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Antipodal Turiyam Neutrosophic Graphs

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Abstract

Graph theory, a mathematical field, investigates the relationships between entities through vertices and edges [29]. Within this discipline, Uncertain Graph Theory emerges to model uncertainties in realworld networks. This paper presents the concept of the Antipodal Turiyam Neutrosophic Graph. In an Antipodal Graph, two nodes are connected if their shortest path distance equals the graph's diameter, emphasizing connections between the farthest nodes. Turiyam Neutrosophic Graphs extend traditional graphs by introducing four membership values-truth, indeterminacy, falsity, and liberal state-assigned to each vertex and edge, enabling a more nuanced representation of complex relationships.

Keywords: Neutrosophic Graph; Fuzzy Graph; Plithogenic Graph; Turiyam Neutrosophic Graph; Antipodal Graph.

1 | Introduction

1.1 | Uncertain Graph Theory

Graph theory is a foundational branch of mathematics that models networks using vertices (nodes) and edges (connections) to represent relationships between entities [19, 29, 85, 91, 123]. In graph theory, parameters such as shortest path distance [7, 35, 93, 98] and diameter [8, 18, 34, 67] are often studied to analyze the mathematical structure of a given graph.

This paper investigates several uncertain graph models, including Fuzzy, Intuitionistic Fuzzy, Neutrosophic, and Plithogenic Graphs, which enhance classical graph theory by introducing different layers of uncertainty. These models offer a comprehensive framework for analyzing complex and imprecise relationships, making them applicable to various real-world contexts. Consequently, a variety of related graph classes and applications have emerged [38, 39, 41-43, 46, 48-52]. Foundational concepts such as Fuzzy Sets and Neutrosophic Sets have also been extensively studied and documented in the literature [10, 14, 26] Similarly, parameters like shortest path distance and diameter are actively studied within uncertain graphs as well [75, 115, 116]. For a more comprehensive overview, readers are encouraged to refer to existing survey papers [44, 46, 48].

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1.2 | Contributions

This paper introduces the concept of the Antipodal Turiyam Neutrosophic Graph. Antipodal graph is a graph where two nodes are connected if their shortest path distance equals the graph's diameter, highlighting farthest node connections [9, 36, 53, 59, 63, 92]. Turiyam Neutrosophic Graphs expand the traditional graph framework by assigning four membership values-truth, indeterminacy, falsity, and liberal state-to each vertex and edge, allowing for a more comprehensive representation of complex relationships [44, 46, 54, 103]. Note that Turiyam Neutrosophic Set is actually a particular case of the quadripartitioned Neutrosophic Set, by replacing "Contradiction" with "Liberal" [104]. The corresponding graph concept known as quadripartitioned neutrosophic graphs is well-documented [69, 70].

While Antipodal Graphs have been extensively studied in contexts like Fuzzy [4, 58, 87, 97], Vague[81], and Neutrosophic Graphs [78-80], the concept of the Turiyam Neutrosophic Antipodal Graph has not been thoroughly explored. This paper aims to address this gap by defining and analyzing the properties of Antipodal Turiyam Neutrosophic Graphs.

1.3 | The Structure of the Paper

The format of this paper is described below. Section 2 provides the Preliminaries and Definitions. Section 3 introduces results of the antipodal single valued Turiyam neutrosophic graph while and future directions in Section 4.

2 | Preliminaries and Definitions

This section provides an overview of the fundamental definitions and notations used throughout the paper.

2.1 | Basic Graph Concepts

Below are some of the foundational concepts in graph theory. For more comprehensive information on graph theory and its notations, refer to [27-29, 64, 119].

Definition 1 (Graph). [29] A graph G is a mathematical structure that represents relationships between objects. It consists of a set of vertices $V(G)$ and a set of edges $E(G)$, where each edge connects a pair of vertices. Formally, a graph is represented as $G = (V, E)$, where V is the set of vertices and E is the set of edges.

Definition 2 (Degree). [29] Let $G = (V, E)$ be a graph. The degree of a vertex $v \in V$, denoted $\deg(v)$, is defined as the number of edges connected to v. For undirected graphs, the degree is given by:

$$\deg(v) = |\{e \in E \mid v \in e\}|$$

For directed graphs, the in-degree $\deg^-(v)$ refers to the number of edges directed towards v, while the out-degree $\deg^+(v)$ represents the number of edges directed away from v.

2.2 | Uncertain Graph

This paper addresses Fuzzy, Intuitionistic Fuzzy, Neutrosophic, Turiyam, and Plithogenic concepts. Note that Turiyam Neutrosophic Set is actually a particular case of the Quadruple Neutrosophic Set, by replacing "Contradiction" with "Liberal" [104].

Definition 3 (Unified Uncertain Graphs Framework). (cf.[47]) Let $G = (V, E)$ be a classical graph with a set of vertices V and a set of edges E. Depending on the type of graph, each vertex $v \in V$ and edge $e \in E$ is assigned membership values to represent various degrees of truth, indeterminacy, falsity, and other nuanced measures of uncertainty.

1. Fuzzy Graph [17, 40, 57, 60, 74, 82, 86, 99, 100, 112, 118]

- Each vertex $v \in V$ is assigned a membership degree $\sigma(v) \in [0,1]$.
- Each edge $e = (u, v) \in E$ is assigned a membership degree $\mu(u, v) \in [0,1]$.

2. Intuitionistic Fuzzy Graph (IFG) [41,16,24,72,83,114,117,124]:
   - Each vertex $v \in V$ is assigned two values: $\mu_A(v) \in [0,1]$ (degree of membership) and $v_A(v) \in [0,1]$ (degree of non-membership), such that $\mu_A(v) + v_A(v) \leq 1$.
   - Each edge $e = (u, v) \in E$ is assigned two values: $\mu_B(u, v) \in [0,1]$ and $v_B(u, v) \in [0,1]$, with $\mu_B(u, v) + v_B(u, v) \leq 1$.
3. Neutrosophic Graph [5,6,23,44,49,52,65,68,73,101,109,110] :
   - Each vertex $v \in V$ is assigned a triplet $\sigma(v) = (\sigma_T(v), \sigma_I(v), \sigma_F(v))$, where $\sigma_T(v), \sigma_I(v), \sigma_F(v) \in [0,1]$ and $\sigma_T(v) + \sigma_I(v) + \sigma_F(v) \leq 3$.
   - Each edge $e = (u, v) \in E$ is assigned a triplet $\mu(e) = (\mu_T(e), \mu_I(e), \mu_F(e))$.
4. Turiyam Neutrosophic Graph 54 56]:
   - Each vertex $v \in V$ is assigned a quadruple $\sigma(v) = (t(v), iv(v), fv(v), lv(v))$, where each component is in $[0,1]$ and $t(v) + iv(v) + fv(v) + lv(v) \leq 4$.
   - Each edge $e = (u, v) \in E$ is similarly assigned a quadruple.
5. Vague Graph [2, 3, 20, 22, 95, 96, 102]:
   - Each vertex $v \in V$ is assigned a pair $(\tau(v), \phi(v))$, where $\tau(v) \in [0,1]$ is the degree of truthmembership and $\phi(v) \in [0,1]$ is the degree of false-membership, with $\tau(v) + \phi(v) \leq 1$.
   - The grade of membership is characterized by the interval $[\tau(v), 1 - \phi(v)]$.
   - Each edge $e = (u, v) \in E$ is assigned a pair $(\tau(e), \phi(e))$, satisfying:

$$\tau(e) \leq \min\{\tau(u), \tau(v)\},\ \phi(e) \geq \max\{\phi(u), \phi(v)\}$$

6. Hesitant Fuzzy Graph [15, 62, 88, 20]:
   - Each vertex $v \in V$ is assigned a hesitant fuzzy set $\sigma(v)$, represented by a finite subset of $[0,1]$, denoted $\sigma(v) \subseteq [0,1]$.
   - Each edge $e = (u, v) \in E$ is assigned a hesitant fuzzy set $\mu(e) \subseteq [0,1]$.
   - Operations on hesitant fuzzy sets (e.g., intersection, union) are defined to handle the hesitation in membership degrees.
7. Single-Valued Pentapartitioned Neutrosophic Graph [25, 69, 71, 94]:
   - Each vertex $v \in V$ is assigned a quintuple $\sigma(v) = (T(v), C(v), R(v), U(v), F(v))$, where:
     - $T(v) \in [0,1]$ is the truth-membership degree.
     - $C(v) \in [0,1]$ is the contradiction-membership degree.
     - $R(v) \in [0,1]$ is the ignorance-membership degree.
     - $U(v) \in [0,1]$ is the unknown-membership degree.
     - $F(v) \in [0,1]$ is the false-membership degree.
     - $T(v) + C(v) + R(v) + U(v) + F(v) \leq 5$.

- Each edge $e = (u, v) \in E$ is assigned a quintuple $\mu(e) = (T(e), C(e), R(e), U(e), F(e))$, satisfying:

$$\begin{cases} T(e) \leq \min\{T(u), T(v)\} \\ C(e) \leq \min\{C(u), C(v)\} \\ R(e) \geq \max\{R(u), R(v)\} \\ U(e) \geq \max\{U(u), U(v)\} \\ F(e) \geq \max\{F(u), F(v)\} \end{cases}$$

Definition 4. [61, 106, 107 111 113] Let $G = (V, E)$ be a crisp graph where V is the set of vertices and $E \subseteq V \times V$ is the set of edges. A Plithogenic Graph PG is defined as:

$$PG = (PM, PN)$$

where:

1. Plithogenic Vertex Set $PM = (M, l, Ml, adf, aCf)$:
   - $M \subseteq V$ is the set of vertices.
   - l is an attribute associated with the vertices.
   - Ml is the range of possible attribute values.
   - adf : $M \times Ml \rightarrow [0,1]^s$ is the Degree of Appurtenance Function (DAF) for vertices.
   - $aCf: Ml \times Ml \rightarrow [0,1]^t$ is the Degree of Contradiction Function (DCF) for vertices.
2. Plithogenic Edge Set $PN = (N, m, Nm, bdf, bCf)$:
   - $N \subseteq E$ is the set of edges.
   - m is an attribute associated with the edges.
   - Nm is the range of possible attribute values.
   - bdf : $N \times Nm \rightarrow [0,1]^s$ is the Degree of Appurtenance Function (DAF) for edges.
   - bCf: Nm $\times Nm \rightarrow [0,1]^t$ is the Degree of Contradiction Function (DCF) for edges.

The Plithogenic Graph PG must satisfy the following conditions:

1. Edge Appurtenance Constraint: For all $(x, a), (y, b) \in M \times Ml$:

$$bdf((xy), (a, b)) \leq \min\{adf(x, a), adf(y, b)\}$$

   where $xy \in N$ is an edge between vertices x and y, and $(a, b) \in Nm \times Nm$ are the corresponding attribute values.

2. Contradiction Function Constraint: For all $(a, b), (c, d) \in Nm \times Nm$:

$$bCf((a, b), (c, d)) \leq \min\{aCf(a, c), aCf(b, d)\}$$

3. Reflexivity and Symmetry of Contradiction Functions:

$$\begin{aligned} aCf(a, a) &= 0, & \forall a \in Ml \\ aCf(a, b) &= aCf(b, a), & \forall a, b \in Ml \\ bCf(a, a) &= 0, & \forall a \in Nm \\ bCf(a, b) &= bCf(b, a), & \forall a, b \in Nm \end{aligned}$$

Example 5. (cf. [44, 47]) The following examples are provided.

- When $s = t = 1, PG$ is called a Plithogenic Fuzzy Graph.
- When $s = 2, t = 1, PG$ is called a Plithogenic Intuitionistic Fuzzy Graph.
- When $s = 3, t = 1, PG$ is called a Plithogenic Neutrosophic Graph.
- When $s = 4, t = 1, PG$ is called a Plithogenic Turiyam Neutrosophic Graph.

The General Plithogenic Graph is a relax definition of the Plithogenic Graph (cf.[37,44, 84]).

Definition 6 (General Plithogenic Graph). [44] Let $G = (V, E)$ be a classical graph, where V is a finite set of vertices, and $E \subseteq V \times V$ is a set of edges.

A General Plithogenic Graph $G^{GP} = (PM, PN)$ consists of:

1. General Plithogenic Vertex Set PM:

$$PM = (M, l, Ml, adf, aCf)$$

where:

- $M \subseteq V$: Set of vertices.
- l : Attribute associated with the vertices.
- Ml : Range of possible attribute values.
- $adf: M \times Ml \to [0,1]^s$: Degree of Appurtenance Function (DAF) for vertices.
- $aCf: Ml \times Ml \to [0,1]^t$: Degree of Contradiction Function (DCF) for vertices.

2. General Plithogenic Edge Set PN:

$$PN = (N, m, Nm, bdf, bCf)$$

where:

- $N \subseteq E$: Set of edges.
- m : Attribute associated with the edges.
- Nm: Range of possible attribute values.
- bdf: $N \times Nm \to [0,1]^s$: Degree of Appurtenance Function (DAF) for edges.
- $bCf: Nm \times Nm \to [0,1]^t$: Degree of Contradiction Function (DCF) for edges.

The General Plithogenic Graph G^{GP} only needs to satisfy the following Reflexivity and Symmetry properties of the Contradiction Functions:

- Reflexivity and Symmetry of Contradiction Functions:

$$\begin{aligned} aCf(a,a) &= 0, & \forall a \in Ml \\ aCf(a,b) &= aCf(b,a), & \forall a,b \in Ml \\ bCf(a,a) &= 0, & \forall a \in Nm \\ bCf(a,b) &= bCf(b,a), & \forall a,b \in Nm \end{aligned}$$

2.3 | Antipodal Fuzzy Graph and Neutrosophic Graph

In Uncertain Graph Theory, Antipodal Fuzzy Graphs [58, 87, 97] and Neutrosophic Graphs [78-80] are well-known concepts. Their definitions are presented below.

Definition 7 (Antipodal Fuzzy Graph). [58] Let $G = (\sigma, \mu)$ be a fuzzy graph with the underlying set V, where:

- $\sigma: V \to [0,1]$ is a fuzzy subset of V,
- $\mu: V \times V \to [0,1]$ is a symmetric fuzzy relation on σ, satisfying $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$, where $\wedge$ denotes the minimum operation.

The Antipodal Fuzzy Graph of G, denoted as $A(G) = (\sigma_{A(G)}, \mu_{A(G)})$, is defined as follows:

1. The node set of $A(G)$ is the same as the node set of G, i.e., V.
2. The fuzzy subset $\sigma_{A(G)}$ is defined as:

$$\sigma_{A(G)}(u) = \sigma(u), \quad \text{for all } u \in V$$

3. The fuzzy relation $\mu_{A(G)}$ is defined based on the μ-distance between nodes:

$$\mu_{A(G)}(u,v) = \begin{cases} \mu(u,v), & \text{if } \delta(u,v) = \text{diam}(G) \text{ and } u,v \text{ are neighbors in } G, \\ \sigma(u) \wedge \sigma(v), & \text{if } \delta(u,v) = \text{diam}(G) \text{ and } u,v \text{ are not neighbors in } G, \\ 0, & \text{otherwise.} \end{cases}$$

Here:

- $\delta(u,v)$ denotes the μ-distance between nodes u and v in the fuzzy graph G, defined as the smallest μ-length of any $u - v$ path.
- $\text{diam}(G)$ is the diameter of G, calculated as the maximum eccentricity of any node $v \in V$, where the eccentricity $e(v)$ is given by:

$$e(v) = \max_{u \in V} \delta(u,v)$$

The Antipodal Fuzzy Graph $A(G)$ is a fuzzy graph because:

1. $\sigma_{A(G)}(u) = \sigma(u)$ for all $u \in V$, ensuring that $\sigma_{A(G)}$ is a valid fuzzy subset on V.
2. By the definition of $\mu_{A(G)}$, it remains a fuzzy relation on $\sigma_{A(G)}$, satisfying:

$$\mu_{A(G)}(u,v) \leq \sigma_{A(G)}(u) \wedge \sigma_{A(G)}(v), \quad \text{for all } u,v \in V$$

Thus, the graph $A(G)$ is called the Antipodal Fuzzy Graph of G, as it connects pairs of nodes whose μ-distance equals the diameter of G.

Definition 8 (Antipodal Single Valued Neutrosophic Graph (ASVNG)). 79] Let $G = (A, B)$ be a single-valued neutrosophic graph (SVNG), where:

- A is the neutrosophic membership function on the vertex set V, with:

$$A(v) = (T_A(v), I_A(v), F_A(v)), \ \forall v \in V,$$

  where $T_A(v), I_A(v)$, and $F_A(v)$ represent the truth, indeterminacy, and falsity memberships of vertex v, respectively.

- B is the neutrosophic relation on the edge set E, with:

$$B(e) = (T_B(e), I_B(e), F_B(e)), \ \forall e \in E,$$

  where $T_B(e), I_B(e)$, and $F_B(e)$ represent the truth, indeterminacy, and falsity memberships of edge e, respectively.

The Antipodal Single Valued Neutrosophic Graph $(ASVNG)$, denoted as $A(G) = (Q, R)$, is defined as follows:

1. Node Set Q:

$$Q=A, \quad \text{on } V$$

2. Edge Set R : Let $\delta(p, q)$ be the neutrosophic distance between nodes p and q, and $d(G)$ be the diameter of G. For nodes $p, q \in V$:

$$\text{If } \delta(p, q)=d(G), \text{ then:}$$

a) If p and q are adjacent in G :

$$R=B, \quad \text{on } E.$$

b) If p and q are not adjacent in G :

$$\begin{gathered} T_R(p, q)=\min \left(T_A(p), T_A(q)\right) \\ I_R(p, q)=\max \left(I_A(p), I_A(q)\right) \\ F_R(p, q)=\max \left(F_A(p), F_A(q)\right) \end{gathered}$$

3 | Result: Antipodal Single Valued Turiyam Neutrosophic Graph

We consider the Antipodal Single Valued Turiyam Neutrosophic Graph. The following sections present its definition and theorems.

Definition 9 (Antipodal Single Valued Turiyam Neutrosophic Graph (ASVTG)). Let $G=(A, B)$ be a Single Valued Turiyam Neutrosophic Graph (SVTG), where:

- V is a non-empty finite set of vertices.
- $E \subseteq V \times V$ is the set of edges.
- A is the Turiyam Neutrosophic membership function on V, defined as:

$$A(v)=\left(T_A(v), I_A(v), F_A(v), L_A(v)\right), \forall v \in V$$

where:

- $T_A(v)$ is the truth-membership degree of vertex v.
- $I_A(v)$ is the indeterminacy-membership degree of vertex v.
- $F_A(v)$ is the falsity-membership degree of vertex v.
- $L_A(v)$ is the latent-membership degree of vertex v.
- Each $T_A(v), I_A(v), F_A(v), L_A(v) \in[0,1]$.
- The sum satisfies $0 \leq T_A(v)+I_A(v)+F_A(v)+L_A(v) \leq 1$.

- B is the Turiyam Neutrosophic relation on E, defined as:

$$B(e)=\left(T_B(e), I_B(e), F_B(e), L_B(e)\right), \forall e \in E.$$

The Antipodal Single Valued Turiyam Neutrosophic Graph $A(G)=(Q, R)$ is defined as follows:

1. Node Set Q:

$$Q=A, \quad \text{on } V$$

2. Edge Set R:

- Let $\delta(p, q)$ be the Turiyam Neutrosophic distance between nodes p and q in G.
- Let $\operatorname{diam}(G)$ denote the diameter of G.

For nodes $p, q \in V$:

$$\text{If } \delta(p,q) = \operatorname{diam}(G), \text{ then:}$$

a) If p and q are adjacent in G :

$$R(p,q) = B(p,q)$$

b) If p and q are not adjacent in G :

$$T_R(p,q) = \min(T_A(p), T_A(q))$$
$$I_R(p,q) = \max(I_A(p), I_A(q))$$
$$F_R(p,q) = \max(F_A(p), F_A(q))$$
$$L_R(p,q) = \max(L_A(p), L_A(q))$$

Theorem 10. The Antipodal Single Valued Turiyam Neutrosophic Graph $A(G) = (Q,R)$ can be transformed into a standard Turiyam Neutrosophic Graph by removing the antipodal conditions.

Proof.

1. Observation: The ASVTG $A(G)$ includes all vertices V with Turiyam Neutrosophic memberships A and edges defined based on antipodal conditions.
2. Transformation Process:
   - Remove the antipodal condition by considering all possible edges.
   - Define the edge set R' where:

     $$R'(p,q) = B(p,q), \quad \text{for all } (p,q) \in E$$

   - For $(p,q) \notin E$, define $R'(p,q)$ using Turiyam Neutrosophic edge definitions.
3. Resulting Graph: The graph $G' = (A, R')$ is a Turiyam Neutrosophic Graph without antipodal constraints. Thus, the ASVTG transforms into a Turiyam Neutrosophic Graph.

Theorem 11. By mapping the Turiyam Neutrosophic memberships to neutrosophic memberships, the ASVTG $A(G)$ can be transformed into an Antipodal Single Valued Neutrosophic Graph (ASVNG).

Proof.

1. Mapping of Memberships:
   - For each vertex $v \in V$:

     $$T'_A(v) = T_A(v)$$
     $$I'_A(v) = I_A(v) + L_A(v)$$
     $$F'_A(v) = F_A(v)$$

   - Normalize $I'_A(v)$ to ensure it lies within [0, 1]:

     $$I''_A(v) = \frac{I'_A(v)}{\max(1, I'_A(v))}$$

   - Similar mappings for edges.
2. Adjusted $ASVNG$: The new memberships satisfy neutrosophic conditions.
3. Edge Definitions: Redefine the edge set R using the neutrosophic memberships.

   Therefore, the ASVTG transforms into an ASVNG.

Theorem 12. By reducing the Turiyam Neutrosophic memberships to single membership degrees, the ASVTG $A(G)$ can be transformed into an Antipodal Fuzzy Graph.

Proof.

1. Mapping of Memberships:

   - For each vertex $v \in V$:

$$\sigma(v) = T_A(v)$$

   - For each edge $e \in E$:

$$\mu(e) = T_B(e)$$

2. Edge Definitions: Redefine the edge set using fuzzy memberships.
3. Antipodal Conditions: Use the fuzzy distance and diameter analogous to the Turiyam Neutrosophic case. Hence, the ASVTG reduces to an Antipodal Fuzzy Graph.

4 | Future Research

Future research will focus on extending this graph class to Hypergraphs, Directed Hypergraphs, and Superhypergraphs [45,66,108,109]. This extension will involve investigating the mathematical structures and exploring potential applications of these generalized forms.

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Author Contributions

All authors contributed equally to this work.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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