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# Fuzzy Inference Quintuple Implication Method Based on Single Valued Neutrosophic t-representable t-norm

**Minxia Luo [1,*], Ziyang Sun [1], and Lixian Wu [1]**

[1] Department of Data Science, China Jiliang University, Hangzhou 310018, PR China.
Emails: mxluo@cjlu.edu.cn; s21080701014@cjlu.edu.cn; s1708070109@cjlu.edu.cn.

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## Abstract

In this paper, we study the single-valued neutrosophic fuzzy inference quintuple implication method. Firstly, single valued neutrosophic fuzzy inference quintuple implication principles for fuzzy modus ponens and fuzzy modus tollens are given. Then, single-valued neutrosophic R-type quintuple implication solutions for fuzzy modus ponens (FMP) and fuzzy modus tollens (FMT) are given. Finally, the robustness of the quintuple implication method based on a left-continuous single valued neutrosophic t-representable t-norm is investigated. As a special case of the main results, the sensitivity of quintuple implication solutions based on special single-valued neutrosophic residual implications is given.

**Keywords:** Single Valued Neutrosophic Set, Single Valued Neutrosophic t-representable t-norm, Quintuple Implication Method.

## 1 | Introduction

Fuzzy sets theory has been widely used in various fields. Specially, fuzzy reasoning plays an important role in fuzzy sets theory. In fuzzy reasoning, the most basic reasoning models are Fuzzy Modus Ponens (FMP) and Fuzzy Modus Tollens (FMT), which can be respectively shown as follows: [1, 2].

FMP$(A, B, A^*)$: for given a fuzzy rule $A \to B$ and premise $A^*$, attempt to deduce a reasonable conclusion $B^*$.

FMT$(A, B, B^*)$: for given a fuzzy rule $A \to B$ and premise $B^*$, attempt to deduce a reasonable conclusion $A^*$.

In the above reasoning models, $A, A^* \in F(X)$ and $B, B^* \in F(Y)$, where $F(X)$ and $F(Y)$ respectively denote fuzzy subsets of the universes $X$ and $Y$.

As the pioneer of fuzzy inference method, Zadeh [1] first proposed the Compositional Rule of Inference (CRI for short). However, there are some shortcomings for the CRI method, for example: the CRI method lacks logic sense and is not reducible. Based on these situations, Wang [2] proposed the fuzzy reasoning triple implication method ( triple I method for short). And the fuzzy reasoning triple I method was established a strict logical basis [3]. Many research results on fuzzy reasoning triple I method have been obtained. Pei [4] constructed unified $\alpha$-triple I method and gave some special results based on four important residual

Corresponding Author: mxluo@cjlu.edu.cn

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implication. Luo and Yao [5] studied the triple I method based on Schweizer-Sklar residual implications. Although the triple I method makes up for some of the shortcomings of the CRI method, it ignores the comparison of $A^*$ and $A$ ( or $B$ and $B^*$) in the inference process. The consideration of proximity will make the calculation results appear unreasonable due to trivial solutions under certain data. In order to better promote the development of reasoning algorithms, Zhou [6] gave the quintuple implication algorithm for fuzzy reasoning, which considers the closeness of $A^*$ and $A$ (or $B$ and $B^*$) in the inference process. Based on the monoidal t-norm based logic, Luo and Zhou [7] expressed the predicate form of the quintuple implication algorithm solution, bringing the quintuple implication algorithm into a strict logical framework.

Although fuzzy sets have been successfully applied in many fields, there are some defects in describing fuzzy and uncertain information. Intuitionistic fuzzy set was introduced by Atanassov [8]. Moreover, Atanassov and Gargov [9] showed that intuitionistic fuzzy sets and interval-valued fuzzy sets are equipotent. Zheng et al. [10] studied fuzzy reasoning triple I method based on intuitionistic fuzzy sets. Li et al. [11] extended CRI method on interval-valued fuzzy set. Luo et al. [12, 13, 14, 15, 16] researched fuzzy reasoning triple I methods based on interval-value associated t-norm and t-representable t-norm, respectively. Li et al. [17] proposed the five implication principles based on interval-value S-implications. Luo and Zhou [18] studied the interval-value quintuple implication method based on interval-value associated t-norm.

Although intuitionistic fuzzy set has some advantages in dealing with fuzzy and incomplete information, it has defects in dealing with fuzzy, incomplete and inconsistent information. In order to deal with these issues, Smarandache [19] proposed neutrosophic set, which is represented by a truth-membership function, an indeterminacy-membership function and a falsity-membership function. However, truth-membership, indeterminacy-membership and falsity-membership function are nonstandard fuzzy subsets, which is difficult to apply in practice. Wang et al [20] proposed single valued neutrosophic set (SVNS for short), its the truth-membership, indeterminacy-membership and falsity-membership degree are real number in unit interval [0, 1]. Single valued neutrosophic set can be considered as a generalization intuitionistic fuzzy set. In recent years, Scholars have paid attention to the study for single valued neutrosophic set. Smarandache [19] studied a unifying field in logics. Smarandache [21] proposed n-norm and n-conorm in neutrosophic logic. Zhang et al. [22] gave new inclusion relation for neutrosophic sets. Hu and Zhang [23] constructed the residuated lattices based on the neutrosophic t-norms and neutrosophic residual implications. Zhao et al. [24] study reverse triple I algorithms based on single valued neutrosophic set. Luo et al. [25] studied fuzzy reasoning triple I method based on single valued neutrosophic t-representable t-norm. However, there are some defects for fuzzy reasoning triple I method based on single valued neutrosophic t-representable t-norm, which can not to solve the following problem.

Example 1. Let $X$ and $Y$ be no-empty sets. Suppose small, medium and large are three single valued neutrosophic sets on $SVNS(X)$, which can be denoted as follows:

[small]$=< 1,0,0 >, < 0.4,0.5,0.3 >, < 0,1,1 >, < 0,1,1 >, < 0,1,1 >$,

[medium]$=< 0,1,1 >, < 0.4,0.5,0.3 >, < 1,0,0 >, < 0.4,0.5,0.3 >, < 0,1,1 >$,

[large]$=< 0,1,1 >, < 0,1,1 >, < 0,1,1 >, < 0.4,0.5,0.3 >, < 1,0,0 >$.

**Table 1.** Model of FMP.

| **Rule** | **If $x$ is small, then $y$ is large** |
|---|---|
| **Premise** | $x$ is medium |
| **Calculate** | $y$ is ? |

The problem of FMP is described as in Table 1. Let $A(x)$, $B(y)$ and $A^*(x)$ denote $x$ is small, $y$ is large and $x$ is medium, respectively. The goal is to calculate $B^*$.

Using the triple I method for FMP [25], we have $B^* =< 1,0,0 >, < 1,0,0 >, < 1,0,0 >, < 1,0,0 >, < 1,0,0 >$ for each of the three implications: $Gödel$ implication, $L$ukasiewicz implication and Gougen implication. In other words, we can get triple I solution for FMP is trivial.

**Table 2.** Model of FMT.

| **Rule** | **If $x$ is small, then $y$ is large** |
|---|---|
| **Premise** | $y$ is medium |
| **Calculate** | $x$ is ? |

The problem of FMT is described as in Table 2. Let $(x)$ , $B(y)$ , $B^*(y)$ denote $x$ is small, $y$ is large and $y$ is medium, respectively. The goal is to calculate $A^*$. Using the triple I method for FMT [25], we have $A^* = \{< 0,1,1 >, < 0,1,1 >, < 0,1,1 >, < 0,1,1 >, < 0,1,1 >\}$ for each of the three implications: $Gödel$ implication, $L$ukasiewicz implication and Gougen implication. In other words, we can get triple I solution for FMT is trivial.

Through analyzing Example 1, we have the following results: let $A \to B$ is a fuzzy rule, if there exists an element $x_0 \in X$ such that $A(x_0) =< 0,1,1 >$ and $A^*(x_0) =< 1,0,0 >$, then the triple I solution for FMP is trivial, i.e., $B^*(y) =< 1,0,0 >$ for every $y \in Y$. If there exists an element $y_0 \in Y$ such that $B(y_0) =< 1,0,0 >$ and $B^*(y_0) =< 0,1,1 >$, then the triple I solution for FMT is trivial, i.e., $A^*(x) =< 0,1,1 >$ for every $x \in X$.

In order to solve the above problem, we study fuzzy reasoning quintuple implication method based on left-continuous single valued neutrosophic t-representable t-norms. The rest of this paper is organized as follows. In Section 2, some basic concepts for single valued neutrosophic sets are reviewed. In Section 3, we give quintuple implication principles for fuzzy inference based on left-continuous single valued neutrosophic t-representable t-norms for fuzzy modus ponens and fuzzy modus tollens, and the corresponding solutions of single valued neutrosophic $\mathcal{R}$-type quintuple implication methods. In Section 4, the robustness of quintuple implication method based on left-continuous single valued neutrosophic t-representable t-norm is investigated. Finally, the conclusions are given in Section 5.

## 2 | Preliminaries

In this section, we review some basic concepts for single valued neutrosophic sets, which will be used in this article.

**Definition 2.1.** [26] A mapping T: $[0,1]^2 \to [0,1]$ is called a triangular norm (t-norm), if it satisfied associativity, commutativity, monotonicity and boundary condition $T(x,1)$=$x$ for any $x \in [0,1]$.

A mapping S: $[0,1]^2 \to [0,1]$ is called a triangular conorm (t-conorm), if it satisfied associativity, commutativity, monotonicity and boundary condition $S(x,0)$=$x$ for any $x \in [0,1]$. A t-norm is called the dual t-norm of the t-conorm, if $T(x,y) = 1 - S(1-x, 1-y)$. Similarly, a t-conorm is called the dual t-conorm of the t-norm, if $S(x,y) = 1 - T(1-x, 1-y)$.

**Definition 2.2.** [26]A t-norm $T$ is called left-continuous (resp., right-continuous), if for any $(x_0, y_0) \in [0,1]^2$, and for each $\varepsilon > 0$ there is a $\delta > 0$ such that

$T(x,y) > T(x_0,y_0) - \varepsilon$, whenever $(x,y) \in (x_0 - \delta, x_0] \times (y_0 - \delta, y_0]$ (resp., $T(x,y) < T(x_0,y_0) + \varepsilon$, whenever $(x,y) \in [x_0, x_0 + \delta] \times [y_0, y_0 + \delta]$) .

**Proposition 2.1.** [26]A t-norm $T$ is a left-continuous t-norm if and only if there exists a binary operation $R_T$ such that $(T, R_T)$ satisfies the residual principle, i.e., $T(x,z) \le y$ iff $z \le R_T(x,y)$ for all $x, y, z \in [0,1]$, where

$$R_T(x,y) = \bigvee\{z \mid T(x,z) \leq y\}$$

is called a residual implication ($R$-implication for short) induced by t-norm $T$.

**Proposition 2.2.** [27]A t-conorm $S$ is a right-continuous t-conorm if and only if there exists a binary operation $R_S$ such that $(S, R_S)$ forms a co-adjoint pair, i.e., $x \leq S(y,z)$ iff $R_S(x,y) \leq z$ for all $x,y,z \in [0,1]$, where

$$R_S(x,y) = \bigwedge\{z \mid x \leq S(y,z)\}$$

is called a coresidual implication ($R_S$-implication for short) induced by t-conorm $S$.

**Example 2.** Three important t-norms and their residuum, t-conorms and their coresiduum [26, 27].

**Table 3.** t-norms and its residuum, t-conorms and its coresiduum.

| Name | t-norms | Residual implications | t-conorms | Coresidual implications |
|---|---|---|---|---|
| **Łukasiewicz** | $T_L(x,y)$ $=0 \vee (x+y-1)$ | $R_{T_L}(x,y)$ $= 1 \wedge (1-x+y)$ | $S_L(x,y)$ $= (x+y) \wedge 1$ | $R_{S_L}(x,y)$ $= (x-y) \vee 0$ |
| **Gougen** | $T_{Go}(x,b) = xy$ | $R_{T_{Go}}(x,y) = 1 \wedge \frac{y}{x}$ | $S_{Go}(x,y)= x+y-xy$ | $R_{S_{Go}}(x,y) = \frac{x-y}{1-y} \vee 0$ |
| **Gödel** | $T_G(x,y) = x \wedge y$ | $R_{T_G}(x,y) = \begin{cases}1, & if \quad x \leq y, \\ y, & if \quad x > y.\end{cases}$ | $S_G(x,y) = x \vee y$ | $R_{S_{Go}}(x,y) = \begin{cases}0, & if \quad x \leq y, \\ x, & if \quad x > y.\end{cases}$ |

**Definition 2.3.** [20] Let $X$ is a universal set. A single valued neutrosophic set $A$ on $X$ is characterized by three functions, i.e., truth-membership function $t_A(x)$, indeterminacy-membership function $i_A(x)$, and falsity-membership function $f_A(x)$. A single valued neutrosophic set $A$ can be defined as follows:

$$A = \{< x, t_A(x), i_A(x), f_A(x) > \mid x \in X\},$$

where $t_A(x), i_A(x), f_A(x) \in [0,1]$ and satisfy the condition $0 \leq t_A(x) + i_A(x) + f_A(x) \leq 3$ for each $x$ in $X$.

The family of all single valued neutrosophic sets on $X$ is denoted by $SVNS(X)$.

**Definition 2.4.** [24] Let $A$, $B$ be two single valued neutrosophic sets on universal $X$, the following relations are defined as follows:

1) $A \subseteq B$ if and only $t_A(x) \leq t_B(x)$, $i_A(x) \geq i_B(x)$ and $f_A(x) \geq f_B(x)$ for all $x \in X$;
2) $A = B$ if and only $A \subseteq B$ and $B \subseteq A$;
3) $A \cap B = \langle \min(t_A(x), t_B(x)), \max(i_A(x), i_B(x)), \max(f_A(x), f_B(x)) \rangle$ for all $x \in X$ for all $x \in X$;
4) $A \cup B = \langle \max(t_A(x), t_B(x)), \min(i_A(x), i_B(x)), \min(f_A(x), f_B(x)) \rangle$ for all $x \in X$;
5) $A^c = \{\langle f_A(x), 1 - i_A(x), t_A(x) \rangle \mid x \in X\}$.

The set of all single valued neutrosophic numbers is denoted by $SVNN$, i.e. $SVNS = \{< t, i, f > \mid t, i, f \in [0,1]\}$. For $\alpha =< t_\alpha, i_\alpha, f_\alpha >$, $\beta =< t_\beta, i_\beta, f_\beta > \in SVNN$, an ordering on $SVNN$ as $\alpha \leq \beta$ if and only if $t_\alpha \leq t_\beta, i_\alpha \geq i_\beta, f_\alpha \geq f_\beta$. $\alpha = \beta$ iff $\alpha \leq \beta$ and $\beta \leq \alpha$ [20].

Obviously, $\alpha \wedge \beta =< t_\alpha \wedge t_\beta, i_\alpha \vee i_\beta, f_\alpha \vee f_\beta >, \alpha \vee \beta =< t_\alpha \vee t_\beta, i_\alpha \wedge i_\beta, f_\alpha \wedge f_\beta >$, $\bigwedge_{i \in I} \alpha_i =< \bigwedge_{i \in I} t_{\alpha_i}, \bigvee_{i \in I} i_{\alpha_i}, \bigvee_{i \in I} f_{\alpha_i} >, \bigvee_{i \in I} \alpha_i =< \bigvee_{i \in I} t_{\alpha_i}, \bigwedge_{i \in I} i_{\alpha_i}, \bigwedge_{i \in I} f_{\alpha_i} >, 0^* =< 0,1,1 >$ and $1^* =< 1,0,0 >$ are the smallest element and the greatest element in the set $SVNN$, respectively. It is easy to verify that ($SVNN$, $\leq$) is a complete lattice.

**Definition 2.5.** [24]A binary operator $\mathcal{T}$: $SVNN \times SVNN \rightarrow SVNN$ defined by $\mathcal{T}(\alpha,\beta) =< T(t_\alpha, t_\beta), S(i_\alpha, i_\beta), S(f_\alpha, f_\beta) >$ is a single valued neutrosophic t-norm, which is called a single valued neutrosophic t-representable t-norm, where $T$ is a t-norm and $S$ is its dual t-conorm on [0, 1]. $\mathcal{T}$ is called a left-continuous single valued neutrosophic t-representable t-norm if $T$ is left-continuous and $S$ is right-continuous.

**Definition 2.6.** [24]A single valued neutrosophic residual implication ($\mathcal{R}$-implication for short) is defined by $\mathcal{R}_\mathcal{T}(\alpha,\beta) = \vee\{\gamma \in SVNN | \mathcal{T}(\gamma,\alpha) \le \beta\}$, $\forall \alpha,\beta \in SVNN$, where $\mathcal{T}$ is a left-continuous single valued neutrosophic t-representable t-norm.

**Proposition 2.3.** [24] Let $\mathcal{T}$ be a single valued neutrosophic t-representable t-norm, the following statements are equivalent:

1) $\mathcal{T}$ is left-continuous;
2) $\mathcal{T}$ and $\mathcal{R}_\mathcal{T}$ form an adjoint pair, i.e., they satisfy the following residual principle

$$\mathcal{T}(\gamma,\alpha) \le \beta \Leftrightarrow \gamma \le \mathcal{R}_\mathcal{T}(\alpha,\beta), \alpha,\beta,\gamma \in SVNS.$$

**Proposition 2.4.** [24]Let $\alpha,\beta \in SVNS$, $\alpha =< t_\alpha, i_\alpha, f_\alpha >$, $\beta =< t_\beta, i_\beta, f_\beta >$, then single valued neutrosophic residual implication $\mathcal{R}_\mathcal{T}(\alpha,\beta) = \text{big} < R_T(t_\alpha, t_\beta)), R_S(i_\beta, i_\alpha), R_S(f_\beta, f_\alpha)\text{big} >$, where $R_T$ is residual implication induced by left-continuous t-norm $T$, $R_S$ is coresidual implication induced by right-continuous t-conorm $S$.

**Proposition 2.5.** Let $\mathcal{R}_\mathcal{T}$ be single valued neutrosophic residual implication induced by left-continuous single valued neutrosophic t-representable t-norm $\mathcal{T}$, then

1) $\mathcal{R}_\mathcal{T}(\alpha,\beta) = 1^*$ iff $\alpha \le \beta$;
2) $\gamma \le \mathcal{R}_\mathcal{T}(\alpha,\beta)$ iff $\alpha \le \mathcal{R}_\mathcal{T}(\gamma,\beta)$;
3) $\mathcal{R}_\mathcal{T}(1^*,\alpha) = \alpha$;
4) $\mathcal{R}_\mathcal{T}(\alpha, \mathcal{R}_\mathcal{T}(\mathcal{R}_\mathcal{T}(\alpha,\beta),\beta) = 1^*$;
5) $\mathcal{R}_\mathcal{T}(\vee_{i\in I} \beta_i, \alpha) = \wedge_{i \in I} \mathcal{R}_\mathcal{T}(\beta_i, \alpha)$;
6) $\mathcal{R}_\mathcal{T}(\beta, \wedge_{i\in I} \alpha) = \wedge_{i \in I} \mathcal{R}_\mathcal{T}(\beta, \alpha_i)$;
7) $\mathcal{R}_\mathcal{T}$ is antitone in the first variable and isotone in the second variable.

Example 3. [24] The following are three important single valued neutrosophic t-representable t-norms and their residual implications.

1) The single valued neutrosophic Łukasiewicz t-norm and its residual implication:

$$\mathcal{T}_L(\alpha,\beta) = \langle (t_\alpha + t_\beta - 1) \vee 0, (i_\alpha + i_\beta) \wedge 1, (f_\alpha + f_\beta) \wedge 1 \rangle$$

$$\mathcal{R}_{\mathcal{T}_L}(\alpha,\beta) = \langle 1 \wedge (1 - t_\alpha + t_\beta), (i_\beta - i_\alpha) \vee 0, (f_\beta - f_\alpha) \vee 0 \rangle.$$

2) The single valued neutrosophic Gougen t-norm and its residual implication:

$$\mathcal{T}_{Go}(\alpha,\beta) = \langle t_\alpha t_\beta, i_\alpha + i_\beta - i_\alpha i_\beta, f_\alpha + f_\beta - f_\alpha f_\beta \rangle.$$

$$\mathcal{R}_{\mathcal{T}_{Go}}(\alpha,\beta)=\begin{cases}\langle 1,0,0\rangle, & if \quad t_\alpha\le t_\beta, i_\beta\le i_\alpha, f_\beta\le f_\alpha,\\ \langle 1,0,\frac{f_\beta-f_\alpha}{1-f_\alpha}\rangle, & if \quad t_\alpha\le t_\beta, i_\beta\le i_\alpha, f_\alpha< f_\beta,\\ \langle 1,\frac{i_\beta-i_\alpha}{1-i_\alpha},0\rangle, & if \quad t_\alpha\le t_\beta, i_\alpha< i_\beta, f_\beta\le f_\alpha,\\ \langle 1,\frac{i_\beta-i_\alpha}{1-i_\alpha},\frac{f_\beta-f_\alpha}{1-f_\alpha}\rangle, & if \quad t_\alpha\le t_\beta, i_\alpha< i_\beta, f_\alpha< f_\beta,\\ \langle \frac{t_\beta}{t_\alpha},0,0\rangle, & if \quad t_\beta< t_\alpha, i_\beta\le i_\alpha, f_\beta\le f_\alpha,\\ \langle \frac{t_\beta}{t_\alpha},0,\frac{f_\beta-f_\alpha}{1-f_\alpha}\rangle, & if \quad t_\beta< t_\alpha, i_\beta\le i_\alpha, f_\alpha< f_\beta,\\ \langle \frac{t_\beta}{t_\alpha},\frac{i_\beta-i_\alpha}{1-i_\alpha},0\rangle, & if \quad t_\beta< t_\alpha, i_\alpha< i_\beta, f_\beta\le f_\alpha,\\ \langle \frac{t_\beta}{t_\alpha},\frac{i_\beta-i_\alpha}{1-i_\alpha},\frac{f_\beta-f_\alpha}{1-f_\alpha}\rangle, & if \quad t_\beta< t_\alpha, i_\alpha< i_\beta, f_\alpha< f_\beta.\end{cases}$$

3) The single valued neutrosophic t-norm and its residual implication:

$\mathcal{T}_G(\alpha,\beta)=\langle t_\alpha\wedge t_\beta, i_\alpha\vee i_\beta, f_\alpha\vee f_\beta\rangle$.

$$\mathcal{R}_{\mathcal{T}_G}(\alpha,\beta)=\begin{cases}\langle 1,0,0\rangle, & if \quad t_\alpha\le t_\beta, i_\beta\le i_\alpha, f_\beta\le f_\alpha,\\ \langle 1,0,f_\beta\rangle, & if \quad t_\alpha\le t_\beta, i_\beta\le i_\alpha, f_\alpha< f_\beta,\\ \langle 1,i_\beta,0\rangle, & if \quad t_\alpha\le t_\beta, i_\alpha< i_\beta, f_\beta\le f_\alpha,\\ \langle 1,i_\beta,f_\beta\rangle, & if \quad t_\alpha\le t_\beta, i_\alpha< i_\beta, f_\alpha< f_\beta,\\ \langle t_\beta,0,0\rangle, & if \quad t_\beta< t_\alpha, i_\beta\le i_\alpha, f_\beta\le f_\alpha,\\ \langle t_\beta,0,f_\beta\rangle, & if \quad t_\beta< t_\alpha, i_\beta\le i_\alpha, f_\alpha< f_\beta,\\ \langle t_\beta,i_\beta,0\rangle, & if \quad t_\beta< t_\alpha, i_\alpha< i_\beta, f_\beta\le f_\alpha,\\ \langle t_\beta,i_\beta,f_\beta\rangle, & if \quad t_\beta< t_\alpha, i_\alpha< i_\beta, f_\alpha< f_\beta.\end{cases}$$

## 3 | Single Valued Neutrosophic Fuzzy Inference Quintuple Implication Method

In this section, we discuss fuzzy inference quintuple implication method based on left-continuous single valued neutrosophic t-representable t-norm.

**Definition 3.1.** Let $X$ and $Y$ be no-empty sets. $B^*$ is called single valued neutrosophic $\mathcal{R}$-type quintuple implication solution for FMP, if it is the smallest single valued neutrosophic set on $SVNS(Y)$ such that the following inference formula equal to the maximum:

$$\mathcal{R}\left(\mathcal{R}\big(A(x),B(y)\big),\mathcal{R}\left(\mathcal{R}\big(A^*(x),A(x)\big),\mathcal{R}\big(A^*(x),B^*(y)\big)\right)\right)$$

**Definition 3.2.** Let $X$ and $Y$ be no-empty sets. $A^*$ is called single valued neutrosophic $\mathcal{R}$-type quintuple implication solution $A^*$ for FMT, if it is the smallest single valued neutrosophic set on $SVNS(Y)$ such that the following inference formula equal to the maximum:

$$\mathcal{R}(\mathcal{R}(A(x),B(y)),\mathcal{R}(\mathcal{R}(B(y),B^*(y)),\mathcal{R}(A(x),A^*(x))))$$

**Theorem 3.1.** Let $, A^* \in SVNS(X)$ , $B \in SVNS(Y)$. Suppose $\mathcal{R}$ is a single valued neutrosophic residual implication induced by a left-continuous single valued neutrosophic t-representable t-norm $\mathcal{T}$, then the single valued neutrosophic $\mathcal{R}$-type quintuple implication solution $B^*$ of FMP is as follows:

$$B^*(y) = \bigvee_{x \in X} \mathcal{T}\,(\mathcal{T}(\mathcal{R}(A(x), B(y)), \mathcal{R}(A^*(x), A(x))), A^*(x))(\forall y \in Y) \quad (1)$$

**Proof:** For all $, A^* \in SVNS(X)$ , $B \in SVNS(Y)$. Fistly, we prove $\mathcal{R}(\mathcal{R}(A(x), B(y)), \mathcal{R}(\mathcal{R}(A^*(x), A(x)), \mathcal{R}(A^*(x), B^*(y)))) = 1^*$. It follows from Eq. (1), we have $\mathcal{T}(\mathcal{T}(\mathcal{R}(A(x), B(y)), \mathcal{R}(A^*(x), A^*(x))), A^*(x)) \leq B^*(y)$. By the residuation property, then $\mathcal{R}(A(x), B(y)) \leq \mathcal{R}(\mathcal{R}(A^*(x), A(x)), \mathcal{R}(A^*(x), B^*(y)))$. Therefore, $\mathcal{R}(\mathcal{R}(A(x), B(y)), \mathcal{R}(\mathcal{R}(A^*(x), A(x)), \mathcal{R}(A^*(x), B^*(y)))) = 1^*$.

Second, we show that $B^*$ is the smallest single valued neutrosophic fuzzy subset such that $\mathcal{R}(\mathcal{R}(A(x), B(y)), \mathcal{R}(\mathcal{R}(A^*(x), A(x)), \mathcal{R}(A^*(x), B^*(y)))) = 1^*$. Suppose $C$ is a arbitrary single valued neutrosophic fuzzy subset such that $\mathcal{R}(\mathcal{R}(A(x), B(y)), \mathcal{R}(\mathcal{R}(A^*(x), A(x)), \mathcal{R}(A^*(x), C(y)))) = 1^*$. By the residuation property, $\mathcal{R}(A(x), B(y)) \leq \mathcal{R}(\mathcal{R}(A^*(x), A(x)), \mathcal{R}(A^*(x), C(y)))$. Then $\mathcal{T}(\mathcal{T}(\mathcal{R}(A(x), B(y)), \mathcal{R}(A^*(x), A(x))), A^*(x)) \leq C(y)$. Thus, $B^* \leq C$.

Therefore, $B^*$ for Eq. (1) is the single valued neutrosophic $\mathcal{R}$-type quintuple implication solution of FMP .

**Corollary 3.1.** Suppose $\mathcal{R}$ is a single valued neutrosophic residual implication induced by a left-continuous single valued neutrosophic t-representable t-norm $\mathcal{T}$, then the single valued neutrosophic $\mathcal{R}$-type quintuple implication solution $B^* = \{< t_{B^*}(y), i_{B^*}(y), f_{B^*}(y) > \mid y \in Y\}$ for FMP is expressed as follows:

$t_{B^*}(y) = \bigvee_{x \in X} T\,(T(R_T(t_A(x), t_B(y)), R_T(t_{A^*}(x), t_A(x))), t_{A^*}(x))(\forall y \in Y)$,

$i_{B^*}(y) = \bigwedge_{x \in X} S\,(S(R_S(i_B(y), i_A(x)), R_S(i_A(x), i_{A^*}(x))), i_{A^*}(x))(\forall y \in Y)$,

$f_{B^*}(y) = \bigwedge_{x \in X} S\,(S(R_S(f_B(y), f_A(x)), R_S(f_A(x), f_{A^*}(x))), f_{A^*}(x))(\forall y \in Y)$.

**Corollary 3.2.** The $\mathcal{R}_{\mathcal{T}_L}$-type quintuple implication solution $B^*(y) =< t_{B^*}(y), i_{B^*}(y), f_{B^*}(y) > (y \in Y)$ for FMP is expressed as follows:

$t_{B^*}(y) = \bigvee_{x \in X}\{\,[t_{A^*}(x) + ((((1 - t_A(x) + t_B(y)) \wedge 1) + ((1 - t_{A^*}(x) + t_A(x)) \wedge 1) - 1) \vee 0) - 1] \vee 0\}(\forall y \in Y)$,

$i_{B^*}(y) = \bigwedge_{x \in X}\{\,[i_{A^*}(x) + ((((i_B(y) - i_A(x)) \vee 0) + ((i_A(x) - i_{A^*}(x)) \vee 0)) \wedge 1)] \wedge 1\}(\forall y \in Y)$,

$f_{B^*}(y) = \bigwedge_{x \in X}\{\,[f_{A^*}(x) + ((((f_B(y) - f_A(x)) \vee 0) + ((f_A(x) - f_{A^*}(x)) \vee 0)) \wedge 1)] \wedge 1\}(\forall y \in Y)$.

**Corollary 3.3.** The $\mathcal{R}_{\mathcal{T}_{Go}}$-type quintuple implication solution $B^*(y) =< t_{B^*}(y), i_{B^*}(y), f_{B^*}(y) > (y \in Y)$ for FMP is expressed as follows:

$$t_{B^*}(y) = \bigvee_{x \in X} \left\{ t_{A^*}(x) \cdot \left( \frac{t_{B(y)}}{t_A(x)} \wedge 1 \right) \cdot \left( \frac{t_{A(x)}}{t_{A^*}(x)} \wedge 1 \right) \right\} (\forall y \in Y)$$

$$i_{B^*}(y) = \bigwedge_{x\in X}\Bigg\{i_{A^*}(x) + \Bigg[\left(\frac{i_B(y)-i_A(x)}{1-i_A(x)}\vee 0\right)+\left(\frac{i_A(x)-i_{A^*}(x)}{1-i_{A^*}(x)}\vee 0\right)-\left(\frac{i_B(y)-i_A(x)}{1-i_A(x)}\vee 0\right)\cdot\left(\frac{i_A(x)-i_{A^*}(x)}{1-i_{A^*}(x)}\vee 0\right)\Bigg] - i_{A^*}(x)\cdot\Bigg[\left(\frac{i_B(y)-i_A(x)}{1-i_A(x)}\vee 0\right)+\left(\frac{i_A(x)-i_{A^*}(x)}{1-i_{A^*}(x)}\vee 0\right)-\left(\frac{i_B(y)-i_A(x)}{1-i_A(x)}\vee 0\right)\cdot\left(\frac{i_A(x)-i_{A^*}(x)}{1-i_{A^*}(x)}\vee 0\right)\Bigg]\Bigg\}(\forall y\in Y)$$

s$f_{B^*}(y) = \wedge_{x\in X}\{ f_{A^*}(x) + [(\frac{f_B(y)-f_A(x)}{1-f_A(x)}\vee 0)+(\frac{f_A(x)-f_{A^*}(x)}{1-f_{A^*}(x)}\vee 0)-(\frac{f_B(y)-f_A(x)}{1-f_A(x)}\vee 0)\cdot(\frac{f_A(x)-f_{A^*}(x)}{1-f_{A^*}(x)}\vee 0)] - f_{A^*}(x)\cdot[(\frac{f_B(y)-f_A(x)}{1-f_A(x)}\vee 0)+(\frac{f_A(x)-f_{A^*}(x)}{1-f_{A^*}(x)}\vee 0)-(\frac{f_B(y)-f_A(x)}{1-f_A(x)}\vee 0)\cdot(\frac{f_A(x)-f_{A^*}(x)}{1-f_{A^*}(x)}\vee 0)]\}(\forall y\in Y)$.

**Corollary 3.4.** The $\mathcal{R}_{\mathcal{T}_G}$-type quintuple implication solution $B^*(y) =< t_{B^*}(y), i_{B^*}(y), f_{B^*}(y) > (y\in Y)$ for FMP is expressed as follows:

$t_{B^*}(y) = \vee_{x\in X}\{ t_{A^*}(x) \wedge [R_{T_G}(t_A(x), t_B(y)) \wedge R_{T_G}(t_{A^*}(x), t_A(x))]\}(\forall y\in Y)$,

$i_{B^*}(y) = \wedge_{x\in X}\{ i_{A^*}(x) \vee [R_{S_G}(i_B(y), i_A(x)) \vee R_{S_G}(i_A(x), i_{A^*}(x))]\}(\forall y\in Y)$,

$f_{B^*}(y) = \wedge_{x\in X}\{ f_{A^*}(x) \vee [R_{S_G}(f_B(y), f_A(x)) \vee R_{S_G}(f_A(x), f_{A^*}(x))]\}(\forall y\in Y)$.

**Theorem 3.2.** Let $\in SVNS(X)$ , $B, B^* \in SVNS(Y)$, $\mathcal{R}$ be single valued neutrosophic residual implication induced by a left-continuous single valued neutrosophic t-representable t-norm $\mathcal{T}$, then the single valued neutrosophic $\mathcal{R}$-type quintuple implication solution $A^*$ for FMT is as follows:

$$A^*(x) = \bigvee_{y\in Y}\mathcal{T}\,(\mathcal{T}(\mathcal{R}(A(x),B(y)),\mathcal{R}(B(y),B^*(y))),A(x))(\forall x\in X) \tag{2}$$

**Proof:** For all $\in SVNS(X)$ , $B, B^* \in SVNS(Y)$. Fistly, we prove $\mathcal{R}(\mathcal{R}(A(x),B(y)),\mathcal{R}(\mathcal{R}(B(y),B^*(y)),\mathcal{R}(A(x),A^*(x)))) = 1^*$. It follows from equation (2), we have $\mathcal{T}(\mathcal{T}(\mathcal{R}(A(x),B(y)),\mathcal{R}(B(y),B^*(y))),A(x)) \le A^*(x)$. By the residuation property, then $\mathcal{R}(A(x),B(y)) \le \mathcal{R}(\mathcal{R}(B(y),B^*(y)),\mathcal{R}(A(x),A^*(x)))$. Therefore, we have $\mathcal{R}(\mathcal{R}(A(x),B(y)),\mathcal{R}(\mathcal{R}(B(y),B^*(y)),\mathcal{R}(A(x),A^*(x)))) = 1^*$.

Second, we show that $A^*$ is the smallest single valued neutrosophic fuzzy subset such that $\mathcal{R}(\mathcal{R}(A(x),B(y)),\mathcal{R}(\mathcal{R}(B(y),B^*(y)),\mathcal{R}(A(x),A^*(x)))) = 1^*$. Suppose $D$ is a arbitrary single valued neutrosophic fuzzy subset such that

$\mathcal{R}(\mathcal{R}(A(x),B(y)),\mathcal{R}(\mathcal{R}(B(y),B^*(y)),\mathcal{R}(A(x),D(x)))) = 1^*$ . By the residuation property, we have $\mathcal{R}(A(x),B(y)) \le \mathcal{R}(\mathcal{R}(B(y),B^*(y)),\mathcal{R}(A(x),D(x)))$.
Then, $\mathcal{T}(\mathcal{T}(\mathcal{R}(A(x),B(y)),\mathcal{R}(B(y),B^*(y))),A(x)) \le D(x)$. Thus, $A \le D$.

Therefore, $A^*$ is the single valued neutrosophic $\mathcal{R}$-type quintuple implication solution for FMT.

**Corollary 3.5.** Let $\mathcal{R}$ is a single valued neutrosophic residual implication induced by a left-continuous single valued neutrosophic t-representable t-norm $\mathcal{T}$, then the single valued neutrosophic $\mathcal{R}$-type quintuple implication solution $A^* = \{< t_{A^*}(x), i_{A^*}(x), f_{A^*}(x) >| x\in X\}$ for FMT is expressed as follows:

$t_{A^*}(x) = \vee_{y\in Y} T\,(T(R_T(t_A(x), t_B(y)), R_T(t_B(y), t_{B^*}(y))), t_A(x))(\forall x\in X)$,

$i_{A^*}(x) = \wedge_{y\in Y} S\,(S(R_S(i_B(y), i_A(x)), R_S(i_{B^*}(y), i_B(y))), i_A(x))(\forall x \in X)$,

$f_{A^*}(x) = \wedge_{y\in Y} S\,(S(R_S(f_B(y), f_A(x)), R_S(f_{B^*}(y), f_B(y))), f_A(x))(\forall x \in X)$.

**Corollary 3.6.** The $\mathcal{R}_{\mathcal{T}_L}$-type quintuple implication solution $A^*(x) =< t_{A^*}(x), i_{A^*}(x), f_{A^*}(x) > (x \in X)$ for FMT is expressed as follows:

$t_{A^*}(x) = \vee_{y\in Y}\{\,[t_A(x) + ((((1 - t_A(x) + t_B(y)) \wedge 1) + ((1 - t_B(y) + t_{B^*}(y)) \wedge 1) - 1) \vee 0) - 1] \vee 0\}(\forall x \in X)$,

$i_{A^*}(x) = \wedge_{y\in Y}\{\,[i_A(x) + ((((i_B(y) - i_A(x)) \vee 0) + ((i_{B^*}(y) - i_B(y)) \vee 0)) \wedge 1)] \wedge 1\}(\forall x \in X)$,

$f_{A^*}(x) = \wedge_{y\in Y}\{\,[f_A(x) + ((((f_B(y) - f_A(x)) \vee 0) + ((f_{B^*}(y) - f_B(y)) \vee 0)) \wedge 1)] \wedge 1\}(\forall x \in X)$.

**Corollary 3.7.** The $\mathcal{R}_{\mathcal{T}_{Go}}$-type quintuple implication solution $A^*(x) =< t_{A^*}(x), i_{A^*}(x), f_{A^*}(x) > (x \in X)$ for FMT is expressed as follows:

$t_{A^*}(x) = \wedge_{y\in Y}\{\, t_A(x) \cdot (\frac{t_{B(y)}}{t_A(x)} \wedge 1) \cdot (\frac{t_{B^*(y)}}{t_B(y)} \wedge 1)\}(\forall x \in X)$,

$i_{A^*}(x) = \vee_{y\in Y}\{\, i_A(x) + [(\frac{i_B(y)-i_A(x)}{1-i_A(x)} \vee 0) + (\frac{i_{B^*}(y)-i_B(y)}{1-i_B(y)} \vee 0) - (\frac{i_B(y)-i_A(x)}{1-i_A(x)} \vee 0) \cdot (\frac{i_{B^*}(y)-i_B(y)}{1-i_B(y)} \vee 0)] - i_A(x) \cdot [(\frac{i_B(y)-i_A(x)}{1-i_A(x)} \vee 0) + (\frac{i_{B^*}(y)-i_B(y)}{1-i_B(y)} \vee 0) - (\frac{i_B(y)-i_A(x)}{1-i_A(x)} \vee 0) \cdot (\frac{i_{B^*}(y)-i_B(y)}{1-i_B(y)} \vee 0)]\}(\forall x \in X)$,

$f_{A^*}(x) = \vee_{y\in Y}\{\, f_A(x) + [(\frac{f_B(y)-f_A(x)}{1-f_A(x)} \vee 0) + (\frac{f_{B^*}(y)-f_B(y)}{1-f_B(y)} \vee 0) - (\frac{f_B(y)-f_A(x)}{1-f_A(x)} \vee 0) \cdot (\frac{f_{B^*}(y)-f_B(y)}{1-f_B(y)} \vee 0)] - f_A(x) \cdot [(\frac{f_B(y)-f_A(x)}{1-f_A(x)} \vee 0) + (\frac{f_{B^*}(y)-f_B(y)}{1-f_B(y)} \vee 0) - (\frac{f_B(y)-f_A(x)}{1-f_A(x)} \vee 0) \cdot (\frac{f_{B^*}(y)-f_B(y)}{1-f_B(y)} \vee 0)]\}(\forall x \in X)$.

**Corollary 3.8.** The $\mathcal{R}_{\mathcal{T}_G}$-type quintuple implication solution $A^*(x) =< t_{A^*}(x), i_{A^*}(x), f_{A^*}(x) > (x \in X)$ for FMT is expressed as follows:

$t_{A^*}(x) = \wedge_{y\in Y}\{\, t_A(x) \wedge [R_{T_G}(t_A(x), t_B(y)) \wedge R_{T_G}(t_B(y), t_{B^*}(y))]\}(\forall x \in X)$,

$i_{A^*}(x) = \vee_{y\in Y}\{\, i_A(x) \vee [R_{S_G}(i_B(y), i_A(x)) \vee R_{S_G}(i_{B^*}(y), i_B(y))]\}(\forall x \in X)$,

$f_{A^*}(x) = \vee_{y\in Y}\{\, f_A(x) \vee [R_{S_G}(f_B(y), f_A(x)) \vee R_{S_G}(f_{B^*}(y), f_B(y))]\}(\forall x \in X)$.

**Example 4.** We use the quintuple implication method as stated in Eq.(1) to deal with the FMP problem shown in Example 1. For the three implications: $G\ddot{o}del$ implication, Lukasiewicz implication and Gougen implication, we have the same result $B^* = \{< 0,1,1 >, < 0,1,1 >, < 0,1,1 >, < 0.4,0.5,0.3 >, < 0.4,0.5,0.3 >\}$, which is close to the statement ``$y$ is large''. Therefore, it is consistent with human thinking.

Moreover, we use the quintuple implication method as stated in Eq.(2) to deal with the FMT problem shown in Example 1. For the three implications: $G\ddot{o}del$ implication, Lukasiewicz implication and Gougen implication, we have the same result $A^* = \{< 0.4,0.5,0.3 >, < 0.4,0.5,0.3 >, < 0,1,1 >, < 0,1,1 >, < 0,1,1 >\}$, which is close to the statement ``$x$ is small''. Hence, it is consistent with human thinking.

As for fuzzy reasoning, the reductivity of an inference method is a significant subject. Therefore, we consider the reductivity of single valued neutrosophic fuzzy modus ponens and single valued neutrosophic fuzzy modus tollens.

**Definition 3.3.** ([3] A method for FMP is said recoverable if $A^* = A$ implies $B^* = B$. Similarly, a method for FMT is recoverable if $B^* = B$ implies $A^* = A$.

**Theorem 3.3.** The single valued neutrosophic fuzzy inference quintuple implication method for FMP is recoverable if $A$ is normal single valued neutrosophic set (there is $x_0 \in X$ such that $A(x_0) =< 1,0,0 >= 1^*$).

**Proof:** Suppose $A^{*}=A$ and there exists $x_0\in X$ such that $A(x_0)=A^{*}(x_0)=\langle 1,0,0\rangle=1^{*}$. Then we have

$$\begin{aligned} B(y) &\geq B^{*}(y) \\ &= \vee_{x\in X}\mathcal{T}\left(\mathcal{T}(\mathcal{R}(A(x),B(y)),\mathcal{R}(A^{*}(x),A(x))),A^{*}(x)\right) \\ &\geq \mathcal{T}(\mathcal{T}(\mathcal{R}(A(x_0),B(y)),\mathcal{R}(A^{*}(x_0),A(x_0))),A^{*}(x_0)) \\ &= \mathcal{T}(\mathcal{T}(\mathcal{R}(1^{*},B(y)),1^{*}),1^{*}) \\ &= B(y) \end{aligned}$$

Therefore $B^{*}=B$. This shows that the quintuple implication method for FMP is recoverable.

**Theorem 3.4.** The single valued neutrosophic fuzzy inference quintuple I method for FMT is reductive if single valued neutrosophic residual implication $\mathcal{R}$ satisfy $\mathcal{R}(\mathcal{R}(A,0^{*}),0^{*})=A$, and $B$ is co-normal single valued neutrosophic set (there is $y_0\in Y$ such that $B(y_0)=<0,1,1>=0^{*}$).

**Proof:** Suppose $B^{*}=B$ is co-normal single valued neutrosophic set, i.e. there exists $y_0\in Y$ such that $B^{*}(y_0)=B(y_0)=<0,1,1>=0^{*}$, then we have

$$\begin{aligned} A(x) &\geq A^{*}(x) \\ &= \bigvee_{y\in Y}\mathcal{T}\left(\mathcal{T}(\mathcal{R}(A(x),B(y)),\mathcal{R}(B(y),B^{*}(y))),A(x)\right) \\ &\geq \mathcal{T}(\mathcal{T}(\mathcal{R}(A(x),B(y_0)),\mathcal{R}(B(y_0),B^{*}(y_0))),A(x)) \\ &= \mathcal{T}(\mathcal{T}(\mathcal{R}(A(x),1^{*}),1^{*}),A(x)) \\ &= A(x) \end{aligned}$$

Therefore $A^{*}=A$. This shows that the quintuple I method for FMT is recoverable.

# 4 | Robustness of Single Valued Neutrosophic Fuzzy Inference Quintuple I Method

In this section, we introduce a new distance between single valued neutrosophic sets. We study robustness of quintuple I method based on left-continuous single valued neutrosophic t-representable t-norms with this new distance.

**Theorem 4.1.** [25] Let $X=\{x_1,x_2,\ldots,x_n\}$, for all $\alpha,\beta\in SVNS(X)$, then

$$d(\alpha,\beta)=\max\{\bigvee_{x_i\in X}|t_\alpha(x_i)-t_\beta(x_i)|,\bigvee_{x_i\in X}|i_\alpha(x_i)-i_\beta(x_i)|,\bigvee_{x_i\in X}|f_\alpha(x_i)-f_\beta(x_i)|\}$$

is a metric on $SVNS(X)$ and $(SVNS(X),d)$ is a metric space. $d$ is called a distance on $SVNS(X)$.

**Definition 4.1.** [25] Suppose that $\mathfrak{F}$ is a n-tuple mapping form $SVNN^n$ to $SVNN$, $\forall\varepsilon\in(0,1)$. For any $<\mathrm{t},\mathrm{i},\mathrm{f}>=(<t_1,i_1,f_1>,<t_2,i_2,f_2>,\ldots,<t_n,i_n,f_n>)\in SVNN^n$,

$$\begin{aligned} &\Delta_{\mathfrak{F}}(<\mathrm{t},\mathrm{i},\mathrm{f}>,\varepsilon) \\ &=\vee\{d(\mathfrak{F}<\mathrm{t},\mathrm{i},\mathrm{f}>,\mathfrak{F}<\mathrm{t}',\mathrm{i}',\mathrm{f}'>)\mid<\mathrm{t}',\mathrm{i}',\mathrm{f}'>\in SVNN^n,d(<\mathrm{t},\mathrm{i},\mathrm{f}>,<\mathrm{t}',\mathrm{i}',\mathrm{f}'>)\leq\varepsilon\} \end{aligned}$$

is called $\varepsilon$ sensitivity of $\mathfrak{F}$ at point $<\mathrm{t},\mathrm{i},\mathrm{f}>$, where $d(<\mathrm{t},\mathrm{i},\mathrm{f}>,<\mathrm{t}',\mathrm{i}',\mathrm{f}'>)=\max\{\vee_j|t_j-t_j'|,\vee_j|i_j-i_j'|,\vee_j|f_j-f_j'|\}$.

**Definition 4.2.** [25] The biggest $\varepsilon$ sensitivity of $\mathfrak{F}$ denoted by

$$\Delta_{\mathfrak{F}}(\varepsilon)=\bigvee_{<\mathrm{i},\mathrm{t},\mathrm{f}>\in SVNS^n}\Delta_{\mathfrak{F}}(<\mathrm{t},\mathrm{i},\mathrm{f}>,\varepsilon)$$

is called $\varepsilon$ sensitivity of $\mathfrak{F}$.

**Definition 4.3.**[25] Let $\mathfrak{F}$ and $\mathfrak{F}'$ be two n-tuple single valued neutrosophic fuzzy connectives. We say that $\mathfrak{F}$ at least as robust as $\mathfrak{F}'$ at point $<\mathrm{t,i,f}>$, if $\forall\varepsilon \in (0,1)$, $\Delta_{\mathfrak{F}}(<\mathrm{t,i,f}>,\varepsilon) \le \Delta_{\mathfrak{F}'}(<\mathrm{t,i,f}>,\varepsilon)$. We say that $\mathfrak{F}$ is more robust than $\mathfrak{F}'$ at point $<\mathrm{t,i,f}>$, if there exists $\varepsilon > 0$ such that $\Delta_{\mathfrak{F}}(<\mathrm{t,i,f}>,\varepsilon) < \Delta_{\mathfrak{F}'}(<\mathrm{t,i,f}>,\varepsilon)$.

**Definition 4.4.** [25] Let $\mathfrak{F}$ and $\mathfrak{F}'$ be two n-tuple single valued neutrosophic fuzzy connectives. We say that $\mathfrak{F}$ at least as robust as $\mathfrak{F}'$, if $\forall\varepsilon \in (0,1)$, $\Delta_{\mathfrak{F}}(\varepsilon) \le \Delta_{\mathfrak{F}'}(\varepsilon)$. We say that $\mathfrak{F}$ is more robust than $\mathfrak{F}'$ if there exists $\varepsilon > 0$ such that $\Delta_{\mathfrak{F}}(\varepsilon) < \Delta_{\mathfrak{F}'}(\varepsilon)$.

**Definition 4.5.**[25] Let $A$ and $A'$ be two single valued neutrosophic fuzzy sets on universal $X$. If $\| A - A' \| = \bigvee_{x\in X} d\,(A(x), A'(x)) \le \varepsilon$ for all $x \in X$, then $A'$ is called $\varepsilon$-perturbation of $A$ denoted by $A' \in O(A,\varepsilon)$.

**Proposition 4.1.**[25] For a binary single valued neutrosophic fuzzy connectives $\mathfrak{F}: SVNN \times SVNN \to SVNN$, we can obtain:

1) Let $\mathfrak{F}$ be a left-continuous single valued neutrosophic t-representable t-norm on $SVNS, \mathcal{T}(\alpha,\beta) = <T(t_\alpha,t_\beta), S(i_\alpha,i_\beta), S(f_\alpha,f_\beta)>$ for all $\alpha =< t_\alpha, i_\alpha, f_\alpha >$, $\beta =< t_\beta, i_\beta, f_\beta > \in SVNN$, then

$$\begin{aligned}
\triangle_{\mathcal{T}}(\varepsilon) &= \bigvee_{(\alpha,\,\beta)\in SVNN^2} \triangle_{\mathcal{T}}((\alpha,\beta),\varepsilon)\\
&= \bigvee_{(\alpha,\,\beta)\in SVNN^2} \{ \bigvee \{|T(t_\alpha,t_\beta)-T(t'_\alpha,t'_\beta)|,\ |S(i_\alpha,i_\beta)-S(i'_\alpha,i'_\beta)|,\ |S(f_\alpha,f_\beta)-S(f'_\alpha,f'_\beta)|\ \ |d((\alpha,\beta),(\alpha',\beta'))\le\varepsilon\}\}\\
&= \bigvee_{(\alpha,\,\beta)\in SVNN^2} \{|T(t_\alpha,t_\beta)-T(t_\alpha+\varepsilon,t_\beta+\varepsilon)|,\ |S(i_\alpha,i_\beta)-S(i_\alpha+\varepsilon,i_\beta+\varepsilon)|,\ |S(f_\alpha,f_\beta)-S(f_\alpha+\varepsilon,f_\beta+\varepsilon)|,\\
&\qquad |T(t_\alpha,t_\beta)-T(t_\alpha+\varepsilon,t_\beta-\varepsilon)|,\ |S(i_\alpha,i_\beta)-S(i_\alpha+\varepsilon,i_\beta-\varepsilon)|,\ |S(f_\alpha,f_\beta)-S(f_\alpha+\varepsilon,f_\beta-\varepsilon)|,\\
&\qquad |T(t_\alpha,t_\beta)-T(t_\alpha+\varepsilon,t_\beta)|,\ |S(i_\alpha,i_\beta)-S(i_\alpha+\varepsilon,i_\beta)|,\ |S(f_\alpha,f_\beta)-S(f_\alpha+\varepsilon,f_\beta)|,\\
&\qquad |T(t_\alpha,t_\beta)-T(t_\alpha,t_\beta+\varepsilon)|,\ |S(i_\alpha,i_\beta)-S(i_\alpha,i_\beta+\varepsilon)|,\ |S(f_\alpha,f_\beta)-S(f_\alpha,f_\beta+\varepsilon)|,\\
&\qquad |T(t_\alpha,t_\beta)-T(t_\alpha,t_\beta-\varepsilon)|,\ |S(i_\alpha,i_\beta)-S(i_\alpha,i_\beta-\varepsilon)|,\ |S(f_\alpha,f_\beta)-S(f_\alpha,f_\beta-\varepsilon)|,\\
&\qquad |T(t_\alpha,t_\beta)-T(t_\alpha-\varepsilon,t_\beta+\varepsilon)|,\ |S(i_\alpha,i_\beta)-S(i_\alpha-\varepsilon,i_\beta+\varepsilon)|,\ |S(f_\alpha,f_\beta)-S(f_\alpha-\varepsilon,f_\beta+\varepsilon)|,\\
&\qquad |T(t_\alpha,t_\beta)-T(t_\alpha-\varepsilon,t_\beta-\varepsilon)|,\ |S(i_\alpha,i_\beta)-S(i_\alpha-\varepsilon,i_\beta-\varepsilon)|,\ |S(f_\alpha,f_\beta)-S(f_\alpha-\varepsilon,f_\beta-\varepsilon)|,\\
&\qquad |T(t_\alpha,t_\beta)-T(t_\alpha-\varepsilon,t_\beta)|,\ |S(i_\alpha,i_\beta)-S(i_\alpha-\varepsilon,i_\beta)|,\ |S(f_\alpha,f_\beta)-S(f_\alpha-\varepsilon,f_\beta)|\}
\end{aligned}$$

2) Let $\mathfrak{F}$ be single valued neutrosophic residuated implication $\mathcal{R}_{\mathcal{T}}$ induced by left-continuous single valued neutrosophic t-representable t-norm $\mathcal{T}, \mathcal{R}_{\mathcal{T}}(\alpha,\beta) = \mathrm{big} < R_T(t_\alpha,t_\beta)), R_S(i_\beta,i_\alpha), R_S(f_\beta,f_\alpha)\mathrm{big} >$ for all $\alpha =< t_\alpha, i_\alpha, f_\alpha >$, $\beta =< t_\beta, i_\beta, f_\beta > \in SVNN$, then

$$\begin{aligned}
\triangle_{\mathcal{R}_{\mathcal{T}}}(\varepsilon) &= \bigvee_{(\alpha,\,\beta)\in SVNN^2} \triangle_{\mathcal{R}_{\mathcal{T}}}((\alpha,\beta),\varepsilon)\\
&= \bigvee_{(\alpha,\,\beta)\in SVNN^2} \{ \bigvee \{|R_T(t_\alpha,t_\beta)-R_T(t'_\alpha,t'_\beta)|,\ |R_S(i_\beta,i_\alpha)-R_S(i'_\beta,i'_\alpha)|,\ |R_S(f_\beta,f_\alpha)-R_S(f'_\beta,f'_\alpha)|\ \ |d((\alpha,\beta),(\alpha',\beta'))\le\varepsilon\}\}\\
&= \bigvee_{(\alpha,\,\beta)\in SVNN^2} \{|R_T(t_\alpha,t_\beta)-R_T(t_\alpha+\varepsilon,t_\beta+\varepsilon)|,\ |R_S(i_\beta,i_\alpha)-R_S(i_\beta+\varepsilon,i_\alpha+\varepsilon)|,\ |R_S(f_\beta,f_\alpha)-R_S(f_\beta+\varepsilon,f_\alpha+\varepsilon)|,\\
&\qquad |R_T(t_\alpha,t_\beta)-R_T(t_\alpha+\varepsilon,t_\beta-\varepsilon)|,\ |R_S(i_\beta,i_\alpha)-R_S(i_\beta+\varepsilon,i_\alpha-\varepsilon)|,\ |R_S(f_\beta+\varepsilon,f_\alpha)-R_S(f_\beta,f_\alpha-\varepsilon)|,\\
&\qquad |R_T(t_\alpha,t_\beta)-R_T(t_\alpha+\varepsilon,t_\beta)|,\ |R_S(i_\beta,i_\alpha)-R_S(i_\beta+\varepsilon,i_\alpha)|,\ |R_S(f_\beta,f_\alpha)-R_S(f_\beta+\varepsilon,f_\alpha)|,\\
&\qquad |R_T(t_\alpha,t_\beta)-R_T(t_\alpha,t_\beta+\varepsilon)|,\ |R_S(i_\beta,i_\alpha)-R_S(i_\beta,i_\alpha+\varepsilon)|,\ |R_S(f_\beta,f_\alpha)-R_S(f_\beta,f_\alpha+\varepsilon)|,\\
&\qquad |R_T(t_\alpha,t_\beta)-R_T(t_\alpha,t_\beta-\varepsilon)|,\ |R_S(i_\beta,i_\alpha)-R_S(i_\beta,i_\alpha-\varepsilon)|,\ |R_S(f_\beta,f_\alpha)-R_S(f_\beta,f_\alpha-\varepsilon)|,\\
&\qquad |R_T(t_\alpha,t_\beta)-R_T(t_\alpha-\varepsilon,t_\beta+\varepsilon)|,\ |R_S(i_\beta,i_\alpha)-R_S(i_\beta-\varepsilon,i_\alpha+\varepsilon)|,\ |R_S(f_\beta,f_\alpha)-R_S(f_\beta-\varepsilon,f_\alpha+\varepsilon)|,\\
&\qquad |R_T(t_\alpha,t_\beta)-R_T(t_\alpha-\varepsilon,t_\beta-\varepsilon)|,\ |R_S(i_\beta,i_\alpha)-R_S(i_\beta-\varepsilon,i_\alpha-\varepsilon)|,\ |R_S(f_\beta,f_\alpha)-R_S(f_\beta-\varepsilon,f_\alpha-\varepsilon)|,\\
&\qquad |R_T(t_\alpha,t_\beta)-R_T(t_\alpha-\varepsilon,t_\beta)|,\ |R_S(i_\beta,i_\alpha)-R_S(i_\beta-\varepsilon,i_\alpha)|,\ |R_S(f_\beta,f_\alpha)-R_S(f_\beta-\varepsilon,f_\alpha)|\}
\end{aligned}$$

where $R_T$ is residual implication induced by left-continuous t-norm $T$, $R_S$ is coresidual implication induced by right-continuous t-conorm $S$.

**Corollary 4.1.** [25] The $\varepsilon$ sensitivity of the single valued neutrosophic Lukasiewicz t-representable t-norm is $\Delta_{\mathcal{T}_L}(\varepsilon) = 2\varepsilon \wedge 1$.

**Corollary 4.2.** [25] The $\varepsilon$ sensitivity of the single valued neutrosophic Lukasiewicz residual implication is $\Delta_{\mathcal{R}_{\mathcal{T}_L}} = 2\varepsilon \wedge 1$.

**Definition 4.6.** Let $A$, $A'$, $B$, $B'$, $A^*$ and $A'^*$ be single valued neutrosophic fuzzy sets. $B^*$ and $B'^*$ are the single valued neutrosophic $\mathcal{R}$-type quintuple I solution of FMP($A$, $B$, $A^*$) and FMP($A'$, $B'$, $A'^*$),respectively. If $\| A - A' \| \leq \varepsilon$, $\| B - B' \| \leq \varepsilon$, $\| A^* - A'^* \| \leq \varepsilon$, then the single valued neutrosophic $\mathcal{R}$-type quintuple I solution of FMP $\varepsilon$-sensitivity $\Delta_{B^*}(\varepsilon)$:

$$\Delta_{B^*}(\varepsilon) = \| B^* - B'^* \| = \bigvee_{y\in Y} d\left(B^*(y), B'^*(y)\right)$$

**Definition 4.7.** Let $A$, $A'$, $B$, $B'$, $B^*$ and $B'^*$ be single valued neutrosophic fuzzy sets. $A^*$ and $A'^*$ are the single valued neutrosophic $\mathcal{R}$-type quintuple I solution of FMT($A$,$B$,$B^*$) and FMT($A'$,$B'$,$B'^*$), respectively. If $||A - A'|| \leq \varepsilon$, $||B - B'|| \leq \varepsilon$, $||B^* - B'^*|| \leq \varepsilon$, then the single valued neutrosophic $\mathcal{R}$-type quintuple I solution of FMT $\varepsilon$-sensitivity $\Delta_{A^*}(\varepsilon)$:

$$\Delta_{A^*}(\varepsilon) = \left||A^* - A'^*|\right| = \bigvee_{x\in X} d\left(A^*(x), A'^*(x)\right)$$

**Theorem 4.2.** Let $A, A', B, B', A^*$ and $A'^*$ be single valued neutrosophic fuzzy sets. $B^*$ and $B'^*$ are the single valued neutrosophic $\mathcal{R}$-type quintuple I solution of FMP($A$, $B$, $A^*$) and FMP($A'$, $B'$, $A'^*$), respectively. If $\| A - A' \| \leq \varepsilon$, $\| B - B' \| \leq \varepsilon$, $\| A^* - A'^* \| \leq \varepsilon$, then the single valued neutrosophic $\mathcal{R}$-type quintuple I solution of FMP $\varepsilon$-sensitivity $\Delta_{B^*}(\varepsilon) = \| B^* - B'^* \| \leq \Delta_{\mathcal{T}}\left(\Delta_{\mathcal{T}}\left(\Delta_{\mathcal{R}}(\varepsilon)\right)\right)$.

**Proof:** Let $A, A', A^*, A'^* \in SNVS(X), B, B' \in SNVS(Y)$. If $\| A - A' \| \leq \varepsilon$, $\| B - B' \| \leq \varepsilon$, $\| A^* - A'^* \| \leq \varepsilon$, then

$$\begin{aligned}
\Delta_{B^*}(\varepsilon) &= ||B^* - B'^*|| \\
&= \bigvee_{y\in Y} d(B^*(y), B'^*(y)) \\
&= \bigvee_{y\in Y} d(\bigvee_{x\in X} \mathcal{T}(\mathcal{T}(\mathcal{R}(A(x), B(y)), \mathcal{R}(A^*(x), A(x)), A^*(x)), \\
&\quad \bigvee_{x\in X} \mathcal{T}(\mathcal{T}(\mathcal{R}(A'(x), B'(y)), \mathcal{R}(A'^*(x), A'(x)), A'^*(x))) \\
&\leq \bigvee_{y\in Y} \bigvee_{x\in X} d(\mathcal{T}(\mathcal{T}(\mathcal{R}(A(x), B(y)), \mathcal{R}(A^*(x), A(x)), A^*(x)), \\
&\quad \mathcal{T}(\mathcal{T}(\mathcal{R}(A'(x), B'(y)), \mathcal{R}(A'^*(x), A'(x)), A'^*(x))) \\
&\leq \Delta_{\mathcal{T}}(\Delta_{\mathcal{T}}(\Delta_{\mathcal{R}}(\varepsilon) \vee \varepsilon))
\end{aligned}$$

By corollary 4.1, $\Delta_{\mathcal{R}}(\varepsilon) \geq (2\varepsilon \wedge 1) > \varepsilon$, then $\Delta_{\mathcal{T}}(\Delta_{\mathcal{R}}(\varepsilon) \vee \varepsilon) = \Delta_{\mathcal{T}}\left(\Delta_{\mathcal{R}}(\varepsilon)\right)$, $\Delta_{B^*}(\varepsilon) = \| B^* - B'^* \| \leq \Delta_{\mathcal{T}}\left(\Delta_{\mathcal{T}}\left(\Delta_{\mathcal{R}}(\varepsilon)\right)\right)$.

**Corollary 4.3.** Let $\mathcal{R}$ be single valued neutrosophic residuated implication induced by left-continuous single valued neutrosophic Łukasiewicz t-representable t-norm $\mathcal{T}$, then $\Delta_{B^*}(\varepsilon) \leq \varepsilon + 2\Delta_{\mathcal{R}}(\varepsilon)$.

**Proof:** Let $A^*(x) =< t_1, i_1, f_1 >$, $A(x) =< t_2, i_2, f_2 >$, $B(y) =< t_3, i_3, f_3 >$, $A'^*(x) =< t_1', i_1', f_1' >$, $A'(x) =< t_2', i_2', f_2' >$, $B'(y) =< t_3', i_3', f_3' >$. Suppose $\| A - A' \| \leq \varepsilon$, $\| B - B' \| \leq \varepsilon$, $\| A^* - A'^* \| \leq \varepsilon$. By proposition 4.1, we have

$$d(\mathcal{T}(\mathcal{T}(\mathcal{R}(A(x),B(y)),\mathcal{R}(A^*(x),A(x))),A^*(x)),\mathcal{T}(\mathcal{T}(\mathcal{R}(A'(x),B'(y)),\mathcal{R}(A'^*(x),A'(x))),A'^*(x)))$$
$$=max\{|(0\vee((0\vee(R_T(t_2,t_3)+R_T(t_1,t_2)-1))+t_1-1))-$$
$$(0\vee((0\vee(R_T(t_2',t_3')+R_T(t_1',t_2')-1))+t_1'-1))|,$$
$$|(1\wedge(i_1+(1\wedge(R_S(i_3,i_2)+R_S(i_2,i_1)))))-$$
$$(1\wedge(i_1'+(1\wedge(R_S(i_3',i_2')+R_S(i_2',i_1')))))|,$$
$$|(1\wedge(f_1+(1\wedge(R_S(f_3,f_2)+R_S(f_2,f_1)))))-$$
$$(1\wedge(f_1'+(1\wedge(R_S(f_3',f_2')+R_S(f_2',f_1')))))|\}$$
$$\leq max\{|(0\vee((0\vee(R_T(t_2,t_3)+R_T(t_1,t_2)-1))+t_1-1))-$$
$$(0\vee((0\vee(R_T(t_2,t_3)+R_T(t_1,t_2)-1))+(t_1+\varepsilon)+\triangle_{\mathcal{R}}(\varepsilon)-1))|,$$
$$|(1\wedge(i_1+(1\wedge(R_S(i_3,i_2)+R_S(i_2,i_1)))))-$$
$$(1\wedge(i_1+\varepsilon+(1\wedge(R_S(i_3,i_2)+R_S(i_2,i_1)+\triangle_{\mathcal{R}}(\varepsilon)))))|,$$
$$|(1\wedge(f_1+(1\wedge(R_S(f_3,f_2)+R_S(f_2,f_1)))))-$$
$$(1\wedge(f_1+\varepsilon+(1\wedge(R_S(f_3,f_2)+R_S(f_2,f_1)+\triangle_{\mathcal{R}}(\varepsilon)))))|\}$$
$$\leq\varepsilon+2\triangle_{\mathcal{R}}(\varepsilon)$$

**Theorem 4.3.** Let $A, A', B, B', B^*$ and $B'^*$ be single valued neutrosophic fuzzy sets. $A^*$ and $A'^*$ are the single valued neutrosophic $\mathcal{R}$-type quintuple I solution of FMT ($A$, $B$, $B^*$) and FMT($A'$, $B'$, $B'^*$), respectively. If $\| A-A' \|\leq \varepsilon$, $\| B-B' \|\leq \varepsilon$, $\| B^*-B'^* \|\leq \varepsilon$, then the single valued neutrosophic $\mathcal{R}$-type quintuple I solution of FMT $\varepsilon$-sensitivity

$$\Delta_{A^*}(\varepsilon)=\| A^*-A'^* \|\leq \Delta_{\mathcal{T}}\left(\Delta_{\mathcal{T}}\left(\Delta_{\mathcal{R}}(\varepsilon)\right)\right).$$

**Proof:** Let $A, A' \in SNVS(X), B, B', B^*, B'^* \in SNVS(Y)$. If $\| A-A' \|\leq \varepsilon$, $\| B-B' \|\leq \varepsilon$, $\| B^*-B'^* \|\leq \varepsilon$, then

$$\Delta_{A^*}(\varepsilon)=||A^*-A'^*||$$
$$=\bigvee_{x\in X} d(A^*(x),A'^*(x))$$
$$=\bigvee_{x\in X} d(\bigvee_{y\in Y}\mathcal{T}(\mathcal{T}(\mathcal{R}(A(x),B(y)),\mathcal{R}(B(y),B^*(y)),A(x)),$$
$$\bigvee_{y\in Y}\mathcal{T}(\mathcal{T}(\mathcal{R}(A'(x),B'(y)),\mathcal{R}(B'(y),B'^*(y)),A'(x)))$$
$$\leq\bigvee_{x\in X}\bigvee_{y\in Y} d(\mathcal{T}(\mathcal{T}(\mathcal{R}(A(x),B(y)),\mathcal{R}(B(y),B^*(y)),A(x)),$$
$$\mathcal{T}(\mathcal{T}(\mathcal{R}(A'(x),B'(y)),\mathcal{R}(B'(y),B'^*(y)),A'(x)))$$
$$\leq\triangle_{\mathcal{T}}(\triangle_{\mathcal{T}}(\triangle_{\mathcal{R}}(\varepsilon)\vee\varepsilon))$$

**Corollary 4.4.** Let $\mathcal{R}$ be single valued neutrosophic residuated implication induced by left-continuous single valued neutrosophic Łukasiewicz t-representable t-norm $\mathcal{T}$, then $\Delta_{A^*}(\varepsilon)\leq\varepsilon+2\Delta_{\mathcal{R}}(\varepsilon)$.

**Proof:** Let $B^*(y)=<t_1,i_1,f_1>$, $A(x)=<t_2,i_2,f_2>$, $B(y)=<t_3,i_3,f_3>$, $B'^*(y)=<t_1',i_1',f_1'>$, $A'(x)=<t_2',i_2',f_2'>$, $B'(y)=<t_3',i_3',f_3'>$. Suppose $\| A-A' \|\leq \varepsilon$, $\| B-B' \|\leq \varepsilon$, $\| A^*-A'^* \|\leq \varepsilon$, By proposition 4.1, we have :

$$
\begin{aligned}
& d(\mathcal{T}(\mathcal{T}(\mathcal{R}(A(x),B(y)),\mathcal{R}(B(y),B^*(y))),A(x)),\\
& \mathcal{T}(\mathcal{T}(\mathcal{R}(A'(x),B'(y)),\mathcal{R}(B'(y),B'(y))),A'(x)))\\
& = max\{|(0\vee((0\vee(R_T(t_2,t_3)+R_T(t_3,t_1)-1))+t_2-1))-\\
& \quad (0\vee((0\vee(R_T(t_2',t_3')+R_T(t_3',t_1')-1))+t_2'-1))|,\\
& \quad |(1\wedge(i_2+(1\wedge(R_S(i_3,i_2)+R_S(i_1,i_3)))))-\\
& \quad (1\wedge(i_2'+(1\wedge(R_S(i_3',i_2')+R_S(i_1',i_3')))))|,\\
& \quad |(1\wedge(f_2+(1\wedge(R_S(f_3,f_2)+R_S(f_1,f_3)))))-\\
& \quad (1\wedge(f_2'+(1\wedge(R_S(f_3',f_2')+R_S(f_1',f_3')))))|\}\\
& \leq max\{|(0\vee((0\vee(R_T(t_2,t_3)+R_T(t_3,t_1)-1))+t_2-1))-\\
& \quad (0\vee((0\vee(R_T(t_2,t_3)+R_T(t_3,t_1)-1))+(t_2+\varepsilon)+\triangle_{\mathcal{R}}(\varepsilon)-1))|,\\
& \quad |(1\wedge(i_2+(1\wedge(R_S(i_3,i_2)+R_S(i_1,i_3)))))-\\
& \quad (1\wedge(i_2+\varepsilon+(1\wedge(R_S(i_3,i_2)+R_S(i_1,i_3)+\triangle_{\mathcal{R}}(\varepsilon)))))|,\\
& \quad |(1\wedge(f_2+(1\wedge(R_S(f_3,f_2)+R_S(f_1,f_3)))))-\\
& \quad (1\wedge(f_2+\varepsilon+(1\wedge(R_S(f_3,f_2)+R_S(f_1,f_3)+\triangle_{\mathcal{R}}(\varepsilon)))))|\}\\
& \leq \varepsilon+2\triangle_{\mathcal{R}}(\varepsilon)
\end{aligned}
$$

# 5 | Conclusions

In this paper, we propose quintuple I method based on left-continuous single valued neutrosophic t-representable t-norms. Single valued neutrosophic fuzzy inference quintuple I Principle for FMP and FMT are proposed. Moreover, the single valued neutrosophic $\mathcal{R}$-type quintuple I solutions for FMP and FMT are given respectively. We prove that single valued neutrosophic fuzzy inference quintuple I methods are recoverable and robust. The logical basis of a fuzzy inference method is very important. In future, we will consider to build the strict logic foundation for quintuple I method based on left-continuous single valued neutrosophic t-representable t-norms, and to bring the single valued neutrosophic fuzzy inference method within the framework of logical semantic.

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## Author Contribution

All authors contributed equally to this work.

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## Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

## Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

## Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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