Multicriteria Algorithms with Applications

Journal Homepage: sciencesforce.com/mawa

SCIENCES FORCE
International Academic Publisher

Multicriteria Algo. Appl. Vol. 5 (2024) 32–51

Paper Type: Original Article

Note for Hypersoft Filter and Fuzzy Hypersoft Filter

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Received: 03 Sep 2024 **Revised:** 01 Nov 2024 **Accepted:** 15 Nov 2024 **Published:** 18 Nov 2024

Abstract

In set theory, a filter is a collection of sets that is closed under intersection and includes all supersets of its elements. Within the framework of soft sets mathematical tools developed to handle uncertainty through parameterized subsets-filters and ultrafilters have also been introduced, with further expansions made in fuzzy set theory and Neutrosophic set theory. In this short paper, we introduce the novel concept of a hypersoft filter and provide a brief discussion of its implications.

Keywords: Filter; Soft Filter; Hypersoft Set; Soft Set; Fuzzy Set.

1 | Introduction

1.1 | Soft Sets and Hypersoft Sets

Set theory serves as a foundational area in mathematics, with significant research dedicated to concepts like filters and ultrafilters. In set theory, a filter is a collection of sets closed under intersection and containing all supersets of its elements [12, 19, 45]. The maximal form of a filter, known as an ultrafilter, is a fundamental concept with applications across various disciplines in mathematics [19, 26, 48, 54, 98, 113] and extends even into fields like social choice theory [78].

A soft set provides a mathematical framework for handling uncertainty, defined as a set with associated parameters to enable a flexible representation of information [77]. Filters and ultrafilters for soft sets have emerged as extensions of classical filters, particularly within fuzzy set theory [124] and Neutrosophic set theory [103], to better manage real-world uncertainties [16] Hypersoft sets further develop soft sets by incorporating multi-attribute parameterization, allowing the modeling of complex, multi-dimensional relationships across distinct attribute groups [107]. These uncertainty-management concepts have also found applications in graph theory and related fields [41-44].

1.2 | Contributions

Despite the importance of research on ultrafilters in Soft Theory, ultrafilters within Hypersoft Theory remain largely unexplored. In this paper, we introduce the concept of a hypersoft filter and examine its potential implications. Additionally, we define the fuzzy hypersoft filter and Neutrosophic hypersoft filter, both of which extend the hypersoft filter by integrating concepts from fuzzy set theory and Neutrosophic set theory.

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https://doi.org/10.61356/j.mawa.2024.5424

1.3 | The Structure of the Paper

The structure of this paper is outlined as follows. Section 2 provides an overview of existing definitions, Section 3 presents the main results, and Section 4 discusses future tasks and directions.

2 | Preliminaries and Definitions

Some foundational concepts from set theory are applied in parts of this work. For further details, please consult relevant references as needed [38, 50, 57, 60, 72].

2.1 | Crisp Sets and Filter

The concepts of Fuzzy Sets,[124–129, 131] Soft Sets, Neutrosophic Sets,1[03, 104] Vague Sets, [4, 20, 24, 56, 130] and Rough Sets [84, 85] are often discussed alongside Crisp Sets, [25, 81, 102, 120] which serve as a fundamental basis for these set concepts. The definition of a Universe Set and a Crisp Set is provided below.

Definition 2.1 (Universe Set). (cf [79]) A universe set, often denoted by U, is a set that contains all the elements under consideration for a particular discussion or problem domain. Formally, U is defined as a set that encompasses every element within the scope of a given context or framework, so that any subset of interest can be regarded as a subset of U.

In set theory, the universe set U is typically assumed to contain all elements relevant to the discourse, meaning that for any set A, if $A \subseteq U$, then all elements of A are elements of U. **Definition 2.2** (Crisp Set). [81] Let X be a universe set, and let $P(X)$ denote the power set of X, which represents all subsets of X. A crisp set $A \subseteq X$ is defined by a characteristic function $\chi_A: X \to \{0,1\}$, where:

$$\chi_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A. \end{cases}$$

This function χ_A Assigns a value of 1 to elements within the set A and 0 to those outside it, creating a clear boundary. Crisp sets are thus bivalent and follow the principle of binary classification, where each element is either a member of the set or not. This discrete nature contrasts with the gradual membership levels of fuzzy sets, where membership can take any value in the interval $[0,1]$.

Crisp sets with operations $\cup$,$\cap$, and $(\cdot)^c$ (complement) on $P(X)$ form a Boolean algebra. In this context, crisp sets can be regarded as a subset of fuzzy sets that exhibit binary discontinuity.

The definition of an Ultrafilter in Set Theory is described as follows (cf [19, 26, 45]).

Definition 2.3 (Filter on a Set). Let X be a set. A collection $\mathcal{F} \subseteq 2^X$ is called a filter on X if it satisfies the following conditions:

(1) If A and B are both in $\mathcal{F}$, then their intersection $A \cap B$ is also in $\mathcal{F}$.

(2) If A is in $\mathcal{F}$ and $A \subseteq B \subseteq X$, then B is also in $\mathcal{F}$.

(3) The empty set $\emptyset$ is not in $\mathcal{F}$.

Example 2.4 (Filter on a Set). Let $X = \{1,2,3,4\}$. Define a collection $\mathcal{F} \subseteq 2^X$ by

$$\mathcal{F} = \{A \subseteq X \mid 3 \in A\}$$

Then $\mathcal{F}$ is a filter on X, as it satisfies the following conditions:

1. Non-emptiness of the Filter: The collection $\mathcal{F}$ is non-empty because $\{3\} \in \mathcal{F}$.
2. Closed under Intersection: If $A, B \in \mathcal{F}$, then both sets A and B contain the element 3 . Therefore, their intersection $A \cap B$ also contains 3, so $A \cap B \in \mathcal{F}$.

3. Upward Closure: If $A \in \mathcal{F}$ and $A \subseteq B \subseteq X$, then B must also contain the element 3 (since $A \subseteq B$ and $3 \in A$). Hence, $B \in \mathcal{F}$.
4. Exclusion of the Empty Set: The empty set $\emptyset$ does not belong to $\mathcal{F}$ because $\emptyset$ does not contain 3 .

Thus, $\mathcal{F}$ satisfies all the properties of a filter on X.

Definition 2.5 (Ultrafilter). A maximal filter, which cannot be extended any further while still being a filter, is called an ultrafilter. An ultrafilter satisfies an additional condition:

(4) For any subset $A \subseteq X$, either A is in $\mathcal{F}$ or its complement $X \setminus A$ is in $\mathcal{F}$, but not both.

2.2 | Soft Set

This subsection explains the concept of a Soft Set. A soft set over a universe U assigns subsets of U to parameters from a parameter set A, allowing for flexible representations of uncertain data. The formal definition of a Soft Set is provided below [8, 9, 74, 77, 121]. For readers interested in details on operations within Soft Set theory, please refer to as needed [77].

Definition 2.6. Let U be a non-empty finite set, called the universe of discourse, and let E be a non-empty set of parameters. A soft set over U is defined as follows:

$$F = (F, A) \text{ over } U \text{ is an ordered pair, where } A \subseteq E \text{ and } F: A \rightarrow P(U)$$

where $F(a) \subseteq U$ for each $a \in A$ and $P(U)$ denotes the power set of U. The set of all soft sets over U is denoted by $S(U)$.

1. Soft Subset: Let $F = (F, A)$ and $G = (G, B)$ be two soft sets over the common universe U. We say that F is a soft subset of G, denoted $F \subseteq G$, if:
   - $A \subseteq B$
   - $F(a) \subseteq G(a)$ for all $a \in A$.
2. Union of Soft Sets: The union of two soft sets $F = (F, A)$ and $G = (G, B)$ over U is defined as $H = (H, C)$ where $C = A \cup B$ and

$$H(e) = \begin{cases} F(e), & e \in A - B \\ G(e), & e \in B - A \\ F(e) \cup G(e), & e \in A \cap B \end{cases}$$

3. Intersection of Soft Sets: The intersection of two soft sets $F = (F, A)$ and $G = (G, B)$ with disjoint parameter sets $A \cap B = \emptyset$ is defined as $H = (H, C)$, where $C = A \cap B$ and

$$H(e) = F(e) \cap G(e), \forall e \in C$$

In this Soft Theory framework, Soft Filters [16] and Soft Ultrafilters [16] have been actively studied. Since filters are defined across various mathematical fields, several definitions exist. Here, we provide the definition specific to a universe set.

Definition 2.7 (Soft Filter). [16] Let U be an initial universe set, and let $\mathrm{PS}(U)$ denote the family of all soft sets over U. A soft filter on U is a non-empty subset $\mathcal{F} \subseteq \mathrm{PS}(U)$ that satisfies the following properties:

i). For any $(f, A), (g, B) \in \mathcal{F}$, the intersection $(f, A) \cap (g, B) \in \mathcal{F}$.

ii). If $(f, A) \in \mathcal{F}$ and $(g, B) \in \mathrm{PS}(U)$ such that $(f, A) \subseteq (g, B)$, then $(g, B) \in \mathcal{F}$.

iii). The null soft set $\emptyset_A \notin \mathcal{F}$ for any set of parameters $A \subseteq E$.

Here:

- The intersection of two soft sets (f, A) and (g, B), denoted $(f, A) \cap (g, B)$, is defined as the soft set (h, C), where $C = A \cap B$ and for each $c \in C, h(c) = f(c) \cap g(c)$.
- $(f, A) \subseteq (g, B)$ if $A \subseteq B$ and for all $c \in A, f(c) \subseteq g(c)$.

Definition 2.8 (Soft Ultrafilter). [16] A soft ultrafilter over X is a soft filter $\mathcal{F}$ that is maximal concerning inclusion, meaning there is no soft filter strictly containing $\mathcal{F}$.

The following proposition is valid.

Proposition 2.9. Every soft filter can be transformed into a filter on an appropriately defined set, thus retaining the essential properties of a filter.

Proof. Let U be an initial universe set and let $\mathrm{PS}(U)$ denote the family of all soft sets over U. Let $\mathcal{F} \subseteq \mathrm{PS}(U)$ be a soft filter on U. We aim to construct a corresponding filter $\mathcal{F}' \subseteq 2^X$, where $X \subseteq U$, such that $\mathcal{F}'$ Satisfies the conditions of a standard filter.

Define $X = \bigcup_{(f,A)\in\mathcal{F}} A$, which is the union of all parameter sets A associated with soft sets in $\mathcal{F}$. Now construct the collection $\mathcal{F}' \subseteq 2^X$ as follows:

$$\mathcal{F}' = \left\{ \bigcup_{(f,A)\in\mathcal{F}} f(a) \mid a \in A \text{ for some } (f, A) \in \mathcal{F} \right\}$$

Where $f(a) \subseteq U$ denotes the subset of U associated with the parameter $a \in A$. Each element of $\mathcal{F}'$ is thus a union of subsets of U drawn from soft sets in $\mathcal{F}$. We now verify that $\mathcal{F}'$ satisfies the conditions of a filter on :

1. Non-emptiness: Since $\mathcal{F}$ is a soft filter, it is non-empty, and therefore $\mathcal{F}'$ also contains at least one non-empty set.
2. Closure under finite intersection: Let $S, T \in \mathcal{F}'$. Then, by construction, there exist soft sets $(f, A), (g, B) \in \mathcal{F}$ and parameters $a \in A, b \in B$ such that $S = f(a)$ and $T = g(b)$. Since $\mathcal{F}$ is closed under intersection, we know $(f, A) \cap (g, B) \in \mathcal{F}$, where the intersection of these soft sets is defined as:

   $$(f, A) \cap (g, B) = (h, C),$$

   with $C = A \cap B$ and $h(c) = f(c) \cap g(c)$ for each $c \in C$. Consequently,

   $$S \cap T = f(a) \cap g(b) = h(c) \text{ for some } c \in C,$$

   and thus $S \cap T \in \mathcal{F}'$, showing that $\mathcal{F}'$ is closed under the intersection.
3. Closure under supersets: Suppose $S \in \mathcal{F}'$ and $S \subseteq T \subseteq X$ for some $T \in 2^X$. Then by the definition of $\mathcal{F}'$, there exists $(f, A) \in \mathcal{F}$ and $a \in A$ such that $S = f(a)$. Since $\mathcal{F}$ satisfies the soft superset condition, any soft set $(g, B) \in \mathrm{PS}(U)$ with $(f, A) \subseteq (g, B)$ must also be in $\mathcal{F}$. Therefore, $T \in \mathcal{F}'$ as required, showing that $\mathcal{F}'$ satisfies the superset condition.
4. Exclusion of the empty set: Since $\mathcal{F}$ does not contain any null soft set $\emptyset_A$, the empty set cannot appear in any union forming $\mathcal{F}'$. Thus, $\emptyset \notin \mathcal{F}'$.

Since $\mathcal{F}'$ satisfies all the filter properties on X, it is a standard filter on X. This completes the proof that every soft filter can indeed be transformed into a filter on an appropriate set X.

2.3 | Fuzzy Filter

This subsection discusses the concept of a fuzzy filter. The fuzzy filter [27, 28, 55, 73, 114–116] and fuzzy ultrafilter [114] are types of filters defined within the framework of fuzzy sets, as detailed in the definitions below. For further information on operations and related concepts in fuzzy set theory, please refer to [124] as needed.

Definition 2.10 (Fuzzy Set). [124] Let X be a non-empty set. A fuzzy set A in X is defined as a function

$$A: X \to [0,1]$$

where $A(x)$ represents the degree of membership of x in A, with $A(x) = 1$ indicating full membership and $A(x) = 0$ indicating no membership. The set of all fuzzy sets on X is denoted by $\mathcal{F}(X)$.

Definition 2.11 (Fuzzy Filter). [114] Let X be a non-empty set. A fuzzy filter $\mathcal{F}$ on X is a non-empty collection of fuzzy sets in X that satisfies the following conditions:

1. Closure under Minimum: If $A, B \in \mathcal{F}$, then $A \cap B \in \mathcal{F}$, where $(A \cap B)(x) = \min\{A(x), B(x)\}$ for all $x \in X$.
2. Upward Closure: If $A \in \mathcal{F}$ and $A(x) \le B(x)$ for all $x \in X$, then $B \in \mathcal{F}$.
3. Non-Triviality: The fuzzy set defined by $A(x) = 0$ for all $x \in X$ (the null fuzzy set) is not in $\mathcal{F}$.

Definition 2.12 (Fuzzy Ultrafilter). [114] A fuzzy ultrafilter $\mathcal{U}$ on X is a fuzzy filter on X that is maximal concerning inclusion, meaning that there is no fuzzy filter $\mathcal{F}$ on X such that $\mathcal{U} \subsetneq \mathcal{F}$. Equivalently, a fuzzy ultrafilter on X satisfies the following property:

1. For every fuzzy set $A \in \mathcal{F}(X)$, either $A \in \mathcal{U}$ or $A^c \in \mathcal{U}$, where A^c is the complement of A defined by $A^c(x) = 1 - A(x)$ for all $x \in X$.

The following proposition is clearly valid.

Proposition 2.13. Every fuzzy filter can be transformed into a filter on a suitably defined set.

Proof. Let X be a universe set, and let $\mathcal{F} \subseteq \mathcal{F}(X)$ be a fuzzy filter on X, where $\mathcal{F}(X)$ denotes the set of all fuzzy sets on X. We aim to construct a corresponding filter $\mathcal{G} \subseteq 2^X$ (the power set of) that retains the essential properties of a filter, using the concept of level sets.

For a fuzzy set $A \in \mathcal{F}(X)$ and any $\alpha \in (0,1]$, define the α-cut of A as:

$$A_\alpha = \{x \in X \mid A(x) \ge \alpha\}.$$

The α-cut A_α is a classical subset of X and represents the set of elements that belong to A with a membership degree of at least α.

Now define the collection $\mathcal{G} = \{A_\alpha \mid A \in \mathcal{F}, \alpha \in (0,1]\}$. We will show that $\mathcal{G}$ satisfies the conditions of a filter on X.

1. Non-emptiness and Exclusion of the Empty Set: Since $\mathcal{F}$ is a fuzzy filter, it does not contain the null fuzzy set (where all memberships are zero). Thus, for every $A \in \mathcal{F}$ and $\alpha \in (0,1]$, the α-cut A_α is non-empty. Consequently, $\emptyset \notin \mathcal{G}$.
2. Closure under Finite Intersection: Let $A, B \in \mathcal{F}$ and $\alpha, \beta \in (0,1]$. Consider the α-cut A_α and the β cut B_β. Since $\mathcal{F}$ is closed under the minimum operation, $A \cap B \in \mathcal{F}$ where $(A \cap B)(x) = \min\{A(x), B(x)\}$ for all $x \in X$. Then, we have:

$$(A \cap B)_\gamma = \{x \in X \mid (A \cap B)(x) \ge \gamma\} \subseteq A_\alpha \cap B_\beta,$$

   where $\gamma = \min\{\alpha, \beta\}$. Since $A \cap B \in \mathcal{F}$, it follows that $(A \cap B)_\gamma \in \mathcal{G}$, so $A_\alpha \cap B_\beta \in \mathcal{G}$. Therefore, $\mathcal{G}$ is closed under intersection.
3. Upward Closure: Let $A_\alpha \in \mathcal{G}$ and suppose $A_\alpha \subseteq B \subseteq X$ for some $B \in 2^X$. We aim to show that $B \in \mathcal{G}$. Define a fuzzy set $C \in \mathcal{F}(X)$ by

$$C(x) = \begin{cases} A(x), & x \in A_\alpha, \\ 1, & x \in B \setminus A_\alpha \\ 0, & \text{otherwise} \end{cases}$$

By the upward closure of $\mathcal{F}$, since $A \in \mathcal{F}$ and $A \leq C$, we have $C \in \mathcal{F}$. Now, for any $x \in B, C(x) \geq \alpha$, so $B = C_\alpha \in \mathcal{G}$, proving upward closure.

Since $\mathcal{G}$ satisfies all the conditions of a filter on X, it follows that every fuzzy filter can be transformed into a filter on an appropriate set of α-cuts of the fuzzy sets in the fuzzy filter $\mathcal{F}$.

2.4 | Fuzzy Soft Filters and Ultrafilters

The definition of a fuzzy soft filter is provided below [89]. Due to its conceptual flexibility, the fuzzy soft set has been widely studied for various applications, including decision-making, as well as in mathematical structures and derived concepts [3, 10, 11, 21, 22, 37, 64, 65, 119, 121, 122]. For further details on operations related to fuzzy soft sets, refer to [89] as needed.

Definition 2.14 (Fuzzy Soft Set). [89] A fuzzy soft set F over X is a mapping $F: E \to I^X$. That is, for each $e \in E, F(e)$ is a fuzzy subset of X.

Alternatively, a fuzzy soft set can be represented as a set of ordered pairs:

$$F = \{(e, F_e) \mid e \in E, F_e \in I^X\}$$

Definition 2.15 (Operations on Fuzzy Soft Sets). [89] Let F, G be fuzzy soft sets over X.

1. The null fuzzy soft set, denoted by $\emptyset$, is defined by $\emptyset(e)(x) = 0$ for all $e \in E$ and $x \in X$.
2. The absolute fuzzy soft set, denoted by X, is defined by $X(e)(x) = 1$ for all $e \in E$ and $x \in X$.
3. The complement of F, denoted by F^c, is defined by $F^c(e)(x) = 1 - F(e)(x)$ for all $e \in E$ and $x \in X$.
4. The union of F and G, denoted by $F \cup G$, is defined by $(F \cup G)(e)(x) = \max\{F(e)(x), G(e)(x)\}$.
5. The intersection of F and G, denoted by $F \cap G$, is defined by $(F \cap G)(e)(x) = \min\{F(e)(x), G(e)(x)\}$.
6. F is a fuzzy soft subset of G, denoted $F \subseteq G$, if $F(e)(x) \leq G(e)(x)$ for all $e \in E$ and $x \in X$.

In this Soft Theory framework, Fuzzy Soft Filters [31, 46, 58] and Fuzzy Soft Ultrafilters [31] have been actively studied.

Definition 2.16 (Fuzzy Soft Filter). [31, 46, 58] A fuzzy soft filter $\mathcal{F}$ over X is a non-empty collection of fuzzy soft sets over X satisfying:

1. Non-emptiness: $\emptyset \notin \mathcal{F}$.
2. Closure under Intersection: If $F, G \in \mathcal{F}$, then $F \cap G \in \mathcal{F}$.
3. Upward Closure: If $F \in \mathcal{F}$ and $F \subseteq H$, then $H \in \mathcal{F}$.

Definition 2.17 (Fuzzy Soft Ultrafilter). [31] A fuzzy soft ultrafilter over X is a fuzzy soft filter $\mathcal{F}$ that is maximal with respect to inclusion, meaning there is no fuzzy soft filter strictly containing $\mathcal{F}$.

Equivalently, $\mathcal{F}$ is a fuzzy soft ultrafilter if for every fuzzy soft set F over X, either $F \in \mathcal{F}$ or $F^c \in \mathcal{F}$.

The following can be seen to hold clearly.

Proposition 2.18. Every fuzzy soft filter can be transformed into a fuzzy filter.

Proof. Let $\mathcal{F}$ be a fuzzy soft filter over a universe X with a parameter set E. By definition, a fuzzy soft filter $\mathcal{F}$ consists of fuzzy soft sets $F: E \rightarrow I^X$, where each $F(e)$ is a fuzzy subset of X (i.e., a mapping $F(e): X \rightarrow [0,1]$).

To construct a corresponding fuzzy filter, we define a mapping Φ that associates each fuzzy soft set $F \in \mathcal{F}$ with a fuzzy set in X. Specifically, for each fuzzy soft set F, define the fuzzy set $\Phi(F)$ on X by:

$$\Phi(F)(x) = \sup_{e \in E} F(e)(x)$$

For all $x \in X$. This transformation aggregates the membership values from all parameters in E, creating a single fuzzy set on X for each fuzzy soft set in $\mathcal{F}$.

Let $\mathcal{G} = \{\Phi(F) \mid F \in \mathcal{F}\}$. We will show that $\mathcal{G}$ is a fuzzy filter on X.

1. Non-Triviality: Since $\mathcal{F}$ is a fuzzy soft filter, it does not contain the null fuzzy soft set, which would map every element of X to 0 across all parameters. Consequently, for each $F \in \mathcal{F}$, there exists some $e \in E$ and some $x \in X$ such that $F(e)(x) > 0$. Therefore, $\Phi(F)(x) > 0$ for some $x \in X$, implying that the null fuzzy set (which maps every $x \in X$ to 0) is not in $\mathcal{G}$.
2. Closure under Minimum: Let $F, G \in \mathcal{F}$, so $\Phi(F), \Phi(G) \in \mathcal{G}$. Then, for each $x \in X$,

   $$(\Phi(F) \cap \Phi(G))(x) = \min\{\Phi(F)(x), \Phi(G)(x)\}$$

   Since $\Phi(F)(x) = \sup_{e \in E} F(e)(x)$ and $\Phi(G)(x) = \sup_{e \in E} G(e)(x)$, it follows that

   $$(\Phi(F) \cap \Phi(G))(x) = \min\left\{\sup_{e \in E} F(e)(x), \sup_{e \in E} G(e)(x)\right\}$$

   Define $H: E \rightarrow I^X$ by $H(e)(x) = F(e)(x) \cap G(e)(x)$ for each $e \in E$ and $x \in X$. Since $\mathcal{F}$ is closed under intersections, $H \in \mathcal{F}$, and we have $\Phi(H) \in \mathcal{G}$. Noting that $\Phi(H)(x) = \sup_{e \in E} H(e)(x) = \min\{\Phi(F)(x), \Phi(G)(x)\}$, it follows that $\Phi(F) \cap \Phi(G) = \Phi(H) \in \mathcal{G}$.
3. Upward Closure: Let $\Phi(F) \in \mathcal{G}$ and suppose $\Phi(F)(x) \leq \Phi(H)(x)$ for all $x \in X$ for some fuzzy set H on X. Define a fuzzy soft set $G: E \rightarrow I^X$ by $G(e)(x) = \max\{F(e)(x), H(x)\}$ for all $e \in E$ and $x \in X$. Since $G(e)(x) \geq F(e)(x)$ for all $e \in E$ and $x \in X, G \in \mathcal{F}$ by the upward closure of $\mathcal{F}$. Then $\Phi(G) = H \in \mathcal{G}$, proving that $\mathcal{G}$ is upward closed.

Thus, $\mathcal{G}$ is a fuzzy filter on X, demonstrating that every fuzzy soft filter can be transformed into a fuzzy filter.

Proposition 2.19. Every fuzzy soft filter can be transformed into a soft filter on an appropriately defined set, thereby retaining the essential properties of a soft filter.

Proof. Let U be an initial universe set, and let I^X represent the set of all fuzzy subsets of X (i.e., mappings $\mu: X \rightarrow [0,1]$). Let $\mathrm{PS}(U)$ denote the family of all soft sets over U, and let $\mathcal{F} \subseteq \mathrm{PS}(U)$ be a fuzzy soft filter over U. We aim to construct a corresponding soft filter $\mathcal{F}' \subseteq \mathrm{PS}(U)$, such that $\mathcal{F}'$ satisfies the properties of a soft filter.

Define a mapping that assigns to each fuzzy soft set $F = \{(e, F_e) \mid e \in E, F_e \in I^X\}$ a soft set $S_F = \{(e, S_e) \mid e \in E, S_e = \{x \in X \mid F_e(x) > \alpha\}$ for some $\alpha \in (0,1]\}$, where each S_e is the support of F_e, corresponding to the elements of X with non-zero membership in F_e. Let $\mathcal{F}' = \{S_F \mid F \in \mathcal{F}\}$, where each S_F is the "crisp" version of F obtained by taking the support of each fuzzy subset in F.

We now verify that $\mathcal{F}'$ satisfies the conditions of a soft filter on :

1. Non-emptiness: Since $\mathcal{F}$ is a fuzzy soft filter, it does not contain the null fuzzy soft set, which implies that for any $F \in \mathcal{F}$, the corresponding support-based soft set S_F is also non-empty. Hence, $\emptyset \notin \mathcal{F}'$.

2. Closure under Intersection: Let $F, G \in \mathcal{F}$. Then their intersection $F \cap G$ is also in $\mathcal{F}$ by the fuzzy soft filter properties. For each parameter $e \in E$, the intersection $(F \cap G)_e = F_e \cap G_e$ corresponds to the fuzzy subset $\min\{F_e(x), G_e(x)\}$. The support of $F \cap G$ under parameter e is thus:

$$S_{F\cap G}(e) = \{x \in X \mid \min\{F_e(x), G_e(x)\} > 0\} = S_F(e) \cap S_G(e).$$

   Consequently, $S_{F\cap G} = S_F \cap S_G$, and since $F \cap G \in \mathcal{F}$, it follows that $S_F \cap S_G \in \mathcal{F}'$, showing closure under the intersection.

3. Upward Closure: Let $F \in \mathcal{F}$ and $S_F \subseteq S_H$ for some $H \in \mathrm{PS}(U)$. By definition, $S_F(e) \subseteq S_H(e)$ for each $e \in E$, which implies that $F_e(x) > 0$ for all $x \in S_F(e)$, and there exists a fuzzy subset H_e such that $F_e \subseteq H_e$. Since $\mathcal{F}$ is upward closed and $F \subseteq H$, it follows that $H \in \mathcal{F}$, thus $S_H \in \mathcal{F}'$.

Since $\mathcal{F}'$ satisfies all properties of a soft filter on U, it follows that every fuzzy soft filter can be transformed into a soft filter by taking the support of each fuzzy subset. This completes the proof.

2.5 | Hypersoft Set and Fuzzy Hypersoft Set

A hypersoft set builds on soft set theory by introducing multiple attributes for each element, enabling the representation of complex, multi-dimensional relationships. The formal definition of a hypersoft set is provided below [66, 92-95]. For more detailed information and operations, refer to sources such as [107] as needed.

Definition 2.20 (Hypersoft Set). [107] Let X be a non-empty finite universe, and let $T_1, T_2, \dots, T_n$ be n-distinct attributes with corresponding disjoint sets $J_1, J_2, \dots, J_n$. A pair (F, J) is called a hypersoft set over the universal set X, where F is a mapping defined by

$$F: J \to \mathcal{P}(X)$$

with $J = J_1 \times J_2 \times \dots \times J_n$.

The concept of a Fuzzy Hypersoft Set, which extends the Hypersoft Set by incorporating fuzzy logic, has been defined and widely researched [5, 13, 29, 90, 91, 123]. Below, we present the formal definition of a Fuzzy Hypersoft Set.

Definition 2.21 (Fuzzy Hypersoft Set). [123] Let U be a universal set and let $F_P(U)$ denote the family of all fuzzy sets over U. Suppose $E_1, E_2, \dots, E_n$ are pairwise disjoint parameter sets with $A_i \subseteq E_i$ for each $i = 1,2, \dots, n$. A fuzzy hypersoft set is defined as the pair $(\Theta, A_1 \times A_2 \times \dots \times A_n)$, where

$$\Theta: A_1 \times A_2 \times \dots \times A_n \to F_P(U)$$

and

$$\Theta(A_1 \times A_2 \times \dots \times A_n) = \{\langle u, \Theta(\alpha)(u)\rangle : u \in U, \alpha \in A_1 \times A_2 \times \dots \times A_n \subseteq E_1 \times E_2 \times \dots \times E_n\}$$

For convenience, we denote $\Sigma = E_1 \times E_2 \times \dots \times E_n, \Gamma = A_1 \times A_2 \times \dots \times A_n$, and let α represent an element of the set Γ. The collection of all fuzzy hypersoft sets over U is denoted by $FHS(U, \Sigma)$, and henceforth, FHS will refer to fuzzy hypersoft sets.

2.6 | Neutrosophic Filter and Neutrosophic Soft Filter

In this subsection, we introduce the concepts of the Neutrosophic Filter and Neutrosophic Soft Filter. The Neutrosophic Set extends the Fuzzy Set and Intuitionistic Fuzzy Set by assigning three membership degrees-truth, indeterminacy, and falsity-to each element, allowing for the representation of incomplete or ambiguous information [103-106]. A Neutrosophic Filter is a filter concept defined specifically for the Neutrosophic Set [80, 96].

The precise definitions for each are provided below. Readers interested in further details or operations of the Neutrosophic Set may refer to relevant literature, such as, [103, 104] as needed.

Definition 2.22. [103] Let X be a given set. A Neutrosophic Set A on X is characterized by three membership functions:

$$T_A: X \to [0,1],\ I_A: X \to [0,1],\ F_A: X \to [0,1]$$

where for each $x \in X$, the values $T_A(x), I_A(x)$, and $F_A(x)$ represent the degree of truth, indeterminacy, and falsity, respectively. These values satisfy the following conditions:

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3.$$

Definition 2.23. Let U be a universe set. A Neutrosophic Filter F on the collection of single-valued neutrosophic sets, denoted as $\mathrm{SVN}(U)$, is defined as a nonempty family $F \subseteq \mathrm{SVN}(U)$ satisfying the following conditions:

1. Non-emptiness of finite intersections: For any finite subset $\{A_1, A_2, \dots, A_n\} \subset F$, there exists $H \in F$ such that $H \subseteq A_1 \cap A_2 \cap \dots \cap A_n$.
2. Containment of supersets: If $A \in F$ and $A \subseteq B$ for some $B \in SVN(U)$, then $B \in F$.

Thus, the neutrosophic filter includes finite intersections and all supersets within the neutrosophic framework.
Proposition 2.24. Every Neutrosophic Filter $\mathcal{F}_N$ on a universe set X can be transformed into a Fuzzy Filter $\mathcal{F}_F$ on X.

Proof. Let $\mathcal{F}_N$ be a Neutrosophic Filter on X. We will construct a Fuzzy Filter $\mathcal{F}_F$ on X using the truth membership functions of the Neutrosophic Sets in $\mathcal{F}_N$.

Define a mapping $\Phi: \mathcal{F}_N \to F(X)$ by:

$$\Phi(A)(x) = T_A(x),\ \forall x \in X,\ \forall A \in \mathcal{F}_N.$$

Let $\mathcal{F}_F = \Phi(\mathcal{F}_N) = \{\Phi(A) \mid A \in \mathcal{F}_N\}$.

We will show that $\mathcal{F}_F$ is a Fuzzy Filter on X.

Non-emptiness: Since $\mathcal{F}_N$ is non-empty, $\mathcal{F}_F$ is also non-empty.

Closure under Intersection: Let $\mu_1, \mu_2 \in \mathcal{F}_F$, where $\mu_i = \Phi(A_i)$ for some $A_i \in \mathcal{F}_N$. Then, for all $\in X$:

$$\mu_i(x) = T_{A_i}(x),\ i = 1,2$$

Consider the intersection $\mu = \mu_1 \cap \mu_2$, defined by $\mu(x) = \min\{\mu_1(x), \mu_2(x)\}$.

Since $\mathcal{F}_N$ is a Neutrosophic Filter, there exists $B \in \mathcal{F}_N$ such that $B \subseteq A_1 \cap A_2$. By the properties of Neutrosophic Sets and the subset relation:

$$T_B(x) \leq \min\{T_{A_1}(x), T_{A_2}(x)\} = \mu(x),\ \forall x \in X$$

Therefore, $\Phi(B)(x) = T_B(x) \leq \mu(x)$.

Since $\Phi(B) \in \mathcal{F}_F$ and $\Phi(B) \leq \mu$, it follows that $\mu \in \mathcal{F}_F$.

Upward Closure: Let $\mu \in \mathcal{F}_F$ and $\mu \leq v$ for some $v \in F(X)$. Since $\mu = \Phi(A)$ for some $A \in \mathcal{F}_N$, and $v(x) \geq T_A(x)$, we define a Neutrosophic Set C with $T_C(x) = v(x), I_C(x) = I_A(x)$, and $F_C(x) = F_A(x)$ for all $x \in X$.

Since $A \subseteq C$ in $\mathcal{F}_N, C \in \mathcal{F}_N$, and $\Phi(C) = v \in \mathcal{F}_F$.

Non-Triviality: Since $\emptyset_N \notin \mathcal{F}_N$, and for any $A \in \mathcal{F}_N, T_A(x) > 0$ for some $x \in X$, it follows that $\Phi(A) \neq \emptyset_F$. Thus, $\emptyset_F \notin \mathcal{F}_F$.

Therefore, $\mathcal{F}_F$ is a Fuzzy Filter on X.

Building on this, the Neutrosophic Soft Set introduces parameterization into the Neutrosophic Set, allowing for more precise categorization and an improved approach to managing uncertainty through a parameter set [6, 7, 30, 62, 63]. Furthermore, the Neutrosophic Soft Filter generalizes the classical concept of a filter to Neutrosophic Soft Sets, facilitating the creation of a topological structure [32, 33].

Definition 2.25. [63] Let X be an initial universe set, and E a set of parameters. A Neutrosophic $\mathrm{SoftSet}(\tilde{F}, E)$ over X is defined by a set-valued function

$$\tilde{F}: E \to P(X)$$

where $\tilde{F}(e)$ is a Neutrosophic Set corresponding to each parameter $e \in E$. The Neutrosophic $\mathrm{SoftSet}(\tilde{F}, E)$ can be expressed as a collection of ordered pairs:

$$(\tilde{F}, E) = \{(e, \{(x, T_{\tilde{F}(e)}(x), I_{\tilde{F}(e)}(x), F_{\tilde{F}(e)}(x)) : x \in X\}) : e \in E\}$$

where $T_{\tilde{F}(e)}(x), I_{\tilde{F}(e)}(x)$, and $F_{\tilde{F}(e)}(x)$ denote the truth, indeterminacy, and falsity degrees for each element $x \in X$. These degrees satisfy:

$$0 \leq T_{\tilde{F}(e)}(x) + I_{\tilde{F}(e)}(x) + F_{\tilde{F}(e)}(x) \leq 3.$$

Definition 2.26. [32, 33] Let $\mathcal{N} \subset NSS(X, E)$ be a family of Neutrosophic Soft Sets over X. Then, $\mathcal{N}$ is called a Neutrosophic Soft Filter on X if it satisfies the following properties:

(N1) $0_{(X,E)} \notin \mathcal{N}$, where $0_{(X,E)}$ denotes the null Neutrosophic Soft Set.

(N2) For any $(\tilde{F}, E), (\tilde{G}, E) \in \mathcal{N}$, we have $(\tilde{F}, E) \cap (\tilde{G}, E) \in \mathcal{N}$.

(N3) If $(\tilde{F}, E) \in \mathcal{N}$ and $(\tilde{F}, E) \subseteq (\tilde{G}, E)$ for some $(\tilde{G}, E)$, then $(\tilde{G}, E) \in \mathcal{N}$.

Proposition 2.27. Every Neutrosophic Soft Filter N on a universe set X can be transformed into a Fuzzy Soft Filter F on X.

Proof. Let N be a Neutrosophic Soft Filter on X. For each Neutrosophic Soft Set $(\tilde{F}, E) \in N$, define a corresponding Fuzzy Soft Set (F, E) by

$$F(e)(x) = T_{\tilde{F}(e)}(x), \ \forall x \in X, \forall e \in E.$$

Let $F = \{(F, E) \mid (\tilde{F}, E) \in N\}$. We will show that F is a Fuzzy Soft Filter on X.

Non-emptiness: Since N is non-empty, F is also non-empty.

Closure under Intersection: Let $(F, E), (G, E) \in F$, corresponding to $(\tilde{F}, E), (\tilde{G}, E) \in N$. Then, for each $e \in E$ and $x \in X$,

$$F(e)(x) = T_{\tilde{F}(e)}(x), \ G(e)(x) = T_{\tilde{G}(e)}(x)$$

Consider the intersection $(H, E) = (F, E) \cap (G, E)$, where for each $e \in E$ and $x \in X$,

$$H(e)(x) = \min\{F(e)(x), G(e)(x)\}$$

Since $(\tilde{F}, E), (\tilde{G}, E) \in N$, their intersection $(\tilde{H}, E) = (\tilde{F}, E) \cap (\tilde{G}, E)$ is in N. For each $e \in E$ and $x \in X$,

$$T_{\tilde{H}(e)}(x) = \min\{T_{\tilde{F}(e)}(x), T_{\tilde{G}(e)}(x)\} = H(e)(x).$$

Therefore, $(H, E) \in F$, and F is closed under intersection.

Upward Closure: Let $(F, E) \in F$ and $(F, E) \subseteq (K, E)$, where (K, E) is a Fuzzy Soft Set over X. Then, for each $e \in E$ and $x \in X$,

$$F(e)(x) \leq K(e)(x)$$

Define $(\tilde{K}, E)$ by

$$T_{\tilde{K}(e)}(x) = K(e)(x),\ I_{\tilde{K}(e)}(x) = I_{\tilde{F}(e)}(x),\ F_{\tilde{K}(e)}(x) = F_{\tilde{F}(e)}(x)$$

Since $T_{\tilde{F}(e)}(x) \leq T_{\tilde{K}(e)}(x)$ for all $e \in E$ and $x \in X$, and $(\tilde{F}, E) \subseteq (\tilde{K}, E)$, the upward closure property of N implies $(\tilde{K}, E) \in N$. Therefore, $(K, E) \in F$.

Non-Triviality: Since $0 \notin N$, for each $(\tilde{F}, E) \in N$, there exists $e \in E$ and $x \in X$ such that $T_{\tilde{F}(e)}(x) > 0$. Hence, $F(e)(x) > 0$, and $0 \notin F$.

Thus, F is a Fuzzy Soft Filter on X.

2.7 | Neutrosophic Hypersoft Set

A Neutrosophic Hypersoft Set is a mathematical structure that employs multiple attributes along with three values truth, indeterminacy, and falsity- to effectively capture uncertainty and complex information. Due to its versatility in applications, the Neutrosophic Hypersoft Set has been the subject of extensive research [1, 51, 59, 75, 87, 99-101]. The formal definition is provided below [2].

Definition 2.28 (Neutrosophic Hypersoft Set). [2] Let U be a universal set, and let $P(U)$ denote the power set of U. Assume there are n distinct attributes, represented as $\ell_1, \ell_2, \ldots, \ell_n$, each with a corresponding set of values $L_1, L_2, \ldots, L_n$. These attribute sets satisfy the conditions. $L_i \cap L_j = \emptyset$ for $i \neq j$ and $i, j \in \{1,2, \ldots, n\}$.

The pair (ψ, Λ) is defined to be a Neutrosophic Hypersoft Set $(NHSS)$ over U if there exists a relation $\Lambda = L_1 \times L_2 \times \cdots \times L_n$. The mapping ψ is a function from Λ to $P(U)$, such that.

$$\psi_\Lambda(L_1 \times L_2 \times \cdots \times L_n) = \{\langle u, T_\Lambda(u), I_\Lambda(u), F_\Lambda(u)\rangle : u \in U\},$$

where $T_\Lambda(u), I_\Lambda(u)$, and $F_\Lambda(u)$ are neutrosophic membership values for truth, indeterminacy, and falsity, respectively, for each $u \in U$.

These membership functions satisfy

$$T, I, F: U \to [0,1]$$

and

$$0 \leq T_\Lambda(u) + I_\Lambda(u) + F_\Lambda(u) \leq 3.$$

3 | Results

The results of this paper are described below.

3.1 | Hypersoft Filter

We define a Hypersoft Filter by extending the concept of a Soft Filter in the Hypersoft context as follows.
Definition 3.1 (Hypersoft Filter). A hypersoft filter $\mathcal{H}$ over X is a non-empty collection of hypersoft sets satisfying:

1. $\emptyset \notin \mathcal{H}$.
2. If $(F, E), (G, E) \in \mathcal{H}$, then $(F \cap G, E) \in \mathcal{H}$, where $(F \cap G)(e) = F(e) \cap G(e)$ for all $e \in E$.
3. If $(F, E) \in \mathcal{H}$ and $(F, E) \subseteq (H, E)$, then $(H, E) \in \mathcal{H}$.

Definition 3.2 (Hypersoft Ultrafilter). A hypersoft ultrafilter over X is a hypersoft filter $\mathcal{H}$ that is maximal concerning inclusion.

Theorem 3.3. Every hypersoft filter (ultrafilter) can be transformed into a soft filter (ultrafilter).

Proof. Let $\mathcal{H}$ be a hypersoft filter over X. Define a mapping Φ from hypersoft sets to soft sets as follows: For each hypersoft set (F, E), define the corresponding soft set (f, E) by setting $f(e) = F(e)$ for all $e \in E$. Now, consider the collection $\mathcal{F} = \{(f, E) \mid (F, E) \in \mathcal{H}\}$.

We will show that $\mathcal{F}$ is a soft filter over X.

1. Non-emptiness: Since $\mathcal{H}$ is non-empty and $\emptyset \notin \mathcal{H}, \mathcal{F}$ is non-empty and $\emptyset \notin \mathcal{F}$.
2. Closure under Intersection: If $(f, E), (g, E) \in \mathcal{F}$, corresponding to $(F, E), (G, E) \in \mathcal{H}$, then $(F \cap G, E) \in \mathcal{H}$, so $(f \cap g, E) \in \mathcal{F}$.
3. Upward Closure: If $(f, E) \in \mathcal{F}$ and $(f, E) \subseteq (h, E)$, then $(F, E) \subseteq (H, E)$. Since $\mathcal{H}$ is upward closed, $(H, E) \in \mathcal{H}$, so $(h, E) \in \mathcal{F}$.

Therefore, $\mathcal{F}$ is a soft filter over X. If $\mathcal{H}$ is a hypersoft ultrafilter, then $\mathcal{F}$ is a soft ultrafilter.

3.2 | Fuzzy Hypersoft Filters and Ultrafilters

We define a Fuzzy Hypersoft Filter by extending the concept of a Soft Filter in the Fuzzy Hypersoft context as follows.

Definition 3.4 (Fuzzy Hypersoft Filter). A fuzzy hypersoft filter $\mathcal{H}$ over X is a non-empty collection of fuzzy hypersoft sets satisfying:

1. $\emptyset \notin \mathcal{H}$.
2. If $(F, E), (G, E) \in \mathcal{H}$, then $(F \cap G, E) \in \mathcal{H}$,where $(F \cap G)(e)(x) = \min\{F(e)(x), G(e)(x)\}$.
3. If $(F, E) \in \mathcal{H}$ and $(F, E) \subseteq (H, E)$, then $(H, E) \in \mathcal{H}$.

Definition 3.5 (Fuzzy Hypersoft Ultrafilter). A fuzzy hypersoft ultrafilter over X is a fuzzy hypersoft filter $\mathcal{H}$ that is maximal with respect to inclusion.

Theorem 3.6. Every fuzzy hypersoft filter (ultrafilter) can be transformed into a fuzzy soft filter (ultrafilter).

Proof. Let $\mathcal{H}$ be a fuzzy hypersoft filter over X. Define a mapping Φ from fuzzy hypersoft sets to fuzzy soft sets as follows:

For each fuzzy hypersoft set (F, E), define the corresponding fuzzy soft set f over X with parameter set E, where $f(e) = F(e)$ for all $e \in E$.

Consider the collection $\mathcal{F} = \{f \mid (F, E) \in \mathcal{H}\}$.

We need to show that $\mathcal{F}$ is a fuzzy soft filter over X.

1. Non-emptiness: $\mathcal{F}$ is non-empty since $\mathcal{H}$ is non-empty, and $\emptyset \notin \mathcal{F}$ because $\emptyset \notin \mathcal{H}$.
2. Closure under Intersection: If $f, g \in \mathcal{F}$, corresponding to $(F, E), (G, E) \in \mathcal{H}$, then $(F \cap G, E) \in \mathcal{H}$, so $f \cap g \in \mathcal{F}$.
3. Upward Closure: If $f \in \mathcal{F}$ and $f \subseteq h$, then $(F, E) \subseteq (H, E)$. Since $\mathcal{H}$ is upward closed, $(H, E) \in \mathcal{H}$, so $h \in \mathcal{F}$.

Therefore, $\mathcal{F}$ is a fuzzy soft filter over X. If $\mathcal{H}$ is a fuzzy hypersoft ultrafilter, then $\mathcal{F}$ is a fuzzy soft ultrafilter.

Theorem 3.7. Every fuzzy hypersoft filter can be transformed into a hypersoft filter on an appropriately defined set, preserving the essential properties of a hypersoft filter.

Proof. Let U be an initial universe set, and let I^X denote the set of all fuzzy subsets of X (i.e., mappings: $X \to [0,1]$). Let $\mathcal{H}$ represent a fuzzy hypersoft filter over U, defined as a non-empty collection of fuzzy hypersoft sets satisfying the following properties:

1. $\emptyset \notin \mathcal{H}$.
2. If $(F, E), (G, E) \in \mathcal{H}$, then $(F \cap G, E) \in \mathcal{H}$, where $(F \cap G)(e)(x) = \min\{F(e)(x), G(e)(x)\}$.
3. If $(F, E) \in \mathcal{H}$ and $(F, E) \subseteq (H, E)$, then $(H, E) \in \mathcal{H}$.

To transform $\mathcal{H}$ into a hypersoft filter, define a mapping that converts each fuzzy hypersoft set (F, E) in $\mathcal{H}$ to a hypersoft set (S_F, E), where $S_F(e) = \{x \in X \mid F(e)(x) > 0\}$ for each $e \in E$. Here, $S_F(e)$ represents the support of the fuzzy subset $F(e)$. Define the collection $\mathcal{H}' = \{(S_F, E) \mid (F, E) \in \mathcal{H}\}$, where each S_F is the "crisp" support-based representation of F.

We now verify that $\mathcal{H}'$ satisfies the properties of a hypersoft filter:

1. Non-emptiness: Since $\mathcal{H}$ does not contain the null fuzzy hypersoft set, each fuzzy hypersoft set $(F, E) \in \mathcal{H}$ has $F(e)(x) > 0$ for some $x \in X$ and $e \in E$. Thus, $S_F(e) \neq \emptyset$ for all $(S_F, E) \in \mathcal{H}'$, implying $\emptyset \notin \mathcal{H}'$.
2. Closure under Intersection: Let $(S_F, E), (S_G, E) \in \mathcal{H}'$, corresponding to fuzzy hypersoft sets $(F, E), (G, E) \in \mathcal{H}$. Since $\mathcal{H}$ is closed under intersections, $(F \cap G, E) \in \mathcal{H}$ where $(F \cap G)(e)(x) = \min\{F(e)(x), G(e)(x)\}$. The support of $F \cap G$ under each $e \in E$ is given by:

$$S_{F\cap G}(e) = \{x \in X \mid \min\{F(e)(x), G(e)(x)\} > 0\} = S_F(e) \cap S_G(e).$$

Therefore, $(S_F \cap S_G, E) = (S_{F\cap G}, E) \in \mathcal{H}'$, establishing closure under intersection.

3. Upward Closure: Let $(S_F, E) \in \mathcal{H}'$ and suppose $(S_F, E) \subseteq (S_H, E)$ for some hypersoft set (S_H, E). By definition of support, this implies that $F(e)(x) > 0$ for all $x \in S_F(e)$ and there exists a fuzzy subset $H(e)$ such that $F(e) \subseteq H(e)$. Since $\mathcal{H}$ is upward closed, $(H, E) \in \mathcal{H}$, so $(S_H, E) \in \mathcal{H}'$.

Therefore, $\mathcal{H}'$ satisfies the conditions of a hypersoft filter over X. If $\mathcal{H}$ is a fuzzy hypersoft ultrafilter, then $\mathcal{H}'$ will also be a hypersoft ultrafilter by construction.

3.3 | Neutrosophic Hypersoft Filters

We define the Neutrosophic Hypersoft Filter and demonstrate that it can be transformed into a Fuzzy Hypersoft Filter, a Hypersoft Filter, and a Neutrosophic Soft Filter. We provide precise mathematical definitions and detailed proofs of these transformations.

Definition 3.8 (Neutrosophic Hypersoft Filter). Let U be a universal set, and $\Lambda = L_1 \times L_2 \times \cdots \times L_n$. A neutrosophic hypersoft filter $\mathcal{N}$ over U is a non-empty collection of neutrosophic hypersoft sets (ψ, Λ) satisfying:

1. Non-Triviality: The null neutrosophic hypersoft set is not in $\mathcal{N}$.
2. Closure under Intersection: If $(\psi_1, \Lambda), (\psi_2, \Lambda) \in \mathcal{N}$, then $(\psi_1 \cap \psi_2, \Lambda) \in \mathcal{N}$, where for all $\lambda \in \Lambda$ and $u \in U$:

$$T_{\psi_1\cap\psi_2}(\lambda)(u) = \min\{T_{\psi_1}(\lambda)(u), T_{\psi_2}(\lambda)(u)\},$$
$$I_{\psi_1\psi_2}(\lambda)(u) = \max\{I_{\psi_1}(\lambda)(u), I_{\psi_2}(\lambda)(u)\},$$
$$F_{\psi_1\cap\psi_2}(\lambda)(u) = \max\{F_{\psi_1}(\lambda)(u), F_{\psi_2}(\lambda)(u)\}.$$

3. Upward Closure: If $(\psi, \Lambda) \in \mathcal{N}$ and $(\psi, \Lambda) \subseteq (\varphi, \Lambda)$, then $(\varphi, \Lambda) \in \mathcal{N}$. The subset relation $\subseteq$ is defined component-wise:

$$\psi(\lambda)(u) \subseteq \varphi(\lambda)(u) \Leftrightarrow \begin{cases} T_\psi(\lambda)(u) \leq T_\varphi(\lambda)(u) \\ I_\psi(\lambda)(u) \geq I_\varphi(\lambda)(u) \\ F_\psi(\lambda)(u) \geq F_\varphi(\lambda)(u) \end{cases}$$

Theorem 3.9. Every neutrosophic hypersoft filter $\mathcal{N}$ on a universe U can be transformed into a fuzzy hypersoft filter $\mathcal{F}$ on U.

Proof. Let $\mathcal{N}$ be a neutrosophic hypersoft filter over U. For each $(\psi, \Lambda) \in \mathcal{N}$, define a fuzzy hypersoft set (θ, Λ) by:

$$\theta(\lambda)(u) = T_\psi(\lambda)(u),\ \forall\lambda \in \Lambda, \forall u \in U.$$

Let $\mathcal{F} = \{(\theta, \Lambda) \mid (\psi, \Lambda) \in \mathcal{N}\}$. We will show that $\mathcal{F}$ satisfies the properties of a fuzzy hypersoft filter.

1. Non-Triviality: Since $\mathcal{N}$ does not contain the null set, there exists $\lambda \in \Lambda$ and $u \in U$ such that $T_\psi(\lambda)(u) > 0$ for each $(\psi, \Lambda) \in \mathcal{N}$. Therefore, $\theta(\lambda)(u) > 0$, and $\mathcal{F}$ does not contain the null fuzzy hypersoft set.
2. Closure under Intersection: For $(\theta_1, \Lambda), (\theta_2, \Lambda) \in \mathcal{F}$, corresponding to $(\psi_1, \Lambda), (\psi_2, \Lambda) \in \mathcal{N}$, define (θ, Λ) by:

   $$\theta(\lambda)(u) = \min\{\theta_1(\lambda)(u), \theta_2(\lambda)(u)\}$$

   Since $(\psi_1 \cap \psi_2, \Lambda) \in \mathcal{N}$ and

   $$T_{\psi_1 \cap \psi_2}(\lambda)(u) = \min\{T_{\psi_1}(\lambda)(u), T_{\psi_2}(\lambda)(u)\} = \theta(\lambda)(u),$$

   it follows that $(\theta, \Lambda) \in \mathcal{F}$. Thus, $\mathcal{F}$ is closed under the intersection.
3. Upward Closure: Let $(\theta, \Lambda) \in \mathcal{F}$ and $(\theta, \Lambda) \subseteq (\varphi, \Lambda)$, where (φ, Λ) is a fuzzy hypersoft set. Define (ψ', Λ) by:

$$T_{\psi'}(\lambda)(u) = \varphi(\lambda)(u),\ I_{\psi'}(\lambda)(u) = I_\psi(\lambda)(u),\ F_{\psi'}(\lambda)(u) = F_\psi(\lambda)(u).$$

Since $(\psi, \Lambda) \subseteq (\psi', \Lambda)$ and $(\psi, \Lambda) \in \mathcal{N}$, by upward closure, $(\psi', \Lambda) \in \mathcal{N}$. Therefore, $(\varphi, \Lambda) \in \mathcal{F}$. Hence, $\mathcal{F}$ is a fuzzy hypersoft filter over U.

Theorem 3.10. Every neutrosophic hypersoft filter $\mathcal{N}$ on a universe U can be transformed into a hypersoft filter $\mathcal{H}$ on U.

Proof. For each $(\psi, \Lambda) \in \mathcal{N}$, define a hypersoft set (F, Λ) by:

$$F(\lambda) = \{u \in U \mid T_\psi(\lambda)(u) > 0\},\ \forall\lambda \in \Lambda.$$

Let $\mathcal{H} = \{(F, \Lambda) \mid (\psi, \Lambda) \in \mathcal{N}\}$.

Non-Triviality: Since $T_\psi(\lambda)(u) > 0$ for some $u \in U, F(\lambda) \neq \emptyset$, ensuring $\mathcal{H}$ does not contain the null hypersoft set.

Closure under Intersection: For$(F_1, \Lambda), (F_2, \Lambda) \in \mathcal{H}$, define (F, Λ) by:

$$F(\lambda) = F_1(\lambda) \cap F_2(\lambda)$$

Since $(\psi_1 \cap \psi_2, \Lambda) \in \mathcal{N}$, it follows that $(F, \Lambda) \in \mathcal{H}$.

3. Upward Closure: If $(F, \Lambda) \subseteq (G, \Lambda)$, define (ψ', Λ) by:

$$T_{\psi'}(\lambda)(u) = \begin{cases} 1, & u \in G(\lambda) \\ 0, & \text{otherwise} \end{cases}$$

As $(\psi, \Lambda) \subseteq (\psi', \Lambda)$ and $(\psi, \Lambda) \in \mathcal{N}$, we have $(\psi', \Lambda) \in \mathcal{N}$, so $(G, \Lambda) \in \mathcal{H}$.

Therefore, $\mathcal{H}$ is a hypersoft filter over U.

Theorem 3.11. Every neutrosophic hypersoft filter $\mathcal{N}$ on a universe U can be transformed into a neutrosophic soft filter S on U.

Proof. Let $E = \Lambda$. For each $(\psi, \Lambda) \in \mathcal{N}$, define a neutrosophic soft set $(\tilde{F}, E)$ by:

$$\tilde{F}(e) = \psi(e),\ \forall e \in E.$$

Let $\mathcal{S} = \{(\tilde{F}, E) \mid (\psi, \Lambda) \in \mathcal{N}\}$.

1. Non-Triviality: Since $T_{\tilde{F}}(e)(u) = T_{\psi}(e)(u) > 0$ for some $u \in U, \mathcal{S}$ does not contain the null neutrosophic soft set.
2. Closure under Intersection: For $(\tilde{F}, E), (\tilde{G}, E) \in \mathcal{S}$, define $(\tilde{H}, E)$ by:

$$\tilde{H}(e) = \tilde{F}(e) \cap \tilde{G}(e)$$

Since $(\psi_1 \cap \psi_2, \Lambda) \in \mathcal{N}, (\tilde{H}, E) \in \mathcal{S}$.

3. Upward Closure: If $(\tilde{F}, E) \subseteq (\tilde{K}, E)$, define (φ, Λ) by $\varphi(e) = \tilde{K}(e)$. Since $(\psi, \Lambda) \subseteq (\varphi, \Lambda)$ and $(\psi, \Lambda) \in \mathcal{N}, (\varphi, \Lambda) \in \mathcal{N}$, so $(\tilde{K}, E) \in \mathcal{S}$.

Hence, $\mathcal{S}$ is a neutrosophic soft filter over U.

4 | Future Tasks: Weak Filter, Quasi-Filter, and Tangle

The following are future research directions.

First, filters and ultrafilters on general sets possess notable properties, such as the Axiom of Choice Property [26, 52, 53, 61, 69] and the Finite Intersection Property [26]. It is of interest to determine whether these properties apply to Neutrosophic Hypersoft Filters and other core filter definitions.

Next, we aim to explore extensions of Weak Filters [15, 66-68] and Quasi Filters, [23] including Soft Weak Filters, Soft Quasi Filters, HyperSoft Weak Filters, and HyperSoft Quasi Filters. Additionally, we plan to conduct research on grills and primals within the framework of soft theory [39].

Additionally, we will investigate how established concepts, such as Tangles and Brambles from graph theory [17, 34-36, 47, 49, 82, 83, 88] as well as the complementary concept of an ideal to filters, might be extended to Fuzzy Sets and Neutrosophic Sets. Notably, ultrafilters and tangles exhibit a profound relationship, specifically as complements of one another [45]. For instance, exploring the extension of matroid tangles [47] to Fuzzy Sets and Neutrosophic Sets may reveal novel mathematical properties.

Moreover, we intend to expand the concepts of filters and ultrafilters to super hypersoft sets, [40, 76, 109) hesitant fuzzy sets, [117, 118]]spherical fuzzy sets, [14, 7, 71]) Vague Sets, [4, 18, 20, 24, 56, 97, 130]) Neutrosophic SuperHypersoft sets, [109, 110, 112] and plithogenic sets, [108, 111]) with a particular focus on their mathematical structures and properties.

Acknowledgments

The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Funding

This research has no funding source.

Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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