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Some Aspects of Neutrosophic Set Theory

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Abstract

This article aims to introduce my work on constructing neutrosophic set theory. Neutrosophy is a new branch of mathematical systems proposed by Smarandache in 1995 as an extension of an intuitionistic fuzzy set according to three-degree membership functions. The new perspective of neutrosophic sets consists of two paths, the first path depends on the three degrees including the degree of truth-membership, degree of indeterminacy-membership, and degree of false-non-membership respectively, while the second path depends on the generated classical set by three types of neutrosophic sets and study the concepts of neutrosophic theory. In previous work, I Presented the concepts of neutrosophic sets including universal, empty, compliment, and subsets, denoted as $H_i^t[I\ ]$, where i equals to $1,2$ or 3. I also explored neutrosophic operations and their properties, such as neutrosophic unions and neutrosophic intersections. In this article, I shall delve into additional materials and theorems related to these concepts and discuss neutrosophic and symmetric differences, including their properties. Furthermore, I shall present a generalization of De Morgan's theorem.

Keywords: Neutrosophic sets $H_i^t[I\ ]i = 1,2,3$; Neutrosophic Complement of Sets; Neutrosophic Difference of Sets; Neutrosophic Symmetric Difference of Sets.

1 | Introduction

In 1995, Smarandache introduced a new branch of philosophy, Neutrosophy Science. In this philosophy, the mathematical system neutrosophic includes indeterminacy in the set such $I_A(x)$ is called the indeterminacy for $T_A(x)$ or $F_A(x)$ on nonstandard interval $]^{0-}, 1^{+}[$ or single-valued [0,1] as a generalization of an intuitionistic fuzzy set. Neutrosophy is the base of neutrosophic logic, a multiple-value logic that generalizes intuitionistic fuzzy logic, and fuzzy logic respectively. For more information about the principle of neutrosophic philosophy, and extend the debate including; the thesis $\langle A \rangle$ and antithesis $\langle AntiA \rangle$ to get a synthesis, thesis $\langle A \rangle$ and antithesis $\langle AntiA \rangle$ to get a retrosynthesis $\langle NeutA \rangle$, neutrosophic system, and neutrosophic dynamic system. I refer to [1, 2]. In addition, for neutrosophic over/under/off/ sets from the point view degree of membership function [3]. The neutrosophic algebra structures based on the neutrosophic number of the form $N_1 = a + bI$, I is literal in indeterminacy, $I^2 = I$, and $0I = 0$ used by scholars such as [4, 5]. In [6], I have tried to provide the necessary initial concepts for constructing the neutrosophic set theory as a generalization of set theory. Furthermore, I tried to construct from a classical set

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a neutrosophic set of three types $H_i^t[I]$, where $i = 1,2,3$, and a neutrosophic number in my work represents the first type that comes from any classical set, while $N_2 = \{a\} \cup aI$ and $N_3 = (a + bI) \cup \{a\}$ Represents the second and third types respectively. This article aims to complement and expand the previous work in [6] by presenting the complements of the neutrosophic set, the difference, and the symmetric difference of the neutrosophic set and presenting some of their properties. This work establishes the theory of neutrosophic set theory with [6] and reinforces previous work in [7-11].

2 | More Information on Subsets, Union, Intersection, and Power Neutrosophic Sets

In this section, I present more information on subsets, union, intersection, and power sets of neutrosophic sets of three types with some theorems and examples as extension concepts in [4].

Definition 2.1 [6] Let $H \neq \emptyset \subset U$ be a non-empty set, then;

i). $H_1^t[I] = \{h_1 + h_2 I: h_1, h_2 \in H\}$ be a neutrosophic set of type 1,

ii). $H_2^t[I] = \{aI \cup \{a\}: a \in H\}$ be a neutrosophic set of type 2, and

iii). $H_3^t[I] = \{(h_1 + h_2 I) \cup \{h_1\}: h_1, h_2 \in H\}$ be a neutrosophic set of type 3, where I is an indeterminacy

Theorem 2.1 Let $H_i^t[I]$, $N_i^t[I]$, and $M_i^t[I]$ be three neutrosophic sets of type i, and i equals 1, 2, or 3. If $H_i^t[I] \subset M_i^t[I]$, and $H_i^t[I] \subset N_i^t[I]$, then

i). $H_i^t[I] \subset M_i^t[I] \cap N_i^t[I]$, and

ii). $H_i^t[I] \subset M_i^t[I] \cup N_i^t[I]$.

Proof (1). Let $H_i^t[I] \subset M_i^t[I]$, and $H_i^t[I] \subset N_i^t[I]$. Assume that $x \in H_i^t[I]$.

$\because x \in H_i^t[I] \Rightarrow x \in M_i^t[I] \wedge x \in N_i^t[I]$

$\Rightarrow (\exists x_1, x_2 \in M) \wedge (\exists x_1, x_2 \in N)$, indeterminacy I such that $x = x_1 + x_2 I$

$\Rightarrow \big(\exists x_1, x_2 \in (M \wedge N)\big)$, indeterminacy I such that $x = x_1 + x_2 I$

$\Rightarrow \Big(x \in \big(M_i^t[I] \wedge N_i^t[I]\big)\Big)$

$\Rightarrow \Big(x \in \big(M_i^t[I] \cap N_i^t[I]\big)\Big)$

$\Rightarrow H_i^t[I] \subset M_i^t[I] \cap N_i^t[I]$. ■

(2). By the same method.

Theorem 2.2 Let $H_i^t[I]$, $N_i^t[I]$, $M_i^t[I]$, and $O_i^t[I]$ be four neutrosophic sets of type i, and i equals 1,2 or 3. If $H_i^t[I] \subset N_i^t[I]$, and $M_i^t[I] \subset O_i^t[I]$, then:

i). $H_i^t[I] \cup M_i^t[I] \subset N_i^t[I] \cup O_i^t[I]$, and

ii). $H_i^t[I] \cap M_i^t[I] \subset N_i^t[I] \cap O_i^t[I]$.

Proof (1). Suppose that $H_i^t[I] \subset N_i^t[I]$, and $M_i^t[I] \subset O_i^t[I]$, for any $i = 1,2,3$.

Assume that, $x \in \big(H_i^t[I] \cup M_i^t[I]\big) \Rightarrow \big(x \in H_i^t[I]\big) \vee \big(x \in M_i^t[I]\big)$

$\Rightarrow \big(x \in N_i^t[I]\big) \vee \big(x \in O_i^t[I]\big)$

$\Rightarrow x \in \big(N_i^t[I] \cup O_i^t[I]\big)$

$$\Rightarrow H_i^t[I] \cup M_i^t[I] \subset N_i^t[I] \cup O_i^t[I]. \blacksquare$$

(2) By a similar method.

Definition 2.2 [4] Let $H_i^t[I]$ be three neutrosophic sets of type i, and i equals 1,2 or 3.

The neutrosophic complement sets of type i are denoted by $\overbrace{H_i^t[I]}^{c}$, and defined by: $\overbrace{H_i^t[I]}^{c} = \{x: x \notin H_i^t[I] \wedge x \in U_i^t[I]\} = \{x: x \notin H \wedge x \in U\}$,

Definition 2.3 [6] Let $H_i^t[I]$ be three neutrosophic sets of type i. The neutrosophic power sets of type i defined by $\Im\left(H_i^t[I]\right) = \{N_i^t[I]: N_i^t[I] \subseteq H_i^t[I], i = 1,2,3\}$.

Theorem 2.3 Let $H_i^t[I]$ and $N_i^t[I]$ be two neutrosophic sets of type i, consider $\Im\left(H_i^t[I]\right)$ and $\Im\left(N_i^t[I]\right)$ are neutrosophic power sets of type i. Then

i). $2^{H_i^t[I]} \cap 2^{N_i^t[I]} = 2^{H_i^t[I] \cap N_i^t[I]}$, and

ii). $2^{H_i^t[I]} \cup 2^{N_i^t[I]} \subset 2^{H_i^t[I] \cap N_i^t[I]}$, for any $i = 1,2,3, H$ and N are classical sets.

Furthermore,

$\Im\left(H_i^t[I]\right)$ or $2^{H_i^t[I]}$ represents the same notation of neutrosophic power sets of type i.

Proof (1). Assume that $E_i^t[I] \in \left(2^{H_i^t[I]} \cap 2^{N_i^t[I]}\right)$,

$$\Leftrightarrow \left(E_i^t[I] \in 2^{H_i^t[I]}\right) \wedge \left(E_i^t[I] \in 2^{N_i^t[I]}\right),$$

$$\Leftrightarrow \left(E_i^t[I] \subseteq H_i^t[I]\right) \wedge \left(E_i^t[I] \subseteq N_i^t[I]\right),$$

$$\Leftrightarrow E_i^t[I] \subseteq \left(H_i^t[I] \cap N_i^t[I]\right),$$

$$\Leftrightarrow E_i^t[I] \in 2^{H_i^t[I] \cap N_i^t[I]}$$

Hence, $2^{H_i^t[I]} \cap 2^{N_i^t[I]} = 2^{H_i^t[I] \cap N_i^t[I]}$, for any $i = 1,2,3. \blacksquare$

(2). Suppose that $E_i^t[I] \in \left(2^{H_i^t[I]} \cup 2^{N_i^t[I]}\right)$

$$\Rightarrow \left(E_i^t[I] \in 2^{H_i^t[I]}\right) \vee \left(E_i^t[I] \in 2^{N_i^t[I]}\right)$$

$$\Rightarrow \left(E_i^t[I] \subseteq H_i^t[I]\right) \vee \left(E_i^t[I] \subseteq N_i^t[I]\right)$$

$$\Rightarrow E_i^t[I] \subseteq \left(H_i^t[I] \cup N_i^t[I]\right)$$

$$\Rightarrow E_i^t[I] \in 2^{H_i^t[I] \cup N_i^t[I]}$$

$$\Rightarrow 2^{H_i^t[I]} \cup 2^{N_i^t[I]} \subset 2^{H_i^t[I] \cap N_i^t[I]}, \text{ for any } i = 1,2,3. \blacksquare$$

Example 2.1 Let $H = \{a, b\}$ and $N = \{c\}$ be two classical sets, then the neutrosophic sets of type-1, type-2, and type-3 are given by:

$H_1^t[I] = \begin{Bmatrix} a + aI, & a + bI, \\ b + aI, & b + bI, \end{Bmatrix}$, $H_2^t[I] = \begin{Bmatrix} a, & aI, \\ b, & bI, \end{Bmatrix}$, and $H_3^t[I] = \begin{Bmatrix} a, & a + aI, & a + bI, \\ b, & b + aI, & b + bI, \end{Bmatrix}$. While $N_1^t[I] = \{c + cI\}$, $N_2^t[I] = \{c, \quad cI\}$, and $N_3^t[I] = \{c, \quad c + cI\}$, we see that

$2^{H_2^t[I]}$: 0 neutrosophic- element: $\underbrace{\emptyset_2^t[I]}_{0}$, 1 neutrosophic- element: $\underbrace{H_2^t[I]}_{1} = \{a\}$, $\underbrace{H_2^t[I]}_{2}\{aI\}$, $\underbrace{H_2^t[I]}_{3}\{b\}$, and $\underbrace{H_2^t[I]}_{4}\{bI\}$,

2 neutrosophic- elements: $\underbrace{H_2^t[I]}_{5} = \{a, aI\}$, $\underbrace{H_2^t[I]}_{6}\{a, b\}$, $\underbrace{H_2^t[I]}_{7}\{a, bI\}$,

$\underbrace{H_2^t[I]}_{8}\{aI, b\}$, $\underbrace{H_2^t[I]}_{9}\{aI, bI\}$, and $\underbrace{H_2^t[I]}_{10}\{aI, bI\}$,

3 neutrosophic- elements: $\underbrace{H_2^t[I]}_{11} = \{a, aI, b\}$, $\underbrace{H_2^t[I]}_{12}\{a, aI, bI\}$, $\underbrace{H_2^t[I]}_{13}\{aI, b, bI\}$, and

$\underbrace{H_2^t[I]}_{14} = \{a, b, bI\}$

4 neutrosophic- elements: $\underbrace{H_2^t[I]}_{15} = \begin{Bmatrix} a, & aI, \\ b, & bI, \end{Bmatrix}$. Also, we have,

$2^{N_2^t[I\,]}$: 0 neutrosophic- element: $\underbrace{\emptyset_2^t[I]}_{0}$,

1 neutrosophic- element: $\underbrace{N_2^t[I]}_{1} = \{c\}$, $\underbrace{H_2^t[I]}_{2}\{cI\}$, and

2 neutrosophic- elements: $\underbrace{N_2^t[I]}_{3} = \{c, cI\}$. Now, $2^{H_2^t[I]} \cup 2^{N_2^t[I]}$ Consists of 24 neutrosophic elements. While $H_2^t[I] \cup N_2^t[I] = \begin{Bmatrix} a, & aI, \\ b, & bI \\ c, & cI \end{Bmatrix}$, and the neutrosophic power set $2^{H_2^t[I] \cup N_2^t[I]}$ consists of 64 neutrosophic elements, and consequently, $2^{H_2^t[I\,]} \cup 2^{N_2^t[I\,]} \neq 2^{H_2^t[I\,] \cup N_2^t[I\,]}$.

3 | More Difference and Symmetric Difference of Neutrosophic Sets of Type-1, ype-2, and Type-3 with their Properties

This section addresses to study of the difference between neutrosophic sets, and the symmetric difference between neutrosophic sets of three types with their properties.

Definition 3.1 Let $H_i^t[I]$, $N_i^t[I]$ be six neutrosophic sets of type i. Then the difference of neutrosophic sets of type i, defined by

$$H_i^t[I] \ominus N_i^t[I] = \{x: (x \in H_i^t[I]) \wedge (x \notin N_i^t[I]), i = 1,2,3\}$$
$$= \{(\exists\, x_1, x_2 \in H) \wedge (\exists\, x_1, x_2 \notin N),\ \text{indeterminacy } I \text{ such that } x = x_1 + x_2 I\}$$

Theorem 3.1 Let $U_i^t[I]$ be a neutrosophic universal set of type i, $H_i^t[I]$, and $N_i^t[I] \subset U_i^t[I]$. Then:

1. $H_i^t[I] \ominus \emptyset_i^t[I] = H_i^t[I]$,
2. $H_i^t[I] \ominus N_i^t[I] \subset H_i^t[I]$ and $N_i^t[I] \ominus H_i^t[I] \subset N_i^t[I]$,
3. $H_i^t[I] \ominus N_i^t[I] = \overbrace{N_i^t[I]}^{c} \ominus \overbrace{H_i^t[I]}^{c}$,
4. $H_i^t[I\,] \ominus N_i^t[I] = H_i^t[I] \cap \overbrace{N_i^t[I]}^{c}$,
5. $H_i^t[I] \cap N_i^t[I] = H_i^t[I] \ominus (H_i^t[I] \ominus N_i^t[I])$,
6. $H_i^t[I] \cup N_i^t[I] = H_i^t[I] \cup (N_i^t[I] \ominus H_i^t[I])$, and
7. $H_i^t[I] \ominus N_i^t[I] = H_i^t[I] \ominus (H_i^t[I] \cap N_i^t[I])$.

Proof.

1. Consider $x \in \left(H_i^t[I] \ominus \emptyset_i^t[I]\right) \Leftrightarrow \left(x \in H_i^t[I]\right) \wedge \left(x \notin \emptyset_i^t[I]\right)$

$$\Leftrightarrow \left(x \in H_i^t[I]\right) \wedge \left(x \in \overbrace{\emptyset_i^t[I]}^{c}\right)$$

$$\Leftrightarrow \left(x \in H_i^t[I]\right) \wedge \left(x \in U_i^t[I]\right)$$

$$\Leftrightarrow \left(x \in \left(H_i^t[I] \cap U_i^t[I]\right)\right)$$

$\Leftrightarrow \left(x \in H_i^t[I]\right)$, hence $H_i^t[I] \ominus \emptyset_i^t[I] = H_i^t[I]$. ■

2. Assume that $x \in \left(H_i^t[I] \ominus N_i^t[I]\right)$, for any, $i = 1,2,3$

$$\Rightarrow \left(x \in H_i^t[I]\right) \wedge \left(x \notin N_i^t[I]\right)$$

$\Rightarrow (\exists\, x_1, x_2 \in H) \wedge (\exists\, x_1, x_2) \notin N$, indeterminacy I such that $x = x_1 + x_2 I$

$\Rightarrow (\exists\, x_1, x_2 \in H)$, indeterminacy I such that $x = x_1 + x_2 I$

$\Rightarrow \left(x \in H_i^t[I]\right) \Rightarrow H_i^t[I] \ominus N_i^t[I] \subset H_i^t[I]$. ■ The second part by the same argument.

3. Assume that $x \in \left(H_i^t[I] \ominus N_i^t[I]\right)$, for any, $i = 1,2,3$

$\Leftrightarrow \left(x \in H_i^t[I]\right) \wedge \left(x \notin N_i^t[I]\right)$,

$\Leftrightarrow (\exists\, x_1, x_2 \in H) \wedge (\exists\, x_1, x_2) \notin N$, indeterminacy I such that $x = x_1 + x_2 I$

$\Leftrightarrow \left(\exists\, x_1, x_2 \notin \overbrace{H}^{c}\right) \wedge \left(\exists\, x_1, x_2 \in \overbrace{N}^{c}\right)$, indeterminacy I such that $x = x_1 + x_2 I$

$\Leftrightarrow \left(\exists\, x_1, x_2 \in \overbrace{N}^{c}\right) \wedge \left(\exists\, x_1, x_2 \notin \overbrace{H}^{c}\right)$, indeterminacy I such that $x = x_1 + x_2 I$

$\Leftrightarrow \left(x \in \overbrace{N_i^t[I]}^{c}\right) \wedge \left(x \notin \overbrace{N_i^t[I]}^{c}\right)$,

$\Leftrightarrow \overbrace{N_i^t[I]}^{c} \ominus \overbrace{N_i^t[I]}^{c}$, therefore, $H_i^t[I] \ominus N_i^t[I] = \overbrace{N_i^t[I]}^{c} \ominus \overbrace{N_i^t[I]}^{c}$. ■

4. Suppose that $x \in \left(H_i^t[I] \ominus N_i^t[I]\right)$

$\Leftrightarrow \left(x \in H_i^t[I]\right) \wedge \left(x \notin N_i^t[I]\right), i = 1,2,3$

$\Leftrightarrow (\exists\, x_1, x_2 \in H) \wedge (\exists\, x_1, x_2 \notin N)$, indeterminacy I such that $x = x_1 + x_2 I$

$\Leftrightarrow (\exists\, x_1, x_2 \in H) \wedge \left(\exists\, x_1, x_2 \in \overbrace{N}^{c}\right)$, indeterminacy I such that $x = x_1 + x_2 I$

$\Leftrightarrow \left(x \in H_i^t[I]\right) \wedge \left(x \in \overbrace{N_i^t[I]}^{c}\right)$,

$\Leftrightarrow x \in \left(H_i^t[I] \cap \overbrace{N_i^t[I]}^{c}\right)$, therefore, $H_i^t[I] \ominus N_i^t[I] = H_i^t[I] \cap \overbrace{N_i^t[I]}^{c}$. ■

5. Suppose that, $x \in \left(H_i^t[I] \ominus \left(H_i^t[I] \ominus N_i^t[I]\right)\right)$, for any, $i = 1,2,3$

$\Leftrightarrow \left(x \in H_i^t[I]\right) \wedge \left(x \notin \left(H_i^t[I] \ominus N_i^t[I]\right)\right)$

$\Leftrightarrow \left(x \in H_i^t[I]\right) \wedge \left(x \notin H_i^t[I] \vee x \in N_i^t[I]\right)$,

$\Leftrightarrow \left(x \in H_i^t[I]\right) \wedge \left(x \in \overbrace{H_i^t[I]}^{c} \vee x \in N_i^t[I]\right)$,

$\Leftrightarrow \left(x \in H_i^t[I] \wedge x \in \overbrace{H_i^t[I]}^{c}\right) \vee \left(x \in H_i^t[I] \wedge x \in N_i^t[I]\right)$,

$\Leftrightarrow (F_N) \vee \left(x \in H_i^t[I] \wedge x \in N_i^t[I]\right)$,

$\Leftrightarrow \left(x \in H_i^t[I] \wedge x \in N_i^t[I]\right)$,

$\Leftrightarrow\ x \in (H_i^t[I] \cap N_i^t[I])$, therefore, $H_i^t[I] \cap N_i^t[I] = H_i^t[I] \ominus (H_i^t[I] \ominus N_i^t[I])$. ■

6. Presume that, $x \in \left(H_i^t[I] \cup (N_i^t[I] \ominus H_i^t[I])\right)$, for any, $i = 1,2,3$

$$\Leftrightarrow (x \in H_i^t[I]) \vee \left(x \in (N_i^t[I] \ominus H_i^t[I])\right)$$
$$\Leftrightarrow (x \in H_i^t[I]) \vee (\,x\, N_i^t[I]\ \wedge\ x \notin H_i^t[I]),$$
$$\Leftrightarrow (x \in H_i^t[I]) \vee \left(x \in N_i^t[I\,]\ \wedge\ x \in \overbrace{H_i^t[I]}^{c}\right),$$
$$\Leftrightarrow (x \in H_i^t[I] \vee x\ \in N_i^t[I]) \wedge \left(x\ \in H_i^t[I]\ \wedge x \in \overbrace{H_i^t[I]}^{c}\right),$$
$$\Leftrightarrow (x \in H_i^t[I] \vee x\ \in N_i^t[I]) \wedge (F_N),$$
$$\Leftrightarrow (x \in H_i^t[I] \vee x\ \in N_i^t[I]),$$

$\Leftrightarrow x \in (H_i^t[I]\ \cup\ N_i^t[I])$, hence $H_i^t[I] \cup N_i^t[I] = H_i^t[I] \cup (N_i^t[I] \ominus H_i^t[I])$. ■

7. Assume that $x \in \left(H_i^t[I] \ominus (H_i^t[I] \cap N_i^t[I])\right)$, for any, $i = 1,2,3$

$$\Leftrightarrow (x \in H_i^t[I]) \wedge \left(x \notin (H_i^t[I] \cap N_i^t[I])\right)$$
$$\Leftrightarrow (x \in H_i^t[I]) \wedge \left((x\ \notin H_i^t[I\,]) \vee (x \notin N_i^t[I\,])\right),$$
$$\Leftrightarrow (x \in H_i^t[I\,]) \wedge \left(\left(x \in \overbrace{H_i^t[I\,]}^{c}\right) \vee (x \notin N_i^t[I])\right),$$
$$\Leftrightarrow \left(x \in H_i^t[I] \wedge x \in \overbrace{H_i^t[I]}^{c}\right) \vee (x \in H_i^t[I] \wedge x\ \notin N_i^t[I]),$$
$$\Leftrightarrow F_N \vee (x \in H_i^t[I] \wedge x\ \notin N_i^t[I]),$$
$$\Leftrightarrow\ (x \in H_i^t[I] \wedge x \notin N_i^t[I]),$$
$$\Leftrightarrow\ (x \in H_i^t[I] \wedge\ x\ \notin N_i^t[I]),$$

$\Leftrightarrow H_i^t[I] \ominus\ N_i^t[I]$, hence

$H_i^t[I] \ominus N_i^t[I] = H_i^t[I] \ominus (H_i^t[I] \cap N_i^t[I])$. ■

The following theorem gives us the generalization of De-Morgan's theorem in classical set theory in neutrosophic classical set theory.

Theorem 3.2 Let $U_i^t[I]$ be a neutrosophic universal set of type i, $H_i^t[I]$, and $N_i^t[I] \subset U_i^t[I]$. Then:

i). $\overbrace{(H_i^t[I] \cup N_i^t[I])}^{c} = \overbrace{H_i^t[I\,]}^{c} \cap\ \overbrace{N_i^t[I]}^{c}$, and

ii). $\overbrace{(H_i^t[I] \cap N_i^t[I])}^{c} = \overbrace{H_i^t[I]}^{c} \cup\ \overbrace{N_i^t[I]}^{c}$.

Notation. It is appropriate to express of $\overbrace{H_i^t[I]}^{c} = 1 \ominus H_i^t[I]$, for any $i = 1,2,3$, where

$H_i^t[I\,] \subset 1$, because all neutrosophic sets belong to considered space with full or 100% degree membership of 1.

Proof (1). Suppose that $x \in \overbrace{(H_i^t[I] \cup N_i^t[I])}^{c}\ \Leftrightarrow x \in \left(1 \ominus (H_i^t[I] \cup N_i^t[I])\right)$

$$\Leftrightarrow \left(x \in 1 \wedge\ x \notin (H_i^t[I] \cup N_i^t[I])\right)$$
$$\Leftrightarrow \left(x \in 1 \wedge\ (x \notin H_i^t[I] \wedge x \notin N_i^t[I])\right)$$

$$\Leftrightarrow \left(x \in 1 \wedge \left(x \in \overbrace{H_i^t[I]}^{c} \wedge x \in \overbrace{N_i^t[I]}^{c}\right)\right)$$

$$\Leftrightarrow \left(\left(x \in 1 \wedge x \in \overbrace{H_i^t[I]}^{c}\right) \wedge x \in \overbrace{N_i^t[I]}^{c}\right)$$

$$\Leftrightarrow \left(x \in \overbrace{H_i^t[I]}^{c} \wedge x \in \overbrace{N_i^t[I]}^{c}\right)$$

$$\Leftrightarrow x \in \left(\overbrace{H_i^t[I]}^{c} \cap x \in \overbrace{N_i^t[I]}^{c}\right), \text{ hence}$$

$$\overbrace{\left(H_i^t[I] \cup N_i^t[I]\right)}^{c} = \overbrace{H_i^t[I]}^{c} \cap \overbrace{N_i^t[I]}^{c}. \blacksquare$$

(2). By a similar method.

Definition 3.2 Let $H_i^t[I\ ]$, $N_i^t[I\ ]$ be six neutrosophic sets of type i. Then the symmetric difference of neutrosophic sets of type i defined by:

$$H_i^t[I] \odot N_i^t[I] = \{x: x \in H_i^t[I] \oplus x \in N_i^t[I], i = 1,2,3\}$$

$$= \{(\exists\ x_1, x_2 \in H) \oplus (\exists\ x_1, x_2) \in N,\ \text{ indeterminacy } I\ such\ that\ x = x_1 + x_2 I\}$$

The symbol $\oplus$ means that, the exclusive or, that is $x \in H_i^t[I]$ or $x \in N_i^t[I]$ but not both. In other words, $H_i^t[I] \odot N_i^t[I] = \left(H_i^t[I] \cup N_i^t[I]\right) \ominus \left(H_i^t[I] \cap N_i^t[I]\right)$ or

$$= \left(H_i^t[I] \ominus N_i^t[I]\right) \cup \left(N_i^t[I] \ominus H_i^t[I]\right)$$

Theorem 3.3 Let $U_i^t[I]$ be a neutrosophic universal set of type i. Consider $H_i^t[I\ ]$, $N_i^t[I\ ]$, and $M_i^t[I\ ] \subset U_i^t[I\ ]$. Then for any $i = 1,2,3$, we have

1. $H_i^t[I] \odot \emptyset_i^t[I] = H_i^t[I]$,
2. $H_i^t[I] \odot H_i^t[I] = \emptyset_i^t[I]$,
3. $H_i^t[I] \odot N_i^t[I] = N_i^t[I] \odot H_i^t[I]$,
4. $\left(H_i^t[I] \odot N_i^t[I]\right) \odot M_i^t[I] = H_i^t[I] \odot \left(H_i^t[I] \odot M_i^t[I]\right)$, and
5. $H_i^t[I] \odot N_i^t[I] = \emptyset_i^t[I] \Leftrightarrow H_i^t[I] = N_i^t[I]$.

Proof.

1. $H_i^t[I] \odot \emptyset_i^t[I] = \left(H_i^t[I] \cup \emptyset_i^t[I]\right) \ominus \left(H_i^t[I] \cap \emptyset_i^t[I]\right)$
$= H_i^t[I] \ominus \emptyset_i^t[I]$
$= H_i^t[I]. \blacksquare$
2. It is clear that by definition $H_i^t[I] \odot H_i^t[I] = \emptyset_i^t[I]. \blacksquare$
3. $(H_i^t[I] \odot N_i^t[I] = \left(H_i^t[I] \cup N_i^t[I]\right) \ominus \left(H_i^t[I] \cap N_i^t[I]\right)$
$= \left(N_i^t[I] \cup H_i^t[I]\right) \ominus \left(N_i^t[I] \cap H_i^t[I]\right)$
$= N_i^t[I] \odot H_i^t[I]. \blacksquare$

The parts (4) and (5) by the same method.

4 | Conclusion

In previous work, I Presented the concepts of neutrosophic sets including universal, empty, compliment, and subsets, denoted as $H_i^t[I\ ]$, where i equals to $1,2$ or 3. I also explored neutrosophic operations and their properties, such as neutrosophic unions and neutrosophic intersections. In this article, I shall delve into additional materials and theorems related to these concepts and discuss neutrosophic and symmetric differences, including their properties.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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