Neutrosophic Optimization and Intelligent Systems

Journal Homepage: sciencesforce.com/nois

Neutrosophic Opt. Int. Syst. Vol. 4 (2024) 39–50

Paper Type: Original Article

An Attempt on Investment Selection Problem under Neutrosophic Environment based on Weighted Value Method

Manas Karak [1], Animesh Mahata [2,*], Subrata Paul [3], Supriya Mukherjee [4], Santosh Biswas [5], and Banamali Roy [6]

[1] Department of Mathematics, Umeschandra College, Kolkata-700012, West Bengal, India; karakmanas659@gmail.com.
[2] Department of Mathematics, Sri RamkrishnaSaradaVidyaMahapitha, Kamarpukur, Hooghly -712612, West Bengal, India; animeshmahata8@gmail.com.
[3] Department of Mathematics, Arambagh Govt. Polytechnic, Arambagh, West Bengal, India; paulsubrata564@gmail.com.
[4] Department of Mathematics, Gurudas College,1/1 Suren Sarkar Road,Kolkata-700054, West Bengal, India; supriyasbku2013@gmail.com.
[5] Department of Mathematics, Jadavpur university, 188 Raja S.C. Mallik Road Kolkata-700032, West Bengal, India; sbiswas.math@jadavpuruniversity.in.
[6] Department of Mathematics, Bangabasi Evening College, Kolkata-700009, West Bengal, India; banamaliroy@yahoo.co.in.

Received: 28 May 2024 **Revised:** 25 Aug 2024 **Accepted:** 27 Sep 2024 **Published:** 01 Oct 2024

Abstract

The main aim of this study is to develop a ranking algorithm in neutrosophic environment and applying this algorithm to choose the best option for investment selection problems. Firstly, the concept of single valued triangular neutrosophic numbers (SVTrN-numbers) and it's characteristics are presented briefly. Here we define the λ-weighted value of SVTrN-numbers and applied it to the best selection of investment selection problem (ISP). Finally, to determine the feasibility and validity of this algorithm, we compared the resultant ranking by the proposed method with other existing approaches.

Keywords: SVTrN-numbers, λ-weighted Value, Ranking Method, Investment Selection Problem.

1 | Introduction

1.1 | The Multi-Criteria Decision Making

Recently, the multi-criteria decision-making (MCDM) method for investment selection problem (ISP) has been completely momentous and is an important part of operation research, decision science, and management science, which is classified by multi-attribute alternative sets. MCDM is the method of recognizing the problem, creating the preference, valuing the alternative, and selecting the best alternatives. To determine the most suitable alternative among the available alternatives, several standards make the

Corresponding Author: animeshmahata8@gmail.com
https://doi.org/10.61356/j.nois.2024.4339

selection markers pick the most suitable alternative. In this article, MCDM plays a vital role in choosing the perfect selection. The ranking method is the most suitable option for choosing the best alternative.

1.2 | Neutrosophic Set with MCDM Problem

However, in realistic applications, because of the inconsistent and inherent uncertainty of the obtainable data, selection makers may not be sufficient to accurately evaluate the values of the MCDM problem, and as a result, it is impossible to reach the optimal alternative. In this situation, to handle uncertainty inconsistent values of MCDM problem fuzzy set (FS), intuitionistic fuzzy set (IFS), and neutrosophic set (NS) perform a vital role. The law of hypothesis of crisp theory is inappropriate to manage those problems because it only implies that whatever elements either 0 or 1 belong to a set or not. In this situation, FS [1] and IFS [2] are suitable to handle this situation. Here, FS is classified through only the membership component, and IFS is classified through membership and non-membership components simultaneously. For this reason, in the FS and IFS environments, MCDM problems are developed. To handle uncertainty more accurately, Smarandache [3], in philosophical sense, presented the NS, which is a generalization of IFS. Each object in NS is classified by three independent components, namely truth, indeterminacy, and falsity. For the applications of real fields, Wang [4] described the idea of single-valued neutrosophic set (SVN-set). The indeterminacy part plays a significant role in creating proper decisions which not considered in FS and IFS. Due to this, we introduced MCDM for investment selection problems in a neutrosophic environment.

1.3 | Motivation and Novelties of the Study

Ranking of SVN numbers plays an important role in MCDM. Several ranking methods have been developed by many researchers [9-11, 16, 18, 21, 24, 36 - 38]. Recently, applying the ranking of SVN numbers, MCDM, and optimization problems have been extensively studied. The MCDM method under a neutrosophic environment is applied in several fields, although the literature review implies that there is just a little work completed on the investment selection problem. Hence, the study faces the problem, and the goal of the article is to fill up the gap in the literature survey. It gives optimistic outcomes. Here we developed de-neutrosophication of SVTrN-numbers by the weighted value method in a generalized way, and using this method, we established a new ranking technique. Applying this method, we select the best optimal alternative. To check the steadiness of this method, we introduced a comparison analysis to determine the changes in ranking.

1.4 | Structure of the Paper

The structure of the article is given in the following. First of all, in Section 1, we described the introduction briefly. In Section 2, we reviewed the literature. In Section 3, some necessary definitions like NS, SVTrN-numbers, (α, β, γ)-cut, and arithmetic operation of SVTrN-numbers are given. In the next Section, we define the λ-weighted value of SVTrN-numbers in a generalized way and apply this concept, we present an ordering system of neutrosophic numbers that is highly reasonable and one of the easiest methods. We give several properties and theorems of SVTrN-numbers that are linked to its operation and relations. In Section 5, with the help of the ordering system, we introduce an ISP with SVTrN-numbers, and after that, in the next section, we compare the result of the investment selection problem by the proposed technique with other existing approaches to check the stability and feasibility of the algorithm. Last of all, the conclusion and upcoming investigation field are recommended by the given work in Section 7.

2 | Literature Review

In this section, we shall review some articles related to the proposed method from the last few years in below Table 1 that are essential to developing the main idea of the proposed work.

Table 1. Literature review of some articles related to the proposed method.

Authors	Years	Environment	Methodology	Applications Area
[5]	2016	Neutrosophic	Value and ambiguity	MCDM
[7]	2021	Neutrosophic	Graded mean integration	Thermal power plants focussing on air pollution
[8]	2018	Intuitionistic fuzzy	Group decision method	Critical path selection
[16]	2024	Neutrosophic	Fractional method	Linear Programming Problem(LPP)
[17]	2023	Neutrosophic	Sign distance method	Transportation problem (TP)
[19]	2019	Neutrosophic	Q-Neutrosophic soft method	Decision-making issue
[26]	2023	Neutrosophic	Neutrosophic Analytic Hierarchy Process (AHP)	Women's University site selection
[35]	2016	Fuzzy	Fuzzy AHP	
[27]	2020	Neutrosophic	Possibilistic mean	Landfill Site
[28]	2021	Neutrosophic	Distance formula	Neutrosophic LPP
[30]	2018	Neutrosophic	Ranking method	Pattern Recognition
[34]	2022	Fuzzy and Crisp	Different MADM methodology	Information systems quality
[25]	2018	Neutrosophic	Zero suffix method	Different types of site
[20]	2019	Neutrosophic	Weighted value method	Assignment problem
[22]	2023	Neutrosophic	Graded mean integration	LPP
[29]	2017	Intuitionistic fuzzy	Geometry average operator	LPP
[24]	2023	Neutrosophic		MCDM
[23]	2022	Neutrosophic	Ranking method	TP
[31]	2020	Neutrosophic	Mellin's method	LPP
[32-33]	2023	Neutrosophic	Score function	MCDM-problem

3 | Pre-requisite Concept

Here we recall some fundamental definitions that are most important to developing the main concept of the article.

3.1 | Neutrosophic Set [3]

Let U be the universe of discourse and $\xi \epsilon U$. Then $\tilde{N}$ is called NS over X if it is classified through three independent components namely $T_{\tilde{N}}$, $I_{\tilde{N}}$, and $F_{\tilde{N}}$ which are called truth, indeterminacy, and falsity neutrosophic components, respectively. These components are maps from U to $]0^-,1^+[$ i.e., $T_{\tilde{N}}(\xi), I_{\tilde{N}}(\xi), F_{\tilde{N}}(\xi) \epsilon\,]0^-,1^+[$, where $]0^-,1^+[$ is called the non-standard unit interval. Thus, $\tilde{N}$ is described by $\tilde{N} = \{< x;\ T_{\tilde{N}}(\xi), I_{\tilde{N}}(\xi), F_{\tilde{N}}(\xi) >: \xi \epsilon\, U\}$, with the condition

$$0^- \leq \sup T_{\tilde{N}}(\xi) + \sup_{\tilde{N}}(\xi) + \sup F_{\tilde{N}}(\xi) \leq 3^+.$$

3.2 | Single Valued Neutrosophic Set [4]

In real applications, neutrosophic components on $]0^-,1^+[$ are difficult. So for real applications in the real field, neutrosophic components take on [0, 1], and NS is said to be SVN-set. Hence, an NS is said to be an SVN-set. Hence, a NS $\tilde{N}$ is defined as $\tilde{N} = \{< \xi;\ T_{\tilde{N}}(\xi), I_{\tilde{N}}(\xi), F_{\tilde{N}}(\xi) >: \xi \epsilon\, [0,1]\}$.

3.3 | Single Valued Triangular Neutrosophic Number [5]

Let us take an NS $\widetilde{M} =< ([l, m, n];\ t_{\widetilde{M}}, i_{\widetilde{M}}, f_{\widetilde{M}}) >\ on\ \mathbb{R}$where$l, m, n\epsilon\mathbb{R}$, and$l\ \leq m\ \leq nandt_{\widetilde{M}}, i_{\widetilde{M}}, f_{\widetilde{M}}\epsilon[0, 1]$is calledSVTrN-numbers whose components are designed by $T_{\widetilde{M}}: \mathbb{R} \rightarrow [0, t_{\widetilde{M}}]$, $I_{\widetilde{M}}: \mathbb{R} \rightarrow [i_{\widetilde{M}}, 1]$, $F_{\widetilde{M}}: \mathbb{R} \rightarrow [f_{\widetilde{M}}, 1]$ as described below:

$$T_{\widetilde{N}}(\xi) = \begin{cases} \frac{(\xi - l)t_{\widetilde{M}}}{(m-l)}, l \leq t_{\widetilde{M}} \leq m, \\ t_{\widetilde{M}}, \qquad \xi = m, \\ \frac{(n-\xi)\ t_{\widetilde{M}}}{(n-m)}, m \leq \xi \leq n, \\ 0, \qquad \text{otherwise.} \end{cases}$$

$$I_{\widetilde{N}}(\xi) = \begin{cases} \frac{(m-\xi) + i_{\widetilde{M}}(\xi - l)}{(m-l)}, l \leq \xi \leq m, \\ i_{\widetilde{M}}, \qquad \xi = m, \\ \frac{(\xi - m) + i_{\widetilde{M}}(n-\xi)}{(n-m)}, m \leq \xi \leq n, \\ 1, \qquad \text{otherwise.} \end{cases}$$

$$F_{\widetilde{N}}(\xi) = \begin{cases} \frac{(m-\xi) + f_{\widetilde{M}}(\xi - l)}{(n-m)}, l \leq \xi \leq m, \\ f_{\widetilde{M}}, \qquad \xi = m, \\ \frac{(\xi - m) + i_{\widetilde{M}}(n-\xi)}{(n-m)}, m \leq \xi \leq n, \\ 1, \qquad \text{otherwise.} \end{cases}$$

respectively.

Figure 1 depicts the graphical representation of an SVTrN-number $< ([3, 5, 8];\ 0.9, 0.5, 0.3) >$.

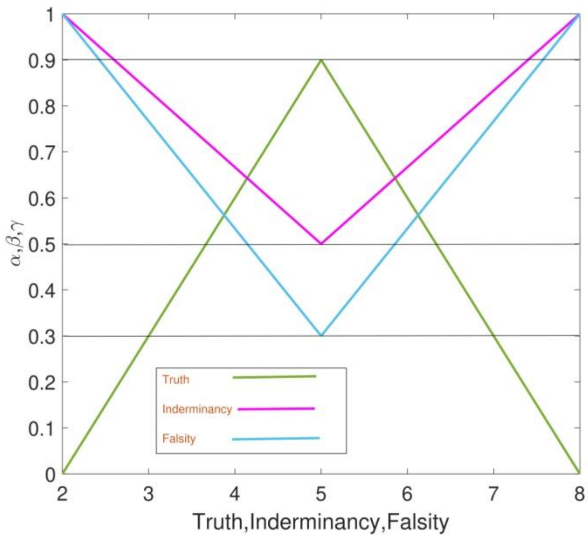

Figure 1. Graphical representation of generalized SVTrN-number <([3, 5, 8]; 0.9, 0.5, 0.3)>.

3.4 | (α, β, γ)-cut of SVTrN-number [9]

The (α, β, γ)-cut of SVTrN-number $\widetilde{M}$are defined in the below:

$[\widetilde{M}_L(\alpha), \widetilde{M}_R(\alpha)]$=$[\frac{(t_{\widetilde{M}}-\alpha)l+\alpha m}{t_{\widetilde{M}}}, \frac{(t_M-\alpha)n+\alpha m}{t_{\widetilde{M}}}]$,

$[\widetilde{M}_{L'}(\beta), \widetilde{M}_{R'}(\beta)]=[\frac{(1-\beta)m+(\beta-i_{\widetilde{M}})l}{1-i_{\widetilde{M}}}, \frac{(1-\beta)m+(\beta-i_{\widetilde{M}})n}{1-i_{\widetilde{M}}}]$

$[\widetilde{M}_{L''}(\gamma), \widetilde{M}_{R''}(\gamma)]= [\frac{(1-\gamma)m+(\gamma-i_{\widetilde{M}})l}{1-f_{\widetilde{M}}}, \frac{(1-\gamma)m+(\gamma-i_{\widetilde{M}})n}{1-f_{\widetilde{M}}}]$.

3.5 | Arithmetic Operation of SVTrN-numbers [9]

For two SVTrN-numders $\widetilde{M} =< ([l, m, n];\ t_{\widetilde{M}}, i_{\widetilde{M}}, f_{\widetilde{M}}) >$ and $\widetilde{N} =< ([u, v, w];\ t_{\widetilde{N}}, i_{\widetilde{N}}, f_{\widetilde{N}}) >$, the addition, scalar multiplications, and subtraction are as follows:

1. $\widetilde{M}\oplus\widetilde{N} =< (l + u, m + v, n + w);\ t_{\widetilde{M}}\wedge t_{\widetilde{\widetilde{N}}}, i_{\widetilde{M}}\vee i_{\widetilde{N}},\ f_{\widetilde{M}}\vee f_{\widetilde{\widetilde{N}}} >$.
2. $\xi\widetilde{M}=< (\xi l, \xi m, \xi n,); t_{\widetilde{M}}, i_{\widetilde{M}}, f_{\widetilde{M}} >$ if $\xi>0$ and $\xi\widetilde{M}=< (\xi n, \xi m, \xi l); t_{\widetilde{M}}, i_{\widetilde{M}}, f_{\widetilde{M}} >$ if $\xi<0$.
3. $\widetilde{M}\ominus\widetilde{N} =< (l - w, m - v, n - u);\ t_{\widetilde{M}}\wedge t_{\widetilde{\widetilde{N}}}, i_{\widetilde{M}}\vee i_{\widetilde{N}},\ f_{\widetilde{M}}\vee f_{\widetilde{\widetilde{N}}} >$.

4 | Value of SVTrN-number and its Characteristics

In this section, we explain the value of SVTrN-numbers, and using this value function, we establish an algorithm for ordering between SVTrN-numbers in a new direction. We also discuss various properties and theorems of this ranking methodology.

4.1 | Model in Imprecise Environment

Definition 4.1. Let $\widetilde{M} =< ([l, m, n];\ t_{\widetilde{M}}, i_{\widetilde{M}}, f_{\widetilde{M}}) >$be any SVTrN- number. Then,

i). For the corresponding truth component, the value of $\widetilde{M}$ described as $V_T(\widetilde{M}) = \frac{1}{12}\int_0^{t_{\widetilde{M}}}\{\widetilde{M}_L(\alpha) + \widetilde{M}_R(\alpha)\}f(\alpha)d\alpha = \frac{1}{12}(l+4m+n)t_{\widetilde{M}}^2$, for $f(\alpha)\epsilon[0, 1]$, and f(α) are non decreasing function of α and f(0)=0.

ii). For the corresponding indeterminacy component, the value of $\widetilde{M}$ described as $V_I(\widetilde{M}) = \frac{1}{12}\int_{i_{\widetilde{M}}}^{1}\{\widetilde{M}_{L'}(\beta) + \widetilde{M}_{R'}(\beta)\}g(\beta)d\beta = \frac{1}{12}(l+4m+n)(1 - i_{\widetilde{M}})^2$, for $g(\beta)\epsilon[0, 1]$, and g(β) are non increassing function of β and g(1)=0.

iii). For the corresponding falsity component, the value of $\widetilde{M}$ described as $V_F(\widetilde{M}) = \{\widetilde{M}_{L''}(\gamma) + \widetilde{M}_{R''}(\gamma)\}h(\gamma)d\gamma = \frac{1}{12}(l+4m+n)(1 - f_{\widetilde{M}})^2$, for $h(\gamma)\epsilon[0, 1]$, and $h(\gamma)$ are non increassing function of γ and h(1)=0.

Let us take for all SVTrN-number $\widetilde{M}$, f(α)=α, α ϵ[0, $t_{\widetilde{M}}$], g(β)=1-β, βϵ[$i_{\widetilde{M}}$, 1], and h(γ)=1-γ, γϵ[$f_{\widetilde{M}}$, 1] in the remaining entire paper.

Definition 4.2. For λϵ[0, 1], the λ-weighted value of SVTrN-number $\widetilde{M}$ is designed by $V_\lambda(\widetilde{M})$ and described as $V_\lambda(\widetilde{M})= \lambda^k V_T(\widetilde{M}) + (1-\lambda^k)V_I(\widetilde{M}) + (1-\lambda^k)V_F(\widetilde{M}) = \frac{1}{12}[(l + 4m + n)\{\lambda^k t_{\widetilde{M}}^2 + (1 - \lambda^k)(1 - i_{\widetilde{M}})^2 + +(1 - \lambda^k)(1 - f_{\widetilde{M}})^2]$, kϵ ℕ (set of natural numbers).

Where λ ϵ[0, 1] is the weight of the value function, which indicates the choice of information by the decision-makers. If λϵ[0, 0.5), then the decision maker's behavior indicates pessimistic behavior in the direction of negativity and uncertainty; if λϵ[0.5, 1], then the decision maker's behavior indicates optimistic behavior in the direction of positivity and certainty, and if λ=0.5, then the decision maker's behavior indicates indifference between certainty and uncertainty.

Property 3.1. For two SVTrN-numbers $\widetilde{M}$and $\widetilde{N}$, the weighted values $V_\lambda(\widetilde{M})$ and $V_\lambda(\widetilde{N})$ fulfill the following disciplines:

i). $V_\lambda(\widetilde{M}) \sim V_\lambda(\widetilde{N}) \leq V_\lambda(\widetilde{M} \oplus \widetilde{N}) \leq V_\lambda(\widetilde{M}) + V_\lambda(\widetilde{N})$,

ii). $V_\lambda(\widetilde{M} \sim \widetilde{N}) \leq V_\lambda(\widetilde{M}) + V_\lambda(\widetilde{N})$,

iii). $V_\lambda(\widetilde{\mu M}) = \mu V_\lambda(\widetilde{M})$, $\mu \in \mathbb{R}$.

Proof: Using Definitions 4.1 and 4.2, we can easily improve the above disciplines.

Definition 4.3. For λ ϵ[0, 1], and two SVTrN-numbers $\widetilde{M}$and $\widetilde{N}$, we define the ranking of $\widetilde{M}$ and $\widetilde{N}$ by

i). $V_\lambda(\widetilde{M})$ -$V_\lambda(\widetilde{N})$>0 iff$V_\lambda(\widetilde{M})$>$V_\lambda(\widetilde{N})$ iff $\widetilde{M} \prec \widetilde{N}$.

ii). $V_\lambda(\widetilde{M})$ -$V_\lambda(\widetilde{N})$ =0 iff$V_\lambda(\widetilde{M})$ =$V_\lambda(\widetilde{N})$ iff $\widetilde{M} \approx \widetilde{N}$.

Note 1: This ranking function is constructed in a generalized way. Mainly, the formula is used for the transformation of SVTrN-numbers into a crisp number. Similarly, we may also create the ranking of others, including single-valued trapezoidal numbers, and hexagonal and pentagonal numbers.

Property 4.2. For λ ϵ[0, 1], and any SVTrN-numbers $\widetilde{M}$, the λ-weighted value$V_\lambda(\widetilde{M})$ is

i). Increasing if $V_T(\widetilde{M})$>$V_I(\widetilde{M})$+$V_F(\widetilde{M})$,

ii). Decreasing if $V_T(\widetilde{M})$<$V_I(\widetilde{M})$+$V_F(\widetilde{M})$,

iii). Constant if $V_T(\widetilde{M})$=$V_I(\widetilde{M})$+$V_F(\widetilde{M})$.

Proof. By the Definition 4.2, $V_\lambda(\widetilde{M}) = \frac{1}{12}[(l + 4m + n)\{\lambda^k t_{\widetilde{M}}{}^2 + (1-\lambda^k)(1-i_{\widetilde{M}})^2 + +(1-\lambda^k)(1-f_{\widetilde{M}})^2]$.

Now, $\frac{d}{d\lambda}(V_\lambda(\widetilde{M}))$=k$\lambda^{k-1}$[$V_T(\widetilde{M})$ -$V_I(\widetilde{M})$+$V_F(\widetilde{M})$].

Since, λ ϵ[0, 1], $\frac{d}{d\lambda}(V_\lambda(\widetilde{M}))$>,<,=0 according as $V_T(\widetilde{M})$ >,<,= [$V_I(\widetilde{M})$+$V_F(\widetilde{M})$] respectively.Hence the proof.

Example 4.1. Let's examine the three SVTrN-numbers $\widetilde{U}$=<([1,2,3]; 1.0,0.2,0.4)>, $\widetilde{V}$=<([2,6,8]; 0.8,0.3,0.2)>, and $\widetilde{W}$=<([2,4,6]; 0.5,0.7,0.8)>. According to Definition 4.2, $V_\lambda(\widetilde{U})$=1.00, for all λ ϵ[0, 1].

Then for λ ϵ[0, 1], the weighted values of SVTrN-numbers $\widetilde{V}$ and $\widetilde{W}$ are monotone increasing and decreasing respectively. By the Definition 4.2 and 4.3, $\widetilde{W} \prec \widetilde{U} \prec \widetilde{V}$, for all λ ϵ[0, 1].

Below is the graphical representation of the values of SVTrN-numbers$\breve{U}$, $\widetilde{V}$, and $\widetilde{W}$ for λ ϵ[0, 1]are shown in Figure 2 and Figure 3.

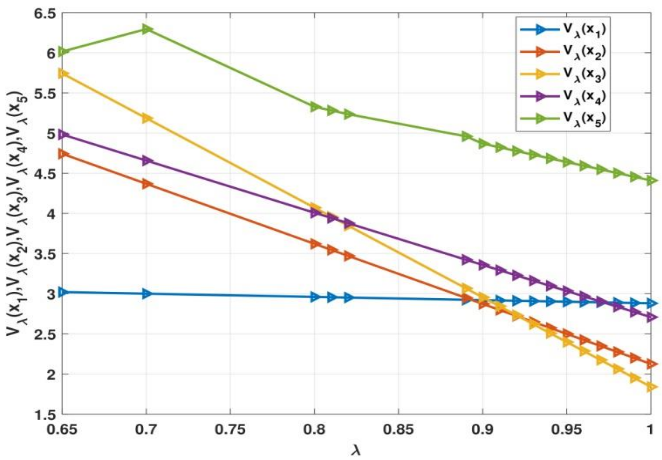

Figure 2. Constant, decreasing, and increasing values of SVTrN-numbers $\breve{U}$, $\breve{V}$, and $\widetilde{W}$ for λ ϵ[0, 1] and k=1.

Example 4.2. Let $\widetilde{A}$ =<([5,6,7]; 0.6,0.3,0.4)>, and $\widetilde{B}$=<(3,4,5); 0.5,0.2,0.3)> be SVTrN-numbers. By weighted value method.

$V_\lambda(\widetilde{A})= \frac{5+4*6+7}{12}[\lambda^k 0.6+(1-\lambda^k)(1-0.3)+(1-\lambda^k)(1-0.4)=3.3-2.1\lambda^k$,

$V_\lambda(\widetilde{B})= \frac{3+4*4+5}{12}[\lambda^k 0.5+(1-\lambda^k)(1-0.2)+(1-\lambda^k)(1-0.3)=6.625-4.417\lambda^k$.

Now $V_\lambda(\widetilde{A})$-$V_\lambda(\widetilde{B})$=-3.325+2.317λ^k<0 for all $k\epsilon\mathbb{N}$ and $\lambda\ \epsilon[0, 1]$.

This implies $V_\lambda(\widetilde{A})<V_\lambda(\widetilde{B})$.

Hence, by ranking method, $\widetilde{A} \prec \breve{B}$.

Property 4.3. For any three SVTN numbers $\widetilde{M}$,$\widetilde{N}$, $\widetilde{P}$, the relation$\preccurlyeq$satisfies the following properties

i). Reflexive if $\widetilde{M} \preccurlyeq \widetilde{M}$,

ii). Anti-symmetric if $\widetilde{M} \preccurlyeq \widetilde{N}$and $\widetilde{N} \preccurlyeq \widetilde{M} \Longrightarrow \widetilde{M} \approx \widetilde{N}$,

iii). Transitive if $\widetilde{M} \preccurlyeq \widetilde{N}$ and $\widetilde{N} \preccurlyeq \widetilde{P} \Longrightarrow \widetilde{M} \preccurlyeq \widetilde{P}$.

Proof. (i), (ii), (iii) are obvious by Definition 4.2.

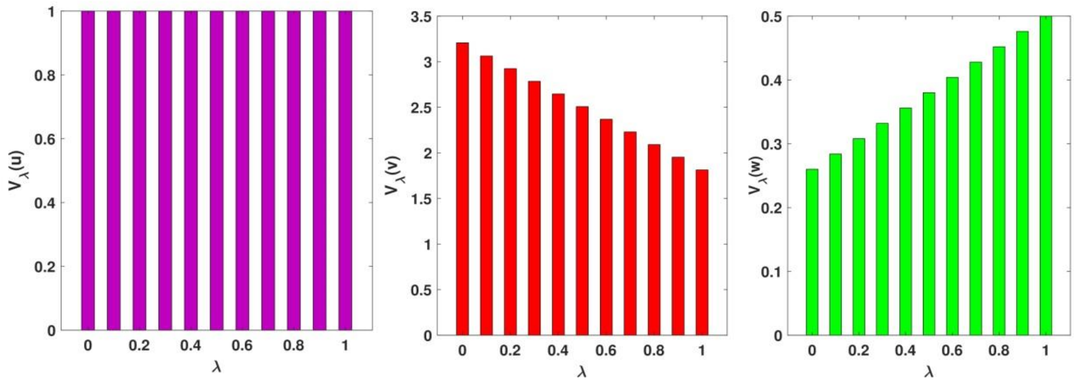

Figure 3. Bar graph of the values of the SVTrN- numbers $\breve{U}$, $\breve{V}$, and $\widetilde{W}$ for $\lambda\ \epsilon[0, 1]$ and k=1.

Remark 4.1. The relations $\preccurlyeq$ fulfill all the conditions of total ordering on a set of all SVTrN numbers.

Remark 4.2. $\widetilde{M} = \widetilde{N}$implies$\widetilde{M} \approx \widetilde{N}$ hold for any two SVTrN-numbers $\widetilde{M}$and $\widetilde{N}$ but the condition $\widetilde{M} \approx \widetilde{N}$ may not imply the relation $\widetilde{M} = \widetilde{N}$, and hence the relation $\approx$ may not be similar to the relation =.

Theorem 4.1. Let $\widetilde{M}$,$\widetilde{N}$ *and* $\widetilde{P}$ be three SVTrN numbers with $t_{\widetilde{M}}=t_{\widetilde{N}}$, $i_{\widetilde{M}}=i_{\widetilde{N}}$, $f_{\widetilde{M}}=f_{\widetilde{N}}$. If $\widetilde{M} \prec \widetilde{N}$, then

i). $\widetilde{M}\oplus\breve{P} \prec \widetilde{N}\oplus\breve{P}$,

ii). $\widetilde{M} \ominus \breve{P} \prec \widetilde{N} \ominus \breve{P}$.

Proof. (i), (ii), and (iii) are obvious (see Definitions 4.2, 4.3).

Theorem 4.2. Let $\widetilde{M}$,$\widetilde{N}$, $\widetilde{P}$ and $\widetilde{Q}$ be four SVTrN numbers with $t_{\widetilde{M}}=t_{\widetilde{N}}$, $i_{\widetilde{M}}=i_{\widetilde{N}}$, $f_{\widetilde{M}}=f_{\widetilde{N}}$, and $t_{\widetilde{P}}=t_{\widetilde{Q}}$, $i_{\widetilde{P}}=i_{\widetilde{Q}}$, $f_{\widetilde{P}}=f_{\widetilde{Q}}$. If $\widetilde{M} \prec \widetilde{N}$ and $\widetilde{P} \prec \widetilde{Q}$ then

i). $\widetilde{M}\oplus\breve{P} \prec \widetilde{N}\oplus\breve{Q}$,

ii). $\widetilde{M} \ominus \breve{Q} \prec \widetilde{N} \ominus \breve{P}$.

iii). $\widetilde{P} \ominus \widetilde{N} \prec \widetilde{Q} \ominus \breve{M}$.

Proof. (i), (ii), (iii) are obvious (see Definitions 4.2, 4.3).

5 | Applications of SVTrN-numbers to an Investment Selection Problem by its Value

Funding organizations desire to invest cash in the best option. There is a panel with five feasible ways to invest: C_1 is an IT enterprise, C_2 is a vehicle enterprise, C_3 is meals enterprise, C_4 is cement enterprise and C_5 is a furniture enterprise. The funding organization has to make choices consistent with the subsequent four attitudes: S_1 is the surroundings effect evaluation; S_2 is the growth evaluation; S_3 is the economic overall performance evaluation; and S_4 are the records envelope evaluations. The four attributes are benefit attributes, and the four attributes of each funding company are taken in terms of SVTrN-numbers because all the attributes change from time to time and are shown in the following Table 2 given below.

Table2. The several attitudes of different organizations.

	S_1	S_2	S_3	S_4
C_1	<([2,3,4]; 0.6,0.1,0.2)>	<([1,4,6]; 0.8,0.5,0.6)>	<([2,4,7]; 0.9,0.2,0.3)>	<([3,5,7]; 0.8,0.3,0.4)>
C_2	<([1,4,7]; 0.8,0.2,0.3)>	<([2,5,8]; 0.7,0.3,0.4)>	<([2,4,6]; 0.5,0.1,0.2)>	<([3,4,5]; 0.6,0.1,0.2)>
C_3	<([3,5,7]; 0.5,0.2,0.3)>	<([4,6,8]; 0.4,0.1,0.2)>	<([5,7,9]; 0.6,0.1,0.2)>	<([2,5,7]; 0.8,0.1,0.3)>
C_4	<([5,6,7]; 0.5,0.1,0.1)>	<([5,7,8]; 0.5,0.2,0.3)>	<([3,4,5]; 0.7,0.1,0.2)>	<([3,5,6]; 0.7,0.4,0.3)>
C_5	<([1,3,4]; 0.7,0.1,0.2)>	<([4,5,8]; 0.8,0.1,0.3)>	<([4,7,9]; 0.8,0.1,0.3)>	<([2,3,4]; 0.8,0.2,0.4)>

Since all the attributes are gain attributes, we convert all the attributes of each funding company into single attributes by using the addition of SVTrN numbers and the comprehensive values of the companies C_i (i=1,2,3,4,5) are given below as follows:

$\widetilde{x_1}$= <([8,16,24]; 0.6,0.5,0.6)>,

$\widetilde{x_2}$= <([8,17,26]; 0.5,0.2,0.3)>,

$\widetilde{x_3}$= <([15,23,31]; 0.4,0.2,0.3)>,

$\widetilde{x_4}$= <([16,22,26]; 0.5,0.4,0.3)>.

$\widetilde{x_5}$= <([11,18,25]; 0.7,0.2,0.4)> respectively.

For simplicity, let us take k=1 and λ ϵ[0.65, 1], because in this interval, the decision maker's behavior indicates optimistic behavior in the direction of positivity and certainty. Now, by Definition 4.2, we calculated the values of SVTrN-numbers x_i (j=1,2,3,4,5) for different values of λ ϵ[0.65, 1], which are shown in the following table given below:

Table3. The values of SVTrN-numbers for different values of λ ϵ[0.65, 1].

λ	$V_\lambda(\widetilde{x_1})$	$V_\lambda(\widetilde{x_2})$	$V_\lambda(\widetilde{x_3})$	$V_\lambda(\widetilde{x_4})$	$V_\lambda(\widetilde{x_5})$
0.65	3.020	4.743	5.744	4.983	6.016
0.70	3.000	4.369	5.186	4.658	6.296
0.80	2.960	3.621	4.071	4.008	5.328
0.81	2.956	3.546	3.959	3.943	5.282
0.82	2.952	3.471	3.848	3.878	5.236
0.89	2.924	2.948	3.067	3.423	4.960
0.90	2.920	2.873	2.955	3.358	4.869
0.91	2.916	2.798	2.844	3.293	4.823
0.92	2.912	2.723	2.732	3.228	4.777
0.93	2.908	2.649	2.621	3.163	4.731
0.94	2.904	2.574	2.509	3.098	4.685
0.95	2.900	2.499	2.398	3.033	4.639

0.96	2.896	2.424	2.286	2.968	4.954
0.97	2.892	2.349	2.175	2.903	4.548
0.98	2.888	2.275	2.063	2.838	4.502
0.99	2.884	2.200	1.952	2.773	4.456
1.00	2.880	2.125	1.840	2.708	4.410

From the above Table 3, we see that for any λ ϵ[0.65, 0.81], $V_\lambda(\widetilde{x_5})>V_\lambda(\widetilde{x_3})>V_\lambda(\widetilde{x_4})>V_\lambda(\widetilde{x_2})>V_\lambda(\widetilde{x_1})$. Then by Definition 4.3, the ordering of the SVTrN-numbers $\widetilde{x_1} \prec \widetilde{x_2} \prec \widetilde{x_4} \prec \widetilde{x_3} \prec \widetilde{x_5}$, and hence the ranking of the corresponding funding organisation is $C_1<C_2<C_4<C_3<C_5$. In this case, the best selection of the funding organization is C_5. However, if λ ϵ[0.82, 0.89], then $V_\lambda(\widetilde{x_5})>V_\lambda(\widetilde{x_4})>V_\lambda(\widetilde{x_3})>V_\lambda(\widetilde{x_2})>V_\lambda(\widetilde{x_1})$. The ordering of the SVTrN-numbers $\widetilde{x_1} \prec \widetilde{x_2} \prec \widetilde{x_3} \prec \widetilde{x_4} \prec \widetilde{x_5}$, and hence the ranking of the corresponding funding organisation is $C_1<C_2<C_3<C_4<C_5$. In this case, the best selection of the funding organization is C_5. Similarly, if λϵ[0.9, 1.0], in all cases, we see that from Table 3 and Figure 4, the best selection of the funding company is C_5.

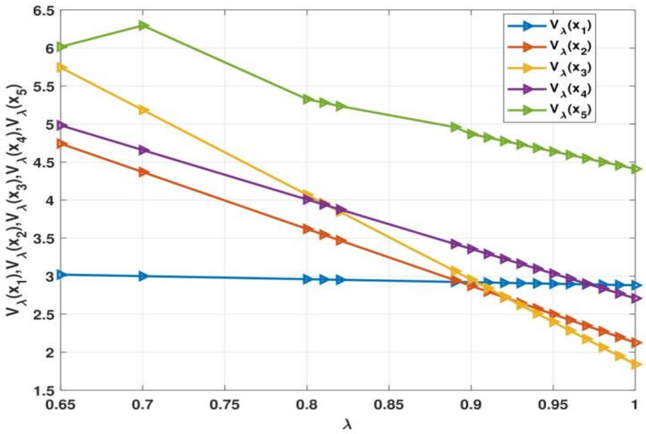

Figure 4. The change of $V_\lambda(\widetilde{x_1}), V_\lambda(\widetilde{x_2}), V_\lambda(\widetilde{x_3}), V_\lambda(\widetilde{x_4})$ and $V_\lambda(\widetilde{x_5})$ with respect to λϵ[0, 1] and k=1.

6 | Comparison Analysis

In this section, to justify the superiority and validity of the weighted value method proposed, we compare the ordering of given SVTrN-numbers by the proposed method for k=1 and λ ϵ[0.65, 1] against other existing approaches [5, 6, 12-16]. We must discard the indeterminacy part when we determine the result of ranking on the IFS. The ranking orders of SVTrN-numbers in each existing approach are shown in the following Table 4, which selects the best funding organization.

Table 4. The selection of the best funding organization by the ranking of given SVTrN-numbers in different approaches.

Authors	Method	Ranking	Best funding organization
Deli and Subas [6]	Value and Ambiguity	$\widetilde{x_3} \prec \widetilde{x_2} \prec \widetilde{x_1} \prec \widetilde{x_4} \prec \widetilde{x_5}$	C_5
Peng et al. [13]	Score, Accuracy, and Certainty	$\widetilde{x_1} \prec \widetilde{x_4} \prec \widetilde{x_3} \prec \widetilde{x_2} \prec \widetilde{x_5}$	C_5
Ye, J. [12]	Score	$\widetilde{x_1} \prec \widetilde{x_3} \prec \widetilde{x_4} \prec \widetilde{x_2} \prec \widetilde{x_5}$	C_5
Manas et al. [16]	Value and Ambiguity	$\widetilde{x_1} \prec \widetilde{x_2} \prec \widetilde{x_5} \prec \widetilde{x_3} \prec \widetilde{x_4}$	C_4
Deli and Subas [5]	Score and Accuracy	$\widetilde{x_1} \prec \widetilde{x_2} \prec \widetilde{x_5} \prec \widetilde{x_4} \prec \widetilde{x_3}$	C_3
Qiang and Zhong [14]	Score and Accuracy	$\widetilde{x_1} \prec \widetilde{x_3} \prec \widetilde{x_2} \prec \widetilde{x_4} \prec \widetilde{x_5}$	C_5
Suresh et al. [15]	Magnitude	$\widetilde{x_2} \prec \widetilde{x_1} \prec \widetilde{x_5} \prec \widetilde{x_3} \prec \widetilde{x_4}$	C_4

Now to establish the re-ability of this method, we consider all the analyses of ranking evaluation stated in the above table and compare these rankings by value method with existing approaches given in the following:

i). Deli and Subas [6] designed the ordering system of SVN numbers by weighted value and ambiguity method, and the result is nearly identical to the result of the proposed approach and the same selection of funding organization for different values of parameters.

ii). Peng et al. [13] described the score, certainty, and accuracy and applied this notion to find out the ordering of SVTrN-numbers, which is close to the result of the mentioned method and the result of funding organization is the same.

iii). Ye. [12] established a ranking algorithm by score function, and applying the ranking method, the ordering and best funding organization is given in the above tale, which is almost equal to the introduced method.

iv). Manas [16] introduced a fractional method using value and ambiguity and applied this method to determine the ranking of SVTrN numbers. In this method, the best selection is C_4.

v). Deli and Subas [5] designed the accuracy and score to find out the ordering SVTrN-numbers, but they did not use the weight of the ranking function, and hence the ordering result is unequal to the results of ranking by the mentioned method.

vi). Qiang and Zhong [14] determine the ordering of SVTrN-numbers by score and accuracy and the result of ranking nearly as proposed method and the result of funding organization is the same.

vii). Suresh Mohan et al. [15] determine the ordering of neutrosophic numbers by magnitude method, and the ordering result is nearing the ordering results of the proposed method for $\lambda \in [0.65,1]$ and has few differences because it has no hesitancy part.

7 | Conclusion

In this article, we establish a decision-based ranking method with the help of the value of SVTrN-numbers and apply it to solve the MCDM for investment selection problem. The proposed method is simple, appreciated, and applied to real-world activities, giving the selection ability to solve this issue at different levels of decision-making. In the future, ranking methods will be investigated more effectively, and we will apply this notion to optimization, algebraic structure, game theory, and so on.

Acknowledgments

The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Author Contributions

All authors contributed equally to this work.

Funding

This research has no funding source.

Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

References

[1] Zadeh L.A. Fuzzy sets. Information and control, 8: 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X(1965)

[2] Atanassov, K.T. Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20(1), 87-96. https://doi.org/10.1016/S0165-0114(86)80034-3.(1986).

[3] Smarandache, F. Neutrosophy, neutrosophic probability, set and logic. Amer. Res. Press, Rehoboth, USA. p. 105. http://fs.gallup.unm.edu/eBook-neutrosophics6.pdf (sixth version)., (1998). DOI: 10.5281/zenodo.57726

[4] Wang, H., Zhang, Y., Sunderraman, R., and Smarandache, F. Single valued neutrosophic sets. Fuzzy Sets, Rough Sets and Multivalued Operations and Applications, 3(1): 33-39. (2011).

[5] Deli, I., Subas, Y. A ranking method of single valued neutrosophic numbers and its application to multi-attribute decision making problems, Int. J. Mach. Learn.and Cyber, 8(4):1309–1322, (2016). DOI:10.100713042-016-0505-3

[6] Deli, I., Subas, Y.Single valued neutrosophic numbers and their applications to multicriteria decision making problem, Neural computing and applications. (2014).

[7] Bera, T., Mahapatra, N.K. (2021) Ranking of thermal power plants focusing on air pollution: A Neutrosophic assessment, Journal of Cleaner Production, 316, 128250, https://doi.org/10.1016/j.jclepro.2021.128250.

[8] Mehlawat, M.K. and Grover, N. Intuitionistic fuzzy multi-criteria group decision making with an application to critical path selection. Annals of Operations Research, 269(1-2), 505-520. (2018). https://doi.org/10.1007/s10479-017-2477-4

[9] Subas, Y. Neutrosophic numbers and their application to Multi-attribute decision making problems (In Turkish Masters Thesis, Kilis 7 Aralık University, Graduate School of Natural and Applied Science. (2015).DOI:10.1007/s13042-016-0505-3

[10] De, P.K., Das, D. Ranking of trapezoidal intuitionistic fuzzy numbers,National Institute of Technology Silchar, India, (2012).DOI: 10.1109/ISDA.2012.6416534

[11] Bera, T. and Mahapatra, N.K. (α,β,γ)-cut neutrosophic soft set and it's application neutrosophic soft groups, Asian Journal of Math. and Compt. Research, 12(3),160-178. (2016). http://go7publish.com/id/eprint/3898

[12] Ye, J. A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. J. Intell. Fuzzy Syst. 26, 2459–2466. [Google Scholar] [CrossRef]. (2014). DOI: 10.3233/IFS-130916

[13] Peng, J.-J.; Wang, J.-Q.; Wang, J.; Zhang, H.-Y.; Chen, X.-H. Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems. Int. J. Syst. Sci., 47, 1–17. [Google Scholar] [CrossRef]. (2016). DOI: 10.1080/00207721.2014.994050

[14] W.J.Qiang,Z.Zhong,Aggregation operators on intuitionistic trapezoidal fuzzy number and its application to multi-criteria decision making problems, Journel of System Engineering and Electronics, 20, 2, 321-326(2009).

[15] Suresh Mohan, Arun Prakash Kannusamy, VengataasalamSamiappan, A new Approach for Ranking of Intuitionistic Fuzzy Numbers, J. Fuzzy. Ext. Appl. Vol. 1, 15-26 (2020). https://doi.org/10.22105/jfea.2020.247301.1003

[16] Karak M., Bharati P., Mahata A., paul S., Biswas S., Mukherjee S., Broumi S., Rong M. and Roy B., Ranking of Neutrosophic number based on values and ambiguities and its application to linear programming problem,Neutrosophic Sets and Systems, Vol. 63,(2024). https://digitalrepository.unm.edu/nss_journal/vol63/iss1/13.

[17] KarakManas, Mahata. A, Rong. M, Mukherjee. S, Mondal S.P, Broumi. S, Roy. B (2023), A solution technique of Transportation Problem in Neutrosophic Environment, Neutrosophic System with applications, Vol.3(2023). https://doi.org/10.61356/j.nswa.2023.5

[18] Maissam Jdid, Huda E Khalid, Mysterious Neutrosophic Linear Models, International Journal of Neutrosophic Science, Vol.81, No. 2(2022). DOI: 10.54216/IJNS.180207

[19] Qamar M.A and Nasruddin Hassan N., An Approach toward a Q-Neutrosophic Soft Set and Its Application in Decision Making, 11(2), 139; https://doi.org/10.3390/sym11020139 (2019)

[20] Bera T., Mahapatra N.K., Neutrosophic linear programming problem and its application to real life, AfrikaMatematika, Volume 31, pages 709–726, (2020). DOI: 10.1007/s13370-019-00754-4

[21] Bera, T., Mahapatra, N.K. On solving linear programming problem by duality approach in neutrosophic environment, Int. J. Mathematics in Operational Research, 18(3). DOI : 10.1504/IJMOR.2020.10029862.(2021).

[22] Bera, T., Mahapatra, N.K. A distinct management of Linear Programming in uncertain atmosphere, IJMOR. DOI: 10.1504/IJMOR.2022.10049822(2022).

[23] Acharya A., Mahata A., Sil N., Mahato S., Mukharjee S., Mahato S.K., Roy B. A prey-refuge harvesting model using intuitionistic fuzzy sets. Decision Analytics Journal, 8 (2023), Article 100308. DOI: 10.1016/j.dajour.20i23.100308

[24] Acharya A., Mahato A., Mukharjee S., Biswas M.A., Das K.P., Mondal S.P., Roy B. A neutrosophic defferential equation approach for modelling glucose distribution in the bloodstream using neutrosophic sets. Decision Analytics Journal, 8 (2023), Arrticle 100264. DOI:10.1016/j.dajour.2023.100264

[25] Mahato A, Mondal SP, Roy B, et al. Influence of impreciseness in designing tritrophic level complex food chain modeling in interval environment. Adv Differ Equ 2020;399. https://doi.org/10.1186/s13662-020-02841-4

[26] G. Tamilarasi and S. Paulraj, An improved solution for the neutrosophic linear programming problems based on Mellin's transform, Fuzzy systems and their mathematics, Volume 26, pages 8497–8507, (2022). https://doi.org/10.1007/s00500-022-07252-z

[27] KalaivaniK.andPalanivelKaliyaperuma, A neutrosophic approach to the transportation problem using single-valued trapezoidal neutrosophic numbers, Proyecciones Journal of Mathematics, Vol. 42, No.2, pp. 533-547, doi. 10.22199/issn.0717-6279-5374(2023).DOI: 10.22199/issn.0717-6279-5374

[28] Mohamed Assarudeen, S. N, Sudhakar, V.J, and Balakrishnan S, A new approach for solving assignment problem in neutrosophic environment by zero suffix method, International Journal of Current Advanced Research, Vol 7, Special Issue: 1, pp 129-132, DOI: http://dx.doi.org/10.24327/IJCAR(2018).

[29] Alzahrani,F.A.;Ghorui,N.;Gazi,K.H.;Giri,B.C.;Ghosg,A.;Mondal,S.P,Optimal site selection for Women university using Neutrosophic multi-crioteria Decision making Approach.Buildings ,13,152(2023). https://doi.org/10.3390/buildings13010152

[30] Khatter, K.: Neutrosophic linear programming using possibilistic mean. Soft. Comput. 24(22), 16,847-16,867 (2020). https://doi.org/10.1007/s00500-020-04980-y

[31] Tongjuan Wang, Distance of Single-Valued Neutrosophic Set and Its Application in Pattern Recognition, Journal of Physics: Conference Series, 2025 (2021) 012019, doi:10.1088/1742-6596/2025/1/012019.

[32] Liang, C., Zhao, S., and Zhang, J. Aggregaton operators on trangularintutonstc fuzzy numbers and its applcation to multicriteria decision making problems. Found Comput DecisSci, 39(3): 189–208 (2014). DOI: 10.2478/fcds-2014-0011

[33] Aal, Samah Ibrahim Abdel, et al. "Two Ranking Methods of Single Valued Triangular Neutrosophic Numbers to Rank and Evaluate Information Systems Quality." Neutrosophic Sets and Systems, vol. 19, Mar. , pp. 132 (2018).

[34] Khalifa, H. A., An approach to the optimization of multi- objective assignment problem with neutrosophic numbers. International Journal of Industrial Engineering and Production Research, 31(2), 287- 294(2020). DOI: 10.22068/ijiepr.31.2.287

[35] SurapatiPramanik, Suman Das, Rakhal Das, and Binod Chandra Tripathy, Neutrosophic BWM-TOPSIS Strategy under SVNS Environment, Neutrosophic Sets and Systems, Vol. 56 (2023). https://digitalrepository.unm.edu/nss_journal/vol56/iss1/13

[36] Nabeeh, N.A.; Abdel-Basset, M.; El-Ghareeb, H.A.; Aboelfetouh, A. Neutrosophic multi-criteria decision making approach for iot-based enterprises. IEEE Access , 7, 59559–59574, 2019. DOI: 10.1109/ACCESS.2019.2908919

[37] Zolfani, S.H.; Hasheminasab, H.; Torkayesh, A.E.; Zavadskas, E.K.; Derakhti, A. A Literature Review of MADM Applications for Site Selection Problems—One Decade Review from 2011 to 2020. Int. J. Inf. Technol. Decis. Mak. 21, 7–57, 2022. DOI: 10.1142/S0219622021300019.

[38] Kharat, M.G.; Kamble, S.J.; Raut, R.D.; Kamble, S.S.; Dhume, S.M. Modeling landfill site selection using an integrated fuzzy MCDM approach. Model. Earth Syst. Environ. 2, 53. 2016. https://doi.org/10.1007/s40808-016-0106-x