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Neutrosophic SuperHyper Bi-Topological Spaces: Extra Topics

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Abstract

This manuscript holds the integral concepts presented by the previously published article called (Neutrosophic SuperHyper Bi-Topological Spaces). This essay contains new definitions related to the notion of SuperHyper Bi-Topological Spaces, such as SuperHyper Supra closure operator, the pairwise neutrosophic interior of the power set $P^{n1}(X)$, three cascade numerical examples demonstrate the NSHBI-TS, pairwise neutrosophic n^{th}-power sets in ($\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n(X)$), as well as extra definitions, theorems, corollaries, and propositions.

Keywords: Neutrosophic; SuperHyper; Bi-Topological Spaces.

1 | Introduction

There is no doubt that the neutrosophic theory has been originated by the Romanian-American scientist Florentin Smarandache on 1995 [1], when he suggested a new kind of philosophy that carry the insight of taking any idea or any issue from three corners or three frames, he claimed existing truth part of the issue, the opposite part is the falsity part of the issue, while the middle part of the issue is the indeterminate part which positioned between the truth and the falsity. F. Smarandache be able to redefined all traditional sets and fields, mathematical operations, mathematical analysis, calculus, geometry, optimization theory, operation research, all theoretical parts of probability and statistics, topological spaces, and other fields of knowledge. As of 1995 till now, F. Smarandache collected dozens of scientists around the globe such as but not limited to (A.A. Salama from Egypt, Huda E. Khalid from Iraq, Said Broumi from Morroco, Ishaani Priyadarshinie from USA, Maikel Yelandi Leyva Vázquez from South of America, M. Ganster from Australia, Xiaohong Zhang from China, Fernando A. F. Ferreira from Portugal, Vasantha Kandasamy from India, Giorgio Nordo from Italy, Sergey Gorbachev from Russia, …etc. [2,3,4,5]) , they working with him under two reputed international journals (Neutrosophic Sets and Systems journal briefly NSS journal which issued by the University of New Mexico and International Journal of Neutrosophic Science briefly IJNS journal) those two journals are indexed by dozens of repositories, such as Scopus, Amazon Kindle (USA), General Science Index, ProQuest (USA), Cengage Thompson Gale (USA), Google Books (USA), Cengage Learning (USA), Google Scholar (USA), DOAJ (Sweden), Index Copernicus (Poland), Engineering Village ,Elsevier, Ei_Compendex source list (Netherland),… etc.

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Neutrosophic logic and theory is more general than fuzzy logic and theory that originated by Azerbaijani - American scientist Lotfi A. Zadeh on 1965 [6]. Neutrosophic logic focuses on redefined the parts of knowledge according membership functions. Also, neutrosophic theory is more general than the intuitionistic fuzzy logic and theory that setup by Bulgarian scientist Krassimir T. Atanassov on 1983 [7], he significantly extended fuzzy sets theory by launching the concept of "Intuitionistic Fuzzy Sets" and investigated their basis properties depending upon membership functions and non- membership functions. However, this chapter sheds the light on the basic concepts of the neutrosophic theory especially the neutrosophic topological spaces with most impacting works in this field of science.

The recent and innovative type of mathematical structure presenting by this essay is depending upon the previous work [8], it was fathomed by the neutrosophic superhyper topological spaces which have the ability to define the neutrosophic universal n^{th}-power set $1_{P^n(X)}$, the neutrosophic empty n^{th}-power set $0_{P^n(X)}$, unmatched mathematical relations (i.e. less than or equal relation, greater than or equal relation, the inclusion relation, the union relation, the intersection relation, and the belonging relation,). However, through the previous related works [8-12], new propositions had been defined, and we redefined De Morgan's theorem, setting up unprecedented of neutrosophic n^{th}-power point. Also, we introduced Neutrosophic SuperHyper Supra Topological Spaces, the definition of the neutrosophic n^{th}-power open sets, the definition of the neutrosophic n^{th}-power closed sets, and dozens of theorems and corollaries. While, this article in our hands, includes novel definitions for the neutrosophic closure and the neutrosophic interior of $P^n(X)$. Furthermore, the definitions of the pairwise neutrosophic closure and the definition of the pairwise neutrosophic interior of $P^n(X)$ have been introduced, the flexibility of the neutrosophic superhyper topological spaces depending upon the neutrosophic n^{th}-power sets enables us to build some new and important numerical examples which had two techniques in representing the same mathematical operation and two different results gained (i.e. examples 3.5 and 3.6). Finally, the pairwise neutrosophic n^{th}-power neighborhood had been defined.

2 | Preliminaries

This section will bring all essencial definitions from the first related work [8] that containig the mathematical structure of NSHTSs, and NSHBI-TSs, and as follow:

Definition 2.1: Let X be a non-empty set, $P^n(X)$ is the neutrosophic n^{th}-power set of a set X, for integer $n \geq 1$. A Neutrosophic SuperHyper Topological space on $P^n(X)$ is a subfamily $\tau^{neutrotopo}$ of $N(P^n(X))$, and satisfying the following axioms:

1. The neutrosophic universal n^{th}-power set $1_{P^n(X)}$,and the neutrosophic empty n^{th}-power set $0_{P^n(X)}$ both are belonging to $\tau^{neutrotopo}$.
2. Any arbitrary (finite on infinite) union of members of $\tau^{neutrotopo}$ belong to $\tau^{neutrotopo}$.
3. $\tau^{neutrotopo}$ is closed under finite intersection of members of $\tau^{neutrotopo}$ (i.e. the intersection of any finite number of members of $\tau^{neutrotopo}$ belongs to $\tau^{neutrotopo}$).

Then ($\tau^{neutrotopo}, P^n(X)$) is called Neutrosophic SuperHyper Topological Spaces (NSHTS). Because of the definition of (NSHTS) via neutrosophic n^{th}-power open sets that commonly used in this manuscript, the family of neutrosophic sets $\tau^{neutrotopo}$ of the n^{th}-power sets are commonly called a (NSHTS) on the neutrosophic n^{th}-power sets $P^n(X)$.

A subpowerset $P^{m1}(C) \subseteq P^{m2}(X)$ for integers $m1 \leq m2$ is to be closed in ($\tau^{neutrotopo}, P^n(X)$) if its complement $P^{m2}(X)/P^{m1}(C)$ is an open set.

Definition 2.2: Let $P^n(X)$ be a neutrosophic n^{th}-power set over a non-empty set X, the neutrosophic interior and the neutrosophic closure of $P^n(X)$ are respectively defined as:

$int^n(P^n(X)) = \cup \{P^m(X): P^m(X) \subseteq P^n(X), P^m(X) \in \tau^{neutrotopo}\}$, this means that for the same collection of the neutrosophic n^{th}-power set $P^n(X)$, all $P^m(X)$ given that $m \leq n$ regarded as interior for $P^n(X)$.

$cl^n(P^n(X)) = \cap \{P^h(X): P^n(X) \subseteq P^h(X), (P^h(X))^c \in \tau^{neutrotopo}\}$.

Definition 2.3: The following mathematical phrases are true for any two neutrosophic $n1^{th}$-power set $P^{n1}(Y1)$ and $n2^{th}$-power set $P^{n2}(Y2)$ on the neutrosophic n^{th}-power set $P^n(X)$, given that $n1, n2 \leq n$, and that there is no restrictions on the relation between n_1 and n_2 :

1. $T_{P^{n1}(Y1)}(\{x\}) \leq T_{P^{n2}(Y2)}(\{x\}), I_{P^{n1}(Y1)}(\{x\}) \leq I_{P^{n2}(Y2)}(\{x\}), and\ F_{P^{n1}(Y1)}(\{x\}) \geq F_{P^{n2}(Y2)}(\{x\})$, for integers $n1, n2 \geq 1$, and for all $\{x\} \subseteq P^n(X)$ iff $P^{n1}(Y1) \subseteq P^{n2}(Y2)$.
2. $P^{n1}(Y1) \subseteq P^{n2}(Y2)$ and $P^{n2}(Y2) \subseteq P^{n1}(Y1)$ iff $P^{n1}(Y1) = P^{n2}(Y2)$, given that $n_1 = n_2$.
3. $P^{n1}(Y1) \cap P^{n2}(Y2) =$
$$\{\langle\{x\}, \min\{T_{P^{n1}(Y1)}(\{x\}), T_{P^{n2}(Y2)}(\{x\})\}, \min\{I_{P^{n1}(Y1)}(\{x\}), I_{P^{n2}(Y2)}(\{x\})\}, \max\{F_{P^{n1}(Y1)}(\{x\}), F_{P^{n2}(Y2)}(\{x\})\}\rangle : \{x\} \subseteq P^n(X)\}$$
4. $P^{n1}(Y1) \cup P^{n2}(Y2) =$
$$\{\langle\{x\}, \max\{T_{P^{n1}(Y1)}(\{x\}), T_{P^{n2}(Y2)}(\{x\})\}, \max\{I_{P^{n1}(Y1)}(\{x\}), I_{P^{n2}(Y2)}(\{x\})\}, \min\{F_{P^{n1}(Y1)}(\{x\}), F_{P^{n2}(Y2)}(\{x\})\}\rangle : \{x\} \subseteq P^n(X)\}$$

In general, the union or the intersection of any arbitrary members of neutrosophic n^{th}-power set $P^{ni}(X)_{i \in I}$ are defined by:

$$\bigcap_{i \in I} P^{ni}(X) = \{\langle\{x\}, \inf\{T_{P^{ni}(\{x\})}\}, \inf\{I_{P^{ni}(\{x\})}\}, \sup\{F_{P^{ni}(\{x\})}\}\rangle : \{x\} \subseteq P^n(X)\},$$

$$\bigcup_{i \in I} P^{ni}(X) = \{\langle\{x\}, \sup\{T_{P^{ni}(\{x\})}\}, \sup\{I_{P^{ni}(\{x\})}\}, \inf\{F_{P^{ni}(\{x\})}\}\rangle : \{x\} \subseteq P^n(X)\}.$$

1. The neutrosophic n^{th}-power universal set $P^n(X)$ is denoted by $1_{P^n(X)}$,and it is exist if and only if the following conditions are holding together:

   $T_{P^n(\{x\})} = 1_{P^n(X)}, I_{P^n(\{x\})} = 1_{P^n(X)}, and\ F_{P^n(\{x\})} = 0_{P^n(X)}$.
2. The neutrosophic n^{th}-power empty set $P^n(X)$ is denoted by $0_{P^n(X)}$, and it is exist if and only if the following conditions are holding together:

   $T_{P^n(\{x\})} = 0_{P^n(X)}, I_{P^n(\{x\})} = 0_{P^n(X)}, and\ F_{P^n(\{x\})} = 1_{P^n(X)}$.
3. Let $P^{n1}(Y1) \subseteq P^{n2}(Y2)$, given that $n_1 \leq n_2$, then the complementary of $P^{n1}(Y1)$ concerning to $P^{n2}(Y2)$ is defined as follow:

   $P^{n1}(Y1) \setminus P^{n2}(Y2) = \{\langle|T_{P^{n1}(Y1)}(\{x\}) - T_{P^{n2}(Y2)}(\{x\})|, |I_{P^{n1}(Y1)}(\{x\}) - I_{P^{n2}(Y2)}(\{x\})|, 1_{P^n(X)} - |F_{P^{n1}(Y1)}(\{x\}) - F_{P^{n2}(Y2)}(\{x\})|\rangle\}$.
4. Clearly, the neutrosophic complement of $1_{P^n(X)}$ and $0_{P^n(X)}$ are defined as:

   $(1_{P^n(X)})^c = \langle T_{P^n(\{x\})} = 0_{P^n(X)}, I_{P^n(\{x\})} = 0_{P^n(X)}, F_{P^n(\{x\})} = 1_{P^n(X)}\rangle = 0_{P^n(X)}$, $(0_{P^n(X)})^c = \langle T_{P^n(\{x\})} = 1_{P^n(X)}, I_{P^n(\{x\})} = 1_{P^n(X)}, F_{P^n(\{x\})} = 0_{P^n(X)}\rangle = 1_{P^n(X)}$.

Definition 2.4: Let X be a non-empty set, $P^n(X)$ is the n^{th}-power neutrosophic set of a set X, for integer $n \geq 1$. If α, β, γ be real standard or non-standard subsets of $]^-0, 1^+[$, then the neutrosophic n^{th}-power set $P^n(x_{\alpha,\beta,\gamma})$ is called a neutrosophic n^{th}- power point, and it is defined by:

$$P^n\left(x_{\alpha,\beta,\gamma}(y)\right) = \begin{cases} \langle \alpha_{P^n(x)}, \beta_{P^n(x)}, \gamma_{P^n(x)} \rangle, & if\ P^n(x) = P^n(y) \\ \langle 0_{P^n(x)}, 0_{P^n(x)}, 1_{P^n(x)} \rangle, & if\ P^n(x) \neq P^n(y) \end{cases}$$

For $x, y \in X, and\ P^n(x_{\alpha,\beta,\gamma}), P^n(y) \subseteq P^n(X)$, here $P^n(y)$ is called the support of $P^n(x_{\alpha,\beta,\gamma})$.

Definition 2.5: Let $P^{n1}(X) \in N(P^n(X))$, the belonging operation of the neutrosophic n^{th}- power point $P^n(x_{\alpha,\beta,\gamma})$ to $P^{n1}(X)$ (i.e. $P^n(x_{\alpha,\beta,\gamma}) \in P^{n1}(X)$) is satisfied if and only if $T_{P^{n1}(\{x\})} \geq \alpha, I_{P^{n1}(\{x\})} \geq \beta, F_{P^{n1}(\{x\})} \leq \gamma$.

Definition 2.6: A sub-collection τ_n^* of neutrosophic n^{th}- power set $P^n(X)$ on a non-empty set X is said to be Neutrosophic SuperHyper Supra Topological Space on X if the n^{th}- power sets $0_{P^n(X)}, 1_{P^n(X)} \in \tau_n^*$, and $\bigcup_{i \in I}^{\infty} P^{ni}(X) \in \tau_n^*$ for $\{P^{ni}(X)\}_{i=1}^{\infty} \in \tau_n^*$. Then $(\tau_n^*, P^n(X))$ is called Neutrosophic SuperHyper Supra Topological Space on X.

3 | New Definitions and Extra Notions for NSHBI-TSs

Definition 3.1: A SuperHyper operator $\psi : \mathrm{N}(P^n(\mathrm{X})) \rightarrow \mathrm{N}(P^n(\mathrm{X}))$ is called a neutrosophic SuperHyper Supra closure operator if it satisfies the following conditions for all $p^{n1}(\mathrm{X}), p^{n2}(\mathrm{X}) \subseteq \mathrm{N}(P^n(\mathrm{X}))$

1. $\psi(0_{P^n}(X)) = 0_{P^n}(X)$.
2. $P^{n1}(X) \subseteq \psi(P^{n1}(X)$.
3. $\psi(P^{n1}(X) \cup \psi(P^{n2}(X)) \subseteq \psi(P^{n1}(X) \cup (P^{n2}(X))$.
4. $\psi(\psi(P^{n1}(X)) = \psi(P^{n1}(X))$.

Theorem 3.2: Let $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n(X))$ be (NSHBI-TS). The operator $cl_p^n : \mathrm{N}(P^n(\mathrm{X})) \rightarrow \mathrm{N}(P^n(\mathrm{X}))$ which defined by:

$$cl_p^n(P^{n1}(\mathrm{X})) = cl_{\tau^{1stpair}}^n(P^{n1}(\mathrm{X})) \cap cl_{\tau^{2ndpair}}^n(P^{n1}(\mathrm{X}))$$

Is the Neutrosophic SuperHyper Supra closure operator and it is induced, a unique Neutrosophic SuperHyper Supra topology is given by:

$$\{ P^{n1}(\mathrm{X}) \in \mathrm{N}(P^n(\mathrm{X})) : cl_p^n((P^{n1}(\mathrm{X}))^c) = (P^{n1}(\mathrm{X}))^c \}$$

Proof: Straight forward

Definition 3.3: Let $(\tau_1^{1st\ pair}, \tau_2^{2nd\ pair}, P^n(X))$ be a Neutrosophic SuperHyper Bi-Topological Space and $P^{n1}(\mathrm{X}) \in \mathrm{N}(P^{n1}(\mathrm{X}))$. The pairwise neutrosophic interior of $P^{n1}(X)$, denoted by $int_p^n(P^{n1}(\mathrm{X}))$ is the neutrosophic union of all pairwise neutrosophic n^{th} -power open subsets of $P^{n1}(X), i.e.$,

$$int_p^n(P^{n1}(X) = U\{P^m(X) \in \mathcal{T}^{neutrotopo} : P^m(X) \subseteq P^{n1}(X)\}.$$

Obviously; $int_p^n(P^n(X))$ is the biggest pairwise neutrosophic n^{th} - power open set contained in $P^{n1}(X)$.

Numerical Example 3.4: Suppose $X = \{a, b, c\}$ with the following neutrosophic n^{th} - power sets :-

$$P^{n1}(X) = \left\{ \{\mathrm{a}, \mathrm{T} = 0.3, \mathrm{I} = 0.1, F = 0.6\}, \{b, c\} \begin{matrix} T = \{0.7, 0.4\} \\ \mathrm{I} = \{0, 0.3\} \\ F = \{0.4, 0.3\} \end{matrix} \right\},$$

$$P^{n2}(X) = \left\{ \{\mathrm{a}, \mathrm{T} = 0.5, \mathrm{I} = 0.2, F = 0.7\}, \{\{b\}, \{c\}\} \begin{matrix} T = \{\{0.6\}, \{0.1\}\} \\ \mathrm{I} = \{\{0\}, \{0.5\}\} \\ F = \{\{0.4\}, \{0.4\}\} \end{matrix} \right\},$$

$$P^{n3}(X) = \left\{ \{\mathrm{a}, \mathrm{b}\} \begin{matrix} T = \{0.3, 0.2\} \\ \mathrm{I} = \{0.4, 0.9\} \\ F = \{0.6, 0.9\} \end{matrix}, \{c, 0.5, 0.1, 0.8\} \right\},$$

$$P^{n4}(X) = \left\{ \{\{\mathrm{a}\}, \{\mathrm{b}\}\} \begin{matrix} T = \{\{0.1\}, \{0.7\}\} \\ \mathrm{I} = \{\{0.2\}, \{0.8\}\} \\ F = \{\{0.6\}, \{0.9\}\} \end{matrix}, \{c, 0.2, 0.5, 0.3\} \right\},$$

$$P^{n5}(X) = \{\{\mathrm{a}, 0.1, 0.1, 0.7\}, \{b, 0.7, 0.6, 0.1\}, \{c, 0.2, 0.4, 0.3\}\},$$

$$P^{n6}(X) = \left\{ \{\mathrm{a}, \mathrm{b}, \mathrm{c}\} \begin{matrix} T = \{0.3, 0.7, 0.4\} \\ \mathrm{I} = \{0.1, 0, 0\} \\ F = \{0.6, 0.5, 0.3\} \end{matrix} \right\}.$$

Then $\left(\tau_1^{1stpair}, \tau_2^{2stpair}, P^n(X) \right)$ is a Neutrosophic SuperHyper Bi-Topological Space (NSHBI-TS), Where $\tau_1^{1stpair} = \{0_{P^n}(X), 1_{P^n}(x), P^{n1}(X), P^{n2}(X), P^{n3}(x), P^{n4}(X)\}$

While, $\tau_2^{2ndpair} = \{0_{P^n}(X), 1_{P^n}(X), P^{n5}(X), P^{n6}(\mathrm{X})\}$.

Obviously, $\tau^{neutrotopo} = \tau_1^{1stpair} \cup \tau_2^{2ndpair} \cup \{P^{n1}(X) \cup P^{n5}(X), P^{n2}(X) \cup P^{n5}(X), P^{n3}(X) \cup P^{n5}(X)\}$

Because all the neutrosophic $n^{th} - power$ sets $P^{n1}(X) \cup P^{n5}(X), P^{n2}(X) \cup P^{n5}(X), P^{n3}(X) \cup P^{n5}(X)$ are not belonging to either $\tau_1^{1stpair}$ nor to $\tau_2^{2ndpair}$.

Numerical Example 3.5: Let $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n(X))$ be the same (NSHBI-TS) as in Example 3.4, and let:-

$$G = \left\{ \{\mathrm{a}, \mathrm{b}\} \begin{matrix} T = \{0.6, 0.4\} \\ \mathrm{I} = \{0.5, 0\} \\ F = \{0.7, 0.3\} \end{matrix}, \{c, 0.4, 0.7, 0.3\} \right\}$$

be a neutrosophic n^{th}-power set over $P^n(X)$. Now to find $cl_p^n(G)$, we need to determine pairwise neutrosophic n^{th}-power closed sets in $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n(X))$. We can conclude that:

$$(P^{n1}(X))^c = \left\{ \{\mathrm{a}, \mathrm{T} = 0.7, \mathrm{I} = 0.9, F = 0.4\}, \{b, c\} \begin{matrix} T = \{0.3, 0.6\} \\ \mathrm{I} = \{1, 0.7\} \\ F = \{0.6, 0.7\} \end{matrix} \right\},$$

$$(P^{n2}(X))^c = \left\{ \{\mathrm{a}, \mathrm{T} = 0.5, \mathrm{I} = 0.8, F = 0.3\}, \{\{b\}, \{c\}\} \begin{matrix} T = \{\{0.4\}, \{0.9\}\} \\ \mathrm{I} = \{\{1\}, \{0.5\}\} \\ F = \{\{0.6\}, \{0.6\}\} \end{matrix} \right\},$$

$$(P^{n3}(X))^c = \left\{ \{a, b\} \begin{matrix} T = \{0.7, 0.8\} \\ I = \{0.6, 0.1\} \\ F = \{0.4, 0.1\} \end{matrix}, \{c, 0.5, 0.9, 0.2\} \right\},$$

$$(P^{n4}(X))^c = \left\{ \{\{a\}, \{b\}\} \begin{matrix} T = \{\{0.9\}, \{0.3\}\} \\ I = \{\{0.8\}, \{0.2\}\} \\ F = \{\{0.4\}, \{0.1\}\} \end{matrix}, \{c, 0.8, 0.5, 0.7\} \right\},$$

$$(P^{n5}(X))^c = \{\{a, 0.9, 0.9, 0.3\}, \{b, 0.3, 0.4, 0.9\}, \{c, 0.8, 0.6, 0.7\}\},$$

$$(P^{n6}(X))^c = \left\{ \{a, b, c\} \begin{matrix} T = \{0.7, 0.3, 0.6\} \\ I = \{0.9, 1, 1\} \\ F = \{0.4, 0.5, 0.3\} \end{matrix} \right\}.$$

And we can write each of the following unions $(P^{n1}(X) \cup P^{n5}(X))^c$, $(P^{n2}(X) \cup P^{n5}(X))^c$, $(P^{n3}(X) \cup P^{n5}(X))^c$ in two different ways as follow:

1. The first solution will go to $(P^{n1}(X) \cup P^{n5}(X))^c$ in two different ways:

   Either by taking the pattern of $P^{n1}(X)$:-

$$P^{n1}(X) \cup P^{n5}(X) = \left\{ \{a, 0.3, 0.1, 0.6\}, \{b, c\} \begin{matrix} T = \{0.7, 0.4\} \\ I = \{0.6, 0.4\} \\ F = \{0.1, 0.3\} \end{matrix} \right\},$$

$$(P^{n1}(X) \cup P^{n5}(X))^c = \left\{ \{a, 0.7, 0.9, 0.4\}, \{b, c\} \begin{matrix} T = \{0.3, 0.6\} \\ I = \{0.4, 0.6\} \\ F = \{0.9, 0.7\} \end{matrix} \right\},$$

   Or by taking the pattern of $P^{n5}(X)$

$$P^{n1}(X) \cup P^{n5}(X) = \{\{a, 0.3, 0.1, 0.6\}, \{b, 0.7, 0.6, 0.1\}, \{c, 0.4, 0.4, 0.3\}\}$$

$$(P^{n1}(X) \cup P^{n5}(X))^c = \{\{a, 0.7, 0.9, 0.4\}, \{b, 0.3, 0.4, 0.9\}, \{c, 0.6, 0.6, 0.7\}\}$$

Here and by the ability and the flexibility of neutrosophic n^{th} -power sets, we saw that the term $(P^{n1}(X) \cup P^{n5}(X))^c$ had written in to different ways

2. The following work goes for finding the term $(P^{n2}(X) \cup P^{n5}(X))^c$ in two different ways:

   Either by taking the pattern of $P^{n2}(X)$:-

$$P^{n2}(X) \cup P^{n5}(X) = \left\{ \{a, 0.5, 0.2, 0.7\}, \{\{b\}, \{c\}\} \begin{matrix} T = \{\{0.7\}, \{0.2\}\} \\ I = \{\{0.6\}, \{0.5\}\} \\ F = \{\{0.1\}, \{0.3\}\} \end{matrix} \right\},$$

$$(P^{n2}(X) \cup P^{n5}(X))^c = \left\{ \{a, 0.5, 0.8, 0.3\}, \{\{b\}, \{c\}\} \begin{matrix} T = \{\{0.3\}, \{0.8\}\} \\ I = \{\{0.4\}, \{0.5\}\} \\ F = \{\{0.9\}, \{0.7\}\} \end{matrix} \right\},$$

   Or by taking the pattern of $P^{n5}(X)$:

$$P^{n2}(X) \cup P^{n5}(X) = \{\{a, 0.5, 0.2, 0.7\}, \{b, 0.7, 0.6, 0.1\}, \{c, 0.2, 0.5, 0.3\}\},$$

$$(P^{n2}(X) \cup P^{n5}(X))^c = \{\{a, 0.5, 0.8, 0.3\}, \{b, 0.3, 0.4, 0.9\}, \{c, 0.8, 0.5, 0.7\}\}.$$

3. The last work goes for finding the $(P^{n3}(X) \cup P^{n5}(X))^c$ in two different ways:

   Either by taking the pattern of $P^{n3}(X)$:

$$P^{n3}(X) \cup P^{n5}(X) = \left\{ \{a, b\} \begin{matrix} T = \{0.3, 0.7\} \\ I = \{0.4, 0.9\} \\ F = \{0.6, 0.1\} \end{matrix}, \{c, 0.5, 0.4, 0.3\} \right\},$$

$$(P^{n3}(X) \cup P^{n5}(X))^c = \left\{ \{a, b\} \begin{matrix} T = \{0.7, 0.3\} \\ I = \{0.6, 0.1\} \\ F = \{0.4, 0.9\} \end{matrix}, \{c, 0.5, 0.6, 0.7\} \right\}.$$

   Or by taking the pattern of $P^{n5}(X)$:

$$P^{n3}(X) \cup P^{n5}(X) = \{\{a, 0.3, 0.4, 0.6\}, \{b, 0.7, 0.9, 0.1\}, \{c, 0.5, 0.4, 0.3\}\},$$

$$(P^{n3}(X) \cup P^{n5}(X))^c = \{\{a, 0.7, 0.6, 0.4\}, \{b, 0.3, 0.1, 0.9\}, \{c, 0.5, 0.6, 0.7\}\}.$$

By the definition of the inclusion (i.e. Def. (2.3), the first point of it), we see the pairwise neutrosophic n^{th}-power sets which contain G are:-

$$(P^{n3}(X))^c \text{ and } 1_{p^n(X)} = \left\{ \{a, b\} \begin{matrix} T = \{1, 1\} \\ I = \{1, 1\} \\ F = \{0, 0\} \end{matrix}, \{c, 1, 1, 0\} \right\}.$$

It follows that: -

$$cl_p^n(G) = (P^{n3}(X))^c \cap 1_{p^n(X)} = (P^{n3}(X))^c.$$

Therefore $cl_p^n(G) = \left(P^{n3}(X)\right)^c$

It is worthy to mention that all neutrosophic n^{th}-power closed sets $[(P^{n1}(X))^c$, $(P^{n2}(X))^c$, $(P^{n4}(X))^c$, $(P^{n5}(X))^c, (P^{n3}(X) \cup P^{n5}(X))^c,, (P^{n2}(X) \cup P^{n5}(X))^c, (P^{n1}(X) \cup P^{n5}(X))^c]$ all are do not contain G.

Numerical Example 3.6: Let $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n(X))$ be the same (NSHBI-TS) as in Example 3.4, and let:

$$M = \left\{ \{a, 0.4, 0.5, 0.2\}, \{b, c\} \begin{matrix} T = \{0.8, 0.8\} \\ I = \{0.7, 0.5\} \\ F = \{0, 0.2\} \end{matrix} \right\},$$

Again, from the definition of inclusion (i.e. Def. 2.3, the first point of it), we see the pairwise neutrosophic n^{th}-power sets which containing in M are:

$0_{P^n(X)}, P^{n\,1}(X), and\ P^{n\,1}(X) \cup P^{n\,5}(X)$ of the first pattern tracking the trace of the pattern of $P^{n\,1}(X)$,

i.e. $$P^{n\,1}(X) \cup P^{n\,5}(X) = \left\{ \{a, 0.3, 0.1, 0.6\}, \{b, c\} \begin{matrix} T = \{0.7, 0.4\} \\ I = \{0.6, 0.4\} \\ F = \{0.1, 0.3\} \end{matrix} \right\},$$

It is clear that $P^{n\,1}(X) \subseteq M, \left(P^{n\,1}(X) \cup P^{n\,5}(X)\right) \subseteq M, 0_{P^n(X)} \subseteq M$.

Therefore,

$int_P^n(M) = \left(P^{n\,1}(X) \cup P^{n\,5}(X)\right) \cup P^{n\,1}(X) \cup 0_{P^n(X)} = P^{n\,1}(X) \cup P^{n\,5}(X)$.

Theorem 3.7: Let $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n\,(X))$ be (NSHBI-TS), and $P^{n\,1}(X), P^{n\,2}(X) \in N(P^n(X))$. Then the following mathematical statements are true:

1. $int_P^n\left(0_{P^n(X)}\right) = 0_{P^n(X)}$ and $int_P^n\left(1_{P^n(X)}\right) = 1_{P^n(X)}$,
2. $int_P^n\left(P^{n1}(X)\right) \subseteq P^{n1}(X)$,
3. $P^{n\,1}(X)$ is a pairwise neutrosophic n^{th}-power open set if and only if $int_P^n\left(P^{n1}(X)\right) = P^{n1}(X)$,
4. $P^{n1}(X) \subseteq P^{n2}(X) \Longrightarrow int_P^n\left(P^{n1}(X)\right) \subseteq int_P^n\left(P^{n2}(X)\right)$,
5. $int_P^n\left(P^{n1}(X) \cap P^{n2}(X)\right) \subseteq int_P^n\left(P^{n1}(X)\right) \cap int_P^n\left(P^{n2}(X)\right)$,
6. $int_P^n[int_P^n(P^{n1}(X))] = int_P^n(P^{n1}(X))$.

The poofs of the six facts mentioned in this theorem are straight forwards.

Theorem 3.8: Let $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n\,(X))$ be (NSHBI-TS), and $P^{n\,1}(X) \in N(P^n(X))$. Then $P^n\left(x_{\alpha,\beta,\gamma}\right) \in int_P^n\left(P^{n1}(X)\right) \Leftrightarrow \exists P^n(z_{\alpha,\beta,\gamma}) \in \tau^{neutrotopo}(P^n\left(x_{\alpha,\beta,\gamma}\right))$ such that $P^n(z_{\alpha,\beta,\gamma}) \subseteq P^{n1}(X)$.

The proof is straight forward.

Theorem 3.9: Let $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n\,(X))$ be (NSHBI-TS). A neutrosophic n^{th}-power set $P^{n1}(X)$ over $P^n(X)$ is a pairwise neutrosophic n^{th}-power open set if and only if $P^{n1}(X) = int_{\tau_1^{1stpair}}^n(P^{n1}(X)) \cup int_{\tau_2^{2ndpair}}^n(P^{n1}(X))$

Proof:

Let $P^{n1}(X)$ be a pairwise neutrosophic n^{th}-power open set. Since $int_{\tau_1^{1stpair}}^n(P^{n1}(X)) \subseteq P^{n1}(X)$ and $int_{\tau_2^{2ndpair}}^n(P^{n1}(X)) \subseteq P^{n1}(X)$ (by Th. 3.7, the second point of it), then $int_{\tau_1^{1stpair}}^n(P^{n1}(X)) \cup int_{\tau_2^{2ndpair}}^n(P^{n1}(X)) \subseteq P^{n1}(X)$.

Now let $P^n\left(x_{\alpha,\beta,\gamma}\right) \in P^{n1}(X)$. Then, either there exists $P^n\left(z_{\alpha,\beta,\gamma}^1\right) \in \tau_1^{1stpair}$ such that $P^n\left(z_{\alpha,\beta,\gamma}^1\right) \subseteq P^{n1}(X)$ or there exists $P^n\left(z_{\alpha,\beta,\gamma}^2\right) \in \tau_2^{2ndpair}$ such that $P^n\left(z_{\alpha,\beta,\gamma}^2\right) \subseteq P^{n1}(X)$, thus $P^n\left(x_{\alpha,\beta,\gamma}\right) \in int_{\tau_1^{1stpair}}^n P^{n1}(X)$ or $P^n\left(x_{\alpha,\beta,\gamma}\right) \in int_{\tau_2^{2ndpair}}^n P^{n1}(X)$, Hence, $P^n\left(x_{\alpha,\beta,\gamma}\right) \in int_{\tau_1^{1stpair}}^n(P^{n1}(X)) \cup int_{\tau_2^{2ndpair}}^n(P^{n1}(X)).\Longrightarrow P^{n1}(X) \subseteq int_{\tau_1^{1stpair}}^n(P^{n1}(X)) \cup int_{\tau_2^{2ndpair}}^n(P^{n1}(X))$, so $P^{n1}(X) = int_{\tau_1^{1stpair}}^n(P^{n1}(X)) \cup int_{\tau_2^{2ndpair}}^n(P^{n1}(X))$.

Conversely,

since $int_{\tau_1^{1stpair}}^n(P^{n1}(X))$ is a neutrosophic n^{th}-power open set in $(\tau_1^{1stpair}, P^n\,(X))$ and $int_{\tau_2^{2ndpair}}^n(P^{n1}(X))$ is a neutrosophic n^{th}-power open set in $(\tau_2^{2ndpair}, P^n\,(X))$, then by definition (2.2) we can conclude that $int^n_{\tau_1^{1stpair}}(P^{n1}(X)) \cup int^n_{\tau_2^{2ndpair}}(P^{n1}(X))$ is a pairwise neutrosophic n^{th}-power open set in $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n(X))$. Thus, $P^{n1}(X)$ is a pairwise neutrosophic n^{th}-power open set.

From subsections (3.1 to 3.9), it is clear that the reasons for delay in the construction project can completely belong to some grades of the truth neutrosophic opinion, simultaneously, the same reasons can belong to some or all grades of indeterminacy neutrosophic opinions, these results, are delighting announcement that in neutrosophic theory, the same reason can have a kind of biasing to the truth opinion side by side to the falsity opinion side by side to the indeterminate opinions in fully consistent and with well definitions of the problems.

Corollary 3.10: Let $\left(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n(X)\right)$ be (NSHBI-TS). Then, $int^n_p P^{n1}(X) = int^n_{\tau_1^{1stpair}}\left(P^{n1}(X)\right) \cup int^n_{\tau_2^{2ndpair}}\left(P^{n2}(X)\right)$

Definition 3.11: An operator $\phi: N(P^n(X)) \to N(P^n(X))$ is called a Neutrosophic SuperHyper Supra Operator (NSHSO) if it satisfies the following conditions:

$\forall\, P^{n1}(X), P^{n2}(X) \in N(P^n(X)$.

1. $\phi(0_X) = 0_X$.
2. $\phi(P^{n1}(X)) = P^{n1}(X)$.
3. $\phi(P^{n1}(X)) \cap (P^{n2}(X)) \subseteq \phi(P^{n1}(X)) \cap \phi(P^{n2}(X))$.
4. $\phi\left(\phi\left(P^{n1}(X)\right)\right) = \phi(P^{n1}(X))$.

Theorem 3.12: Let $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n(X))$ be (NSHBI-TS). Then, the operator $int^n_P : N(P^n(X)) \to N(P^n(X))$ which is defined by : $int^n_P\left(P^{n1}(X)\right) = int^n_{\tau_1^{1stpair}}\left(P^{n1}(X)\right) \cup int^n_{\tau_2^{2ndpair}}\left(P^{n1}(X)\right)$, is Neutrosophic SuperHyper Supra Interior Operator (NSHSIO), and it is induced, a unique neutrosophic SuperHyper supra topology given by

$\{P^{n1}(X) \in N\left(P^n(X)\right): int^n_P\left(P^{n1}(X)\right) = P^{n1}(X)\}$, which is precisely $\tau^{neutrotopo}$ Proof: Straight forward.

Theorem 3.13: Let $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n(X))$ be (NSHBI-TS) and $P^{n1}(X) \in N\left(P^n(X)\right)$. Then,

1. $int^n_P\left(P^{n1}(X)\right) = (cl^n_P(P^n(X))^c)^c$.
2. $cl^n_P\left(P^{n1}(X)\right) = (int^n_P(P^{n1}(X))^c)^c$.

Proof: Straight forward.

Definition 3.14: Let $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n(X))$ be (NSHBI-TS) , $P^{n1}(X) \in N\left(P^n(X)\right)$, and $P^n(X_{(\alpha,\beta,\gamma)}) \in N\left(P^n(X)\right)$. Then $P^{n1}(X)$ is said to be a pairwise neutrosophic n^{th}-power neighborhood of $P^n(x_{\alpha,\beta,\gamma})$, if there exists a pairwise neutrosophic n^{th}-power open set $P^{n2}(X)$ such that $P^n\left(x_{\alpha,\beta,\gamma}\right) \in P^{n2}(X) \subseteq P^{n1}(X)$. The family of pairwise neutrosophic n^{th}-power neighborhood of neutrosophic n^{th}-power point $P^n\left(x_{\alpha,\beta,\gamma}\right)$ denoted by $P^{n1}_{\tau^{neutrotopo}}(x_{\alpha,\beta,\gamma})$.

Theorem 3.15: Let $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n(X))$ be (NSHBI-TS), and let $P^{n1}(X) \in N\left(P^n(X)\right)$. Then $P^{n1}(X)$is a pairwise neutrosophic n^{th}-power open set if and only if $P^{n1}(X)$is a pairwise neutrosophic n^{th}-power neighborhood of its neutrosophic n^{th}-power points.

Proof:

Let $P^{n1}(X)$ be a pairwise neutrosophic n^{th}-power open set, and let $P^n(x_{\alpha,\beta,\gamma}) \in P^{n1}(X)$. Then $P^n(x_{\alpha,\beta,\gamma}) \in P^{n1}(X) \subseteq P^{n1}(X)$, therefore $P^{n1}(X)$ is a pairwise neutrosophic n^{th}-power neighborhood of $P^n(x_{\alpha,\beta,\gamma})$ for each $P^n(x_{\alpha,\beta,\gamma}) \in P^{n1}(X)$.

Conversely, suppose that $P^{n1}(X)$ is a pairwise neutrosophic n^{th}-power neighborhood of its neutrosophic n^{th}-power points, and $P^n(x_{\alpha,\beta,\gamma}) \in P^{n1}(X)$. Then there exist a pairwise neutrosophic n^{th}-power open set $P^{n2}(X)$ such that $P^n(x_{\alpha,\beta,\gamma}) \in P^{n2}(X) \subseteq P^{n1}(X)$. Since

$P^{n1}(X) = \bigcup_{P^n(x_{\alpha,\beta,\gamma}) \in P^{n1}(X)} \{P^n(x_{\alpha,\beta,\gamma})\} \subseteq \bigcup_{P^n(x_{\alpha,\beta,\gamma}) \in P^{n1}(X)} P^{n2}(X) \bigcup_{P^n(x_{\alpha,\beta,\gamma}) \in P^{n1}(X)} P^{n1}(X) =$ $P^{n1}(X)$. It follows that $P^{n1}(X)$is a union of all pairwise neutrosophic n^{th}-power open sets. Hence, $P^{n1}(X)$ is a pairwise neutrosophic n^{th}-power open set.

Proposition 3.16: Let $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n(X))$ be (NSHBI-TS), and $\{P^{n1}_{\tau neutrotopo}(x_{\alpha,\beta,\gamma}) : P^n(x_{\alpha,\beta,\gamma}) \in N(P^n(X))\}$ be a system of pairwise neutrosophic n^{th}-power neighborhoods. Then,

1. For every $P^{n1}(X) \in P^{n1}_{\tau neutrotopo}(x_{\alpha,\beta,\gamma}), P^n(x_{\alpha,\beta,\gamma}) \in P^{n1}(X)$,
2. $P^{n1}(X) \in P^{n1}_{\tau neutrotopo}(x_{\alpha,\beta,\gamma})$ and $P^{n1}(X) \subseteq P^{n3}(X) \Longrightarrow P^{n3}(X) \in P^{n1}_{\tau neutrotopo}(x_{\alpha,\beta,\gamma})$,
3. $P^{n1}(X) \in P^{n1}_{\tau neutrotopo}(x_{\alpha,\beta,\gamma}) \Longrightarrow \exists P^{n3}(X) \in P^{n1}_{\tau neutrotopo}(x_{\alpha,\beta,\gamma})$ such that $P^{n3}(X) \subseteq P^{n1}(X)$ and $P^{n3}(X) \in P^{n1}_{\tau neutrotopo}(y_{\alpha',\beta',\gamma'})$, for every $P^n(y_{\alpha',\beta',\gamma'}) \in P^{n3}(X)$.

The proofs of the first two mathematical statements are straight forwards. For the third statement, let $P^{n1}(X)$ be a pairwise neutrosophic n^{th}-power neighborhood of $P^n(x_{\alpha,\beta,\gamma})$, then there exists a pairwise neutrosophic n^{th}-power open set $P^{n3}(X) \in \tau^{neutrotopo}$ such that $P^n(x_{\alpha,\beta,\gamma}) \in P^{n3}(X) \subseteq P^{n1}(X)$. Since $P^n(x_{\alpha,\beta,\gamma}) \in P^{n3}(X) \subseteq P^{n3}(X)$ can be written, then $P^{n3}(X) \in P^{n1}_{\tau neutrotopo}(x_{\alpha,\beta,\gamma})$. From theorem 3.15, if $P^{n3}(X)$ is a pairwise neutrosophic n^{th}-power open set, then $P^{n1}(X)$ is a pairwise neutrosophic n^{th}-power neighborhood of its neutrosophic points, i.e., $P^{n3}(X) \in P^{n1}_{\tau neutrotopo}(y_{\alpha',\beta',\gamma'})$, for every $P^n(y_{\alpha',\beta',\gamma'}) \in P^{n3}(X)$.

Corollary 3.17: Let $P^{n1}(X), P^{n3}(X) \in P^{n1}_{\tau neutrotopo}(x_{\alpha,\beta,\gamma}) \Longrightarrow P^{n1}(X) \cap P^{n3}(X) \notin P^{n1}_{\tau neutrotopo}(x_{\alpha,\beta,\gamma})$.

Explanation for the above corollary:

Actually, if $P^{n1}(X), P^{n3}(X) \in P^{n1}_{\tau neutrotopo}(x_{\alpha,\beta,\gamma})$, there exist $P^{n4}(X), P^{n5}(X) \in P^{n1}_{\tau neutrotopo}(x_{\alpha,\beta,\gamma})$ such that

$P^n(x_{\alpha,\beta,\gamma}) \in P^{n4}(X) \subseteq P^{n1}(X)$ and $P^n(x_{\alpha,\beta,\gamma}) \in P^{n5}(X) \subseteq P^{n3}(X)$.

But $P^{n4}(X) \cap P^{n5}(X)$ does not need to be a pairwise neutrosophic n^{th}-power open set. Therefore, $P^{n1}(X) \cap P^{n3}(X)$ does not need to be a pairwise neutrosophic n^{th}-power neighborhood of $P^n(x_{\alpha,\beta,\gamma})$.

Theorem 3.18: Let $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^n(X))$ be (NSHBI-TS). Then $P^{n1}_{\tau neutrotopo}(x_{\alpha,\beta,\gamma}) = P^{n1}_{\tau^{1stpair}}(x_{\alpha,\beta,\gamma}) \cup P^{n1}_{\tau^{2ndpair}}(x_{\alpha,\beta,\gamma})$ for each $P^n(x_{\alpha,\beta,\gamma}) \in N(P^n(X))$.

Proof:

Let $P^n(x_{\alpha,\beta,\gamma}) \in N(P^n(X))$ be any neutrosophic n^{th}-power point and $P^{n1}(X) \in P^{n1}_{\tau neutrotopo}(x_{\alpha,\beta,\gamma})$. Then there exists a pairwise neutrosophic n^{th}-power open set , $P^{n3}(X) \in \tau^{neutrotopo}$ such that $P^n(x_{\alpha,\beta,\gamma}) \in P^{n3}(X) \subseteq P^{n1}(X)$. If $P^{n3}(X) \in \tau^{neutrotopo}$ there exist $P^{n31}(X) \in \tau^{1stpair}$ and $P^{n32}(X) \in \tau^{2ndpair}$ such that , $P^{n3}(X) = P^{n31}(X) \cup P^{n32}(X)$. Since $P^n(x_{\alpha,\beta,\gamma}) \in P^{n3}(X) = P^{n31}(X) \cup P^{n32}(X)$, then $P^n(x_{\alpha,\beta,\gamma}) \in P^{n31}(X)$ or $P^n(x_{\alpha,\beta,\gamma}) \in P^{n32}(X)$. So, $P^n(x_{\alpha,\beta,\gamma}) \in P^{n31}(X) \subseteq P^{n3}(X) \subseteq P^{n1}(X)$ or $P^n(x_{\alpha,\beta,\gamma}) \in P^{n32}(X) \subseteq P^{n3}(X) \subseteq P^{n1}(X)$.

In this case, $P^{n1}(X) \in P^{n1}_{\tau^{1stpair}}(x_{\alpha,\beta,\gamma})$ or $P^{n1}(X) \in P^{n1}_{\tau^{2ndpair}}(x_{\alpha,\beta,\gamma})$,

That is, $P^{n1}(X) \in P^{n1}_{\tau^{1stpair}}(x_{\alpha,\beta,\gamma}) \cup P^{n1}_{\tau^{2ndpair}}(x_{\alpha,\beta,\gamma})$.

Conversely,

Suppose that $P^{n1}(X) \in P^{n1}_{\tau^{1stpair}}(x_{\alpha,\beta,\gamma}) \cup P^{n1}_{\tau^{2ndpair}}(x_{\alpha,\beta,\gamma})$. Then $P^{n1}(X) \in P^{n1}_{\tau^{1stpair}}(x_{\alpha,\beta,\gamma})$ or $P^{n1}(X) \in P^{n1}_{\tau^{2ndpair}}(x_{\alpha,\beta,\gamma})$. Hence, there exists $P^{n}(x_{\alpha,\beta,\gamma}) \in P^{n31}(X) \in \tau^{1stpair}$ or $P^{n}(x_{\alpha,\beta,\gamma}) \in P^{n32}(X) \in \tau^{2ndpair}$ such that $P^{n}(x_{\alpha,\beta,\gamma}) \in P^{n31}(X) \subseteq P^{n1}(X)$ and $P^{n}(x_{\alpha,\beta,\gamma}) \in P^{n32}(X) \subseteq P^{n1}(X)$. As a result, $P^{n}(x_{\alpha,\beta,\gamma}) \in P^{n31}(X) \cup P^{n32}(X) = P^{n3}(X) \subseteq P^{n1}(X)$. Such that $P^{n3}(X) \in \tau^{neutrotopo}$, i.e., $P^{n1}(X) \in P^{n1}_{\tau^{neutrotopo}}(x_{\alpha,\beta,\gamma})$.

Definition 3.19: An operator $\#: N(P^{n}(X)) \to N(P^{n}(X))$ is called a neutrosophic Supra n^{th}-power neighborhood operator if it satisfies the following conditions :

$\forall\, P^{n1}(X), P^{n3}(X) \in N(P^{n}(X))$

1. $\forall P^{n1}(X) \in \#(P^{n}(x_{\alpha,\beta,\gamma})), P^{n}(x_{\alpha,\beta,\gamma}) \in P^{n1}(X)$,
2. $P^{n1}(X) \in \#(P^{n}(x_{\alpha,\beta,\gamma}))$ and $P^{n1}(X) \subseteq P^{n3}(X) \Longrightarrow P^{n3}(X) \in \#P^{n}(x_{\alpha,\beta,\gamma})$
3. $P^{n1}(X) \in \#(P^{n}(x_{\alpha,\beta,\gamma})) \Longrightarrow \exists\, P^{n3}(X) \in \#(P^{n}(x_{\alpha,\beta,\gamma}))$, such that $P^{n1}(X) \subseteq P^{n3}(X)$ and $P^{n1}(X) \in \#\left(P^{n}(y_{\alpha',\beta',\gamma'})\right), P^{n}(y_{\alpha',\beta',\gamma'}) \in P^{n3}(X)$.

Theorem 3.20: Let $(\tau_1^{1stpair}, \tau_2^{2ndpair}, P^{n}(X))$ be neutrosophic SuperHyper BI-Topological Space . Then, the operator $P^{n1}_{\tau^{neutrotopo}}: N(P^{n}(X)) \to N(P^{n}(X))$ which is defined by $P^{n1}_{\tau^{neutrotopo}}(P^{n}(x_{\alpha,\beta,\gamma})) = P^{n1}_{\tau^{1stpair}}(P^{n}(x_{\alpha,\beta,\gamma})) \cup P^{n1}_{\tau^{2ndpair}}\left(P^{n}(x_{\alpha,\beta,\gamma})\right)$, is neutrosophic Supra n^{th}-power neighborhood operator and it is induced, a unique neutrosophic Supra topology given by $\{P^{n1}(X) \in N(P^{n}(X)): \forall P^{n}(x_{\alpha,\beta,\gamma}) \in P^{n1}(X)\ for\ P^{n1}(X) \in P^{n1}_{\tau^{neutrotopo}}(x_{\alpha,\beta,\gamma})\}$ which is precisely $\tau^{neutrotopo}$.

4 | Conclusion and Subsequent Works

A swift look to the sections of this article and its previous related work [8] demonstrates that new types and modern topological spaces (i.e. Neutrosophic SuperHyper Topological Spaces $\tau^{neutrotopo}$, and Neutrosophic SuperHyper Bi-Topological Spaces $(\tau_1^{1stpair}, \tau_2^{2ndpair})$) have been defined depending upon n^{th}-power set $P^{n}(X)$, indeed these new types of sets took its importance from its existence in the neutrosophic environment. The n^{th}-power sets $P^{n}(X)$ that defined by the neutrosophic notion adheres to define each element in the set by three membership functions (truth, indeterminate, and falsity) membership functions gave us new insights to define new kinds of topological spaces depending on modern structure of these sets.

The modern type of mathematical frame represented by the neutrosophic superhyper topological spaces had the ability to define the neutrosophic universal n^{th}-power set $1_{P^{n}(X)}$, the neutrosophic empty n^{th}-power set $0_{P^{n}(X)}$, unmatched mathematical relations (i.e. less than or equal relation, greater than or equal relation, the inclusion relation, the union relation, the intersection relation, and the belonging relation,). However, new propositions have been defined, and we redefined the De Morgan's theorem, setting up unprecedented of neutrosophic n^{th}-power point. Also, we introduced Neutrosophic SuperHyper Supra Topological Spaces, the definition of the neutrosophic n^{th}-power open sets, the definition of the neutrosophic n^{th}-power closed sets and dozens of theorems and corollaries. This study include a novel definitions for the neutrosophic closure and the neutrosophic interior of $P^{n}(X)$. Furthermore, the definitions of the pairwise neutrosophic closure and the definition of the pairwise neutrosophic interior of $P^{n}(X)$ have been introduced, the flexibility of the neutrosophic superhyper topological spaces depending upon the neutrosophic n^{th}-power sets enables us to build some new and important numerical examples which had two techniques in representing the same mathematical operation and two different results gained (i.e. examples 3.4, 3.5, 3.6). Finally, the pairwise neutrosophic n^{th}-power neighborhood had been defined.

There are unlimited and broad notions and mathematical aspects to work in the neutrosophic SuperHyper Topological Spaces such as but not limited to:

1. Irreducible sets in view of the Neutrosophic n^{th}-power sets $P^n(X)$.
2. Convergent sequences in Neutrosophic SuperHyper Topological Spaces.
3. Continuity and compactness notions in the Neutrosophic SuperHyper Topological Spaces.
4. Hausdorff distance in the Neutrosophic SuperHyper Topological Spaces.
5. Kuratowski convergence in the Neutrosophic SuperHyper Topological Spaces.
6. Wijsman convergence in the Neutrosophic SuperHyper Topological Spaces.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

References

[1] Smarandache, F. 1995. Neutrosophic logic and set.
[2] Smarandache, F. (Ed.). 2016. The Encyclopedia of Neutrosophic Researchers, pons Editions, ISBN 978-1-59973-468-2.
[3] Smarandache, F. (Ed.). 2018. The Encyclopedia of Neutrosophic Researchers, pons Editions, ISBN 978-1-59973-614-3.
[4] Smarandache, F. 2019. The Encyclopedia of Neutrosophic Researchers, pons Editions, ISBN 978-1-59973-608-2 .
[5] Smarandache, F., & Leyva-Vazquez, M. 2021. The Encyclopedia of Neutrosophic Researchers, Editions, ISBN 978-1-59973-698-3
[6] Zadeh, L. A.1965. Fuzzy sets. Inform. and Control. 8: 338-353.
[7] Atanassov K. 1983. Intuitionistic Fuzzy Sets. VII ITKR Session. Sofia (Deposed in Centr. Sci.-Techn. Library of Bulg. Acad. of Sci., 1697/84) (in Bulgarian).

[8] Huda E. Khalid, Gonca D. G., Muslim A. Noah Zainal " Neutrosophic SuperHyper Bi-Topological Spaces: Original Notions and New Insights" Neutrosophic Sets and Systems, Vol. 51, 2022, pp. 33-45. DOI: 10.5281/zenodo.7135241

[9] F. Smarandache. 2016. SuperHyperAlgebra and Neutrosophic SuperHyperAlgebra, Section into the authors book Nidus Idearum. Scilogs, II: de rerum consectatione, second edition, Bruxelles: Pons, 107.

[10] F. Smarandache. 2022. The SuperFunction and the Neutrosophic SuperHyperFunction. , Neutrosophic Set and Systems, Vol 49.pp. 594-600.

[11] F. Smarandache. 2022. Introduction to SuperHyperAlgebra and Neutrosophic SuperHyperAlgebra , Journal of Algebraic HyperStructures and Algebras, vol 3, No. 2 .pp. 17-24.

[12] F. Smarandache. 2022. Introduction to n-SuperHyperGraph- the most general form of graph today, Neutrosophic Set and Systems, vol 48.pp. 483-485.