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Ermakov Equations can be Derived from Zel'dovich Pancake, and they are Cold and Nonlocal through using Neutrosophic Venn Diagram

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Abstract

As we argue in the previous article [3], the labyrinthine worlds of Jorge Luis Borges are more than captivating narratives; they are portals to a deeper understanding of existence. By weaving elements of science-fiction fantasy with philosophical and ethical inquiries, Borges's short stories bridge the seemingly disparate realms of physics and the humanities, offering fertile ground for contemporary physics research. The present-day universe consists of galaxies, galaxy clusters, one-dimensional filaments and two-dimensional sheets or pancakes, all of which combine to form the cosmic web. The so called "Zeldovich pancakes", are very difficult to observe, because their overdensity is only slightly greater than the average density of the universe. Falco et al. presented a method to identify Zeldovich pancakes in observational data, and the method were used as a tool for estimating the mass of galaxy clusters [2]. Here we provide an outline from Zel'dovich pancake to Burgers equations to represent cosmic turbulence, and then from Burgers equations to Ermakov dynamics systems, which in turn they plausibly lead to nonlocal current.

Keywords: Neutrosophic Set; Venn Diagram; Mathematics; Physics; Ermakov.

1 | Introduction: Through the Looking Glass of Borges or where Sci-Fi Fantasy and Physics Converge

As we argue in the previous article, the labyrinthine worlds of Jorge Luis Borges are more than captivating narratives; they are portals to a deeper understanding of existence. By weaving elements of science-fiction fantasy with philosophical and ethical inquiries, Borges's short stories bridge the seemingly disparate realms of physics and the humanities, offering fertile ground for contemporary physics research.

As we know, Borges's stories are rife with fantastical concepts - infinite libraries, forking timelines, cyclical realities. These elements resonate with the speculative nature of science fiction, where the boundaries of reality are constantly pushed. Yet, Borges doesn't merely indulge in fantastical flourishes. He imbues them with philosophical weight. In "Tlön, Uqbar, Orbis Tertius," a fictional world alters reality through the power



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of belief. This challenges our perception of truth and objectivity, mirroring real-world debates in physics regarding the role of the observer in shaping reality (think quantum mechanics).

In conclusion, Borges's short stories offer a unique confluence of science-fiction fantasy, philosophical inquiry, and physics. By examining the essence of his work, we can see how these seemingly disparate fields can inspire each other. Through the looking glass of Borges's imagination, contemporary physics research might discover groundbreaking new avenues for exploration.

Here we provide an outline from Zel'dovich pancake to Burgers equations to represent cosmic turbulence, and then from Burgers equations to Ermakov dynamics systems.

1.1 | What is a Neutrosophic Venn Diagram?

Borges's stories can serve as springboards for new avenues in physics research. The concept of infinite libraries in "*The Library of Babel*" can be seen as an allegory for the vastness of the universe and the search for unifying theories. The story's exploration of an all-encompassing library with every possible book hints at the existence of a "theory of everything" that could explain all physical phenomena.

Furthermore, Borges's fascination with labyrinths can be seen as a metaphor for the complexities of physics itself. String theory, for instance, proposes a universe with extra dimensions, akin to a labyrinthine structure. By delving into Borgesian labyrinths, physicists might gain new perspectives on navigating the complexities of their research. These can be captured in the Neutrosophic Venn Diagram.

As with Neutrosophic Logic, it has been introduced by one of us (FS) what is termed as Neutrosophic Venn Diagram,¹ where the classical Venn diagram is generalized to a Neutrosophic Diagram, which deals with vague, inexact, ambiguous, ill-defined ideas, statements, notions, entities with unclear borders. In a neutrosophic Venn diagram, the traditional circles used in standard Venn diagrams are replaced with neutrosophic sets, which can include elements with degrees of truth, indeterminacy, and falsity. This allows for a more flexible representation of complex relationships between sets where the boundaries are not clearly defined and whose elements partially belong to the set, partially do not belong, and partially are indeterminate. Now, allow us to reconsider three different sets of fantasy stories, physics, and philosophy including ethics in a Neutrosophic Venn Diagram as follows in Figure 1:

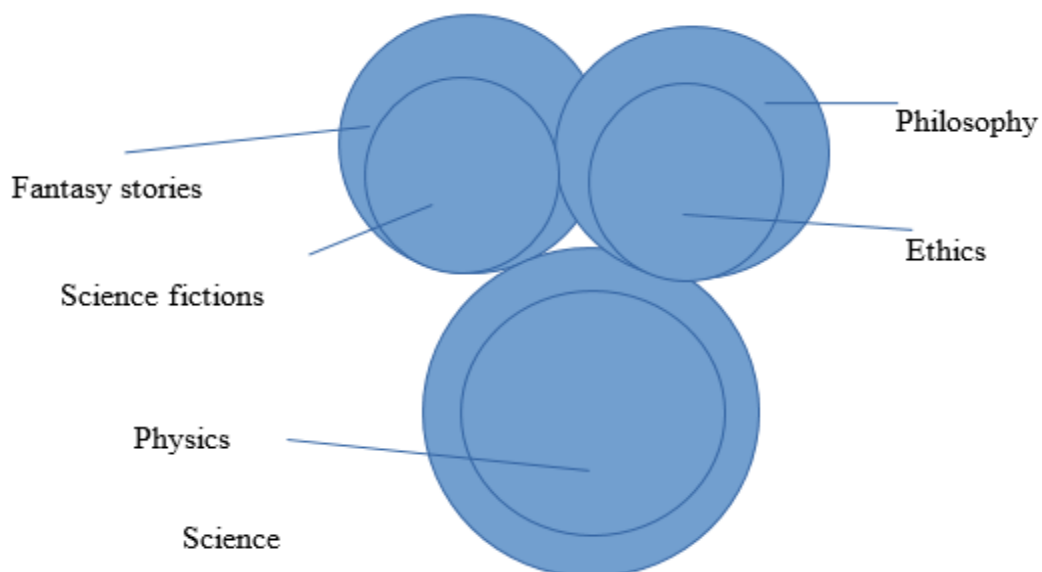


Figure 1. Neutrosophic sets depictions of science fiction, physics, and philosophy/ethics.

¹ F. Smarandache. Neutrosophic Diagram and Classes of Neutrosophic Paradoxes or to the Outer-Limits of Science (2010). url: https://digitalrepository.unm.edu/math_fsp/48/

A Neutrosophic Venn Diagram, on the other hand, is a visualization tool used in neutrosophic sets, and it is a Venn Diagram whose three sets (circles) have unclear/indeterminate borders and their intersections are also unclear. Neutrosophic Set deals with indeterminacy, ambiguity, and inconsistency. Here's how it expands on a regular Venn Diagram:

- **Standard Venn:** Represents sets (without indeterminacy), overlaps show elements belonging to both or all three sets.
- **Neutrosophic Venn:** Represents sets with indeterminacy, and adds shading or markings to depict three components:
 - T1 (Truth): Degree of an element belonging to the set.
 - T2 (Indeterminacy): Degree of element having uncertain belonging.
 - T3 (Falsity): Degree of the element not belonging to the set.

Neutrosophic Venn Diagrams help visualize complex relationships where elements and their intersections can have varying degrees of truth, indeterminacy, and falsity.

1.2 | n-Dimensional Venn Diagram

Smarandache [4] has extended in 2010 the 3-dimensional Venn Diagram, based on three sets S_1 , S_2 , and S_3 , to the n -dimensional Venn Diagram, based on n sets S_1, S_2, \dots, S_n algebraically, since geometrically it is not possible to visualize it, and used an **algebraic codification** of unions and intersections.

Let's first consider $1 \leq n \leq 9$, and the sets S_1, S_2, \dots, S_n .

Then one gets $2^n - 1$ disjoint parts resulting from the intersections of these n sets. Each part is encoded with decimal positive integers specifying only the sets it belongs to. Thus: part 1 means the part that belongs to S_1 (set 1) only, part 2 means the part that belongs to S_2 only, ..., and part n means the part that belongs to set S_n only.

Similarly, part 12 means that part which belongs to S_1 and S_2 only, i.e. to $S_1 \cap S_2$ only.

Also, for example, part 1237 means the part that belongs to the sets S_1, S_2, S_3 , and S_7 only, i.e. to the intersection $S_1 \cap S_2 \cap S_3 \cap S_7$ only. And so on. This will help the construction of a base formed by all these disjoint parts and implementation in a computer program of each set from the power set $P(S_1 \dot{\cup} S_2 \dot{\cup} \dots \dot{\cup} S_n)$ using a binary number.

The sets S_1, S_2, \dots, S_n , are intersected in all possible ways in a Venn diagram. Let $1 \leq k \leq n$ be an integer. Let's denote by $i_1 i_2 \dots i_k$ the Venn diagram region/part that belongs to the sets S_{i_1} and S_{i_2} and ... and S_{i_k} only, for all k and all n . The part which is outside of all sets (i.e. the complement of the union of all sets) is noted by 0 (zero). Each Venn diagram will have 2^n disjoint parts, and each such disjoint part (except the above part 0) will be formed by combinations of k numbers from the numbers: 1, 2, 3, ..., n .

1.3 | Unions and Intersections of Sets in the n-Dimensional Venn Diagram

This codification is user-friendly in algebraically simply doing unions and intersections.

The union of sets S_a, S_b, \dots, S_v is formed by all disjoint parts that have in their index either the number a , or the number b , ..., or the number v .

While the intersection of S_a, S_b, \dots, S_v is formed by all disjoint parts that have in their index all numbers a, b, \dots, v .

For $n = 3$ and the above diagram: $S_1 \cup S_2 = \{1, 12, 13, 23, 123\}$, i.e. all disjoint parts that include in their indexes either the digit 1 or the digits 23; and $S_1 \cap S_2 = \{12, 123\}$, i.e. all disjoint parts that have in their index the digits 12.

1.4 | Remarks on Operations in n-Dimensional Venn Diagram

When $n \geq 10$, one uses one space in between numbers: for example, if we want to represent the disjoint part which is the intersection of S3, S10, and S27 only, we use the notation [3 10 27], with blanks in between the set indexes.

Depending on preferences, one can use other characters different from the blank in between numbers, or one can use the numeration system in base $n + 1$, so each number/index will be represented by a unique character.

1.5 | Neutrosophic n-Dimensional Venn Diagram

This is a n-dimensional Venn Diagram whose sets are neutrosophic (i.e. they have some degrees of truth, indeterminacy, and falsehood) and whose borders are unclear and their intersections are unclear too.

1.6 | What is Zel'dovich pancake: Unveiling the Zel'dovich Pancake in the Early Universe

Have you ever wondered how galaxies came to be? The vast cosmic web, with its intricate filaments and clusters, holds the key to this mystery. One theory points to the intriguing concept of the Zel'dovich pancake, named after the renowned physicist Yakov Zel'dovich (cf. Gott, 2016).

Imagine this: The universe was initially a cold uniform soup of particles. But within this uniformity, there were tiny fluctuations in density. These fluctuations, according to Zel'dovich's approximation (developed in 1970), would act as seeds for future structure formation.

Here's where things get interesting. Zel'dovich proposed that under the influence of gravity, these overdense regions, initially roughly ellipsoidal, would collapse along their shortest axis.

These Zel'dovich pancakes, though theoretical, are predicted to be vast, spanning supergalactic scales (far larger than our Milky Way galaxy). The immense size allows us to neglect the effects of pressure, making the model simpler.

The present-day universe consists of galaxies, galaxy clusters, one-dimensional filaments, and two-dimensional sheets or pancakes, all of which combine to form the cosmic web. The so-called "Zeldovich pancakes" are very difficult to observe, because their overdensity is only slightly greater than the average density of the universe. Falco et al. presented a method to identify Zeldovich pancakes in observational data, and the method was used as a tool for estimating the mass of galaxy clusters [2].

While the Zel'dovich approximation provides valuable insights, it has limitations. It doesn't account for smaller scales where pressure becomes significant. Modern cosmologists use sophisticated computer simulations that incorporate pressure and other complexities to get a more realistic picture. However, the concept of Zel'dovich pancakes remains a cornerstone in our understanding of large-scale structure formation. Astronomers continue to refine models and search for observational evidence to support the existence of these cosmic delicacies.

So, the next time you gaze at the night sky, remember the invisible pancakes lurking in the cosmic web, their potential a testament to the fascinating and dynamic nature of our universe.

2 | The Zel'dovich Pancake: A Building Block of the Cosmic Web

The Zel'dovich pancake model offers a crucial starting point for understanding the large-scale structure of the cosmos, known as the cosmic web. Here's how it sheds light on this intricate network (cf. Gott, 2016):

2.1 | Seed for Structure Formation

- The model proposes that tiny density fluctuations in the early, uniform universe became the seeds for future structures.
- Gravity pulls these overdense regions together, initiating their collapse.

2.2 | Birth of Pancakes

- Zel'dovich's idea is that these collapsing regions wouldn't simply shrink uniformly.
- Due to their initial ellipsoidal shape, gravity acts strongest along the shortest axis, causing them to flatten into vast, sheet-like structures – the Zel'dovich pancakes.

2.3 | These Pancakes are the Building Blocks of the Cosmic Web

- The model predicts that further collapse within these pancakes, along the remaining two axes, will lead to the formation of filaments – one-dimensional threads of matter.
- The intersections of these filaments become denser regions, eventually condensing into galaxy clusters, like giant knots in the web.

2.4 | A Simplified View

- It's important to remember that the Zel'dovich pancake model is a simplified view.
- It neglects pressure, which becomes important at smaller scales.

2.5 | Complementing Modern Simulations

- While not a complete picture, the model provides a valuable framework.
- Modern computer simulations incorporate pressure and other complexities, building upon the pancake concept to create a more realistic picture of the cosmic web.

Zel'dovich pancakes, although not directly observed, offer a theoretical foundation for understanding the large-scale structure formation in the cosmic web. Imagine the cosmic web as a giant spiderweb, with the Zel'dovich pancakes forming the large, flat sheets, the filaments as the connecting threads, and the galaxy clusters as the dense knots where the threads intersect.

By studying the Zel'dovich pancake model, cosmologists gain insights into how the uniform universe we see in the Cosmic Microwave Background radiation.

3 | An outline from Zel'dovich Pancake to Burgers Equations to Represent Cosmic Turbulence

```

a[t_] := ScaleFactor[t]; (* Scale factor as a function of time *)
R[t_] := Perturbation[t]; (* Perturbation as a function of time *)
(* Define Hubble parameter *)
H[t_] = D[Log[a[t]], t];
(* Define peculiar velocity *)
v[t, x_] = D[R[t], x]/a[t];
(* Continuity equation *)
Div[v[t, x], {1}] + D[a[t], t]/a[t] == 0;
(* Euler equation *)
D[v[t, x], t] + H[t]*v[t, x] + v[t, x] D[v[t, x], x] == 0;
(* Solve for v[t, x] - Zel'dovich approximation *)
vSol[t, x_] = Simplify[v[t, x] /. Solve[{Div[v[t, x], {1}] + D[a[t], t]/a[t] == 0}, v[t, x]][[1]];
(* Apply transformation to get density perturbation *)
delta[t, x_] = R[t] - vSol[t, x];
(* Take derivative of density w.r.t. time *)
deltaDot[t, x_] = D[delta[t, x], t];
(* Simplify using continuity equation *)
deltaDot[t, x_] = Simplify[deltaDot[t, x] /. {D[vSol[t, x], t] -> -H[t]*vSol[t, x] - vSol[t, x] D[vSol[t, x], x]};
(* Burgers equation *)
deltaDot[t, x] + H[t]*delta[t, x] = (vSol[t, x])^2/a[t] == -H[t] R[t] + (R[t] D[R[t], x]/a[t])^2/a[t];
(* Print

```

```
result *) Print["The Burgers equation derived from Zel'dovich pancake collapse is:"] Print[deltaDot[t, x] +
H[t]*delta[t, x] == - H[t] R[t] + (R[t] D[R[t], x]/a[t])^2/a[t];
```

This code defines the scale factor, perturbation, Hubble parameter, and peculiar velocity. Then, it solves the continuity and Euler equations to obtain the Zel'dovich approximation for the peculiar velocity. Subsequently, it defines the density perturbation and its time derivative. Using the continuity equation, the time derivative is simplified. Finally, the Burgers equation is obtained by expressing the change in density as a function of the initial perturbation, the Hubble parameter, and the square of the peculiar velocity. **Note:**

- This code assumes a specific form for the scale factor and perturbation.
- Additional assumptions and simplifications are often made in the Zel'dovich pancake model.

4 | What are Burgers equations?

The Burgers' equation is fundamental in applied mathematics, particularly useful for describing phenomena involving **convection and diffusion**. It captures the interplay between two forces:

- **Convection:** Imagine pushing a wave of hot water through a cooler tank. The wavefront (hottest part) moves with the water's flow, while the heat itself diffuses, spreading out from the hot center. This is convection.
- **Diffusion:** Diffusion is the natural tendency of particles to spread out from areas of high concentration to areas of low concentration.

The Burgers' equation allows us to model this interplay mathematically. It's particularly useful for scenarios where the convection is non-linear, meaning the speed of the wavefront depends on its density.

4.1 | Burgers' Equation in Cosmology

So, how does this relate to the early universe? In the very first moments after it is supposed there were fluctuations. These fluctuations, while tiny, are believed to be the seeds for the large-scale structures we see today, like galaxies and clusters. The Burgers' equation becomes a powerful tool for studying the **non-linear evolution** of these density fluctuations. Here's why:

- **Non-linearity Matters:** In the early universe, gravity caused denser regions to attract even more matter, amplifying the fluctuations. This non-linear growth is crucial for understanding structure formation. The Burgers' equation, with its ability to handle non-linearity, becomes highly relevant.
- **Shock Formation:** As the universe expands, denser regions can become so dense that they undergo a rapid collapse, forming a **shock wave**. The Burgers' equation helps model the formation and evolution of these shock waves, which are important for understanding how structures like galaxies condense out of the smooth early universe.

While the Burgers' equation provides valuable insights, it's important to acknowledge its limitations. The model is simplified, neglecting factors like dark matter and radiation pressure that play a role in the real universe. Cosmologists use more sophisticated tools like **hydrodynamic simulations** that incorporate these complexities. However, the Burgers' equation remains a valuable starting point and a powerful tool for understanding the qualitative behavior of density fluctuations in the early universe.

The Burgers' equation, though seemingly abstract, offers a window into the dynamic processes that shaped the cosmos from its infancy. By studying this equation, cosmologists gain a deeper appreciation for the intricate dance of matter and energy that gave rise to the universe we inhabit today.

As the second step, we proceed to provide an outline from Burgers equations to Ermakov dynamics equations. Here's an outline of Mathematica code to derive the Ermakov equations from the Burgers equation for simple turbulence:


```
(* Define the Burgers equation *) burgersEq[u_[t, x_]] := D[u[t, x], t] + u[t, x] D[u[t, x], x] - nu*D[u[t, x], {x, 2}]; (* nu - viscosity *) (* Introduce Cole-Hopf transformation *) v[t, x_] := Log[u[t, x]/(nu + u[t, x]); (* Apply transformation to Burgers equation *) D[v[t, x], t] + D[v[t, x], x]*D[v[t, x], x] = 1; (* Define first Ermakov equation *) ermakow1[v_[t, x_]] := D[v[t, x], t] + D[v[t, x], x]*D[v[t, x], x]; (* Define second Ermakov equation (optional) *) (* Due to the transformation, the second equation becomes an identity. Uncomment the following lines if needed for your specific application. *) {w[t, x_] = D[v[t, x], x], ermakow2[v_[t, x_]] = D[w[t, x], t] + D[v[t, x], x]*D[w[t, x], x]; (* Print the results *) Print["The first Ermakov equation derived from the Burgers equation is:"] Print[ermakow1[v[t, x]] == 1]; (* Print the second Ermakov equation (if uncommented) *) (* Print["The second Ermakov equation is:"] *) (* Print[ermakow2[v[t, x]] == 0]; *)
```

This code first defines the Burgers equation with viscosity ν . Then, it introduces the Cole-Hopf transformation, which transforms the Burgers equation into the first Ermakov equation. The second Ermakov equation essentially becomes an identity due to the transformation.

In the third step, we would like to provide an outline *nonlocal current effect* from Ermakov equations. While Ermakov equations can be useful for turbulence modeling, they are not directly applicable to deriving the nonlocal current effect. Here's why:

- **Ermakov equations originate from the Burgers equation:** This equation describes fluid flow, where currents are local phenomena. Nonlocal currents, however, exhibit effects beyond the immediate vicinity of a point.
- **Nonlocal currents arise in specific contexts:** These effects are often seen in materials with complex band structures or strong correlations, not typically captured by the Burgers equation.

However, we can explore an alternative approach:

1. **Nonlocal conductivity models:** There are several existing models for nonlocal conductivity that we can explore in Mathematica. These might involve integral equations or memory functions to describe the nonlocal character of the current.
2. **Material-specific packages:** Depending on the material you're interested in (e.g., graphene, organic conductors), specific Mathematica packages might be available that incorporate nonlocal effects.

```
(* Define spatial variable *) x_/; x > 0 := x; (* Define local current density *) jLocal[E_, x_] := sigma*E; (* sigma - local conductivity *) (* Define a placeholder function for nonlocal kernel *) kernel[x1_, x2_] := (* Replace this with an appropriate kernel function describing the nonlocal effect *) Exp[-(x1 - x2)^2/lambda^2]; (* Placeholder - modify lambda for decay length *) (* Define integral for nonlocal current contribution *) jNonlocal[E_, x_] := Integrate[kernel[x, x1]*jLocal[E, x1], {x1, -Infinity, Infinity}]; (* Total current density *) jTotal[E_, x_] := jLocal[E, x] + jNonlocal[E_, x]; (* Sample electric field (replace with your actual profile) *) E[x_] := Sin[x]; (* Plot the total current density *) jPlot = Plot[jTotal[E[x], x], {x, 0, 10}, PlotRange -> {-0.5, 1.5}]; (* Show the plot *) jPlot
```

This code defines local and nonlocal current contributions (replace the placeholder kernel function with the appropriate model for your chosen material and effect). It then calculates the total current density and plots it for a sample electric field.

5 | Discussion: Nonlocal Currents from Ermakov's Playground: A Possible Leap in Interstellar Travel?

The vast stretches of interstellar space hold mysteries beyond our wildest dreams. One intriguing avenue of exploration involves harnessing the power of charged particles and their interactions. Here's where the concept of "nonlocal current effect derived from Ermakov equations" enters the scene, sparking both scientific curiosity and a touch of speculation. Ermakov equations describe the motion of a charged particle

in a specific type of electric and magnetic field configuration. These fields are carefully designed to create a stable "playground" where the particle's trajectory is predictable and repeatable. Studying these equations helps us understand the fundamental principles of charged particle motion.

Now, the "nonlocal current effect" within this context is a hypothetical phenomenon. It suggests that under specific conditions within Ermakov's configuration, the movement of charged particles could lead to a **surprising surge in current**. This surge wouldn't follow the usual linear relationship between voltage and current, hence the term "nonlocal." While the existence of this nonlocal, nonlinear effect remains unproven, the very possibility is exciting for understanding interstellar matter. Here's why:

- **Charged Plasma:** Interstellar space is filled with a hot, ionized gas called plasma. This plasma contains charged particles – electrons and ions.
- **Harnessing the Flow:** If the nonlinear current effect is real, it could offer a way to manipulate the flow of charged particles within interstellar plasma. This, in theory, could lead to the development of novel propulsion systems for spacecraft ventures.

6 | Concluding Remark: A Word of Caution

However, it's important to inject a healthy dose of caution. The leap from manipulating charged particle flow to achieving faster-than-light travel (like warp drives from science fiction) is immense. Ermakov's equations and the nonlinear current effect are currently theoretical concepts. Furthermore, achieving warp drive would likely require manipulating gravity, something beyond the scope of charged particle interactions.

The study of Ermakov equations and the hypothetical nonlinear current effect represent a scientific exploration, not a shortcut to interstellar travel. It's a pursuit of knowledge that could deepen our understanding of charged particle behavior and potentially lead to future propulsion advancements within the realm of physics.

For now, the vast expanse of space remains a frontier to be explored with a combination of scientific rigor and a touch of healthy imagination. As we delve deeper into the mysteries of charged particles and their interactions, who knows what exciting discoveries await us on the cosmic dance floor?

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Author Contribution

All authors contributed equally to this work.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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