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# An Efficient Neutrosophic Method for Generating Random Variates from Erlang Distribution

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## Abstract

The simulation process depends on generating a series of random numbers subject to a regular probability distribution in the field [0, 1]. The generation of these numbers is based on the cumulative distribution function of the regular distribution, but we encounter many systems that do not, by nature, follow the regular distribution adopted in the simulation process. Therefore, it is necessary to Convert these random numbers into random variables that follow the probability distribution in which the system to be simulated operates. In classical logic, many techniques can be used in the conversion process, which results in random variables that follow irregular probability distributions. However, the results we obtain are specific results that do not take into account the changes that may occur in the system's operating environment. To obtain more accurate results, we presented in previous research a study to generate neutrosophic random numbers that follow a regular distribution with no specificity that can be enjoyed by both ends of the field [0, 1]. One or both of them together, and for systems that operate according to probability distributions other than the regular distribution defined in the field [0, 1], we have presented some techniques through which we can obtain neutrosophic random variables based on the neutrosophic random numbers that were generated, in this research and using Previous information: We present a neutrosophic study to generate random variables that follow the Erlang distribution, which is one of the most important and widely used distributions in scientific fields.

**Keywords:** Neutrosophic Uniform Distribution; Simulation; Cumulative Distribution Function; Neutrosophic Random Numbers; Neutrosophic Random Variables; Inverse Transformation; Neutrosophic Exponential Distribution; Erlang Distribution.

## 1 | Introduction

Given the great difficulty that we may face when studying the operation of any real system, as well as the high cost of studying it, in addition to the fact that some systems cannot be studied directly, here comes the importance of the simulation process in all branches of science, as it depends on applying the study to... Systems similar to real systems and then projecting these results, if they are appropriate, onto the real system. The main interest in statistical analysis is to generate a series of random variables that follow the probability distribution in which the system under study operates. In almost all simulation tests, we need to generate random variables. Follow a distribution, a distribution that adequately describes and represents the physical

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process involved in the experiment at that point. During the experiment, it may be necessary to generate a random variable from a distribution many times depending on the complexity of the model to be simulated. We can generate random events that simulate any real system by examining probability distributions that apply to Events and properties of this system, there are several techniques for generating random variables from a specific distribution and keeping up with the recent studies that were presented using neutrosophic logic, which included most branches of science [1-11], and provided a new formulation of probability distributions, we found it necessary to reformulate some of the techniques used to generate Random variables follow probability distributions using the concepts of this logic. We presented various research in this field, such as the uniform distribution over the field [a, b], the exponential distribution, the inverse transformation technique - the beta distribution, the gamma distribution, the rejection and acceptance technique - the Poisson distribution, and the mixed technique - the distribution. Natural and Box-Muller technique [12-19]. In continuation of what we did previously, we present in this research a neutrosophic study. To generate neutrosophic random variables that follow the Erlang distribution, based on the relations linking it to the gamma distribution and the exponential distribution. It is one of the distributions for which a neutrosophical study of the process of generating random variables has been presented.

## 2 | Conversation

The Erlang distribution is one of the important distributions in practical applications, and it is a form of the gamma distribution when the value of K in it is equal to a positive integer. It has been proven by statisticians that this distribution is nothing more than the sum of K Asian variables, each of which has a mathematical expectation equal to 1/K , and from there, to generate random variables that follow the Erlang distribution, we only need to collect K exponential random variables, each of which has a mathematical expectation equal to 1/K. We know that the process of generating random variables that follow any probability distribution is preceded by the process of generating random numbers that follow a uniform distribution in the field [0, 1].

### 2.1 | To Generate Classical Random Numbers that Follow a Uniform Distribution in the Interval [0, 1] [20, 21]:

Several methods can be used to obtain a series of classical random numbers $R_1, R_2, \ldots$ that follow a uniform distribution in the range $[0,1]$. In this research, we will use the mean square method defined according to the following equation:

$$R_{i+1} = Mid\left[R_i^2\right]; i = 0,1,2,3. \quad (1)$$

Where $Mid$ is to the middle four ranks of $R_i^2$, and $R_0$ is chosen, i.e., a fractional random number consisting of four ranks (called a seed) that does not contain zero in any of its four ranks.

Since we want to present a neutrosophic study, these classical random numbers must be converted into neutrosophic random numbers as follows:

### 2.2 | To Convert Classical Random Numbers that Follow a Uniform Distribution in the Domain [0, 1] into Neutrosophic Random Numbers [12]

To convert the numbers resulting from (1) into neutrosophic random numbers that follow a uniform distribution in the field $[0, 1]$, we distinguish the following cases for the field $[0, 1]$ with a margin of indeterminacy $\delta$ where $\delta \in [0, m]$ and $0 < m < 1$.

The first case: indeterminacy at the minimum of the field, i.e., $[0 + \delta, 1]$. In this case, we substitute the following equation:

$$NR_i = \frac{R_i - \delta}{1 - \delta} \quad (2)$$

$R_i$ are the random numbers resulting from the Eq. (1).

The second case: Indeterminacy at the upper limit of the field, i.e., $[0,1+\delta]$. In this case, we substitute the following equation:

$$NR_i = \frac{R_i}{1+\delta} \tag{3}$$

*The third case*: Indeterminacy in the upper and lower limits of the field, i.e., $[0+\delta, 1+\delta]$. In this case, we substitute the following equation:

$$NR_i = R_i - \delta \tag{4}$$

We know that to obtain neutrosophic random variables that follow a probability distribution based on a series of classical or neutrosophic random numbers, we distinguish three cases:

The first case: Neutrosophic random numbers and the probability distribution are given in the classical form.

In the second case: the random numbers are classical and the probability distribution is given in the neutrosophic form.

The third case: Neutrosophic random numbers and the probability distribution are given in the Neutrosophic form. Therefore, to generate neutrosophic random variables that follow the exponential distribution, we have the following subsection.

## 2.3 | To Generate Neutrosophic Random Variables that Follow an Exponential Distribution [14]:

To obtain neutrosophic random variables that follow an exponential distribution, we have the following cases:

The First case:

Generate random variables that follow an exponential distribution defined by the following probability density function:

$$f(x) = \lambda e^{-\lambda x} \quad ; \quad x > 0$$

Using a series of neutrosophic random numbers that we obtain from one of the Eqs. (2) or (3) or (4).

Using the inverse transformation method as we found previously, we substitute the following equation:

$$y_{Ni} = -\frac{\ln NR_i}{\lambda} \qquad i = 0,1,2 \tag{5}$$

Accordingly:

By substituting Eq. (2) with Eq. (5), we get the equation:

$$y_{Ni} = -\frac{\ln NR_i}{\lambda} = -\frac{1}{\lambda}\ln\left[\frac{R_i - \delta}{1-\delta}\right] \qquad i = 0,1,2 \tag{6}$$

By substituting Eq. (3) with Eq. (5) we get the equation:

$$y_{Ni} = -\frac{\ln NR_i}{\lambda} = -\frac{1}{\lambda}\ln\left[\frac{R_i}{1+\delta}\right] \qquad i = 0,1,2 \tag{7}$$

By substituting Eq. (4) with Eq. (5) we get the equation:

$$y_{Ni} = -\frac{\ln NR_i}{\lambda} = -\frac{1}{\lambda}\ln[R_i - \delta] \qquad i = 0,1,2 \tag{8}$$

The second case: The series of random numbers is classical, i.e., $R_0, R_1, R_2, \ldots$and follows a uniform distribution over the field $[0,1]$. The exponential distribution is given in the neutrosophic form, i.e., defined by the equation:

$$f(x) = \lambda_N e^{-\lambda_N x} \quad ; \quad x > 0$$

Using the inverse transformation method as we found previously, we substitute the equation :

$$y_{Ni} = -\frac{\ln R_i}{\lambda_N} \quad i = 0,1,2 \tag{9}$$

The Third case

We have the series of neutrosophic random numbers, i.e., $NR_0, NR_1, NR_2, \ldots$and we obtain it from one of the Eqs (2) or (3) or (4), and the exponential distribution is defined in the neutrosophic form, i.e., by the equation:

$$f(x) = \lambda_N e^{-\lambda_N x} \quad ; \quad x > 0$$

Using the inverse transformation method as we found previously, we substitute the equation:

$$y_{Ni} = -\frac{\ln NR_i}{\lambda_N} \quad i = 0,1,2 \tag{10}$$

By replacing Eq. (2) with Eq. (10), we get the following equation:

$$y_{Ni} = -\frac{\ln NR_i}{\lambda_N} = -\frac{1}{\lambda_N} \ln\left[\frac{R_i - \delta}{1-\delta}\right] \quad i = 0,1,2 \tag{11}$$

By replacing Eq. (3) with Eq. (10), we get the following equation:

$$y_{Ni} = -\frac{\ln NR_i}{\lambda_N} = -\frac{1}{\lambda_N} \ln\left[\frac{R_i}{1+\delta}\right] \quad i = 0,1,2 \tag{12}$$

By replacing Eq. (4) with Eq. (10), we get the following equation:

$$y_{Ni} = -\frac{\ln NR_i}{\lambda_N} = -\frac{1}{\lambda_N} \ln[R_i - \delta] \quad i = 0,1,2 \tag{13}$$

## 2.4 | The Classical Study of Generating Random Variables Following the Erlang Distribution [20, 21]:

We found that the Erlang distribution is a special case of the gamma distribution, which is defined by two parameters, $k$ and $\mu$. The Erlang distribution is defined by a probability density function given by the following equation:

$$f(x; k, \mu) = \frac{x^{k-1} e^{-\frac{x}{\mu}}}{\mu^k (k-1)!} \quad ; x, \mu \geq 0$$

Where $k$ is a positive integer, which is a special case of the gamma distribution when the parameter $\lambda$ is integer and positive.

It has been proven that this distribution results from the sum of $k = \lambda$, a random variable subject to the exponential distribution with a uniform mathematical expectation equal to $\frac{1}{K}$. So, to generate random variables that follow the Erlang distribution, we generate $K$, a random variable subject to the exponential distribution, defined by the following probability density function:

$$f(x) = ke^{-kx} \quad ; \quad x > 0$$

Then we take the sum of its logarithm $x$ and thus we get the following equation:

$$x = \sum_{i=1}^{k} y_i \tag{14}$$

## 3 | Generating Neutrosophic Random Variables from the Erlang Distribution

From the above, we can present the following neutrosophic study for generating neutrosophic random variables that follow the Erlang distribution:

Depending on the state of indeterminacy that the problem under study requires, we take the neutrosophic random variables $y_{Ni}$ that follows the exponential distribution that we obtain using one of the Eqs. (6), (7), (8), (9), (11), (12), (13). By replacing it with the Eq. (14), we obtain the variables required for the simulation in the system under study. This is done using one of the following conversion expressions:

By replacing Eq. (6) with Eq. (14), we get the following equation:

$$= x_N \sum_{i=1}^{k} y_{Ni} = -\frac{1}{k}\sum_{i=1}^{k} ln\left[\frac{R_i-\delta}{1-\delta}\right] \tag{15}$$

By replacing Eq. (7) with Eq. (14), we get the following equation:

$$x_N = \sum_{i=1}^{k} y_{Ni} = -\frac{1}{k}\sum_{i=1}^{k} ln\left[\frac{R_i}{1+\delta}\right] \tag{16}$$

By replacing Eq. (8) with Eq. (14), we get the following equation:

$$x_N = \sum_{i=1}^{k} y_{Ni} = -\frac{1}{k}\sum_{i=1}^{k} ln[R_i-\delta] \tag{17}$$

By replacing Eq. (9) with Eq. (14), we get the following equation:

$$x_N = \sum_{i=1}^{k_N} y_{Ni} = -\frac{1}{k_N}\sum_{i=1}^{k_N} lnR_i \tag{18}$$

By replacing Eq. (11) with Eq. (14), we get the following equation:

$$x_N = \sum_{i=1}^{k_N} y_{Ni} = -\frac{1}{k_N}\sum_{i=1}^{k_N} ln\left[\frac{R_i-\delta}{1-\delta}\right] \tag{19}$$

By replacing Eq. (12) with Eq. (14), we get the following equation:

$$x_N = \sum_{i=1}^{k_N} y_{Ni} = -\frac{1}{k_N}\sum_{i=1}^{k_N} ln\left[\frac{R_i}{1+\delta}\right] \tag{20}$$

By replacing Eq. (13) with Eq. (14), we get the following equation:

$$x_N = \sum_{i=1}^{k_N} y_{Ni} = -\frac{1}{k_N}\sum_{i=1}^{k_N} ln[R_i-\delta] \tag{21}$$

Where $k_N = k \pm \varepsilon$ and $\varepsilon$ is the indeterminacy in the $k$ parameter, and it can be any number provided that $k_N$ remains a positive integer.

## 4 | Practical Example

Suppose we have a system that operates according to the Erlang distribution, whose classical probability density function is given by the following formula:

$$f(x;2,3) = \frac{xe^{-\frac{x}{3}}}{9} \quad ; x, \mu \geq 0$$

Since $k = 2$ is a positive integer, then this distribution arises from the sum of $k = 2$, a random variable subject to an exponential distribution with a uniform mathematical expectation equal to $\frac{1}{2}$, and its probability density function if $k$ is a neutrosophic value, i.e., $k_N = k \pm \varepsilon$ where it is indeterminacy. We take it as $\varepsilon \in \{0,1,2\}$, then the probability density function becomes as follows:

$$f(x; k \in \{2,3,4\}, 3) = \frac{x^{\{1,2,3\}} e^{-\frac{x}{3}}}{3^{\{2,3,4\}}(\{2,3,4\} - 1)!} \quad ; x, \mu \geq 0$$

Therefore, to generate random variables that follow the Erlang distribution, we follow the following steps:

We generate random variables that follow a uniform distribution over the field $[0,1]$ using the mean square method, and taking the seed $R_0 = 0.1276$, we obtain the following two classical random numbers:

$$R_1 = 0.6281, R_2 = 0.4509, R_3 = 0.3310, R_4 = 0.9561$$

For the field $[0 + \delta, 1 + \delta]$ we take the indeterminacy $\delta \in [0,0.03]$ and for $k_N = k \pm \varepsilon$ we take $\varepsilon \in \{0,1,2\}$ and then substitute in Eqs. (15), (16), and (21).

- By substituting in Eq. (15) we get:

$$x_N = \sum_{i=1}^{k} y_{Ni} = -\frac{1}{k}\sum_{i=1}^{k} ln\left[\frac{R_{i-}\delta}{1-\delta}\right]$$

$$x_N = -\frac{1}{k}\sum_{i=1}^{k} ln\left[\frac{R_{i-}\delta}{1-\delta}\right] = -\frac{1}{2}\sum_{i=1}^{2} ln\left(\frac{R_i - [0,0.03]}{1 - [0,0.03]}\right)$$

$$x_N = -\frac{1}{2}\left[ln\left(\frac{0.6281 - [0,0.03]}{1 - [0,0.03]}\right) + ln\left(\frac{0.4509 - [0,0.03]}{1 - [0,0.03]}\right)\right]$$

$$x_N = -\frac{1}{2}\left[ln\left(\frac{[0.5981,0.6281]}{[0.97,1]}\right) + ln\left(\frac{[0.4209,0.4509]}{[0.97,1]}\right)\right]$$

$$x_N = -\frac{1}{2}\left[ln([0.6166,0.6281]) + ln([0.4339,0.4509])\right]$$

$$x_N \in [0.2325,0.2418] + [0.3983,0.4175] = [0.6308,0.6593]$$

$$x_N \in [0.6308,0.6593]$$

- By substituting in Eq. (16) we get:

$$x_N = \sum_{i=1}^{k} y_{Ni} = -\frac{1}{k}\sum_{i=1}^{k} ln\left[\frac{R_i}{1+\delta}\right]$$

$$x_N = -\frac{1}{k}\sum_{i=1}^{k} ln\left[\frac{R_i}{1+\delta}\right] = -\frac{1}{2}\sum_{i=1}^{2} ln\left(\frac{R_i}{1 + [0,0.03]}\right)$$

$$x_N = -\frac{1}{2}\left[ln\left(\frac{0.6281}{1 + [0,0.03]}\right) + ln\left(\frac{0.4509}{1 + [0,0.03]}\right)\right]$$

$$x_N = -\frac{1}{2}\left[ln\left(\frac{0.6281}{[1,1.03]}\right) + ln\left(\frac{0.4509}{[1,1.03]}\right)\right]$$

$$x_N \in [0.6308,0.6603]$$

- By substituting in Eq. (17) we get:

$$x_N = \sum_{i=1}^{k} y_{Ni} = -\frac{1}{k}\sum_{i=1}^{k} ln[R_i - \delta]$$

$$x_N = -\frac{1}{k}\sum_{i=1}^{k} ln[R_i - \delta] = -\frac{1}{2}\sum_{i=1}^{2} ln(R_i - [0,0.03])$$

$$x_N = -\frac{1}{2}\ [ln(0.6281\ - [0,0.03]) + ln(0.4509\ - [0,0.03])]$$

$$x_N = -\frac{1}{2}\ [ln(\,[0.5981,0.6281]) + ln([0.4209,0.4509])]$$

$$x_N \in \ \big[[0.2325,0.2370] + [0.3983,0.4327]\big] = [0.6308,0.6697]$$

$$x_N \in \ [0.6308,0.6697]$$

- By substituting into (18), for all values of $k_N$.

$$x_N = \sum_{i=1}^{k_N} y_{Ni} = -\frac{1}{k_N}\sum_{i=1}^{k_N} lnR_i$$

For $k_N = 2 + 0$ we find:

$$x_N = -\frac{1}{k_N}\sum_{i=1}^{k_N} lnR_i = -\frac{1}{2}\sum_{i=1}^{2} ln(R_i)$$

$$x_N = -\frac{1}{2}\ [ln(0.6281\,) + ln(0.4509)]$$

$$x_N = 0.2325 + 0.3983 = 0.6308$$

$$x_N = 0.6308$$

For $k_N = 2 + 1$ we find:

$$x_N = -\frac{1}{k_N}\sum_{i=1}^{k_N} lnR_i = -\frac{1}{3}\sum_{i=1}^{3} ln(R_i)$$

$$x_N = -\frac{1}{3}\ [ln(0.6281\,) + ln(0.4509) + \ln(0.3310)]$$

$$x_N = \ [0.1550 + 0.2655 + 0.3685] = 0.789$$

$$x_N = 0.789$$

For $k_N = 2 + 2$ we find:

$$x_N = -\frac{1}{k_N}\sum_{i=1}^{k_N} lnR_i = -\frac{1}{4}\sum_{i=1}^{4} ln(R_i)$$

$$x_N = -\frac{1}{4}\ [ln(0.6281\,) + ln(0.4509) + \ln(0.3310) + \ln(0.9561)]$$

$$x_N = [0.1163 + 0.1991 + 0.2764 + 0.0112] = 0.603$$

$$x_N = 0.603$$

We calculate the value of $x_N$ in the three cases and put the result in the figure

$$x_N \in \{0.6308,0.789,0.603\}$$

- By substituting into (19), for all values of $k_N$.

$$x_N = \sum_{i=1}^{k_N} y_{Ni} = -\frac{1}{k_N}\sum_{i=1}^{k_N} ln\left[\frac{R_{i-}\delta}{1-\delta}\right]$$

For $k_N = 2 + 0$ we find:

$$x_N = -\frac{1}{k_N}\sum_{i=1}^{k_N} ln\left[\frac{R_{i-}\delta}{1-\delta}\right] = -\frac{1}{2}\sum_{i=1}^{2} ln\left(\frac{R_i - [0,0.03]}{1-[0,0.03]}\right)$$

$$x_N = -\frac{1}{2}\left[ln\left(\frac{0.6281 - [0,0.03]}{1-[0,0.03]}\right) + ln\left(\frac{0.4509 - [0,0.03]}{1-[0,0.03]}\right)\right]$$

$$x_N = -\frac{1}{2}\left[ln\left(\frac{[0.5981,0.6281]}{[0.97,1]}\right) + ln\left(\frac{[0.4209,0.4509]}{[0.97,1]}\right)\right]$$

$$x_N = -\frac{1}{2}\ [ln([0.6166,0.6281]) + ln([0.4339,0.4509])]$$

$$x_N \in [0.2325,0.2418] + [0.3983,0.4175] = [0.6308,0.6593]$$

$$x_N \in [0.6308,0.6593]$$

For $k_N = 2 + 1$ we find:

$$x_N = -\frac{1}{k_N}\sum_{i=1}^{k_N} ln\left[\frac{R_{i-}\delta}{1-\delta}\right] = -\frac{1}{3}\sum_{i=1}^{3} ln\left(\frac{R_i - [0,0.03]}{1-[0,0.03]}\right)$$

$$x_N = -\frac{1}{3}\left[ln\left(\frac{0.6281 - [0,0.03]}{1-[0,0.03]}\right) + ln\left(\frac{0.4509 - [0,0.03]}{1-[0,0.03]}\right) + ln\left(\frac{0.3310 - [0,0.03]}{1-[0,0.03]}\right)\right]$$

$$x_N = -\frac{1}{3}\left[ln\left(\frac{[0.5981,0.6281]}{[0.97,1]}\right) + ln\left(\frac{[0.4209,0.4509]}{[0.97,1]}\right) + \left(\frac{[0.301,0.3310]}{[0.97,1]}\right)\right]$$

$$x_N = -\frac{1}{3}\ [ln([0.6166,0.6281]) + ln([0.4339,0.4509]) + ln([0.3103,0.3310])]$$

$$x_N \in [0.1550,0.1612] + [0.2655,0.2783] + [0.3685,0.3901] = [0.8296,0.789]$$

$$x_N \in [0.789,0.8296]$$

For $k_N = 2 + 2$ we find:

$$x_N = -\frac{1}{k_N}\sum_{i=1}^{k_N} ln\left[\frac{R_{i\_}\delta}{1-\delta}\right] = -\frac{1}{4}\ \sum_{i=1}^{4} ln\left(\frac{R_i - [0,0.03]}{1-[0,0.03]}\right)$$

$$x_N = -\frac{1}{4}\ \left[ln\left(\frac{0.6281\ - [0,0.03]}{1-[0,0.03]}\right) + ln\left(\frac{0.4509\ - [0,0.03]}{1-[0,0.03]}\right) + ln\left(\frac{0.9561 - [0,0.03]}{1-[0,0.03]}\right)\right]$$

$$x_N = -\frac{1}{4}\ \left[ln\left(\frac{[0.5981,0.6281]}{[0.97,1]}\right) + ln\left(\frac{[0.4209,0.4509]}{[0.97,1]}\right) + ln\left(\frac{[0.301,0.3310]}{[0.97,1]}\right) + ln\left(\frac{[0.9261, 0.9561]}{[0.97,1]}\right)\right]$$

$$x_N = -\frac{1}{4}\ [ln([0.6166,0.6281]) + ln([0.4339,0.4509]) + ln([0.3103,0.3310]) + ln([0.9547,0.9561])]$$

$$x_N \in [0.1163,0.1209] + [0.1991,0.2087] + [0.2764,0.2926] + [0.0112,0.0116] = [0.6338,0.603]$$

$$x_N \in [0.603,0.6338]$$

We calculate the value of $x_N$ in the three cases and put the result in the figure

$$x_N \in \{[0.6308,0.6593], [0.789,0.8296], [0.603,0.6338]\}$$

- By substituting into (20), for all values of $k_N$.

$$x_N = \sum_{i=1}^{k_N} y_{Ni} = -\frac{1}{k_N}\sum_{i=1}^{k_N}\ ln\left[\frac{R_i}{1+\delta}\right]$$

For $k_N = 2 + 0$ we find:

$$x_N = -\frac{1}{k_N}\sum_{i=1}^{k_N} ln\left[\frac{R_i}{1+\delta}\right] = -\frac{1}{2}\ \sum_{i=1}^{2} ln\left(\frac{R_i}{1+[0,0.03]}\right)$$

$$x_N = -\frac{1}{2}\ \left[ln\left(\frac{0.6281}{1+[0,0.03]}\right) + ln\left(\frac{0.4509}{1+[0,0.03]}\right)\right]$$

$$x_N = -\frac{1}{2}\ \left[ln\left(\frac{0.6281}{[1,1.03]}\right) + ln\left(\frac{0.4509}{[1,1.03]}\right)\right]$$

$$x_N = -\frac{1}{2}\ [\ln([0.6098,0.6281]) + ln([0.4378,0.4509])]$$

$$x_N \in ([0.2325,0.2473] + [0.3983,0.4130])$$

$$x_N \in [0.6308,0.6603]$$

For $k_N = 2 + 1$ we find:

$$x_N = -\frac{1}{k_N}\sum_{i=1}^{k_N} ln\left[\frac{R_i}{1+\delta}\right] = -\frac{1}{3}\ \sum_{i=1}^{3} ln\left(\frac{R_i}{1+[0,0.03]}\right)$$

$$x_N = -\frac{1}{3}\ \left[ln\left(\frac{0.6281}{1+[0,0.03]}\right) + ln\left(\frac{0.4509}{1+[0,0.03]}\right) + ln\left(\frac{0.3310}{1+[0,0.03]}\right)\right]$$

$$x_N = -\frac{1}{3}\ \left[ln\left(\frac{0.6281}{[1,1.03]}\right) + ln\left(\frac{0.4509}{[1,1.03]}\right) + ln\left(\frac{0.3310}{[1,1.03]}\right)\right]$$

$$x_N = -\frac{1}{3}\ [\ln([0.6098,0.6281]) + ln([0.4378,0.4509]) + ln([0.3214,0.3310])]$$

$$x_N \in ([0.1550,0.1649] + [0.2655,0.2753] + [0.3685,0.3784])$$

$$x_N \in [0.789,0.8186]$$

For $k_N = 2 + 2$ we find:

$$x_N = -\frac{1}{k_N}\sum_{i=1}^{k_N} ln\left[\frac{R_i}{1+\delta}\right] = -\frac{1}{4}\ \sum_{i=1}^{4} ln\left(\frac{R_i}{1+[0,0.03]}\right)$$

$$x_N = -\frac{1}{4}\ \left[ln\left(\frac{0.6281}{1+[0,0.03]}\right) + ln\left(\frac{0.4509}{1+[0,0.03]}\right) + ln\left(\frac{0.3310}{1+[0,0.03]}\right) + ln\left(\frac{0.9561}{1+[0,0.03]}\right)\right]$$

$$x_N = -\frac{1}{4}\ \left[ln\left(\frac{0.6281}{[1,1.03]}\right) + ln\left(\frac{0.4509}{[1,1.03]}\right) + ln\left(\frac{0.3310}{[1,1.03]}\right) + ln\left(\frac{0.9561}{[1,1.03]}\right)\right]$$

$$x_N = -\frac{1}{4}\ [\ln([0.6098,0.6281]) + ln([0.4378,0.4509]) + ln([0.3214,0.3310]) + ln([0.9283,0.9561])]$$

$$x_N \in ([0.1163,0.1237] + [0.1991,0.2065] + [0.2764,0.2838] + [0.0112,0.0186])$$

$$x_N \in [0.603,0.6326]$$

We calculate the value of $x_N$ in the three cases and put the result in the figure:

$$x_N \in \{[0.6308,0.6603], [0.789,0.8186], [0.603,0.6326]\}$$

- By substituting into (21), for all values of $k_N$.

$$x_N = \sum_{i=1}^{k_N} y_{Ni}\ = -\frac{1}{k_N}\sum_{i=1}^{k_N}\ ln[R_i - \delta]$$

For $k_N = 2 + 0$ we find:

$$x_N = -\frac{1}{k_N}\sum_{i=1}^{k_N} ln[R_i - \delta] = -\frac{1}{2}\ \sum_{i=1}^{2} ln(R_i - [0,0.03])$$
$$x_N = -\frac{1}{2}\ [ln(0.6281\ - [0,0.03]) + ln(0.4509\ - [0,0.03])]$$
$$x_N = -\frac{1}{2}\ [ln(\ [0.5981,0.6281]) + ln([0.4209,0.4509])]$$
$$x_N \in\ \big[[0.2325,0.2570] + [0.3983,0.4327]\big]$$
$$x_N \in\ [0.6308,0.6897]$$

For $k_N = 2 + 1$ we find:

$$x_N = -\frac{1}{k_N}\sum_{i=1}^{k_N} ln[R_i - \delta] = -\frac{1}{3}\sum_{i=1}^{3} ln(R_i - [0,0.03])$$

$$x_N = -\frac{1}{3}\ [ln(0.6281\ - [0,0.03]) + ln(0.4509\ - [0,0.03]) + ln(\ 0.3310\ \ - [0,0.03])]$$

$$x_N = -\frac{1}{3}\ [ln(\ [0.5981,0.6281]) + ln([0.4209,0.4509]) + ln([0.301, 0.3310\ ])]$$

$$x_N \in \ \big[[0.1550,0.1713] + [0.2655,0.2885] + [0.3685,0.4002]\big]$$

$$x_N \in\ [0.789,0.86]$$

For $k_N = 2 + 2$ we find:

$$x_N = -\frac{1}{k_N}\sum_{i=1}^{k_N} ln[R_i - \delta] = -\frac{1}{4}\sum_{i=1}^{4} ln(R_i - [0,0.03])$$

$$x_N = -\frac{1}{4}\ [ln(0.6281\ - [0,0.03]) + ln(0.4509\ - [0,0.03]) + ln(\ 0.3310\ \ - [0,0.03]) + ln(0.9561\ - [0,0.03])]$$

$$x_N = -\frac{1}{4}\ [ln(\ [0.5981,0.6281]) + ln([0.4209,0.4509]) + ln([0.301, 0.3310\ ]) + ln([0.9261, 0.9561\ ])]$$

$$x_N \in\ \big[[0.1163,0.1285] + [0.1991,0.2163] + [0.2764,0.3002] + [0.0112,0.0192]\big] = [0.603,0.6642]$$

$$x_N \in\ [0.603,0.6642]$$

We calculate the value of $x_N$ in the three cases and put the result in the figure

$$x_N \in \{[0.6308,0.6897], [0.789,0.86], [0.603,0.6642]\}$$

## 5 | Conclusions

Simulation has become a modern tool that helps us study many systems that could not be studied or predict the results that we can obtain through the operation of these systems over time. The simulation process depends on generating a series of random numbers subject to a regular probability distribution in the field [0,1], and then converting these random numbers into random variables that follow the probability distribution in which the system to be simulated operates. The studies that were presented according to classical logic give the results specific values that suit specific circumstances, and any change that occurs in the work environment makes them inappropriate results and may cause unexpected losses to avoid such losses, we presented in this research a neutrosophic study to generate random variables that follow the Erlang distribution using mathematical relationships that were deduced from the relation of the Erlang distribution to the gamma distribution and the exponential distribution. Accordingly, we benefited from the neutrosophic studies that we presented in previous research for generating neutrosophic random numbers and the neutrosophic study for generating Random variables that follow the exponential distribution, and we obtained neutrosophic mathematical relations that can be used to obtain random variables that follow the Erlang distribution. Through the indeterminacy of the neutrosophic values, we will obtain simulation results suitable for all conditions that the working environment of the system under study can pass through. Thus, simulating the systems Using the concepts of neutrosophic logic provides us with more accurate results than the results provided by the classical study.

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## Author Contribution

All authors contributed equally to this work.

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## Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

## Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

## Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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