\right.}\right. \quad (8)$$

**Definition 3.14.** Suppose $B_{i1} = \{\langle z, Tru_1^{po}(z), In_1^{po}(z), Fa_1^{po}(z), Tru_1^{ne}(z), In_1^{ne}(z), Fa_1^{ne}(z) \rangle\}$ and $B_{i2} = \{\langle z, Tru_2^{po}(z), In_2^{po}(z), Fa_2^{po}(z), Tru_2^{ne}(z), In_2^{ne}(z), Fa_2^{ne}(z)\rangle\}$ be two BNS Then and $w_i \in [0,1]$ be each element's weight x_i for i =1; 2; ...; n as $\sum_{i=1}^{n} w_i = 1$ Then, the weighted hybrid vector similarity measure between BNS $B_{i1}$ and $B_{i2}$, indicated $HybVw(B_{i1}, B_{i2})$ is described as

$$HybV(B_{i1}, B_{i2}) = \lambda . \sum_{i=1}^{n} w_i\left(\frac{Tru_1^{po}(z).Tru_2^{po}(z) + In_1^{po}(z).In_2^{po}(z) + Fa_1^{po}(z).Fa_2^{po}(z) - Tru_1^{ne}(z).Tru_2^{ne}(z) + In_1^{ne}(z).In_2^{ne}(z) + Fa_1^{ne}(z).Fa_2^{ne}(z)}{2\left((Tru_1^{po}(z))^2 + (In_1^{po}(z))^2 + (Fa_1^{po}(z))^2 + (Tru_2^{po}(z))^2 + (In_2^{po}(z))^2 + (Fa_2^{po}(z))^2 - (Tru_1^{ne}(z))^2 + (In_1^{ne}(z))^2 + (Fa_1^{ne}(z))^2 - (T_2^{n}(x))^2 + (I_2^{n}(x))^2 + (F_2^{n}(x))^2\right)}\right.$$

$$+(1-\lambda)\sum_{i=1}^{n} w_i\left(\frac{Tru_1^{po}(z).Tru_2^{po}(z) + In_1^{po}(z).In_2^{po}(z) + Fa_1^{po}(z).Fa_2^{po}(z) - Tru_1^{ne}(z).Tru_2^{ne}(z) + In_1^{ne}(z).In_2^{ne}(z) + Fa_1^{ne}(z).Fa_2^{ne}(z)}{2\left(\sqrt{(Tru_1^{po}(z))^2 + (In_1^{po}(z))^2 + (Fa_1^{po}(z))^2} . \sqrt{(Tru_2^{po}(z))^2 + (In_2^{po}(z))^2 + (Fa_2^{po}(z))^2} - \sqrt{(Tru_1^{ne}(z))^2 + (In_1^{ne}(z))^2 + (Fa_1^{ne}(z))^2} . \sqrt{(Tru_2^{ne}(z))^2 + (In_2^{ne}(z))^2 + (Fa_2^{ne}(z))^2}\right)}\right. \quad (9)$$

# 4 | BNS Proposed Methodology

BNS extends the capability of traditional MCDM by incorporating the bipolar nature of information, effectively handling both positive and negative aspects of decision criteria. This section discusses BNS-MCDM methods using the BNWA operator and similarity measures.

**Table 1.** Linguistic variables of BNS.

| Linguistic variables | $T^P$ | $I^P$ | $F^P$ | $T^n$ | $I^n$ | $F^n$ |
|---|---|---|---|---|---|---|
| Absolutely Significant (AS) | 0.95 | 0.05 | 0.05 | −0.30 | −0.95 | −0.95 |
| Very Highly Significant (VHS) | 0.85 | 0.10 | 0.10 | −0.15 | −0.85 | −0.85 |
| Highly Significant (HS) | 0.75 | 0.15 | 0.15 | −0.25 | −0.75 | −0.70 |
| Significant (S) | 0.65 | 0.20 | 0.20 | −0.35 | −0.65 | −0.60 |
| Moderately Significant (MS) | 0.55 | 0.25 | 0.25 | −0.45 | −0.55 | −0.55 |
| Equally Significant (ES) | 0.50 | 0.30 | 0.30 | −0.50 | −0.50 | −0.40 |
| Almost Significant (ALS) | 0.30 | 0.55 | 0.50 | −0.70 | −0.30 | −0.30 |
| Not Significant (NS) | 0.10 | 0.80 | 0.80 | −0.90 | −0.10 | −0.10 |

## 4.1 | Bipolar Neutrosophic Weighted Average Operator

BNWA operator [16] aggregates BN information by considering the weights of different criteria. This operator is useful for synthesizing multiple criteria into a single evaluation measure. Using definition 3.9 to get the Procedure for BNS-MCDM:

i). Identify Criteria and Alternatives: Determine the criteria $C = \{C_1, C_2, \dots C_n\}$ and alternatives $A = \{A_1, A_2, \dots A_m\}$ for the decision-making problem.

ii). Assign Bipolar Neutrosophic Values as linguistic variables provided in Table 1: For each criterion, assign bipolar neutrosophic values (positive and negative membership degrees) to Build the matrix of choices that the decision provides as

$(B_{ij})_{mxn} = ((Tru_{ij}^{Po}, In_{ij}^{Po}, Fa_{ij}^{Po}, Tru_{ij}^{ne}, In_{ij}^{ne}, Fa_{ij}^{ne}))_{mxn}$, where $Tru_{ij}^{Po}, In_{ij}^{Po}, Fa_{ij}^{Po}, Tru_{ij}^{ne}, In_{ij}^{ne}, Fa_{ij}^{ne} \in [0,1]$

iii). Determine Weights: Assign weights to each criterion based on their importance. Let weight $w = \{w_1, w_2, \dots w_n\}^T$ be the attributes' weight vector, as $\sum_{j=1}^{n} w_j = 1, w_j \geq 0\ (j = 1,2, \dots, n)$.

iv). Calculate BNWA: Use the BNWA operator Eq. (4) to aggregate the bipolar neutrosophic values across all criteria.

v). Determine the score values of $S(B_{ij})(i = 1,2, \dots, m)$ for the BNN collectively using Eq. (1).

vi). Rank the alternatives according to their respective score values.

## 4.2 | Similarity Measures of BNS

Similarity measures [36] are used to contrast BNS and determine the degree of similarity between them. These measures are essential for ranking and selecting alternatives in decision-making.

i). Identify Criteria and Alternatives: Determine the criteria $C = \{C_1, C_2, \dots C_n\}$ and alternatives $A = \{A_1, A_2, \dots A_m\}$ for the decision-making problem.

ii). Assign Bipolar Neutrosophic Values: For each criterion, assign bipolar neutrosophic values (positive and negative membership degrees) to Create the decision matrix that the decision provides as

$(B_{ij})_{mxn} = ((Tru_{ij}^{Po}, In_{ij}^{Po}, Fa_{ij}^{Po}, Tru_{ij}^{ne}, In_{ij}^{ne}, Fa_{ij}^{ne}))_{mxn}$ , ,where $Tru_{ij}^{Po}, In_{ij}^{Po}, Fa_{ij}^{Po}, Tru_{ij}^{ne}, In_{ij}^{ne}, Fa_{ij}^{ne} \in [0,1]$ is called an NB-multi-attribute DM matrix of the decision maker.

iii). Determine Weights: Assign weights to each criterion based on their importance. Let weight $w = \{w_1, w_2, \ldots w_n\}^T$ be the attributes' weight vector, as $\sum_{j=1}^{n} w_j = 1, w_j \geq 0 \ (j = 1,2, \ldots, n)$.

iv). Define positive ideal bipolar neutrosophic solution $u^* = (\widetilde{A_1^*}, \widetilde{A_2^*}, \ldots \widetilde{A_n^*})$ is the solution of the decision matrix $(\widetilde{A_{ij}})_{mxn}$ , where each element has the subsequent form

$$\widetilde{A}_j^* = (\max_i \left\{T_{ij}^p\right\}, \min_i \left\{I_{ij}^p\right\}, \min_i \left\{F_{ij}^p\right\}, \min_i\{T_{ij}^n\}, \max_i\{I_{ij}^n\}, \max_i\{F_{ij}^n\}, j = (1,2,..n)$$

And negative ideal bipolar neutrosophic solution $u^{\rightarrow} = (\widetilde{A_1^{\rightarrow}}, \widetilde{A_2^{\rightarrow}}, \ldots \widetilde{A_n^{\rightarrow}})$ is the solution of the decision matrix $(\widetilde{A_{ij}})_{mxn}$ , where each element has the subsequent form

$$\widetilde{A}_j^{\rightarrow} = (\min_i \left\{T_{ij}^p\right\}, \max_i \left\{I_{ij}^p\right\}, \max_i \left\{F_{ij}^p\right\}, \max_i\{T_{ij}^n\}, \min_i\{I_{ij}^n\}, \min_i\{F_{ij}^n\}, j = (1,2,..n)$$

v). Compute the similarity measure for each pair of alternatives using the similarity measure formula as Eq. (9).

vi). Rank the alternatives based on their similarity measures.

This overview provides a comprehensive understanding of how BNS-MCDM methods can be utilized using the BNWA operator and similarity measures, facilitating better decision-making in complex and uncertain environments.

Figure 2 provides a visual guide to understanding the steps involved in BNS-MCDM using the bipolar neutrosophic weighted average operator and similarity measures, facilitating better decision-making in complex and uncertain environments.

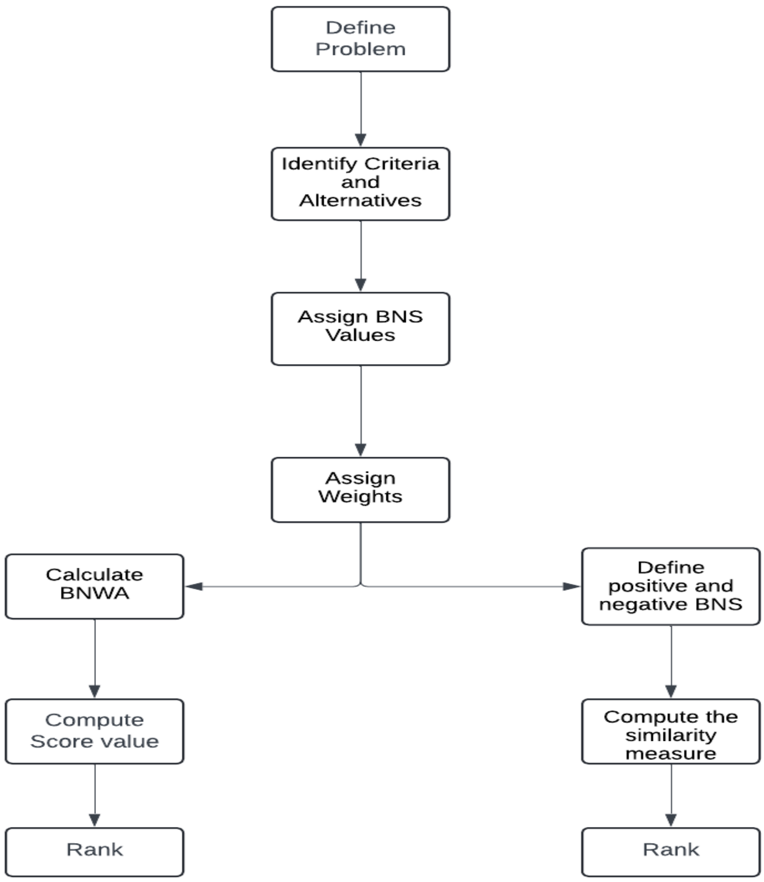


**Figure 2.** Flowchart to represent the process of BNS-MCDM.

# 5 | Case Study

Financial markets in Egypt play a crucial role in the country's financial system. The Egyptian Exchange (EGX) is the main stock exchange in Egypt, providing a platform for trading stocks, bonds, and financial securities. In addition to the stock exchange, money and capital markets are essential components of Egypt's financial system, as banks and other financial institutions rely on them for their operations. This case study focuses on assessing the financial performance of the top companies listed on the EGX using MCDM methods involving BNS. The companies considered are:

1) Alt1: Commercial International Bank-Egypt (CIB)
2) Alt2: T M G Holding
3) Alt3: ELSWEDY ELECTRIC
4) Alt4: Misr Fertilizers Production Company – Mopco
5) Alt5: Abou Kir Fertilizers
6) Alt6: Alexandria Containers and Goods
7) Alt7: Qatar National Bank
8) Alt8: Eastern Company
9) Alt9: Telecom Egypt
10) Alt:10 E-finance For Digital and Financial Investments

Bipolar neutrosophic sets are particularly useful in MCDM problems. In financial markets, investors and analysts often face complex decision-making problems, where they need to evaluate multiple alternatives (e.g., stocks, bonds, or investment portfolios) based on various criteria (e.g. liquidity and volatility).

In a financial performance assessment of financial markets using stock returns with MCDM BNS, the criteria used for assessment is essential. in determining the overall performance of the stocks. Here are some potential criteria that can be used for this assessment:

1) Stock Returns (C1): Measures the returns generated by the stock over a specific period. Calculated as the percentage change in the stock price over the period.
2) Volatility (C2): Measures the risk associated with the stock, typically indicated by the standard deviation of the stock returns. Higher volatility indicates higher risk.
3) Liquidity (C3): Measures the ease with which a stock can be bought or sold without affecting its market price. It can be assessed using the bid-ask spread or trading volume.
4) Market Capitalization (C4): Measures the total market value of the company's outstanding shares. Calculated as the product of the stock price and the number of outstanding shares.
5) Earnings Growth (C5): Measures the growth in the company's earnings over a specific period. Calculated as the percentage change in earnings per share.
6) Revenue Growth (C6): Measures the growth in the company's revenue over a specific period. Calculated as the percentage change in total revenue.
7) Analyst Ratings (C7): Measures the consensus rating of the stock by financial analysts. Calculated as the average rating by a group of analysts.

To assess the financial performance of the top companies listed on the EGX MCDM methods involving BNS, follow these steps:

**Step 1.** Define the criteria and alternatives: Identify the stocks to be evaluated (top 10 companies in this case) and 7 criteria.

**Step 2.** Assign linguistic variables of BNS to each criterion in the decision matrix as Table 2 then assign BNS value for each criterion.

**Step 3.** Establish the criteria's weights: Assign weights to each criterion based on their importance in evaluating financial performance as shown in Table 3.

**Table 2.** Decision matrix with linguistic variables.

| | C1 | C2 | C3 | C4 | C5 | C6 | C7 |
|---|---|---|---|---|---|---|---|
| **Alt1** | AS | NS | MS | MS | HS | HS | ALS |
| **Alt2** | ES | VHS | HS | MS | ES | S | HS |
| **Alt3** | HS | AS | S | ALS | S | VHS | S |
| **Alt4** | ALS | MS | ES | S | AS | ALS | MS |
| **Alt5** | VHS | NS | AS | NS | MS | ALS | VHS |
| **Alt6** | ES | S | ALS | AS | VHS | ES | AS |
| **Alt7** | AS | ES | MS | HS | NS | S | HS |
| **Alt8** | S | MS | VHS | S | AS | AS | S |
| **Alt9** | NS | HS | HS | ES | HS | MS | NS |
| **Alt10** | MS | ALS | NS | VHS | ALS | HS | ES |

**Table 3.** Criteria weights.

| | w1 | w2 | w3 | w4 | w5 | w6 | w7 |
|---|---|---|---|---|---|---|---|
| **W** | 0.15 | 0.20 | 0.10 | 0.27 | 0.08 | 0.10 | 0.10 |

**Step 4.** To rank the 10 alternatives using MCDM with Bipolar neutrosophic sets in two phases.

- **Phase 1: BNWA Operator**

**Step 1.1.** Aggregate Bipolar Neutrosophic Values: Use the BNWA Operator Eq. (4) to combine the values, as shown in Table 4.

**Step 1.2.** Determine each alternative's score values using the BNS by Eq. (1) to get the final rank, represented in Table 5.

- **Phase 2: Similarity Measures of BNS**

**Step 2.1.** Compute the positive and negative ideal Bipolar Neutrosophic: identify C2 and C7 as negative criteria, and the remainder as positive.

**Step 2.2.** Calculate the similarity measures for each alternative. Use Eq. (9) to calculate the similarity measures and obtain the final rank, as shown in Table 6.

**Table 4.** Aggregated BNS values.

| AW | $T^P$ | $I^P$ | $F^P$ | $T^n$ | $I^n$ | $F^n$ |
|---|---|---|---|---|---|---|
| **Alt1** | 0.650 | 0.245 | 0.242 | -0.457 | -0.650 | -0.639 |
| **Alt2** | 0.679 | 0.192 | 0.192 | -0.321 | -0.679 | -0.648 |
| **Alt3** | 0.750 | 0.178 | 0.173 | -0.357 | -0.750 | -0.733 |
| **Alt4** | 0.602 | 0.257 | 0.251 | -0.459 | -0.602 | -0.580 |
| **Alt5** | 0.603 | 0.316 | 0.313 | -0.475 | -0.603 | -0.603 |
| **Alt6** | 0.813 | 0.139 | 0.137 | -0.362 | -0.813 | -0.799 |
| **Alt7** | 0.726 | 0.181 | 0.181 | -0.359 | -0.726 | -0.692 |
| **Alt8** | 0.762 | 0.152 | 0.152 | -0.329 | -0.762 | -0.745 |
| **Alt9** | 0.560 | 0.289 | 0.289 | -0.440 | -0.560 | -0.504 |
| **Alt10** | 0.613 | 0.265 | 0.258 | -0.387 | -0.613 | -0.599 |

**Table 5.** Score values and final rank (Phase 1).

| | Score | RNAK |
|---|---|---|
| **Alt1** | 0.666 | 6 |
| **Alt2** | 0.717 | 5 |
| **Alt3** | 0.754 | 3 |
| **Alt4** | 0.636 | 8 |
| **Alt5** | 0.617 | 9 |
| **Alt6** | 0.798 | 1 |
| **Alt7** | 0.737 | 4 |
| **Alt8** | 0.773 | 2 |
| **Alt9** | 0.601 | 10 |
| **Alt10** | 0.653 | 7 |

**Table 6.** Positive and negative BNS.

| | $T^P$ | $I^P$ | $F^P$ | $T^n$ | $I^n$ | $F^n$ |
|---|---|---|---|---|---|---|
| $\widetilde{A}_1^*$ | 0.95 | 0.05 | 0.05 | $-0.90$ | $-0.10$ | $-0.10$ |
| | 0.10 | 0.80 | 0.80 | $-0.15$ | $-0.95$ | $-0.95$ |
| | 0.95 | 0.05 | 0.05 | $-0.90$ | $-0.10$ | $-0.10$ |
| | 0.95 | 0.05 | 0.05 | $-0.90$ | $-0.10$ | $-0.10$ |
| | 0.95 | 0.05 | 0.05 | $-0.90$ | $-0.10$ | $-0.10$ |
| | 0.95 | 0.05 | 0.05 | $-0.70$ | $-0.30$ | $-0.30$ |
| | 0.10 | 0.80 | 0.80 | $-0.15$ | $-0.95$ | $-0.95$ |

**Table 7.** Similarity measures and final rank (Phase 2).

| $HybV_w$ | $\lambda = 0.25$ | RANK |
|---|---|---|
| **Alt1** | 0.6424 | 1 |
| **Alt2** | 0.20148 | 4 |
| **Alt3** | $-2.239$ | 10 |
| **Alt4** | $-1.3215$ | 8 |
| **Alt5** | 0.43048 | 3 |
| **Alt6** | $-1.6795$ | 9 |
| **Alt7** | $-0.2001$ | 6 |
| **Alt8** | $-0.1516$ | 5 |
| **Alt9** | 0.45115 | 2 |
| **Alt10** | $-1.1077$ | 7 |

By following these detailed steps, the financial performance of the top companies on the Egyptian Exchange (EGX) can be effectively assessed using MCDM methods involving BNS. This approach accommodates the complexity and ambiguity of financial data, aiding stakeholders in making informed investment decisions.

## 6 | Discussion and Analysis

The application of MCDM methods involving BNS provides a nuanced approach to evaluating the financial performance of the top companies listed on the Egyptian Exchange (EGX). The financial performance assessment revealed distinct rankings for the top 10 companies based on various criteria, including stock returns, volatility, liquidity, market capitalization, earnings growth, revenue growth, and analyst ratings. The aggregated scores from the BNWA Operator and the similarity measures provided a comprehensive ranking of the companies as shown in Figure 3. Commercial International Bank-Egypt (CIB) emerged as a leading performer, reflecting its strong stock returns, market capitalization, and positive analyst ratings by similarity measures. The weights assigned to each criterion played a significant role in determining the final rankings.


**Figure 3.** Alternatives and rank.

## 6.1 | Sensitivity Analysis

The sensitivity analysis conducted using different values of $\lambda$ in the Similarity Measures highlights the impact of varying degrees of $\lambda$ on the final rankings of the top companies listed on the Egyptian Exchange. The results from the analysis are detailed in Figure 4 and Table 8. The table presents the rankings of the 10 companies for different values of $\lambda$ (0.25, 0.3, 0.5, 0.8, 0.9), which reflect varying degrees of weight assigned to positive and negative aspects in the decision-making process. Commercial International Bank-Egypt (CIB) (A1) consistently maintains a high rank for $\lambda = 0.25$ and $\lambda = 0.3$ indicating strong performance when positive aspects are given more weight. However, its rank drops significantly when $\lambda$ increases. T M G Holding (A2) performs consistently well across all values of $\lambda$, particularly excelling as the highest-ranked alternative for $\lambda = 0.5$, $\lambda = 0.8$, and $\lambda = 0.9$. This indicates robust performance under varying degrees of $\lambda$.

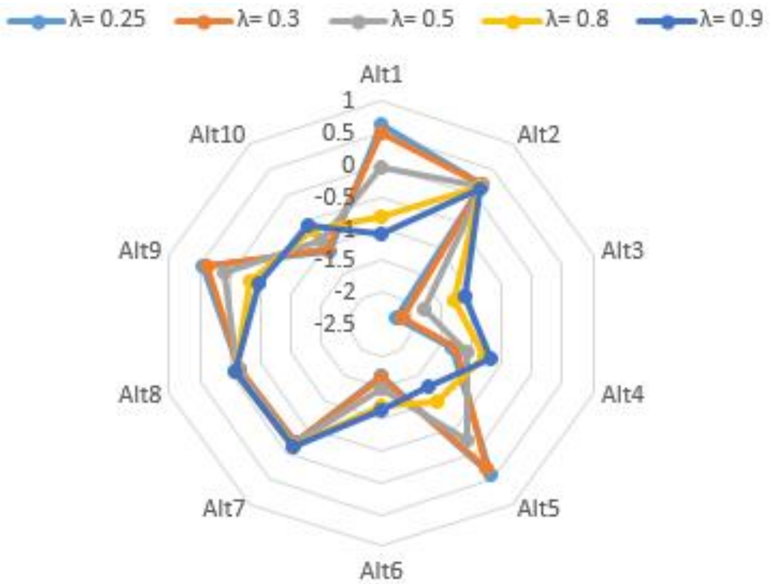


**Figure 4.** Sensitivity analysis.

**Table 8.** Different Values of $\lambda$.

| $HybV_w$ | λ= 0.25 | Rank | λ= 0.3 | RANK | λ= 0.5 | RANK | λ= 0.8 | RANK | λ= 0.9 | RANK |
|---|---|---|---|---|---|---|---|---|---|---|
| **Alt1** | 0.6424 | 1 | 0.50796 | 1 | -0.02980 | 3 | -0.83643 | 7 | -1.10531 | 7 |
| **Alt2** | 0.20148 | 4 | 0.19658 | 4 | 0.17697 | 1 | 0.14756 | 1 | 0.13775 | 1 |
| **Alt3** | -2.239 | 10 | -2.15199 | 10 | -1.80407 | 10 | -1.28218 | 10 | -1.10821 | 8 |
| **Alt4** | -1.3215 | 8 | -1.27274 | 8 | -1.07786 | 8 | -0.78554 | 6 | -0.68810 | 6 |
| **Alt5** | 0.43048 | 3 | 0.30032 | 3 | -0.22033 | 6 | -1.00129 | 8 | -1.26162 | 10 |
| **Alt6** | -1.6795 | 9 | -1.63785 | 9 | -1.47107 | 9 | -1.22090 | 9 | -1.13752 | 9 |
| **Alt7** | -0.2001 | 6 | -0.19364 | 6 | -0.16797 | 5 | -0.12946 | 3 | -0.11663 | 3 |
| **Alt8** | -0.1516 | 5 | -0.14671 | 5 | -0.12714 | 4 | -0.09779 | 2 | -0.08800 | 2 |
| **Alt9** | 0.45115 | 2 | 0.37985 | 2 | 0.09465 | 2 | -0.33315 | 4 | -0.47575 | 4 |
| **Alt10** | -1.1077 | 7 | -1.06642 | 7 | -0.90117 | 7 | -0.65330 | 5 | -0.57068 | 5 |

The sensitivity analysis using different values of $\lambda$ in the similarity measures has provided a deeper understanding of the financial performance and ranking stability of the top companies listed on the EGX. By considering $\lambda$, stakeholders can make more informed and balanced investment decisions. This analysis underscores the importance of flexibility and adaptability in financial decision-making processes.

## 6.2 | Comparative Analysis

This analysis employs a MCDM approach to compare the financial performance of top companies listed on the Egyptian Exchange (EGX) using BNS. The assessment consists of two primary phases: the HybVw and the Jaccard Vector Similarity Measure of BNS [37]. The first phase, HybVw, aggregates the BN information to obtain a comprehensive evaluation of each company. This operator effectively handles both positive and negative information, providing a nuanced understanding of the companies' financial performance. The second phase, the Jaccard Vector Similarity Measure of BNS, compares and ranks the companies based on their BN evaluation values. This measure calculates the similarity between each company's evaluation and the ideal solution, allowing for a detailed comparison of their financial performance as Table 9 and Figure 5. The combined use of HybVw and the Jaccard vector similarity measure of BNS provides a robust and comprehensive framework for evaluating the financial performance of top companies listed on the EGX. This approach can aid stakeholders in making informed decisions in complex financial environments by offering a detailed and nuanced understanding of the companies' strengths and weaknesses.

**Table 9.** Jaccard vector similarity measure.

| | Si | Rank |
|---|---|---|
| **Alt1** | 5.74888 | 1 |
| **Alt2** | -4.12457 | 6 |
| **Alt3** | -2.04309 | 5 |
| **Alt4** | -4.95074 | 8 |
| **Alt5** | -1.74362 | 4 |
| **Alt6** | -5.49313 | 9 |
| **Alt7** | -6.0242 | 10 |
| **Alt8** | -0.078 | 3 |
| **Alt9** | 1.451241 | 2 |
| **Alt10** | -4.64043 | 7 |

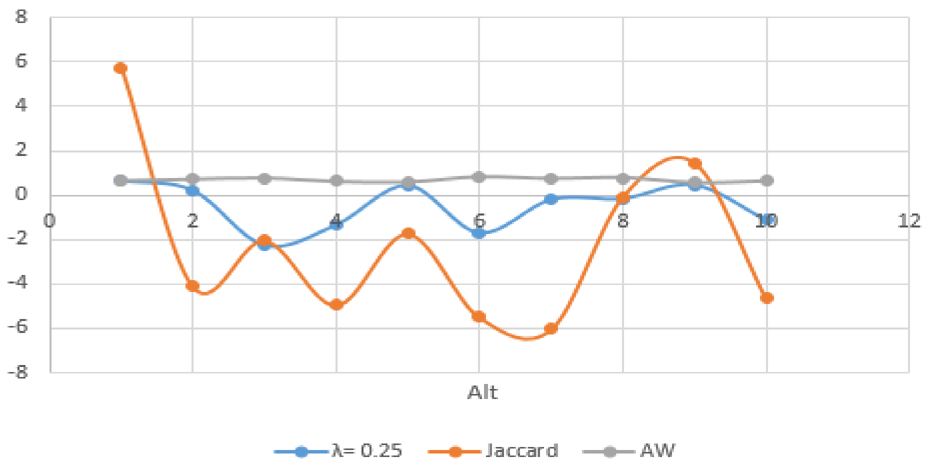


**Figure 5.** Comparative analysis.

## 7 | Conclusions

This paper presents a novel methodology for assessing the financial performance of companies listed on the Egyptian Exchange (EGX) using MCDM methods with BNS. The integration of BNS with MCDM methods addresses the complexity and ambiguity inherent in financial data, providing a robust framework for comprehensive evaluation. The incorporation of BNS into MCDM methods significantly improves the evaluation of financial performance by capturing both positive and negative aspects of the data. This dual consideration allows for a more nuanced and accurate assessment of stock performance, accommodating the inherent uncertainties and inconsistencies in financial data. The approach effectively handles multiple criteria, including stock returns, volatility, liquidity, market capitalization, and earnings growth. By evaluating these criteria simultaneously, the method provides a holistic view of a company's financial health, enabling stakeholders to make well-informed investment decisions. The use of the BNWA operator and similarity measures facilitates the ranking and comparison of investment alternatives. The final rankings of the top companies on the EGX, derived from these methods, reflect a detailed analysis of their financial performance across various dimensions. The findings of this study are valuable for investors and analysts seeking to optimize their portfolios and manage risks. The ability to evaluate and rank companies based on a comprehensive set of criteria allows for more strategic investment decisions and better alignment with investment goals. The findings provide practical insights for investors and financial analysts. The comprehensive evaluation framework allows for a more informed selection of investment opportunities, helping stakeholders navigate complex financial environments with greater confidence. The ability to rank and compare companies based on multiple criteria supports more strategic decision-making and effective portfolio management.

This paper highlights the potential for further research in integrating BNS with other decision-making frameworks or exploring its application in different financial contexts. Future studies could investigate the impact of varying criteria weights or explore the use of advanced similarity measures to refine the evaluation process.

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## Author Contributions

All authors contributed equally to this work.

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## Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

## Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

## Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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