Neutrosophic Optimization and Intelligent Systems

Journal Homepage:https://sciencesforce.com/index.php/nois

Neutrosophic Opt. Int. Syst. Vol. 5 (2024) 29-37.

Paper Type: Original Article

A Compact Exploration of Turiyam Neutrosophic Competition Graphs

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Received: 09 Jun 2024 **Revised:** 14 Dec 2024 **Accepted:** 11 Jan 2025 **Published:** 13 Jan 2025

Abstract

Graph theory, a branch of mathematics, examines relationships between entities using vertices and edges. Within this field, Uncertain Graph Theory has emerged to model uncertainties in real-world networks. A notable concept in this area is the competition graph, which captures interactions by connecting vertices that ”compete” for the same neighbor, represented by directed edges indicating common neighbors in a digraph. This brief paper introduces the concept of the Generalized Turiyam Neutrosophic Competition Graph and explores its relationships with other graph classes.

Keywords: Neutrosophic graph, Fuzzy graph, Turiyam Neutrosophic Graph, Competition Graph, Fuzzy Set

1 | Introduction

1.1 | Uncertain Competition Graph

Graph theory is a fundamental branch of mathematics that models networks through vertices (nodes) and edges (connections), capturing relationships within various systems [10, 48, 34, 15]. Various classes of graphs have been proposed to suit specific mathematical structures and applications [11].

This paper explores different uncertain graph models, including Fuzzy, Intuitionistic Fuzzy, Neutrosophic, Turiyam, and Plithogenic Graphs. These models extend classical graph theory by introducing varying degrees of uncertainty, allowing for a more flexible analysis of complex and ambiguous relationships. Such uncertain graph models have numerous real-world applications, prompting the development of related graph classes [17, 20, 22]. Fundamental concepts such as Fuzzy Sets and Neutrosophic Sets serve as the foundation for these uncertain graph models and are extensively documented in the literature [42, 33, 16].

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https://doi.org/10.61356/j.nois.2025.5465

In the realm of Uncertain Graph Theory, the concept of a Competition Graph has emerged as significant. A competition graph represents interactions where directed edges in a digraph indicate common neighbors, connecting vertices that "compete" for the same neighbor. Notable examples include Fuzzy Competition Graphs [5, 35, 7] and Neutrosophic Competition Graphs [8, 6, 5], which are studied for their intriguing mathematical properties and potential applications.

Given the extensive body of literature and the diverse applications, the study of uncertain graphs holds considerable significance. For a comprehensive overview, readers are encouraged to consult existing survey papers [21, 17, 22].

1.2 | Our Contribution in This Paper

This paper introduces the concept of the Generalized Turiyam Neutrosophic Competition Graph. This model expands the traditional graph framework by assigning four membership values—truth, indeterminacy, falsity, and liberal state—to each vertex and edge, enabling a richer representation of complex relationships [23, 17, 21]. Note that Turiyam Neutrosophic Set is actually a particular case of the quadripartitioned Neutrosophic Set, by replacing "Contradiction" with "Liberal" [41]. The corresponding graph concept known as quadripartitioned neutrosophic graphs is well-documented [29, 30].

2 | Preliminaries and Definitions

This section provides an overview of the fundamental definitions and notations used throughout the paper.

2.1 | Basic Graph Concepts

Below are some of the foundational concepts in graph theory. For more comprehensive information on graph theory and its notations, refer to [15, 26, 47].

Definition 1 (Graph)**.** [15] A *graph* G is a mathematical structure that represents relationships between objects. It consists of a set of vertices $V(G)$ and a set of edges $E(G)$, where each edge connects a pair of vertices. Formally, a graph is represented as $G = (V, E)$, where V is the set of vertices and E is the set of edges.

Definition 2 (Degree)**.** [15] Let $G = (V, E)$ be a graph. The *degree* of a vertex $v \in V$, denoted $\deg(v)$, is defined as the number of edges connected to v. For undirected graphs, the degree is given by:

$$\deg(v) = |\{e \in E \mid v \in e\}|.$$

For directed graphs, the *in-degree* $\deg^-(v)$ refers to the number of edges directed towards v, while the *out-degree* $\deg^+(v)$ represents the number of edges directed away from v.

2.2 | Uncertain Graph

This paper addresses Fuzzy, Intuitionistic Fuzzy, Neutrosophic, Turiyam, and Plithogenic concepts. Note that Turiyam Neutrosophic Set is actually a particular case of the Quadruple Neutrosophic Set, by replacing "Contradiction" with "Liberal" [41].

Definition 3 (Unified Uncertain Graphs Framework)**.** (cf.[17]) Let $G = (V, E)$ be a classical graph with a set of vertices V and a set of edges E. Depending on the type of graph, each vertex $v \in V$ and edge $e \in E$ is assigned membership values to represent various degrees of truth, indeterminacy, falsity, and other nuanced measures of uncertainty.

(1) *Fuzzy Graph* [37, 46]*:*

- Each vertex $v \in V$ is assigned a membership degree $\sigma(v) \in [0, 1]$.
- Each edge $e = (u, v) \in E$ is assigned a membership degree $\mu(u, v) \in [0, 1]$.

(2) *Intuitionistic Fuzzy Graph (IFG)* [45, 49, 1]*:*

- Each vertex $v \in V$ is assigned two values: $\mu_A(v) \in [0, 1]$ (degree of membership) and $\nu_A(v) \in [0, 1]$ (degree of non-membership), such that $\mu_A(v) + \nu_A(v) \leq 1$.

- Each edge $e = (u, v) \in E$ is assigned two values: $\mu_B(u, v) \in [0, 1]$ and $\nu_B(u, v) \in [0, 1]$, with $\mu_B(u, v) + \nu_B(u, v) \leq 1$.

(3) *Neutrosophic Graph* [27, 12, 4, 21]*:*

- Each vertex $v \in V$ is assigned a triplet $\sigma(v) = (\sigma_T(v), \sigma_I(v), \sigma_F(v))$, where $\sigma_T(v), \sigma_I(v), \sigma_F(v) \in [0, 1]$ and $\sigma_T(v) + \sigma_I(v) + \sigma_F(v) \leq 3$.
- Each edge $e = (u, v) \in E$ is assigned a triplet $\mu(e) = (\mu_T(e), \mu_I(e), \mu_F(e))$.

(4) *Turiyam Neutrosophic Graph* [23, 25, 24]*:*

- Each vertex $v \in V$ is assigned a quadruple $\sigma(v) = (t(v), iv(v), fv(v), lv(v))$, where each component is in $[0, 1]$ and $t(v) + iv(v) + fv(v) + lv(v) \leq 4$.
- Each edge $e = (u, v) \in E$ is similarly assigned a quadruple.

(5) *Vague Graph* [3, 38, 2]*:*

- Each vertex $v \in V$ is assigned a pair $(\tau(v), \phi(v))$, where $\tau(v) \in [0, 1]$ is the degree of truth-membership and $\phi(v) \in [0, 1]$ is the degree of false-membership, with $\tau(v) + \phi(v) \leq 1$.
- The grade of membership is characterized by the interval $[\tau(v), 1 - \phi(v)]$.
- Each edge $e = (u, v) \in E$ is assigned a pair $(\tau(e), \phi(e))$, satisfying:

$$\tau(e) \leq \min\{\tau(u), \tau(v)\}, \quad \phi(e) \geq \max\{\phi(u), \phi(v)\}.$$

(6) *Hesitant Fuzzy Graph* [9, 31, 32]*:*

- Each vertex $v \in V$ is assigned a hesitant fuzzy set $\sigma(v)$, represented by a finite subset of $[0, 1]$, denoted $\sigma(v) \subseteq [0, 1]$.
- Each edge $e = (u, v) \in E$ is assigned a hesitant fuzzy set $\mu(e) \subseteq [0, 1]$.
- Operations on hesitant fuzzy sets (e.g., intersection, union) are defined to handle the hesitation in membership degrees.

(7) *Single-Valued Pentapartitioned Neutrosophic Graph* [14, 36, 29]*:*

- Each vertex $v \in V$ is assigned a quintuple $\sigma(v) = (T(v), C(v), R(v), U(v), F(v))$, where:
  - $T(v) \in [0, 1]$ is the truth-membership degree.
  - $C(v) \in [0, 1]$ is the contradiction-membership degree.
  - $R(v) \in [0, 1]$ is the ignorance-membership degree.
  - $U(v) \in [0, 1]$ is the unknown-membership degree.
  - $F(v) \in [0, 1]$ is the false-membership degree.
  - $T(v) + C(v) + R(v) + U(v) + F(v) \leq 5$.
- Each edge $e = (u, v) \in E$ is assigned a quintuple $\mu(e) = (T(e), C(e), R(e), U(e), F(e))$, satisfying:

$$\begin{cases} T(e) \leq \min\{T(u), T(v)\}, \\ C(e) \leq \min\{C(u), C(v)\}, \\ R(e) \geq \max\{R(u), R(v)\}, \\ U(e) \geq \max\{U(u), U(v)\}, \\ F(e) \geq \max\{F(u), F(v)\}. \end{cases}$$

2.3 | Generalized Neutrosophic Competition Graph

The definition of the Generalized Neutrosophic Competition Graph is presented below.

Definition 4 (Generalized Neutrosophic Digraph)**.** [13] A directed graph $\vec{G}' = (V, \vec{E})$, where $\vec{E} \subseteq V \times V$, is called a *generalized neutrosophic digraph* if it includes three membership functions for vertices and edges, as well as transformation functions, defined as follows:

- For each vertex $v_i \in V$, there exist functions:

$$\rho_T : V \to [0,1], \quad \rho_F : V \to [0,1], \quad \rho_I : V \to [0,1],$$

where $\rho_T(v_i), \rho_F(v_i), \rho_I(v_i)$ denote the degrees of true membership, falsity membership, and indeterminacy membership of the vertex v_i, respectively. These functions satisfy:

$$0 \leq \rho_T(v_i) + \rho_F(v_i) + \rho_I(v_i) \leq 3, \quad \text{for all } v_i \in V.$$

- For each directed edge $\vec{e} = (v_i, v_j) \in \vec{E}$, there exist functions:

$$\mu_T : \vec{E} \to [0,1], \quad \mu_F : \vec{E} \to [0,1], \quad \mu_I : \vec{E} \to [0,1],$$

where $\mu_T(v_i, v_j), \mu_F(v_i, v_j), \mu_I(v_i, v_j)$ denote the degrees of true membership, falsity membership, and indeterminacy membership of the directed edge (v_i, v_j), respectively. These functions are defined as:

$$\mu_T(v_i, v_j) = \varphi_T(\rho_T(v_i), \rho_T(v_j)),$$

$$\mu_F(v_i, v_j) = \varphi_F(\rho_F(v_i), \rho_F(v_j)),$$

$$\mu_I(v_i, v_j) = \varphi_I(\rho_I(v_i), \rho_I(v_j)),$$

where $\varphi_T, \varphi_F, \varphi_I$ are transformation functions such that:

$$E_T = \{(\rho_T(v_i), \rho_T(v_j)) : \mu_T(v_i, v_j) \geq 0\}, \quad E_F = \{(\rho_F(v_i), \rho_F(v_j)) : \mu_F(v_i, v_j) \geq 0\},$$

$$E_I = \{(\rho_I(v_i), \rho_I(v_j)) : \mu_I(v_i, v_j) \geq 0\}.$$

Definition 5 (Generalized Neutrosophic Competition Graph)**.** [13] Let $\vec{G}' = (V, \vec{E})$ be a generalized neutrosophic digraph. The *generalized neutrosophic competition graph* $C(\vec{G}')$ of $\vec{G}'$ is a generalized neutrosophic graph that has the same vertex set V. It includes a neutrosophic edge between vertices $u, v \in V$ if and only if:

$$N^+(u) \cap N^+(v) \neq \emptyset,$$

where $N^+(u)$ is the out-neighborhood of vertex u, defined as:

$$N^+(u) = \{(v_j, (\mu_T(v_i, v_j), \mu_F(v_i, v_j), \mu_I(v_i, v_j))) : (v_i, v_j) \in \vec{E}\}.$$

Moreover, there exist sets $S_1 = \{\gamma_T^u : u \in V\}, S_2 = \{\gamma_F^u : u \in V\}, S_3 = \{\gamma_I^u : u \in V\}$ and functions:

$$\varphi_1 : S_1 \times S_1 \to [0,1], \quad \varphi_2 : S_2 \times S_2 \to [0,1], \quad \varphi_3 : S_3 \times S_3 \to [0,1].$$

The edge-membership values of an edge $(u, v) \in E'$ are defined as:

$$\mu_T(u, v) = \varphi_1(\gamma_T^u, \gamma_T^v), \quad \mu_F(u, v) = \varphi_2(\gamma_F^u, \gamma_F^v), \quad \mu_I(u, v) = \varphi_3(\gamma_I^u, \gamma_I^v),$$

where:

$$\gamma_T^u = \min\{\mu_T(u, w) : w \in N^+(u) \cap N^+(v)\}, \quad \gamma_F^u = \max\{\mu_F(u, w) : w \in N^+(u) \cap N^+(v)\},$$

$$\gamma_I^u = \max\{\mu_I(u, w) : w \in N^+(u) \cap N^+(v)\}.$$

3 | Result: Generalized Turiyam Neutrosophic Competition Graph

The definition and mathematical properties of the Generalized Turiyam Neutrosophic Competition Graph are presented below[13].

Definition 6 (Generalized Turiyam Neutrosophic Digraph)**.** A directed graph $\vec{G'} = (V, \vec{E})$, where $\vec{E} \subseteq V \times V$, is called a *Generalized Turiyam Neutrosophic Digraph* if it includes four membership functions for vertices and edges, along with transformation functions, defined as follows:

- For each vertex $v_i \in V$, there exist functions:
$$\rho_T : V \to [0,1], \quad \rho_I : V \to [0,1], \quad \rho_F : V \to [0,1], \quad \rho_L : V \to [0,1],$$
where $\rho_T(v_i), \rho_I(v_i), \rho_F(v_i), \rho_L(v_i)$ denote the degrees of truth-membership, indeterminacy-membership, falsity-membership, and liberal-membership of the vertex v_i, respectively. These functions satisfy:
$$0 \leq \rho_T(v_i) + \rho_I(v_i) + \rho_F(v_i) + \rho_L(v_i) \leq 4, \quad \text{for all } v_i \in V.$$

- For each directed edge $\vec{e} = (v_i, v_j) \in \vec{E}$, there exist functions:
$$\mu_T : \vec{E} \to [0,1], \quad \mu_I : \vec{E} \to [0,1], \quad \mu_F : \vec{E} \to [0,1], \quad \mu_L : \vec{E} \to [0,1],$$
where $\mu_T(v_i, v_j), \mu_I(v_i, v_j), \mu_F(v_i, v_j), \mu_L(v_i, v_j)$ denote the degrees of truth-membership, indeterminacy-membership, falsity-membership, and liberal-membership of the directed edge (v_i, v_j), respectively. These functions are defined as:
$$\mu_T(v_i, v_j) = \varphi_T(\rho_T(v_i), \rho_T(v_j)),$$
$$\mu_I(v_i, v_j) = \varphi_I(\rho_I(v_i), \rho_I(v_j)),$$
$$\mu_F(v_i, v_j) = \varphi_F(\rho_F(v_i), \rho_F(v_j)),$$
$$\mu_L(v_i, v_j) = \varphi_L(\rho_L(v_i), \rho_L(v_j)),$$
where $\varphi_T, \varphi_I, \varphi_F, \varphi_L$ are transformation functions such that:
$$E_T = \{(\rho_T(v_i), \rho_T(v_j)) : \mu_T(v_i, v_j) \geq 0\}, \quad E_I = \{(\rho_I(v_i), \rho_I(v_j)) : \mu_I(v_i, v_j) \geq 0\},$$
$$E_F = \{(\rho_F(v_i), \rho_F(v_j)) : \mu_F(v_i, v_j) \geq 0\}, \quad E_L = \{(\rho_L(v_i), \rho_L(v_j)) : \mu_L(v_i, v_j) \geq 0\}.$$

Definition 7 (Generalized Turiyam Neutrosophic Competition Graph)**.** Let $\vec{G'} = (V, \vec{E})$ be a Generalized Turiyam Neutrosophic Digraph. The *Generalized Turiyam Neutrosophic Competition Graph* $C(\vec{G'})$ of $\vec{G'}$ is a Generalized Turiyam Neutrosophic Graph that has the same vertex set V. It includes a Turiyam Neutrosophic edge between vertices $u, v \in V$ if and only if:
$$N^+(u) \cap N^+(v) \neq \emptyset,$$
where $N^+(u)$ is the out-neighborhood of vertex u, defined as:
$$N^+(u) = \{(v_j, (\mu_T(v_u, v_j), \mu_I(v_u, v_j), \mu_F(v_u, v_j), \mu_L(v_u, v_j))) : (v_u, v_j) \in \vec{E}\}.$$
Moreover, there exist sets $S_T = \{\gamma_T^u : u \in V\}, S_I = \{\gamma_I^u : u \in V\}, S_F = \{\gamma_F^u : u \in V\}, S_L = \{\gamma_L^u : u \in V\}$ and functions:
$$\varphi'_T : S_T \times S_T \to [0,1], \quad \varphi'_I : S_I \times S_I \to [0,1], \quad \varphi'_F : S_F \times S_F \to [0,1], \quad \varphi'_L : S_L \times S_L \to [0,1].$$
The edge-membership values of an edge $(u, v) \in E'$ are defined as:
$$\mu_T(u,v) = \varphi'_T(\gamma_T^u, \gamma_T^v), \quad \mu_I(u,v) = \varphi'_I(\gamma_I^u, \gamma_I^v), \quad \mu_F(u,v) = \varphi'_F(\gamma_F^u, \gamma_F^v), \quad \mu_L(u,v) = \varphi'_L(\gamma_L^u, \gamma_L^v),$$
where:
$$\gamma_T^u = \min\{\mu_T(u,w) : w \in N^+(u) \cap N^+(v)\},$$
$$\gamma_I^u = \max\{\mu_I(u,w) : w \in N^+(u) \cap N^+(v)\},$$
$$\gamma_F^u = \max\{\mu_F(u,w) : w \in N^+(u) \cap N^+(v)\},$$
$$\gamma_L^u = \max\{\mu_L(u,w) : w \in N^+(u) \cap N^+(v)\}.$$

Theorem 8. *Any Generalized Turiyam Neutrosophic Digraph can be transformed into a Generalized Neutrosophic Digraph by appropriate mappings of membership degrees.*

Proof: Given a Generalized Turiyam Neutrosophic Digraph $\vec{G}' = (V, \vec{E})$ with vertex membership functions $\rho_T, \rho_I, \rho_F, \rho_L$ and edge membership functions $\mu_T, \mu_I, \mu_F, \mu_L$, we can construct a Generalized Neutrosophic Digraph $\vec{G}'' = (V, \vec{E})$ by mapping the Turiyam Neutrosophic membership degrees to Neutrosophic membership degrees as follows:

For each vertex $v_i \in V$:

$$\rho'_T(v_i) = \rho_T(v_i) + \rho_L(v_i),$$

$$\rho'_I(v_i) = \rho_I(v_i),$$

$$\rho'_F(v_i) = \rho_F(v_i).$$

Since in Turiyam Neutrosophic graphs $\rho_T(v_i) + \rho_I(v_i) + \rho_F(v_i) + \rho_L(v_i) \leq 4$, it follows that:

$$\rho'_T(v_i) + \rho'_I(v_i) + \rho'_F(v_i) = (\rho_T(v_i) + \rho_L(v_i)) + \rho_I(v_i) + \rho_F(v_i) \leq 4.$$

Similarly, for each edge $(v_i, v_j) \in \vec{E}$:

$$\mu'_T(v_i, v_j) = \mu_T(v_i, v_j) + \mu_L(v_i, v_j),$$

$$\mu'_I(v_i, v_j) = \mu_I(v_i, v_j),$$

$$\mu'_F(v_i, v_j) = \mu_F(v_i, v_j).$$

The transformation functions $\varphi'_T, \varphi'_I, \varphi'_F$ for the Neutrosophic digraph are defined accordingly, ensuring the relationships hold.

This mapping effectively combines the truth and liberal membership degrees of the Turiyam Neutrosophic graph into the truth membership degree of the Neutrosophic graph. The conditions of the Generalized Neutrosophic Digraph are satisfied under this transformation. □

Theorem 9. *Any Generalized Turiyam Neutrosophic Competition Graph can be transformed into a Generalized Neutrosophic Competition Graph via the same mapping of membership degrees.*

Proof: From the previous theorem, the Generalized Turiyam Neutrosophic Digraph $\vec{G}'$ can be transformed into a Generalized Neutrosophic Digraph $\vec{G}''$. Since the competition graph is derived from the digraph by considering the out-neighborhoods $N^+(u)$, the same mapping of membership degrees can be applied.

For each vertex $u \in V$, the values $\gamma^u_T, \gamma^u_I, \gamma^u_F$ in the Turiyam Neutrosophic Competition Graph are transformed to $\gamma'^u_T, \gamma'^u_I, \gamma'^u_F$ in the Neutrosophic Competition Graph as:

$$\gamma'^u_T = \gamma^u_T + \gamma^u_L,$$

$$\gamma'^u_I = \gamma^u_I,$$

$$\gamma'^u_F = \gamma^u_F.$$

The edge membership values in the competition graph are then computed using the transformed γ values and the appropriate transformation functions $\varphi_1, \varphi_2, \varphi_3$. This ensures that the Generalized Turiyam Neutrosophic Competition Graph is transformed into a Generalized Neutrosophic Competition Graph. □

4 | Future tasks:Competition hypergraphs

We outline future research directions. Our goal is to extend the above graph to Competition Hypergraphs, investigating their mathematical properties and potential applications [39, 40, 44]. Additionally, we consider Competition Superhypergraphs—a graph concept that integrates the ideas of Competition Graphs and Superhypergraphs [28, 18, 19, 43].

Funding

This research received no external funding.

Acknowledgments

We humbly extend our heartfelt gratitude to everyone who has provided invaluable support, enabling the successful completion of this paper. We also express our sincere appreciation to all readers who have taken the time to engage with this work. Furthermore, we extend our deepest respect and gratitude to the authors of the references cited in this paper. Thank you for your significant contributions.

Data Availability

This paper does not involve any data analysis.

Ethical Approval

This article does not involve any research with human participants or animals.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Disclaimer

This study primarily focuses on theoretical aspects, and its application to practical scenarios has not yet been validated. Future research may involve empirical testing and refinement of the proposed methods. The authors have made every effort to ensure that all references cited in this paper are accurate and appropriately attributed. However, unintentional errors or omissions may occur. The authors bear no legal responsibility for inaccuracies in external sources, and readers are encouraged to verify the information provided in the references independently. Furthermore, the interpretations and opinions expressed in this paper are solely those of the authors and do not necessarily reflect the views of any affiliated institutions.

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