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Extended Certainty Factors utilizing Neutrosophic Logic

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Abstract

Addressing uncertainty is a significant challenge in the field of artificial intelligence (AI). AI systems often encounter uncertainty due to incomplete data, ambiguous information, or inherent unpredictability in specific situations. To tackle this uncertainty, various solutions have been developed. These solutions range from probabilistic approaches like Bayesian networks and Monte Carlo methods to fuzzy logic and neural networks. These strategies enable AI systems to model and make judgments amid ambiguity by assigning probabilities, dealing with imprecise data, or utilizing learning processes that adapt to changing and uncertain contexts. However, probabilistic methods have limitations when it comes to handling uncertainty. These limitations include assumptions of independence, computational complexity, difficulty in capturing subjectivity, and interpretability. To address these limitations, other methods have been proposed, such as the model of certainty factors. This model offers a framework for reasoning under uncertainty by assigning a numerical value to statements or propositions. Additionally, neutrosophic logic extends classical logic by incorporating the concept of indeterminacy through truth/indeterminacy/falsity-membership functions. In this paper, we propose a method for investigating how certainty factors adapt to a neutrosophic environment. This method contributes to the development of more robust and adaptable decision support systems capable of dealing with diverse uncertainty. Our method, introduced for the first time in related literature, addresses issues such as the limited handling of indeterminacy, the inability to address contradictions, and the limited binary representation of uncertainty that characterize the model of certainty factors. We will provide a clear illustration of these implications through an example that demonstrates the superiority of our suggested method over the traditional technique of certainty factors.

Keywords: Uncertainty, Indeterminacy, Certainty Factors, Neutrosophic Logic, Artificial Intelligence, Decision-making, Approximation Methods, Soft Computing.

1 | Introduction

The absence of specific knowledge in a reasoning process (for example, when making a choice) is referred to as uncertainty. The following are the primary causes of uncertainty encountered when solving problems [1]:

- Inaccurate data, such as that obtained from a low-accuracy sensor.
- Incomplete data, such as when several sensors in a control system fail and an instant judgment must be made using data from the remaining sensors.

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- Subjectivity or deficiencies in the description of knowledge, particularly when using heuristic methods, where subjectivity is frequently incorporated.
- Any limits that render the whole decision-making framework insufficient, such as cost constraints that make some measurements unprofitable and time constraints that need an immediate decision to be taken to defuse dangerous situations.

Handling uncertainty is a significant challenge in the field of Artificial Intelligence (AI). The pursuit of accurate decision-making often grapples with incomplete information, ambiguous data, and unpredictable circumstances. Uncertainty permeates various aspects of AI, stemming from factors like noisy data, imprecise measurements, or the inherent complexity of real-world environments.

AI-enabled decision-making systems rely on computers to gather relevant data, construct modeling algorithms, and arrive at a conclusion. However, the issue of ambiguity in decision-making has not been adequately addressed. This raises two questions: (1) What are the sources of uncertainty? (2) How can uncertainty be tackled in AI-enabled decision-making applications?

Traditional AI methods for dealing with uncertainty have drawbacks, including probabilistic approaches such as Bayesian networks and Monte Carlo methods, as well as fuzzy logic and neural networks. These constraints include assumptions of independence, computational complexity, difficulties capturing subjectivity, and interpretability concerns. The classic model of certainty factors, while useful, suffers from a basic binary representation of uncertainty, insufficient handling of indeterminacy, and an inability to adequately handle contradictions.

To solve these constraints, this paper suggests a novel methodology that combines certainty factors with neutrosophic logic. Neutrosophic logic builds on classical logic by introducing the idea of indeterminacy via truth/indeterminacy/falsity-membership functions. By merging these two models, we hope to improve the resilience and flexibility of decision support systems across a wide range of uncertain scenarios.

Certainty factors (CF) [2-3] play a crucial role in the AI landscape as they provide a formal framework for dealing with uncertainty and making informed decisions in knowledge-based systems. In the field of artificial intelligence, certainty factors are numerical representations of confidence or conviction in the accuracy of claims or hypotheses. These variables, typically ranging between -1 and +1, aid in decision-making by integrating data from multiple sources to reach conclusions in the presence of uncertainty or limited knowledge. Initially rooted in expert systems, certainty factors have significantly enhanced the flexibility and robustness of AI models. They enable these models to handle challenging scenarios by combining diverse data sources and managing conflicting information. Certainty factors, within the realm of AI and decision-making, find various applications in domains such as expert systems [3-8], medical diagnosis [9-16], fault diagnosis in engineering [17-18], financial analysis [19-20], natural language processing [21] and risk assessment and management [22-23].

Certainty factors, often utilized in expert systems and decision support systems for reasoning under uncertainty, are a valuable tool in computational reasoning. They are especially useful in scenarios where uncertainty is common and human judgment or expertise is essential. However, this method, with its simple binary representation, may not fully capture the complexity of real-world uncertainties that exist on a spectrum. Other limitations include difficulties in managing conflicting evidence, a lack of quantitative precision, and an inability to adapt to contextual changes.

Neutrosophic logic, an advanced extension of classical logic, transforms how we perceive and address uncertainty, indeterminacy, and contradictions in complex systems. Neutrosophic logic introduces a trio of functions, namely truth-membership, indeterminacy-membership, and falsity-membership to resolve constraints in classical logic when faced with partial, imprecise, or conflicting information [24]. This innovative paradigm acknowledges not only true and false values but also a third domain of indeterminacy, where items can possess both truth and falsity characteristics simultaneously. The ability of neutrosophic logic to capture and formalize this inherent complexity makes it a crucial tool in various domains, including artificial intelligence, decision sciences, engineering, and philosophy.

Neutrosophic logic's versatility and ability to handle uncertainties, contradictions, and vague information find applications across various domains. It has been successfully applied to decision-making systems [25-30], medical diagnosis and healthcare [31-36], pattern recognition and image processing [37-41], control systems and robotics [42-45], engineering and risk management [46-49] and environmental studies [50-53].

1.1 | Motivation and Research Objectives

- The main motivation concerning our study is the observation that lack of evidence for the truth or falsity of a sentence cannot be expressed rigorously within the certainty factors (CF) model. In such a model this is applied by assigning zero value to the certainty factor of the sentence, i.e. CF=0. This limitation in expressing indeterminacy motivated us to hybridize CF models with neutrosophic logic. Within this framework, lack of evidence aligns with indeterminacy, uncertain membership degrees, and incomplete information, signifying situations where the available evidence is insufficient or inconclusive to determine the truth or falsity of a statement with certainty. In other words, our primary aim was to pursue the extension of an uncertainty theory by which we would be able to recognize our ignorance and indeterminacy without ignoring and overlooking available information.
- CFs are calculated using two independent measurement units: belief and disbelief. The requirement for two unique and independent measures stems from a comprehension of confirmation theory, which states that evidence supporting one hypothesis does not always imply evidence against that hypothesis. In favor of this, many researchers have stated that, although they trust in a hypothesis to some extent, they are unwilling to say that they agree with the hypothesis's negation to some extent [54]. In our study, we integrate neutrosophic logic with certainty factors to get intuitively correct results regarding the aforementioned problem. The idea of complement in neutrosophic logic is similar to that of classical set theory; however, in neutrosophic logic, an element's membership in a set may have a truth degree, an indeterminacy degree, and a falsehood degree, rather than a strict binary membership as in classical sets.
- CF models might struggle with detached and locality reasoning or overlooking uncertainty. By hybridizing CFs with neutrosophic logic, our motivation is to provide a framework that will allow for a clearer representation of uncertainty, improving the accuracy of conclusions. In this manner, an integrated technique can help to reduce mistakes that might occur when relying solely on CF models. The possibility for reasoning mistakes due to detachment or location concerns is reduced by exploiting the capabilities of models like neutrosophic logic and certainty factors.
- CFs are often binary, indicating either certainty (positive) or doubt (negative). However, this simplistic view may not accurately capture the intricacies of uncertainties in the real world, which exist on a spectrum. By incorporating neutrosophic logic, we propose a more suitable framework that can represent uncertainties in a more nuanced manner, encompassing not just true/false but also indeterminate states.
- Many real-world situations involve complex uncertainties that cannot be categorized as either true or false. By utilizing certainty factors within a neutrosophic framework, we can better model uncertainties in relation to the complexities of these real-world scenarios.
- The integration of certainty factors into neutrosophic frameworks enhances the decision-making process. This combination enables systems to make judgments based on a more comprehensive understanding of uncertainty, leading to more informed and adaptable choices.
- To propose a method that combines certainty factors with neutrosophic logic, allowing for a more sophisticated depiction of uncertainty, including indeterminate states.

- To demonstrate the suggested method's advantages over traditional certainty factors using illustrative cases.
- To establish a theoretical foundation and practical implementations for the suggested strategy in a variety of sectors where AI decision-making is crucial.

1.2 | Novelties

- Decision support systems can make better-informed judgments by incorporating neutrosophic certainty factors, which take into account not only certainties and uncertainties but also degrees of indeterminacy. In complicated and unpredictable contexts, this refined approach aids in more accurate decision-making.
- Neutrosophic logic is based on a triadic concept that encompasses truth, indeterminacy, and falsity. The certainty factors within this framework can regulate and convey the extent to which a statement or proposition is true, indeterminate, or false, thereby facilitating a more sophisticated understanding of uncertainty.
- Neutrosophic certainty factors pave the way for the emergence of more complex reasoning mechanisms. They enable systems to reason with insufficient, inconsistent, or inaccurate data, resulting in more resilient and flexible reasoning processes.

1.3 | Contributions

- To the best of the author's knowledge, this is the first research in the related literature that combines neutrosophic logic with certainty factors. In this approach, we aim to present a new formalism that will serve as a strong theoretical foundation in the field of knowledge representation and reasoning, particularly in dealing with uncertainty.
- Our methodology introduces an innovative approach to modeling uncertainties, offering a sophisticated representation that goes beyond binary concepts and addresses situations characterized by indeterminacy.
- Our research not only contributes to the theoretical framework of neutrosophic certainty factors but also demonstrates their practical applicability in various domains. This lays the groundwork for advanced decision support systems and opens doors for future enhancements and integrations within hybrid uncertainty models.

1.4 | Structure of the Paper

The current research work follows the next structure: Section 2 summarises the main concepts and ideas needed to comprehend the fundamental principles of neutrosophic logic and certainty factors to construct our theory and propose our logic formalism, called neutrosophic certainty factors (NCF). Section 3 provides an illustrative example to demonstrate NFC's applicability and expressiveness in a real-world setting. Following that, in section 4, we describe why and where our formalism may find fruitful study ground, and in the last section, we emphasize NEC's utility and relevance from a scientific standpoint, which could lead the way for academics and practitioners. Then, in Section 5 we briefly provide a comparative analysis of our proposed method with other well-known approaches in AI that handle uncertainty, aiming to elucidate the advantages of the proposed NCF approach. Lastly, we provide our concluding remarks about the practical implications and potential applications of our methodology and propose future research work based on the limitations of our study.

2 | Materials and Methods

In this section, we will first introduce the fundamental ideas and definitions of certainty factors and neutrosophic logic, which will provide the foundation for describing our suggested formalism, namely neutrosophic certainty factors (NCF).

2.1 | Certainty Factors

Bayes' formulae are complicated enough and insufficient for human brain reasoning functions. Certainty factors theory is a viable alternative to Bayesian reasoning when trustworthy statistical data is unavailable or evidence independence cannot be assumed. In this manner, certainty factors avoid the problem of computing all simple and conditional probabilities that require using Bayes' law. Furthermore, calculations when combining certainties are simpler, due to the assumption of independence of events [1].

Certainty factors are numerical values that express the certainty of the truth of a sentence or event. They were first introduced into the MYCIN expert system to add some degree of certainty to the conclusions of the various rules [3]. Such rules have a general form:

If *event* then *hypothesis* with *certainty factor CF*

That is, if the fact is true, then we are sure of the hypothetical conclusion to the degree of CF. The certainty factor takes values in the interval [-1, +1]. The value -1 expresses absolute certainty about the falsehood of the proposition, the value +1 absolute certainty about its truth, while the value 0 expresses ignorance.

Since a certainty factor concerning some hypotheses should provide some measure of certainty of our belief in the hypothesis, we could state the following axiom:

Axiom 1. A certainty factor is a function whose range is the interval [-1, +1]. The certainty factor *CF(H,E)* is intended to measure the *change* in belief in *H* given the evidence *E* with 0 indicating no change, +1 indicating that *H* is certain, and -1 indicating that $\neg$ *H* is certain. As *CF(H,E)* increases, so does the change in belief in *H* given *E*. The value of *CF(H,E)* does not depend on the prior belief in *H*.

For example, the following rule:

R1: If fever then flu 0.85

expresses the fact that if a patient has a fever then the case that he has flu can be made with certainty factor 0.85.

In addition to the certainty accompanying the rule, it is possible to assign certainty values to the value of the rule's event (or events). In this case, the final certainty of the hypothetical conclusion is equal to the product of the certainties. For example, consider the following rule:

R2: If $fever_{CF=0.65}$ *then flu 0.85*

The fact that the patient has a fever is recorded with a certainty of 0.65. The latter is possible for example in the case where the fever is not measured with a thermometer but is estimated by the touch. In this case, the certainty of the hypothetical conclusion of the rule will be 0.65 * 0.85 = 0.55.

If there is more than one event in the left part of the rule which are associated with AND (or with OR) then the certainty factor of the left part is considered as the smallest (or largest) value of CF that appears. This is because in such cases certainty is determined by the degree of certainty of the least (or most) possible event. The total certainty factor of the rule is again obtained by the product of the total certainty factor of the left part and the certainty factor of the hypothetical conclusion.

Thus, the certainty factor of a constructed premise A is calculated using the following formulae, starting with the certainty degrees of the components [55]:

$$\text{cf (A)} = \max(\text{cf}(A_1), \text{cf}(A_2)) \quad \text{if } A = A1 \vee A2 \tag{1}$$

$$\text{cf (A)} = \min(\text{cf}(A_1), \text{cf}(A_2)) \quad \text{if } A = A1 \wedge A2 \tag{2}$$

$$\text{cf (A)} = -\text{cf (A)} \quad \text{if } A = \bar{A}. \tag{3}$$

For example in the following rule:

R3: If $fever_{0.8}$ *and* $cold_{0.7}$ *then flu 0.9*

the overall certainty factor of the recorded events i.e. of the left part of the rule is:

CF_{if} = *min* (CF_{fever},CF_{cold}) = *min (0.8, 0.7) = 0.7*

while the hypothetical conclusion that the patient has the flu is inferred with certainty:

CF_{flu} = CF_{if} * *0.9 = 0.7 * 0.9 = 0.63.*

In the case that the certainty of some hypothetical conclusion is already CF_p and the activation of another rule draws the same hypothetical conclusion with certainty CF_n, then the overall certainty of the hypothetical conclusion is determined by the signs of CF_p and CF_n, based on the following relations [55]:

- If $CF_p > 0$ and $CF_n > 0$, then CF = $CF_p + CF_n\,(1 - CF_p) = CF_p + CF_n - CF_nCF_p$ (4)
- If $CF_p < 0$ and $CF_n < 0$, then CF = $CF_p + CF_n\,(1 + CF_p) = CF_p + CF_n + CF_nCF_p$ (5)
- If $CF_p * CF_n < 0$, then CF = $\dfrac{CF_p + CF_n}{1 - \min(|CF_p| * |CF_n|)}$ (6)

2.2 | Neutrosophic Logic

Neutrosophic logic is concerned with three essential elements: truth, falsehood, and indeterminacy. Truth refers to the degree to which a proposition is entirely true, while falsehood refers to the extent to which a statement is absolutely untrue. Indeterminacy, on the other hand, refers to the degree of uncertainty in a proposition, or the idea that it is partially true and partially untrue. By using truth degrees, in neutrosophic logic, a concept A is T% true, I% indeterminate, and F% false, where (T, I, F) ⊂ $||{-0}, 1^+||^3$, $||{-0}, 1^+||$ being an interval of hyperreals.

In this paradigm, truth, falsity, and indeterminacy can all exist concurrently, allowing for a more thorough representation of complicated and uncertain information. Sets that contain neutrosophic components are used in neutrosophic logic, where the elements have degrees of membership in truth, falsity, and indeterminacy. Its ability to handle ambiguity and uncertainty makes it valuable in situations where traditional logic systems might struggle to provide accurate representations.

In this framework, a formula φ is characterized by a triplet of truth values, called the *neutrosophic value* defined as [56]:

$$\text{NL}(\varphi) = (\text{T}(\varphi), \text{I}(\varphi), \text{F}(\varphi)) \tag{7}$$

where $(\text{T}(\varphi), \text{I}(\varphi), \text{F}(\varphi)) \subset ||{-0}, 1^+||^3$

2.3 | Neutrosophic Certainty Factors

To introduce our method we first give basic definitions and operations of single-valued neutrosophic numbers (SVNNs).

Definition 1 [57]. Let A = ($T_A(x)$, $I_A(x)$, $F_A(x)$) and B= ($T_B(x)$, $I_B(x)$, $F_B(x)$) be two any SVNNs. Then the following set operations hold:

$$\bar{A} = (\text{FA}(x), 1\text{-IA}(x), \text{TA}(x)), \text{ for all } x \in X \tag{8}$$

where X is a universal space of points (objects).

$$A \cap B = (\min (TA(x),TB(x)), \max (IA(x),IB(x)), \max (FA(x),FB(x))) \text{ for all } x \in X \tag{9}$$

$$A \cup B = (\max (TA(x),TB(x)), \min (IA(x),IB(x)), \min (FA(x),FB(x))) \text{ for all } x \in X \tag{10}$$

Definition 2 [57]. Let A = ($T_A(x)$, $I_A(x)$, $F_A(x)$) and B= ($T_B(x)$, $I_B(x)$, $F_B(x)$) be two any SVNNs. Then

$$A \oplus B = (TA(x) + TB(x)-TA(x)\cdot TB(x), IA(x)\cdot IB(x), FA(x)\cdot FB(x)) \tag{11}$$

$$A \otimes B = (TA(x)\cdot TB(x), IA(x)+IB(x)- IA(x)\cdot IB(x), FA(x)+FB(x)- FA(x)\cdot FB(x)) \tag{12}$$

Definition 3 [58]. Let A = (T, I, F) be a single-valued neutrosophic number, and let F be a single-valued neutrosophic scoring function. The level of membership, based on the degree of membership, indeterminacy, and falsehood degree of membership of A, is specified by:

$$F(A) = \frac{1+T-2I-F}{2} \tag{13}$$

Definition 4. Let CF_A = (CF_T,CF_I,CF_F) be the NCF for proposition A, where: CF_T is the certainty of the truth of A, CF_I is the certainty of the indeterminacy of A and CF_F is the certainty of the falsity of A.

These values are calculated based on the following operations:

Negation: $\neg A = (CF_F,CF_I,CF_T)$ (14)

This operation swaps the truth and falsity components.

Conjunction(AND):A∩B=(min(CFT(A),CFT(B)),max(CFI(A),CFI(B)),max(CFF(A),CFF(B))) (15)

Disjunction(OR):A∪B=(max(CFT(A),CFT(B)),min(CFI(A),CFI(B)),min(CFF(A),CFF(B))) (16)

Based on Axiom 1, given in subsection 2.1 for certainty factors, we can apply the following axiom that holds in a neutrosophic environment:

Axiom 2. A neutrosophic certainty factor $CF_A(H,E)$ is a function whose range is the interval [-1, +1]. It measures the degree of belief in *H* given the evidence *E*, where:

$CF_A(H,E)$ = +1 indicates that *H* is certain,

$CF_A(H,E))$ = -1 indicates that ¬*H* is certain,

$CF_A(H,E)$ = 0 indicates complete indeterminacy or neutrality (neither *H* nor ¬*H* is certain).

As $CF_A(H,E)$ increases (from -1 to +1), the degree of belief in *H* given *E* changes accordingly. The value of $CF_A(H,E)$ reflects the level of certainty or uncertainty considering all possible truth values of *H* (true, indeterminate, false). Importantly, $CF_A(H,E)$ is independent of the prior belief in *H* and captures the overall impact of evidence *E* on the belief in *H* within the neutrosophic framework.

Now, we can re-examine the rules that were used in subsection 2.1 and use certainty factors in a neutrosophic environment instead. For example, let's consider rule 2:

R4: If $fever_{CF=(0.65,0.1,0.25)}$ then flu (0.7, 0.1, 0.2)

Using this approach, we can express certainty factors more realistically and expressively. We can argue that the evidence of the patient having a fever is 65% true, 25% false, and 10% indeterminate, as it was estimated through touch. Based on this evidence, we can assume that the hypothetical part of rule 2, which states that the patient has the flu (due to the fever), is 70% true, 20% false, and 10% indeterminate, possibly due to a malfunctioning thermometer or other measurement issues.

Let us now examine rule 3 in the neutrosophic framework, thus utilizing neutrosophic certainty factors (NCF).

R5: If $fever_{(0.8,0.1,0.1)}$ and $cold_{(0.7,0.15,0.15)}$ then flu (0.8, 0.1, 0.1)

According to equation (15), the overall certainty factor of the recorded events i.e. of the left part of the rule is:

CF_{if} = $CF_{fever} \cap CF_{cold}$ = (min (0.8, 0.7), max (0.1,0.15), max (0.1,0.15)) = (0.7,0.15,0.15)

while the hypothetical conclusion that the patient has the flu is inferred with neutrosophic certainty by calculating equation (12):

CF_{flu} = $CF_{if} \otimes$ (0.8, 0.1, 0.1) = (0.7,0.15,0.15) $\otimes$ (0.8, 0.1, 0.1) = (0.7*0.8, 0.15+0.1 -0.15*0.1, 0.15+0.1 -0.15*0.1) = (0.56, 0.24, 0.24).

3 | Results

3.1 | Interpretation of Neutrosophic Values

Neutrosophic values are expressed as triplets (T, I, F), representing the degrees of truth, indeterminacy, and falsity, respectively. Each component ranges between 0 and 1.

Truth (T): This value indicates the degree of belief that a statement or hypothesis is true. Higher values signify greater confidence in the truth of the proposition.

Indeterminacy (I): This value represents the degree of uncertainty or ambiguity about the truth or falsity of a proposition. Higher values indicate more uncertainty.

Falsity (F): This value reflects the degree of belief that a statement or hypothesis is false. Higher values signify greater confidence in the falsity of the proposition.

3.2 | Illustrative Examples of Neutrosophic Certainty Factors

Example 1. Let us assume that two rules lead to the same hypothetical conclusion B but under different assumptions. For example:

R6: If $X_{CF=0,45}$ *then B 0.75,*

R7: If $Y_{CF=0.8}$ *and* $Z_{CF=0.7}$ *and* $\Omega_{CF=0.45}$ *then B 0.65*

According to what we have said in subsection 2.1., it follows that for rule R6 it will be:

$$CF_p = 0.45 * 0.75 = 0.34 \quad (17)$$

while for R7 (equation (2)):

$$CF_n = 0.65 * \min(0.8, 0.7, 0.45) = 0.65 * 0.45 = 0.29 \quad (18)$$

Since CF_p and CF_n are both positive, the total certainty of the hypothetical conclusion B will be Eq. (4):

$$CF_B = 0.34 + 0.29\ (1-0.34) = 0.53 \quad (19)$$

Although there is nothing wrong with the above scenario, it does not provide us with a solution when it comes to dealing with indeterminacy often encountered in real-world case studies. This can occur, for example, when multiple experts provide opinions or judgments regarding a specific outcome or decision, and discrepancies among their assigned CFs are expressed. It can also occur when integrating data from various sources, resulting in inconsistencies or contradictions that lead to uncertainty in CF assignments. Sometimes, not all relevant information or evidence is available to accurately assign CFs. In such cases, judgments are made based on limited data, leading to indeterminacy. Lastly, certain situations might present ambiguous evidence or information that can be interpreted in multiple ways, leading to differing CF assignments.

Under this assumption, the above scenario could be easily dealt with by assigning neutrosophic values to the certainty factors as discussed in subsection 2.3:

R8: If $X_{CF=(0.45,0.3,0.2)}$ *then B (0.75, 0.2, 0.1),* *R9: If* $Y_{CF=(0.8,0.2,0.1)}$ *and* $Z_{CF=(0.7,0.2,0.2)}$ *and* $\Omega_{CF=(0.45,0.1,0.3)}$ *then B (0.65, 0.15, 0.25)*

By applying equation (12):

CF_p = (0.45*0.75, 0.3+0.2 – 0.3*0.2, 0.2+0.1-0.2*0.1) = (0.34, 0.44, 0.28) (20)

Now, by applying equations (9) and (12):

CF_n = (0.65, 0.15, 0.25) ⊗ CF_{if} (21)

where CF_{if} = $CF_Y \cap CF_Z \cap CF_\Omega$ = (min(0.8,0.7,0.45), max(0.2,0.2,0.1), max(0.1,0.2,0.3) = (0.45, 0.2, 0.3)

Now equation (21) becomes:

CF_n = (0.65, 0.15, 0.25) ⊗(0.45, 0.2, 0.3) = (0.65*0.45, 0.15+0.2-0.15*0.2, 0.25 +0.3-0.25*0.3) =

(0.29, 0.32, 0.47) (22)

To calculate CF_B in the neutrosophic environment and because equations (4), (5) & (6) do not apply in our framework, as it is always the case that CF_p and $CF_n > 0$ since (T, I, F) ∈ [0,1], we apply the neutrosophic addition operator as expressed in equation (11):

CF_B= CF_p⊕ CF_n= (0.34, 0.44, 0.28)⊕(0.29, 0.32, 0.47) =(0.34+0.29-0.34*0.29, 0.44*0.32, 0.28*0.47) = (0.53, 0.14, 0.13) (23)

In neutrosophic logic/set/probability it's possible to have the sum of components (T, I, F) different from 1. More specifically [59]:

T+I+F>1, for paraconsistent (conflicting) information;

T+I+F=1, for complete information;

T+I+F<1, for incomplete information.

From the results obtained in equation (26), we observe that the truth degree (0.53), when utilizing NCFs, is equal to the value obtained with equation (19) using "traditional" CFs. However, the latter does not indicate that we are dealing with incomplete information in this scenario (as T+I+F<1). Therefore, CFs may give a false sense of confidence if applied to situations with limited data or insufficient information to make definitive judgments. The absence of data or incomplete information can lead to uncertainties that are difficult to represent using a single numerical value. This can pose challenges when assessing hypotheses or making decisions based on incomplete evidence.

The fundamental purpose of a score function is to determine the conversion of a neutrosophic number to a real number. It is used as a systematic approach to solving decision-making problems with neutrosophic information. By applying equation (13) to the result obtained in (23), we have F(A) = 0.56. With the result of the score function, the decision-maker(s) can define a threshold value, which serves as a boundary or limit that helps in evaluating options or situations, according to their judgment and expertise. Depending on the value of the threshold the rules of the model could be activated or rejected thus providing a clear guideline for decision-makers and helping streamline the decision-making process by making it more objective and consistent.

Example 2: Now let us examine a different example to evaluate the robustness of our proposed method against the traditional Bayesian approach.

Let us assume that we have collected the following data from a patient of a hospital including symptoms and test results, to calculate the likelihood of a disease X.

Symptoms and Test Results:

Symptom A (present in 70% of Disease X cases)

Symptom B (present in 60% of Disease X cases)

Test 1 (positive in 80% of Disease X cases)

Test 2 (positive in 75% of Disease X cases)

Patient's Data:

Patient: Symptom A (yes), Symptom B (no), Test 1 (positive), Test 2 (positive)

A) Traditional Bayesian method

P(Disease X∣Symptoms and Tests)∝P(Symptoms and Tests∣Disease X)×P(Disease X)

Assuming P(Disease X)=0.1 (prevalence)

P(Symptom A∣Disease X)=0.7

P(Symptom B∣Disease X)=0.6

P(Test 1∣Disease X)=0.8

P(Test 2∣Disease X)=0.75

Since the patient has Symptom A, positive Test 1 and positive Test 2:

P(Symptoms and Tests∣Disease X)=0.7×0.8×0.75=0.42 (24)

The probability of Disease X given the symptoms and tests for the Patient is proportional to 0.42.

B) Proposed NFC method

Symptoms and Test Results:

Symptom A: 70% true, 20% false, 10% indeterminate

Symptom B: 40% true, 50% false, 10% indeterminate

Test 1: 80% true, 15% false, 5% indeterminate

Test 2: 75% true, 20% false, 5% indeterminate

Patient's Data:

Patient: Symptom A (neutrosophic value), Symptom B (neutrosophic value), Test 1 (neutrosophic-positive), Test 2 (neutrosophic-positive)

According to our conceptual framework, we obtain the following rule based on the symptoms and tests taken on the patient:

Based on the patient's data, he/she shows evidence of symptom A, thus we have the following rule:

R10: If $A_{CF=(0.7,0.1,0.2)}$ *and* $Z_{CF=(0.8,0.05,0.15)}$ *and* $\Omega_{CF=(0.75,0.05,0.2)}$ *then X*

Based on the patient's data, he/she shows evidence of symptom B, thus we have the following rule:

R11: If $B_{CF=(0.4,0.1,05)}$ *and* $Z_{CF=(0.8,0.05,0.15)}$ *and* $\Omega_{CF=(0.75,0.05,0.2)}$ *then X*

Assuming, as in case (A), that the prior probability that the patient, given the symptoms and tests taken, has the disease X is 10% (in neutrosophic formulation (0.1, 0.0, 0.9)), the hypothetical conclusion that the patient has the disease X is inferred with neutrosophic certainty by calculating next equation:

$CF = CF_{if} * CF_{then}$ (25)

For rule R5:

Based on equation (15), the overall certainty factor of the recorded events i.e. of the left part of the rule is:

$CF_P = CF_A \cap CF_Z \cap CF_\Omega$ = (min (0.7, 0.8, 0.75), max (0.1,0.05,0.05), max (0.2,0.15,0.2)) = (0.7,0.1,0.2) (26) and by applying equation (25):

$CF_{DisX1} = CF_P \otimes (0.1, 0.0, 0.9) = (0.7,0.1,0.2) \otimes (0.1, 0.0, 0.9) = (0.7*0.1, 0.0, 0.2+0.9 -0.2*0.9) =$

$(0.07, 0.00, 0.92)$ (27)

For rule R6:

Based on equation (15), the overall certainty factor of the recorded events i.e. of the left part of the rule is:

$CF_N = CF_Y \cap CF_Z \cap CF_\Omega$ = (min (0.4, 0.8, 0.75), max (0.1,0.05,0.05), max (0.5,0.15,0.2)) = (0.4,0.1,0.5) (28)

$CF_{DisX2} = CF_N \otimes (0.1, 0.0, 0.9) = (0.4,0.1,0.5) \otimes (0.1, 0.0, 0.9) = (0.4*0.1, 0.0, 0.5+0.9 -0.5*0.9) =$

$(0.04, 0.00, 0.95)$ (29)

As stated above in the current subsection, the total NCF of Disease X will be (equation (11)):

$CF_{DisX} = CF_{DisX1} \oplus CF_{DisX2} = (0.07, 0.00, 0.92) \oplus (0.04, 0.00, 0.95) = (0.07+0.04–0.07*0.04, 0.00*0.00, 0.92*0.95)$
$= (0.11, 0.00, 0.87)$. (30)

By applying equation (13) to the above result, we have:

F(DisX) = (1.11-0.87)/2= 0.12 (31)

If we interpret the outcome of equation (31), we could indicate that the element or proposition in question (in our context, the proposition of *a patient having Disease X*)has a low overall degree of "certainty" or "truth,"(or in numbers, the proposition is only 12% true), a fact that wasn't quite clear when applying the Bayesian method (see equation (24)). In addition, our method allowed for a more holistic approach since we were able to include both symptoms A & B in our framework with little computational cost, to draw our conclusion regarding the patient's disease in a more thorough manner.

Hence, the numerical case study shows that the NCF model can produce a more nuanced and trustworthy estimate of disease X than the usual Bayesian technique. This improved ability to handle ambiguities and provide explicit certainty factors can have a substantial impact on diagnostic decision-making in medical practice.

4 | Applications

By considering the examples demonstrated in sections 2 and 3 it becomes clear that integrating neutrosophic logic into certainty factors greatly improves the traditional CF model. This enhancement enables us to effectively handle incomplete, indeterminate, and inconsistent information. Although both CFs and NCFs deal with uncertainty, CFs rely on a numerical scale that indicates belief or disbelief, while NCFs, in the context of neutrosophic logic, include a trivalent perspective that encompasses memberships of truth, indeterminacy, and falsity. This comprehensive approach allows for a more thorough treatment of uncertain information.

This increased granularity is extremely useful in decision-making processes, particularly when dealing with highly uncertain or ambiguous data. It allows for a more exact evaluation and management of uncertainty. Furthermore, NCFs display improved flexibility and robustness when modeling complex systems, enabling better analysis and comprehension of circumstances characterized by uncertainty. Furthermore, NCFs excel at dealing with complicated and contradictory information by providing a framework for expressing changing degrees of uncertainty and improving the dependability of decision support systems.

5 | Comparative Analysis

In this section, we briefly conduct a comparative analysis of our proposed methodology compared to other well-established methods used in AI for handling uncertainty, namely Bayesian networks, fuzzy logic, and Dempster-Shafer theory. Our goal is to highlight the advantages of the proposed NCF approach.

5.1 | Bayesian Networks

Bayesian networks [60-64] are graphical models that illustrate the probability connections between variables. They are particularly useful in scenarios with a clear probabilistic structure and sufficient data to estimate probabilities.

Their strengths include a rigorous probabilistic framework that integrates prior knowledge with observed data, as well as an explicit treatment of variable dependencies. However, Bayesian networks sometimes rely on assumptions of conditional independence, which may not always hold true, and the inference process can be computationally intensive, especially for large networks. In contrast, the NCF model does not depend on independence assumptions and offers a more flexible representation by incorporating indeterminacy, making it a simpler and potentially more straightforward approach to handling uncertainty without extensive probability calculations.

5.2 | Fuzzy Logic

Fuzzy logic [65-69] is most suitable for control systems and decision-making processes where precision is not crucial. Its main advantages lie in its ability to perform well in situations with imprecise or ambiguous information, as well as its ease of implementation. However, fuzzy logic lacks a probabilistic interpretation of uncertainty, limiting its applicability in certain AI applications. Additionally, its rules are often static, making adaptation to new data or changing circumstances challenging. The NCF model enhances fuzzy logic by including indeterminacy and enabling more dynamic uncertainty management. While fuzzy logic effectively addresses imprecision, NCF offers a more comprehensive framework for handling both uncertainty and indeterminacy.

5.3 | Dempster-Shafer Theory

Dempster-Shafer theory (DST) [70-74] is a mathematical theory of evidence that combines information from multiple sources to evaluate the likelihood of an event. It establishes a formal framework for evidence combination and explicitly accounts for uncertainty by allowing for varying levels of confidence and plausibility. However, DST calculations can be computationally demanding, especially when dealing with large sets of evidence and interpreting belief functions may be less intuitive than probabilistic alternatives. The NCF model is similar to DST in addressing uncertainty and data combination, but it goes a step further by incorporating indeterminacy, offering a more nuanced approach to handling uncertainty. Additionally, NCF may require fewer computational resources than DST, making it more suitable for certain applications.

Table 1 summarizes the key differences between NCF, Bayesian networks, fuzzy logic, and Dempster-Shafer theory. As observed in Table 1, the NCF model provides a strong and adaptable framework for dealing with uncertainty, especially in situations where indeterminacy is critical. Its capacity to combine levels of belief, skepticism, and indeterminacy enables more sophisticated decision-making than the aforementioned methods.

Table 1. Comparative analysis with other methods.

Feature	Bayesian Networks	Fuzzy Logic	Dempster-Shafer Theory	Neutrosophic Certainty Factors
Probabilistic Framework	Yes	No	Yes	No
Imprecision Handling	Moderate	High	Moderate	High
Indeterminacy Handling	No	No	Yes	Yes
Computational Complexity	High	Low	High	Moderate
Assumptions of Independence	Yes	No	No	No
Adaptability to New Data	High	Low	Moderate	High
Intuitiveness	Moderate	High	Low	High

6 | Concluding Remarks

We are all aware that when studying a situation and forming judgments about it in the real world, we cannot be entirely certain of our conclusions. There is certainly some doubt surrounding it. Uncertainty exists in most tasks that require intellectual behavior, such as planning, reasoning, problem-solving, decision-making, and categorization. The majority of practical thinking comprises ambiguity, partial ignorance, and incomplete or conflicting knowledge, which frequently leads to confusion. This invites the following question: (a) How should uncertainty be represented? (b) How are uncertainty measures evaluated, merged, and modified? (c) How may these metrics be utilized to draw implications and conclusions?

Several other methods have been proposed to handle uncertainty. Among them, some of the most commonly used techniques are fuzzy logic, which utilizes fuzzy sets, Bayesian reasoning with probabilities, and the theory of evidence or belief functions. However, as we have discussed in Section 5, all of the aforementioned methods share a common limitation when dealing with uncertainty: they do not offer a sufficient mechanism to represent incomplete, vague, indeterminate, and contradictory information.

For these reasons, in this paper, we propose a new method that incorporates certainty factors in a neutrosophic environment. This approach provides a versatile and flexible framework with the ability to manage uncertainty, making our method a valuable tool in complex decision-making processes often encountered in the real world.

6.1 | Practical Implications of Proposed Method

Integrating Neutrosophic Certainty Factors (NCF) into AI systems has several important practical implications.

- **Improved Decision-Making Accuracy:** By combining degrees of truth, indeterminacy, and falsity, the NCF model enables AI systems to make more precise and nuanced choices. This decreases the possibility of mistakes generated by simplistic binary or probabilistic models that fail to account for uncertainty completely.

- **Improved Handling of Uncertainty:** The NCF model effectively addresses uncertainty, including missing and ambiguous data. This capacity is crucial in real-world applications because AI systems frequently confront noisy or incomplete datasets.

- **Enhanced Interpretability:** The triplet representation of NCF provides a clear and interpretable approach to comprehending the confidence levels of AI choices. This openness is critical for winning the trust and approval of end users and stakeholders, who need to understand the rationale behind AI-driven choices.

6.2 | Advantages of Proposed Method

Neutrosophic certainty factors (NCFs) offer several advantages in handling uncertain information within the neutrosophic logic framework. First and foremost, it is acknowledged [75] that uncertain reasoning, particularly when employing models like the Certainty Factor (CF) model, can indeed encounter challenges related to detachment and locality, potentially leading to errors in reasoning. By suggesting a hybrid method such as NFC we encounter to present a framework that can enhance the reasoning process and improve the accuracy of conclusions. A hybrid approach, like ours, provides a way to mitigate errors that may arise from overreliance on CF models alone. By leveraging the strengths of multiple models, the potential for errors in reasoning due to detachment or locality issues is minimized. Other advantages of our proposed method include:

- NCFs communicate certainty or uncertainty in a more nuanced and detailed manner, enabling the expression of degrees of truth, falsehood, and indeterminacy. This allows for a more precise assessment of the dependability of information.
- NCFs efficiently handle ambiguity by allowing for the expression of varying degrees of confidence or doubt. This is particularly useful when information cannot be easily categorized as true or false.
- NCFs provide a flexible framework for decision-making in the presence of ambiguity. By incorporating certainty factors into the neutrosophic logic paradigm, they enable judgments that are adaptable and context-aware, taking into account degrees of confidence or uncertainty.
- Compared to standard certainty metrics, NCFs can more effectively handle imprecise, incomplete, or contradictory information. This resilience is especially valuable in sectors where uncertain data is common, such as decision support systems and artificial intelligence.

6.3 | Potential Applications

The suggested NCF model may be implemented across several domains, boosting the performance and dependability of AI systems in real-world applications.

- **Healthcare**: The NCF model may combine clinical data from several sources, including patient symptoms, test findings, and treatment outcomes, to improve medical diagnosis and treatment planning. This results in more accurate diagnosis and personalized treatment approaches.
- **Finance**: The NCF model helps improve financial forecasting and risk management by delivering accurate predictions despite market volatility and insufficient economic data. This allows for better-informed investment decisions and effective risk management.
- **Cybersecurity**: In cybersecurity, the NCF model improves threat detection and response by estimating the confidence of possible threats based on incomplete or ambiguous signals. This increases the resilience of security measures while decreasing the risk of false positives and negatives.

6.4 | Limitations of our Study

Overcoming the limitations of our study, further improvements that could help strengthen the applicability, efficiency, and reliability of our proposed method are:

- Additional empirical research and real-world implementations can confirm the reliability of NCFs in many circumstances. This would reveal information about their practical value, strengths, and effectiveness.
- Developing the theoretical underpinnings of NCFs inside neutrosophic logic might lead to a better understanding of their characteristics, connections, and interactions, hence facilitating their wider acceptance and improvement.
- Efforts may be made to improve the interpretability of NCFs, making them more user-friendly and understandable. This might include visualizations or tools to help in understanding and manipulating these elements.
- It would be interesting to develop a probabilistic interpretation for neutrosophic certainty factors based on the measures of belief and disbelief used in CF models with the use of neutrosophic Bayesian rule [76]. In this way, it could be shown that each combination function imposes conditional independence assumptions on the propositions involved in the combinations. For example, when we use the parallel-combination function to combine CFs for the rules "if e_1 then h" and "if e_2 then h," we implicitly assume that e_1 and e_2 are conditionally independent, given h and NOT h.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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