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# Evaluating Performance and Satisfaction of Bus Raid Transit Using the Hybrid T2NN-APPRESAL Method

Asmaa Elsayed [1,*]

[1] Faculty of computers and informatics, Zagazig University, Egypt; aa.ahmed023@eng.zu.edu.eg

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## Abstract

There are many factors that affect the environment in general, the most important of which is climate change as well as population growth. To preserve the environment, researchers seek to find the best ways and solutions to preserve the quality of the environment through the use of renewable energy to reduce emissions resulting from other energies as well as provide a good life for people in light of population density. The high One of these solutions is bus rapid transit (BRT). It's a public transportation system that uses dedicated lanes to provide fast, efficient bus service. BRT systems often feature stations with elevated platforms for easy boarding, prepayment systems to speed up boarding, and traffic signal priority to keep buses moving smoothly. BRT is designed to offer many of the benefits of light rail or subway systems at a lower cost and with more flexibility. It's commonly implemented in cities around the world as a way to improve public transportation and reduce congestion. The term "BRT" is mainly used in the Americas and China, while in India, it is called "BRTS," and in Europe, it is often referred to as a "busway" or "BHLS." This paper seeks to analyze the BRT system based on its features according to the user's satisfaction using the Approach for Preference, Performance and Ranking Evaluation with SAtisfaction Level (APPRESAL) method to overcome the problems of multi-criteria decision-making (MCDM) methods that lack in dealing with user satisfaction as well as the influence of factors on the analysis under the type-2 neutrosophic numbers (T2NN) environment to overcome ambiguity and uncertainty resulting from the collected data.

**Keywords:** Multi-Criteria Decision Making, Type-2 Neutrosophic Numbers, Bus Rapid Transit, APPRESAL.

# 1 | Introduction

Climate change is one of the most important problems facing the world currently, so many researchers have resorted to studying reducing the impact of climate change on the environment in many fields. One of the most important areas that greatly affect the environment is transportation. The appropriate strategy to overcome the problem of congestion and reduce air pollution is to establish BRT (Bus Rapid Transit). BRT is one of the most important innovations witnessed in the field of buses to solve the transportation problem, according to UITP (Union International des Transports Publics), due to the impact that it can have on cities in terms of congestion and air pollution. BRT has grown fast in the last 25 years, promising low cost, rapid implementation, and large positive impacts [1]. In Brazil, the first BRT emerged in 1974 in Curitiba. The

Corresponding Author: aa.ahmed023@eng.zu.edu.eg

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strength of the system relies on the planning tradition of the city and the idea that public transport is key to urban development [2]. The BRT has become very popular in urban cities due to its cost-effectiveness. The bus system is a new part of public transportation in 191 countries around the world, according to the Global BRT Database (2024) [3]. In 2024, the BRT Global Database Collected statistics tell us that there are BRT systems in 191 cities along 5,842 km to serve about 31.596,612 passengers per day, as represented in Table 1 and Figure 1. After observation, we find that the continent with the largest number of cities, kilometers, and passengers is Latin America. The first rapid bus system was implemented in Runcorn, England, in 1971.

**Table 1.** Data form GBD.

| Regions | Passengers per day | Number of cities | Length(km) |
|---|---|---|---|
| **Africa** | 491578 (1.55%) | 6 (3.14%) | 152 (2.95%) |
| **Asia** | 7987756 (25.28%) | 46 (24.08%) | 1772 (30.33%) |
| **Europe** | 3169846 (10.03%) | 47 (24.6%) | 950 (16.25%) |
| **Latin America** | 8.505.436 (58.56%) | 64 (33.5%) | 2044 (34.98%) |
| **Northern America** | 1005796 (3.18%) | 23 (12.04%) | 815 (13) |
| **Oceania** | 436200 (1.38%) | 5 (2.61%) | 109 (1.86%) |

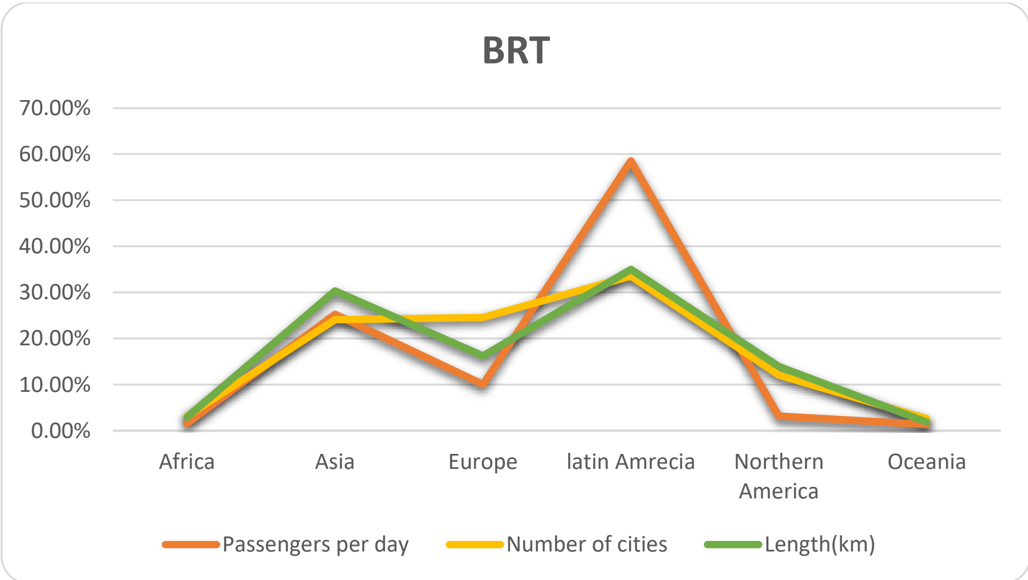


**Figure 1.** Data form GBD.

Moreover, the BRT system operates in a dynamic urban environment marked by rapid population growth and shifting commuter preferences [4]. Bus transportation has features similar to those of the railway or metro system. It is more reliable, convenient, and faster than regular bus services. BRT can avoid delays that usually reduce the quality of regular bus service, such as being stuck in congested traffic or waiting to pay the fare. The BRT consists of large buses that run in specific lanes. These lanes are a section of the road with a length of no less than approximately 3 kilometers. They have dedicated lanes for buses and also contain stations where buses stop. These stations contain technology that enables passengers to pay the fare before boarding to avoid the resulting delay in paying on the bus. The BRT appeared in Latin America as an innovation that could change cities and the way people move, as it constitutes the first step towards integrated public transportation networks (IPTN) as an incentive for civilizational development. Urban transportation systems have become so different that they threaten cities, meaning they reduce the average life expectancy of roads. The increase in the number of private cars has been a major factor in increasing congestion, pollution, and the quality of roads. Some cities in Western Europe, such as London, Paris, Copenhagen, and Berlin, have implemented some plans to reduce the use of cars over time. This is why, between 1993 and 2014, Vienna managed to decrease the number of trips by car by a third [5]. More than half of the world's population lives in urban areas. Due to the ongoing rapid urbanization and growth of the world's population, there will be about 2.5 billion people added to the urban population by 2050, mainly in Africa and Asia [6].


**Figure 2.** New administrative capital.

The New Administrative Capital has a good life vision (Figure 2), which means that the region must be attractive and competitive while at the same time being sustainable. Therefore, the labor market in the New Capital is still growing and thriving, which means that people need to move around, and the government must provide this through public transportation. But with the increase in population, the population density in Cairo reached 21 million and 332 thousand in 2024. This population increase has led to increased ownership of cars and their use, reducing the volume of public transportation as well as daily traffic congestion. The number of cars reached 2.6 million, or 25.8%, according to the latest statistics in 2023, and the number of buses in the governorate reached 4,600. The percentage of buses compared to other transportation is 4%. See Figure 3.

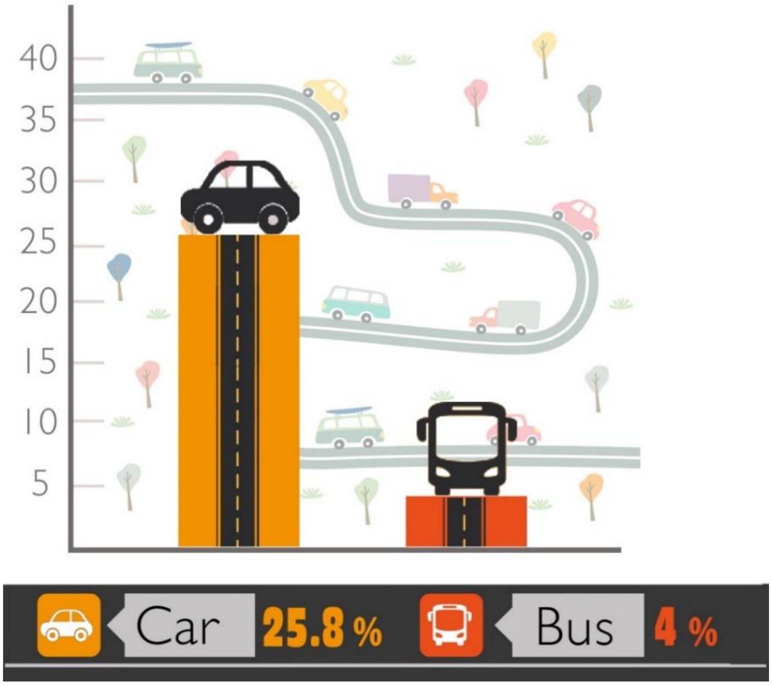


**Figure 3.**

The Brazilian city of Curitiba became known around the world for pioneering BRT in the 1970s [7]. Therefore, the government must provide high-quality public transportation, especially bus services, to encourage people to rely on public transportation. The challenge here is to find additional space for buses and explain to motorists that they will be allocated less road capacity, forcing them to leave their cars. The Egyptian government began implementing the BRT project in 2021 in Cairo in order to facilitate community movement and overcome congestion. The BRT is characterized by shorter journey times, more accurate and reliable services, and increased access to workplaces. The bus runs on electric power as part of the directive for green transportation and preserving the environment. It also contains an ITS system to achieve the highest rates of safety and security on the road and monitor cameras.

There are many countries implementing the rapid bus project, and Egypt is striving to implement and work on this project. To implement any decision, one of the decision-making methods must be used to find the best solutions and implement them, but it lacks dealing with the level of user satisfaction and its impact on the degree of each factor in the analysis. AS It works first by finding weights for the criteria and then classifying them in one of the known ways, such as AHP, ELECTRE, TOPSIS, VIKOR, and others. Manirathinam et al. [7] suggest a novel fuzzy-based MCDM method named Approach for Preference, Performance, and Ranking Evaluation with Satisfaction Level (APPRESAL) [8]. This method uses fuzzy numbers to express the grades that users give to each factor. We here suggest using the type-2 neutrosophic number based on the APPRESAL method. T2NN was improved by Abdel-Basset et al. [9] to allow decision-makers to assess the data that has been collected. The collected data contains uncertain data that can affect the results. So, using the type-2 neutrosophic number is an ideal solution, as T2NN is split into three components: T for truth, I for uncertainty, and F for falsehood. Therefore, T2NN can deal with vagueness and ambiguity very well to get accurate results.

## 1.1 | Contributions of this Study

The primary contributions of this study are summarized below: This paper presents the BRT as the best traffic solution to overcome traffic congestion and reduce accidents and air pollution, as it is economically effective and sustainable. And also the development of a new approach, the APPRESAL method, with a T2NN environment. The proposed approach, T2NN-APPRESAL, improves the performance of decision-making problems. This method is used to evaluate the user's opinions and their impact on alternatives. The goal of this paper is to evaluate the BRT project using T2NN based on the APRESAIL method and choose the best of 3 alternatives.

## 1.2 | Organization of the Paper

This paper is organized as follows: In Section 2, a literature review of the studies used in this paper is presented. Section 3 introduces the concept and methodology for the suggested approach, T2NN-APPRESAL. Section 4 introduces a case study for this method. Finally, Section 5 displays the conclusion of this study.

# 2 | Literature Review

In this section, a simple explanation will be given that contains literature associated with this study. This part consists of three sub-parts. The first one presents studies related to BRT. The second part introduces the studies that explain the neutrosophic numbers T2NN and the APPRESAL method.

## 2.1 | Bus Rapid System

Latin America has the largest share of RBT implementation, Hidalgo et al. [1] explore the BRT industry with a focus on Latin America. (Abdulafis Toliat et al., 2024) explain some problems that face the customer and can threaten the BRT's sustainability in Lagos, Nigeria. Medeiros et al. [11] introduced a study for merging transport network companies and taxis in Curitiba's BRT system.

## 2.2 | T2NN and APPRESAL

MCDM methods are divided into three categories: subjective, objective, and hybrid [6] in A Comprehensive Review of Multiple Criteria Decision-Making (MCDM) Methods: Advancements, Applications, and Future Directions. These methods help to compare alternatives and find the best one [10]. One of the latest methods of MCDM is the APPRESAL method, which was developed by Manirathinam et al. [7] to assess the performance and satisfaction of micro-mobility in smart cities using fuzzy sets [8]. Type-1 neutrosophic number (T1NNS) is a mathematical concept introduced by Florentin Smarandache in the early 1990s as a generalization of fuzzy numbers to capture the nature of human judgments and beliefs, which can be expressed as true, false, or indeterminate. It has been successfully applied in various fields, including building performance optimization, to improve the accuracy and robustness of decision-making processes. The concept of T1NNS is based on three levels of truth: true, false, and indeterminate. The True level represents beliefs that are confirmed, the false level represents beliefs that are refuted, and the indeterminate level represents beliefs that are uncertain or have not been evaluated yet Type-2 neutrosophic number (T2NNS) is an extension of the concept of a T1NN to a higher level of indeterminacy [11]. T2NN is an environment that can deal with ambiguity [9].

# 3 | Methodology

This section introduces the methodology for each study in this paper. This section is also divided into two parts. First, some basic concepts and definitions about T2NN. Second APPERSAL method.

## 3.1 | Preliminaries

First we introduce some concepts and operations associated with T2NN are given below [9]. A Type 2 neutrosophic number set (T2NNS) $\widetilde{U}$ in Z is represented by

$$\widetilde{U} = \left\langle \left(T_{T_{\widetilde{U}}}(z), T_{I_{\widetilde{U}}}(z), T_{F_{\widetilde{U}}}(z)\right), \left(I_{T_{\widetilde{U}}}(z), I_{I_{\widetilde{U}}}(z), I_{F_{\widetilde{U}}}(z)\right), \left(F_{T_{\widetilde{U}}}(z), F_{I_{\widetilde{U}}}(z), F_{F_{\widetilde{U}}}(z)\right) \right\rangle \quad (1)$$

Where $\check{T}_{\widetilde{U}}(z) : Z \to F[0,1]$ , $\check{I}_{\widetilde{U}}(z) : Z \to F[0,1]$ , $\check{F}_{\widetilde{U}}(z) : Z \to F[0,1]$ . The type -2 neutrosophic number set $\check{T}_{\widetilde{U}}(z) = \left(T_{T_{\widetilde{U}}}(z), T_{I_{\widetilde{U}}}(z), T_{F_{\widetilde{U}}}(z)\right)$ , $\check{I}_{\widetilde{U}}(z) = \left(I_{T_{\widetilde{U}}}(z), I_{I_{\widetilde{U}}}(z), I_{F_{\widetilde{U}}}(z)\right)$ , $\check{F}_{\widetilde{U}}(z) =$

$\left(F_{T_{\widetilde{U}}}(z), F_{I_{\widetilde{U}}}(z), F_{F_{\widetilde{U}}}(z)\right)$ defined as the truth , indeterminacy and falsity member-ships of z in $\widetilde{U}$. Suppose that $\widetilde{U}_1 = \left\langle \left(T_{T_{\widetilde{U}1}}(z), T_{I_{\widetilde{U}1}}(z), T_{F_{\widetilde{U}1}}(z)\right), \left(I_{T_{\widetilde{U}1}}(z), I_{I_{\widetilde{U}1}}(z), I_{F_{\widetilde{U}1}}(z)\right), \left(F_{T_{\widetilde{U}1}}(z), F_{I_{\widetilde{U}1}}(z), F_{F_{\widetilde{U}1}}(z)\right)\right\rangle$ and $\widetilde{U}_2 = \left\langle \left(T_{T_{\widetilde{U}2}}(z), T_{I_{\widetilde{U}2}}(z), T_{F_{\widetilde{U}2}}(z)\right), \left(I_{T_{\widetilde{U}2}}(z), I_{I_{\widetilde{U}2}}(z), I_{F_{\widetilde{U}2}}(z)\right), \left(F_{T_{\widetilde{U}2}}(z), F_{I_{\widetilde{U}2}}(z), F_{F_{\widetilde{U}2}}(z)\right)\right\rangle$. Are two T2NNs then the following equations describe some of T2NN operators.

$$\mathbf{1}: \widetilde{U}_1 \oplus \widetilde{U}_2 = \left\langle \begin{matrix} \begin{pmatrix} T_{T_{\widetilde{U}_1}}(z) + T_{T_{\widetilde{U}_2}}(z) - T_{T_{\widetilde{U}_1}}(z).T_{T_{\widetilde{U}2}}(z),\ T_{I_{\widetilde{U}1}}(z) + T_{I_{\widetilde{U}2}}(z) - T_{I_{\widetilde{U}1}}(z).T_{I_{\widetilde{U}2}}(z), \\ T_{F_{\widetilde{U}1}}(z) + T_{F_{\widetilde{U}2}}(z) - T_{F_{\widetilde{U}1}}(z).T_{F_{\widetilde{U}2}}(z) \end{pmatrix}, \\ \left(I_{T_{\widetilde{U}1}}(z).I_{T_{\widetilde{U}2}}(z), I_{I_{\widetilde{U}1}}(z).I_{I_{\widetilde{U}2}}(z), I_{F_{\widetilde{U}1}}(z).I_{F_{\widetilde{U}2}}(z)\right), \\ \left(F_{T_{\widetilde{U}1}}(z).F_{T_{\widetilde{U}2}}(z), F_{I_{\widetilde{U}1}}(z).F_{I_{\widetilde{U}2}}(z), F_{F_{\widetilde{U}1}}(z).F_{F_2}(z)\right) \end{matrix} \right\rangle \quad (2)$$

$$2: \widetilde{U}_1 \otimes \widetilde{U}_2 =$$

$$\left\langle \begin{matrix} \left(\left(T_{T_{\widetilde{U}_1}}(z).T_{T_{\widetilde{U}_2}}(z), T_{I_{\widetilde{U}1}}(z).T_{I_{\widetilde{U}2}}(z), T_{F_{\widetilde{U}1}}(z).T_{F_{\widetilde{U}2}}(z)\right)\right), \\ \left(\left(I_{T_{\widetilde{U}1}}(z) + I_{T_{\widetilde{U}2}}(z) - I_{T_{\widetilde{U}1}}(z).I_{T_{\widetilde{U}2}}(z)\right), \left(I_{I_{\widetilde{U}1}}(z) + I_{I_{\widetilde{U}2}}(z) - I_{I_{\widetilde{U}1}}(z).I_{I_{\widetilde{U}2}}(z)\right), \begin{pmatrix} I_{F_{\widetilde{U}1}}(z) + I_{F_{\widetilde{U}2}}(z) - \\ I_{F_{\widetilde{U}1}}(z).I_{F_{\widetilde{U}2}}(z) \end{pmatrix}\right) \\ \left(\left(F_{T_{\widetilde{U}1}}(z) + F_{T_{\widetilde{U}2}}(z) - F_{T_{\widetilde{U}1}}(z).F_{T_{\widetilde{U}2}}(z)\right), \left(F_{I_{\widetilde{U}1}}(z) + F_{I_{\widetilde{U}2}}(z) - F_{I_{\widetilde{U}1}}(z).F_{I_{\widetilde{U}2}}(z)\right), \begin{pmatrix} F_{F_{\widetilde{U}1}}(z) + F_{F_2}(z) - \\ F_{F_{\widetilde{U}1}}(z).F_{F_2}(z) \end{pmatrix}\right) \end{matrix} \right\rangle \quad (3)$$

$$3: \text{Score Function: } S(\widetilde{U}) = \frac{1}{12}\left\langle 8 + \left(T_{T_{\widetilde{U}1}}(Z) + 2\left(T_{I_{\widetilde{U}1}}(Z)\right) + T_{F_{\widetilde{U}1}}(Z)\right) - \left(I_{T_{\widetilde{U}1}}(Z) + 2\left(I_{I_{\widetilde{U}1}}(Z)\right) + I_{F_{\widetilde{U}1}}(Z)\right) - \left(F_{T_{\widetilde{U}1}}(Z) + 2\left(F_{I_{\widetilde{U}1}}(Z)\right) + F_{F_{\widetilde{U}1}}(Z)\right)\right\rangle \quad (4)$$

## 3.2 | APPERSAL Method [8]

This method works in three stages:

Stage 1 - Evaluation of preference consensus index values.

Stage 2- Evaluating the performance consensus indicator values.

Stage 3 - Analyzing and classifying the level of satisfaction.

The three stages begin by taking into account the group of alternatives ($Alt_i$) where i= 1, 2, …., $m$ and criteria ($\mathcal{C}_j$) where j= 1, 2, …., $n$ then ($\mathcal{U}_k$) where k= 1, 2, …., $\ell$ constitute the group of agents and experts in the decision-making problem. Build the preference matrix represented in Table 2 using data that users express their opinions express based on their favorite level of each criterion. Then build the performance matrix represented in Table3 this is done using the performance level of each alternative under each criterion.

**Table 2.** Preference matrix.

| | $\mathcal{U}_1$ | $\mathcal{U}_2$ | $\mathcal{U}_3$ | …. | $\mathcal{U}_l$ |
|---|---|---|---|---|---|
| $\mathcal{C}_1$ | $r_{11}$ | $r_{12}$ | $r_{13}$ | …. | $r_{1l}$ |
| $\mathcal{C}_2$ | $r_{21}$ | $r_{22}$ | $r_{23}$ | …. | $r_{2l}$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $\mathcal{C}_n$ | $r_{n1}$ | $r_{n2}$ | $r_{n3}$ | …. | $r_{nl}$ |

**Table 3.** Performance matrix.

| | $\text{Alt}_1$ | | | | | | $\text{Alt}_2$ | | | | | … | $\text{Alt}_m$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\mathcal{U}_1$ | $\mathcal{U}_2$ | $\mathcal{U}_3$ | … | $\mathcal{U}_l$ | | $\mathcal{U}_1$ | $\mathcal{U}_2$ | $\mathcal{U}_3$ | … | $\mathcal{U}_l$ | … | $\mathcal{U}_1$ | $\mathcal{U}_2$ | $\mathcal{U}_3$ | … | $\mathcal{U}_l$ |
| $\mathcal{C}_1$ | $\mathcal{P}_{11}^1$ | $\mathcal{P}_{11}^2$ | $\mathcal{P}_{11}^3$ | … | $\mathcal{P}_{11}^l$ | | $\mathcal{P}_{12}^1$ | $\mathcal{P}_{12}^2$ | $\mathcal{P}_{12}^3$ | … | $\mathcal{P}_{12}^l$ | … | $\mathcal{P}_{1m}^1$ | $\mathcal{P}_{1m}^2$ | $\mathcal{P}_{1m}^3$ | … | $\mathcal{P}_{1m}^l$ |
| $\mathcal{C}_2$ | $\mathcal{P}_{21}^1$ | $\mathcal{P}_{21}^2$ | $\mathcal{P}_{21}^3$ | … | $\mathcal{P}_{21}^l$ | | $\mathcal{P}_{22}^1$ | $\mathcal{P}_{22}^2$ | $\mathcal{P}_{22}^3$ | … | $\mathcal{P}_{22}^l$ | … | $\mathcal{P}_{2m}^1$ | $\mathcal{P}_{2m}^2$ | $\mathcal{P}_{2m}^3$ | … | $\mathcal{P}_{2m}^l$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $\mathcal{C}_n$ | $\mathcal{P}_{n1}^1$ | $\mathcal{P}_{n1}^2$ | $\mathcal{P}_{n1}^3$ | … | $\mathcal{P}_{n1}^l$ | | $\mathcal{P}_{n2}^1$ | $\mathcal{P}_{n2}^2$ | $\mathcal{P}_{n2}^3$ | … | $\mathcal{P}_{n2}^l$ | … | $\mathcal{P}_{nm}^1$ | $\mathcal{P}_{nm}^2$ | $\mathcal{P}_{nm}^3$ | … | $\mathcal{P}_{nm}^l$ |

The three stages begin by taking into account the group of alternatives ($Alt_i$) where i= 1, 2, …., $m$ and criteria ($\mathcal{C}_j$) where j= 1, 2, …., $n$ then ($\mathcal{U}_k$) where k= 1, 2, …., $\ell$ constitute the group of agents and experts in the decision-making problem. Build the preference matrix represented in Table 2 using data that users express their opinions express based on their favorite level of each criterion. Then build the performance matrix represented in Table 3 this is done using the performance level of each alternative under each criterion.

**Stage 1: Preference Index**

Step 1: Calculate the normalized index preference matrix using this equation,

$$\mathcal{W}(ri)_{jk} = \sqrt{\frac{1}{k} \cdot \sum_{k=1}^{l}(r_{jk} - \overline{r_{jk}})^2} \quad where \quad \overline{r_{jk}} = \frac{1}{k}\sum_{k=1}^{l} r_{jk} \tag{5}$$

Step 2: Calculate the consensus index of preference by using this equation

$$\mathcal{W}(\mathcal{C}ri)_{jk} = 1 - \frac{1}{\sqrt{k-1}}\left[\frac{(\mathcal{W}(ri)_{jk})}{\overline{r_{jk}}}\right] \tag{6}$$

The consensus index of preference represented in Table 4.

**Table 4.** Consensus index of preference.

| | $\mathcal{C}_1$ | $\mathcal{C}_2$ | $\mathcal{C}_3$ | ….. | $\mathcal{C}_n$ |
|---|---|---|---|---|---|
| $r_j$ | $r_1$ | $r_2$ | $r_3$ | …... | $r_4$ |

Step3: Calculate the similarity index of preference matrices using these equations

The positive similarity $$\mathcal{W}(\mathcal{S}_j)^+ = \sum_{k=1}^{l}\left[\frac{r_{jk} \cdot \mathcal{W}(\mathcal{C}ri)_{jk}}{MIN\ (r_{jk})}\right] \tag{7}$$

The negative similarity $$\mathcal{W}(\mathcal{S}_j)^- = \sum_{k=1}^{l}\left[\frac{r_{jk} \cdot \mathcal{W}(\mathcal{C}ri)_{jk}}{MAX\ (r_{jk})}\right] \tag{8}$$

Step 4: Calculate the consensus preference index by this equation:

$$\mathcal{W}(\mathcal{C}i)_{\mathrm{j}} = \frac{\mathcal{W}(\mathcal{S}_{\mathrm{j}})^+}{\mathcal{W}(\mathcal{S}_{\mathrm{j}})^+ + \mathcal{W}(\mathcal{S}_{\mathrm{j}})^-} \tag{9}$$

**Stage 2: Performance Index**

Step 1: Calculate the normalized index performance matrix using this equation,

$$\mathcal{P}(ri)_{ji} = \sqrt{\frac{1}{k} \cdot \sum_{k=1}^{l}(\mathcal{P}_{ji}^k - \overline{\mathcal{P}_{ji}})^2} \quad where \quad \overline{\mathcal{P}_{ji}} = \frac{1}{k}\sum_{k=1}^{l}\mathcal{P}_{ji}^k \tag{10}$$

Step 2: Calculate the consensus index of performance by using this equation

$$\mathcal{P}(\mathcal{C}ri)_{ji} = 1 - \frac{1}{\sqrt{k-1}}\left[\frac{(\mathcal{P}(ri)_{ji})}{\mathcal{P}_{ji}}\right] \tag{11}$$

The consensus index of performance represented in Table 5.

**Table 5.** Consensus index of performance.

| | $\mathbf{Alt_1}$ | $\mathbf{Alt_2}$ | $\mathbf{Alt_3}$ | ….. | $\mathbf{Alt_m}$ |
|---|---|---|---|---|---|
| $\mathcal{C}_1$ | $\widetilde{\mathcal{P}}_{11}$ | $\widetilde{\mathcal{P}}_{12}$ | $\widetilde{\mathcal{P}}_{13}$ | ….. | $\widetilde{\mathcal{P}}_{1m}$ |
| $\mathcal{C}_2$ | $\widetilde{\mathcal{P}}_{21}$ | $\widetilde{\mathcal{P}}_{22}$ | $\widetilde{\mathcal{P}}_{23}$ | ….. | $\widetilde{\mathcal{P}}_{2m}$ |
| $\mathcal{C}_3$ | $\widetilde{\mathcal{P}}_{31}$ | $\widetilde{\mathcal{P}}_{32}$ | $\widetilde{\mathcal{P}}_{33}$ | ….. | $\widetilde{\mathcal{P}}_{3m}$ |
| ⋮ | ….. | ….. | ….. | ⋱ | ….. |
| $\mathcal{C}_n$ | $\widetilde{\mathcal{P}}_{n1}$ | $\widetilde{\mathcal{P}}_{n2}$ | $\widetilde{\mathcal{P}}_{n3}$ | ….. | $\widetilde{\mathcal{P}}_{nm}$ |

Step 3: calculate the similarity index of performance matrices using these equations.

The positive similarity $\mathcal{P}(\mathcal{S}_{ji})^+ = \sum_{k=1}^{l}\left[\frac{\mathcal{P}_{ji}^{k} \cdot \mathcal{P}(\mathcal{C}ri)_{ji}}{MIN\ (\mathcal{P}_{ji}^{k})}\right]$ (12)

The negative similarity $\mathcal{P}(\mathcal{S}_{ji})^- = \sum_{k=1}^{l}\left[\frac{\mathcal{P}_{ji}^{k} \cdot \mathcal{P}(\mathcal{C}ri)_{ji}}{MAX\ (\mathcal{P}_{ji}^{k})}\right]$ (13)

Step 4: calculate the consensus performance index by this equation:

$$\mathcal{P}(\mathcal{C}i_{ji}) = \frac{\mathcal{P}(\mathcal{S}_{ji})^+}{\mathcal{P}(\mathcal{S}_{ji})^+ + \mathcal{P}(\mathcal{S}_{ji})^-} \quad (14)$$

**Stage 3: Satisfaction index and ranking**

The three stages begin by taking into account the group of alternatives ($Alt_i$) where i= 1, 2, …., $m$ and criteria ($\mathcal{C}_j$) where j= 1, 2, …., $n$ then ($\mathcal{U}_k$) where k = 1, 2, …., $\ell$ constitute the group of agents and experts in the decision-making problem. Build the preference matrix represented in Table 2 using data that users express their opinions express based on their favorite level of each criterion. Then build the performance matrix represented in Table 3 this is done using the performance level of each alternative under each criterion.

Step 1: To analyze the satisfaction level of alternatives in terms of criteria, rank the criteria based on preference values ( $\mathcal{W}(\mathcal{C}i)_j$ ) and performance values ( $\mathcal{P}(\mathcal{C}i_{ji})$. Derive the ordered pairs of rank values as $(x, y)$ and indicate the satisfaction index plots for each alternative. Here, $x$ is the preference rank and $y$ is the performance rank corresponding to each criterion respectively. See (paper for appraisal)

Step 2: Rank the alternatives using

$$\mathcal{R}_i = \sum_{j=1}^{n} \mathcal{W}(\mathcal{C}i)_j \cdot \mathcal{P}(\mathcal{C}i_{ji}). \quad (15)$$

# 4 | Case Study

Egypt is now working on building a rapid bus, so statistics were generated from users to obtain their opinions on the features of the bus to work on, taking into account the importance of users' preferences for each factor of the bus and their impact on its quality.

## 4.1 | Assessment System Indicators

BRT features that result in safer and more convenient trips, More reliable, safer and more convenient. The bus features are several criteria that affect the quality of the bus, as the goal of this method is to obtain the best system that suits all opinions for customer satisfaction. Therefore, the following criteria have been classified based on the opinions of some customers to obtain the best result.

### 4.1.1 | Criteria

- **C1: Bus Way:** The biggest problem that the bus system can face is that there is not enough space, so appropriate spaces must be allocated to provide the best service. Bus Road criteria is divided into several factors: sf1. Dedicated right of way to ensure that buses are able to move quickly and without obstacles that cause congestion. Sf2.Bus-way alignment to ensure that buses stay away from the busy road. It is preferable to separate the lanes from the rest of the other lanes. Sf3. Intersection treatment by preventing traffic turns through the bus lane leads to any reduction resulting from the diversion of traffic. This is done by preventing turns or priority signals at the intersection.

- **C2: Station:** The station must have all the features to provide the best services to passengers. The most important factor of the station is the central station(sf4) There must be central stations that serve both directions, which makes transportation easier and more convenient and also helps reduce the cost of construction. Sf5 a safe and comfortable station The station must be spacious enough to allow ease of transportation. Movement between passengers must be protected from wind or rain, as well as heat or cold, and must be safe and contain cameras and lighting. Sf6 off-board collection Collecting the fare at the station instead of the bus reduces the time of delay in waiting in paying the fare

- **C3: Bus:** The first factor is sf7 the number of doors on the bus There must be at least two wide doors to allow passengers to enter and exit freely. Secondly, sf8 real time information, passengers must be provided with actual information based on GPS and the display of information, whether through electronic panels or audio and digital messages. This can be achieved through working mobile applications. Third, Sf9 minimizing bus emissions the volume of emissions affecting the environment can be reduced by using renewable energies and electrical charging to help reduce air pollution and improve environmental quality.

- **C4: Road:** Sf10: Control centers are increasingly widespread, allowing the operators to monitor the bus, identify the problem, and quickly respond to it, which saves time and improves the quality of bus service. The second factor sf11 is platform-level boarding which that the stop must be on the same level as the bus in order to facilitate boarding and disembarking, especially for people with special needs, children's strollers, and wheelchairs. Finally, Sf12 pavement quality, the pavement must be of high quality for better operation for a longer period while reducing the need for maintenance of the road.

### 4.1.2 | Alternatives

- Lagos, Nigeria The Lagos Metropolitan Area Transport Authority's (LAMATA) BRT corridor is about 22 kilometres long and is the world's most economical BRT.

- Bogotá, Colombia The TransMilenio system in Bogotá is a segregated, four-lane BRT system with a maximum peak-load capacity of 45,000 passengers per hour per direction.

- Kunming, China: Kunming developed the country's first BRT system in 1999.

Experts created two different types of surveys: one to obtain information about user preferences for different factors of BRT service, and the other to collect statistics on the technical performance of individual BRT cities and their services. Twelve different users are surveyed for these surveys. The necessary information is collected using terminology and linguistic variables. Hence, the obtained data matrices are then transformed into T2NN decision matrices using the T2NN methods shown in Tables 6–7. neutrosophic numbers using the linguistic terms shown in Table 6. This Table represents the importance of the weights of each criterion.

We suggest that the users use the linguistic terms shown in Tables 7 and 6 to assess the weight of the criteria and the classification of alternatives. Using score function Eq. (4) for T2NN to convert neutrosophic numbers to crisp numbers.

**Table 6.**

| Linguistic variables | The type 2 neutrosophic number scale for relative importance of each criteria $[(T_t, T_i.T_f),(I_t, I_i.I_f),(F_t, F_i.F_f)]$ | Score function |
|---|---|---|
| Weakly important (WI) | ⟨(0.20, 0.30, 0.20) , (0.60, 0.70, 0.80) , (0.45, 0.75, 0.75)⟩ | 0.3 |
| Equal important (EI) | ⟨(0.40, 0.30, 0.25) , (0.45, 0.55, 0.40) , (0.45, 0.60, 0.55)⟩ | 0.4 |
| Strong important (SI) | ⟨ (0.65, 0.55, 0.55) , (0.40, 0.45, 0.55) , (0.35, 0.40, 0.35)⟩ | 0.6 |
| Very strongly important (VSI) | ⟨(0.80, 0.75, 0.70) , (0.20, 0.15, 0.30) , (0.15, 0.10, 0.20)⟩ | 0.8 |
| Absolutely important (AI) | ⟨(0.90, 0.85, 0.95) , (0.10, 0.15, 0.10) , (0.05, 0.05, 0.10) ⟩ | 0.9 |

**Table 7.**

| Linguistic variables | The type 2 neutrosophic number scale for relative importance of each criteria $[(T_t, T_i.T_f),(I_t, I_i.I_f),(F_t, F_i.F_f)]$ | score function |
|---|---|---|
| Very Bad (VB) | ⟨(0.20, 0.20, 0.10) , (0.65, 0.80, 0.85) , (0.45, 0.80, 0.70)⟩ | 0.2 |
| Bad (b) | ⟨(0.35, 0.35, 0.10) , (0.50, 0.75, 0.80) , (0.50, 0.75, 0.65)⟩ | 0.3 |
| Medium bad (Mb) | ⟨(0.50, 0.30, 0.50) , (0.50, 0.35, 0.45) , (0.45, 0.30, 0.60)⟩ | 0.525 |
| Meduim (m) | ⟨(0.40, 0.45, 0.50) , (0.40, 0.45, 0.50) , (0.35, 0.40, 0.45)⟩ | 0.53 |
| Meduim Good (MG) | ⟨(0.60, 0.45, 0.50) , (0.20, 0.15, 0.25) , (0.10, 0.25, 0.15)⟩ | 0.7 |
| good (g) | ⟨(0.70, 0.75, 0.80) , (0.15, 0.20, 0.25) , (0.10, 0.15, 0.20)⟩ | 0.8 |
| very good (vg) | ⟨(0.95, 0.90, 0.95) , (0.10, 0.10, 0.05) , (0.05, 0.05, 0.05)⟩ | 0.9 |

**Stage 1:** Build preference matrix form user's opinions as shown in Table 8.

**Table 8.**

| | $\mathcal{U}_1$ | $\mathcal{U}_2$ | $\mathcal{U}_3$ | $\mathcal{U}_4$ | $\mathcal{U}_5$ | $\mathcal{U}_6$ | $\mathcal{U}_7$ | $\mathcal{U}_8$ | $\mathcal{U}_9$ | $\mathcal{U}_{10}$ | $\mathcal{U}_{11}$ | $\mathcal{U}_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\mathcal{SF}_1$ | AI | VSI | AI | VSI | AI | SI | VSI | VSI | AI | AI | VSI | VSI |
| $\mathcal{SF}_2$ | EI | EI | EI | SI | VSI | SI | VSI | EI | SI | SI | EI | SI |
| $\mathcal{SF}_3$ | SI | SI | VSI | SI | VSI | EI | VSI | VSI | VSI | SI | SI | VSI |
| $\mathcal{SF}_4$ | WI | WI | EI | EI | WI | SI | EI | EI | WI | SI | EI | WI |
| $\mathcal{SF}_5$ | VSI | VSI | VSI | SI | AI | VSI | AI | VSI | SI | VSI | SI | AI |
| $\mathcal{SF}_6$ | AI | VSI | AI | VSI | SI | VSI | AI | SI | VSI | VSI | VSI | AI |
| $\mathcal{SF}_7$ | AI | VSI | AI | SI | VSI | SI | VSI | VSI | AI | SI | VSI | SI |
| $\mathcal{SF}_8$ | VSI | SI | VSI | SI | VSI | VSI | SI | SI | EI | VSI | EI | EI |
| $\mathcal{SF}_9$ | SI | SI | VSI | VSI | SI | EI | SI | EI | VSI | SI | EI | VSI |
| $\mathcal{SF}_{10}$ | SI | SI | VSI | EI | VSI | SI | SI | EI | EI | SI | WI | SI |
| $\mathcal{SF}_{11}$ | EI | SI | SI | EI | EI | SI | SI | WI | SI | WI | WI | EI |
| $\mathcal{SF}_{12}$ | VSI | VSI | SI | VSI | SI | EI | SI | VSI | AI | AI | SI | VSI |

Then convert these Linguistic terms into crisp numbers using score function Eq. (4) to get Table 9.

**Table 9.**

| | $\mathcal{U}_1$ | $\mathcal{U}_2$ | $\mathcal{U}_3$ | $\mathcal{U}_4$ | $\mathcal{U}_5$ | $\mathcal{U}_6$ | $\mathcal{U}_7$ | $\mathcal{U}_8$ | $\mathcal{U}_9$ | $\mathcal{U}_{10}$ | $\mathcal{U}_{11}$ | $\mathcal{U}_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\mathcal{SF}_1$ | 0.9 | 0.8 | 0.9 | 0.8 | 0.9 | 0.6 | 0.8 | 0.8 | 0.9 | 0.9 | 0.8 | 0.8 |
| $\mathcal{SF}_2$ | 0.4 | 0.4 | 0.4 | 0.6 | 0.8 | 0.6 | 0.8 | 0.4 | 0.6 | 0.6 | 0.4 | 0.6 |
| $\mathcal{SF}_3$ | 0.6 | 0.6 | 0.8 | 0.6 | 0.8 | 0.4 | 0.8 | 0.8 | 0.8 | 0.6 | 0.6 | 0.8 |
| $\mathcal{SF}_4$ | 0.3 | 0.3 | 0.4 | 0.4 | 0.3 | 0.6 | 0.4 | 0.4 | 0.3 | 0.6 | 0.4 | 0.3 |
| $\mathcal{SF}_5$ | 0.8 | 0.8 | 0.8 | 0.6 | 0.9 | 0.8 | 0.9 | 0.8 | 0.6 | 0.8 | 0.6 | 0.9 |
| $\mathcal{SF}_6$ | 0.9 | 0.8 | 0.9 | 0.8 | 0.6 | 0.8 | 0.9 | 0.6 | 0.8 | 0.8 | 0.8 | 0.9 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\mathcal{SF}_7$ | 0.9 | 0.8 | 0.9 | 0.6 | 0.8 | 0.6 | 0.8 | 0.8 | 0.9 | 0.6 | 0.8 | 0.6 |
| $\mathcal{SF}_8$ | 0.8 | 0.6 | 0.8 | 0.6 | 0.8 | 0.08 | 0.6 | 0.6 | 0.4 | 0.8 | 0.4 | 0.4 |
| $\mathcal{SF}_9$ | 0.6 | 0.6 | 0.8 | 0.8 | 0.6 | 0.4 | 0.6 | 0.4 | 0.8 | 0.6 | 0.4 | 0.8 |
| $\mathcal{SF}_{10}$ | 0.6 | 0.6 | 0.8 | 0.4 | 0.8 | 0.6 | 0.6 | 0.4 | 0.4 | 0.6 | 0.3 | 0.6 |
| $\mathcal{SF}_{11}$ | 0.4 | 0.6 | 0.6 | 0.4 | 0.4 | 0.6 | 0.6 | 0.3 | 0.6 | 0.3 | 0.3 | 0.4 |
| $\mathcal{SF}_{12}$ | 0.8 | 0.8 | 0.6 | 0.8 | 0.6 | 0.4 | 0.6 | 0.8 | 0.9 | 0.9 | 0.6 | 0.8 |

**Step 1**: Calculate the normalized matrix using Eq. (5) to obtain Table 10.

**Table 10.**

| $\mathcal{W}(ri)_{jk}$ | $\mathcal{SF}_1$ | $\mathcal{SF}_2$ | $\mathcal{SF}_3$ | $\mathcal{SF}_4$ | $\mathcal{SF}_5$ | $\mathcal{SF}_6$ | $\mathcal{SF}_7$ | $\mathcal{SF}_8$ | $\mathcal{SF}_9$ | $\mathcal{SF}_{10}$ | $\mathcal{SF}_{11}$ | $\mathcal{SF}_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $ri$ | *0.083* | *0.1443* | *0.128* | *0.1038* | *0.109* | *0.1* | *0.119* | *0.159* | *0.1518* | *0.1498* | *0.1256* | *0.146* |

**Step 2:** calculate the consensus index of preference by using Eq. (6) to get Table 11.

**Table 11.**

| $\mathcal{W}(\mathcal{C}ri)_{\text{jk}}$ | $\mathcal{SF}_1$ | $\mathcal{SF}_2$ | $\mathcal{SF}_3$ | $\mathcal{SF}_4$ | $\mathcal{SF}_5$ | $\mathcal{SF}_6$ | $\mathcal{SF}_7$ | $\mathcal{SF}_8$ | $\mathcal{SF}_9$ | $\mathcal{SF}_{10}$ | $\mathcal{SF}_{11}$ | $\mathcal{SF}_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $ri$ | 0.97 | 0.92 | 0.94 | 0.919 | 0.958 | 0.96 | 0.95 | 0.954 | 0.926 | 0.919 | 0.917 | 0.938 |

**Step 3:** Obtain the positive and negative similarity using Eqs. (7) and ,(8) and the result shown in Tables 12 and13.

**Table 12.**

| $\mathcal{W}(\mathcal{S}_j)^+$ | $\mathcal{SF}_1$ | $\mathcal{SF}_2$ | $\mathcal{SF}_3$ | $\mathcal{SF}_4$ | $\mathcal{SF}_5$ | $\mathcal{SF}_6$ | $\mathcal{SF}_7$ | $\mathcal{SF}_8$ | $\mathcal{SF}_9$ | $\mathcal{SF}_{10}$ | $\mathcal{SF}_{11}$ | $\mathcal{SF}_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $ri$ | 15.958 | 15.18 | 19.27 | 14.396 | 14.847 | 15.36 | 14.41 | 17.556 | 17.131 | 20.603 | 16.801 | 20.167 |

**Table 13.**

| $\mathcal{W}(\mathcal{S}_j)^-$ | $\mathcal{SF}_1$ | $\mathcal{SF}_2$ | $\mathcal{SF}_3$ | $\mathcal{SF}_4$ | $\mathcal{SF}_5$ | $\mathcal{SF}_6$ | $\mathcal{SF}_7$ | $\mathcal{SF}_8$ | $\mathcal{SF}_9$ | $\mathcal{SF}_{10}$ | $\mathcal{SF}_{11}$ | $\mathcal{SF}_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $ri$ | 10.656 | 7.95 | 99.635 | 7.1985 | 9.888 | 10.22 | 9.57 | 8.778 | 8.567 | 7.696 | 8.4057 | 7.973 |

**Step 4:** Calculate the consensus preference index by Eq (9) to weights for sub-feature represented in Table 14.

**Table 14.**

| $\mathcal{W}(\mathcal{C}i)_j$ | $\mathcal{SF}_1$ | $\mathcal{SF}_2$ | $\mathcal{SF}_3$ | $\mathcal{SF}_4$ | $\mathcal{SF}_5$ | $\mathcal{SF}_6$ | $\mathcal{SF}_7$ | $\mathcal{SF}_8$ | $\mathcal{SF}_9$ | $\mathcal{SF}_{10}$ | $\mathcal{SF}_{11}$ | $\mathcal{SF}_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $ri$ | 0.6007 | 0.667 | 0.667 | 0.666 | 0.6002 | 0.6005 | 0.6009 | 0.667 | 0.666 | 0.728 | 0.666 | 0.716 |

**Finally,** total weights for each main criterion represented in Table 15.

**Table 15.**

| $\mathcal{W}(\mathcal{C}i)_{\text{j}}$ | $\mathcal{C}_1$ | $\mathcal{C}_2$ | $\mathcal{C}_3$ | $\mathcal{C}_4$ |
|---|---|---|---|---|
| $ri$ | 0.6449 | 0.6222 | 0.6446 | 0.703 |

**Stage 1: Performance index:**

The users express their opinions based on Table 6 which represent variables for alternatives then convert these values using score function to get crisp numbers doing that for our 3 alternatives and result represented in Tables from Table 16 to Table 18.

**Table 16.** Data about first alternatives (Alt1) performance.

| $Alt_1$ | $u_1$ | $u_2$ | $u_3$ | $u_4$ | $u_5$ | $u_6$ | $u_7$ | $u_8$ | $u_9$ | $u_{10}$ | $u_{11}$ | $u_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $SF_1$ | 0.2 | 0.2 | 0.3 | 0.2 | 0.2 | 0.2 | 0.3 | 0.2 | 0.3 | 0.2 | 0.3 | 0.3 |
| $SF_2$ | 0.9 | 0.8 | 0.7 | 0.9 | 0.9 | 0.8 | 0.9 | 0.8 | 0.9 | 0.7 | 0.3 | 0.7 |
| $SF_3$ | 0.53 | 0.53 | 0.8 | 0.7 | 0.8 | 0.3 | 0.8 | 0.8 | 0.8 | 0.53 | 0.53 | 0.8 |
| $SF_4$ | 0.2 | 0.2 | 0.3 | 0.525 | 0.525 | 0.7 | 0.525 | 0.525 | 0.3 | 0.7 | 0.3 | 0.8 |
| $SF_5$ | 0.8 | 0.7 | 0.3 | 0.7 | 0.7 | 0.7 | 0.3 | 0.3 | 0.53 | 0.53 | 0.3 | 0.53 |
| $SF_6$ | 0.9 | 0.9 | 0.9 | 0.525 | 0.8 | 0.525 | 0.525 | 0.8 | 0.525 | 0.525 | 0.525 | 0.7 |
| $SF_7$ | 0.9 | 0.525 | 0.7 | 0.7 | 0.8 | 0.7 | 0.3 | 0.8 | 0.3 | 0.3 | 0.3 | 0.7 |
| $SF_8$ | 0.3 | 0.7 | 0.9 | 0.525 | 0.8 | 0.53 | 0.8 | 0.53 | 0.53 | 0.8 | 0.9 | 0.2 |
| $SF_9$ | 0.9 | 0.525 | 0.525 | 0.8 | 0.525 | 0.7 | 0.525 | 0.7 | 0.3 | 0.7 | 0.525 | 0.525 |
| $SF_{10}$ | 0.8 | 0.7 | 0.9 | 0.7 | 0.7 | 0.7 | 0.53 | 0.7 | 0.9 | 0.8 | 0.7 | 0.8 |
| $SF_{11}$ | 0.3 | 0.8 | 0.8 | 0.9 | 0.53 | 0.9 | 0.8 | 0.53 | 0.9 | 0.8 | 0.8 | 0.8 |
| $SF_{12}$ | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.8 | 0.8 | 0.8 | 0.9 | 0.7 | 0.9 | 0.8 |

**Table 17.** Data about first alternatives (Alt2) performance.

| $Alt_2$ | $u_1$ | $u_2$ | $u_3$ | $u_4$ | $u_5$ | $u_6$ | $u_7$ | $u_8$ | $u_9$ | $u_{10}$ | $u_{11}$ | $u_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $SF_1$ | 0.2 | 0.2 | 0.3 | 0.7 | 0.3 | 0.9 | 0.7 | 0.9 | 0.3 | 0.7 | 0.9 | 0.9 |
| $SF_2$ | 0.9 | 0.2 | 0.7 | 0.8 | 0.8 | 0.7 | 0.8 | 0.9 | 0.8 | 0.8 | 0.8 | 0.8 |
| $SF_3$ | 0.9 | 0.2 | 0.7 | 0.8 | 0.9 | 0.8 | 0.8 | 0.8 | 0.7 | 0.7 | 0.8 | 0.9 |
| $SF_4$ | 0.525 | 0.3 | 0.53 | 0.53 | 0.3 | 0.53 | 0.8 | 0.525 | 0.8 | 0.525 | 0.9 | 0.525 |
| $SF_5$ | 0.3 | 0.3 | 0.53 | 0.525 | 0.525 | 0.525 | 0.53 | 0.9 | 0.525 | 0.9 | 0.8 | 0.8 |
| $SF_6$ | 0.9 | 0.2 | 0.8 | 0.525 | 0.7 | 0.8 | 0.7 | 0.8 | 0.8 | 0.9 | 0.8 | 0.9 |
| $SF_7$ | 0.7 | 0.8 | 0.8 | 0.7 | 0.8 | 0.7 | 0.8 | 0.7 | 0.8 | 0.9 | 0.7 | 0.7 |
| $SF_8$ | 0.8 | 0.525 | 0.7 | 0.525 | 0.8 | 0.9 | 0.525 | 0.9 | 0.9 | 0.8 | 0.9 | 0.9 |
| $SF_9$ | 0.9 | 0.525 | 0.525 | 0.525 | 0.8 | 0.8 | 0.8 | 0.53 | 0.8 | 0.9 | 0.8 | 0.9 |
| $SF_{10}$ | 0.7 | 0.3 | 0.7 | 0.3 | 0.7 | 0.7 | 0.8 | 0.7 | 0.8 | 0.9 | 0.7 | 0.7 |
| $SF_{11}$ | 0.2 | 0.53 | 0.53 | 0.3 | 0.53 | 0.3 | 0.8 | 0.8 | 0.9 | 0.8 | 0.53 | 0.8 |
| $SF_{12}$ | 0.9 | 0.7 | 0.8 | 0.7 | 0.3 | 0.8 | 0.7 | 0.8 | 0.8 | 0.9 | 0.7 | 0.7 |

**Table 18.** Data about first alternatives (Alt3) performance.

| $Alt_3$ | $u_1$ | $u_2$ | $u_3$ | $u_4$ | $u_5$ | $u_6$ | $u_7$ | $u_8$ | $u_9$ | $u_{10}$ | $u_{11}$ | $u_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $SF_1$ | 0.2 | 0.53 | 0.3 | 0.2 | 0.2 | 0.2 | 0.3 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |
| $SF_2$ | 0.9 | 0.8 | 0.8 | 0.9 | 0.9 | 0.9 | 0.8 | 0.9 | 0.9 | 0.8 | 0.8 | 0.525 |
| $SF_3$ | 0.53 | 0.9 | 0.8 | 0.9 | 0.8 | 0.9 | 0.525 | 0.9 | 0.53 | 0.9 | 0.9 | 0.8 |
| $SF_4$ | 0.8 | 0.525 | 0.8 | 0.7 | 0.9 | 0.8 | 0.9 | 0.8 | 0.8 | 0.9 | 0.9 | 0.7 |
| $SF_5$ | 0.2 | 0.2 | 0.53 | 0.3 | 0.3 | 0.3 | 0.53 | 0.8 | 0.3 | 0.8 | 0.8 | 0.8 |
| $SF_6$ | 0.9 | 0.3 | 0.8 | 0.53 | 0.7 | 0.8 | 0.7 | 0.8 | 0.8 | 0.9 | 0.8 | 0.9 |
| $SF_7$ | 0.8 | 0.53 | 0.8 | 0.53 | 0.8 | 0.53 | 0.8 | 0.53 | 0.8 | 0.9 | 0.53 | 0.53 |
| $SF_8$ | 0.7 | 0.3 | 0.53 | 0.3 | 0.7 | 0.7 | 0.525 | 0.7 | 0.7 | 0.8 | 0.7 | 0.8 |
| $SF_9$ | 0.9 | 0.525 | 0.525 | 0.525 | 0.8 | 0.8 | 0.8 | 0.53 | 0.8 | 0.9 | 0.8 | 0.9 |
| $SF_{10}$ | 0.7 | 0.53 | 0.7 | 0.53 | 0.7 | 0.7 | 0.8 | 0.7 | 0.8 | 0.8 | 0.7 | 0.7 |
| $SF_{11}$ | 0.2 | 0.525 | 0.525 | 0.3 | 0.525 | 0.3 | 0.8 | 0.8 | 0.525 | 0.8 | 0.525 | 0.8 |
| $SF_{12}$ | 0.9 | 0.7 | 0.8 | 0.7 | 0.525 | 0.8 | 0.7 | 0.9 | 0.8 | 0.8 | 0.9 | 0.9 |

**Step 1:** use Eq. (10) to calculate the normalized index performance matrix for three alternatives shown in Table 19.

**Step 2:** Using Eq. (11) to calculate the consensus index of performance for 3 alternatives shown in Table 20.

**Table 19.**

| $\mathcal{P}(ri)_{ji}$ | $\mathcal{SF}_1$ | $\mathcal{SF}_2$ | $\mathcal{SF}_3$ | $\mathcal{SF}_4$ | $\mathcal{SF}_5$ | $\mathcal{SF}_6$ | $\mathcal{SF}_7$ | $\mathcal{SF}_8$ | $\mathcal{SF}_9$ | $\mathcal{SF}_{10}$ | $\mathcal{SF}_{11}$ | $\mathcal{SF}_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $Alt_1$ | 0.049 | 0.164 | 0.162 | 0.196 | 0.183 | 0.163 | 0.2185 | 0.2183 | 0.151 | 0.098 | 0.178 | 0.065 |
| $Alt_2$ | 0.258 | 0.1756 | 0.180 | 0.176 | 0.198 | 0.191 | 0.176 | 0.1505 | 0.1519 | 0.1748 | 0.225 | 0.149 |
| $Alt_3$ | 0.0938 | 0.103 | 0.152 | 0.106 | 0.2415 | 0.167 | 0.1457 | 0.1889 | 0.1519 | 0.085 | 0.204 | 0.109 |

**Table 20.**

| $\mathcal{P}(\mathcal{C}ri)_{ji}$ | $\mathcal{SF}_1$ | $\mathcal{SF}_2$ | $\mathcal{SF}_3$ | $\mathcal{SF}_4$ | $\mathcal{SF}_5$ | $\mathcal{SF}_6$ | $\mathcal{SF}_7$ | $\mathcal{SF}_8$ | $\mathcal{SF}_9$ | $\mathcal{SF}_{10}$ | $\mathcal{SF}_{11}$ | $\mathcal{SF}_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $Alt_1$ | 0.939 | 0.936 | 0.926 | 0.874 | 0.896 | 0.928 | 0.887 | 0.895 | 0.925 | 0.960 | 0.927 | 0.977 |
| $Alt_2$ | 0.853 | 0.929 | 0.9276 | 0.906 | 0.900 | 0.922 | 0.929 | 0.943 | 0.938 | 0.921 | 0.884 | 0.938 |
| $Alt_3$ | 0.884 | 0.9625 | 0.941 | 0.994 | 0.851 | 0.932 | 0.9347 | 0.908 | 0.9376 | 0.963 | 0.8886 | 0.958 |

**Step 3:** Calculate the similarity for each alternative using Eqs. (12 and 13) to get the result shown in Table 21 for positive similarity and Table 22 for negative similarity.

**Table 21.**

| $\mathcal{P}(\mathcal{S}_{ji})^+$ | $\mathcal{SF}_1$ | $\mathcal{SF}_2$ | $\mathcal{SF}_3$ | $\mathcal{SF}_4$ | $\mathcal{SF}_5$ | $\mathcal{SF}_6$ | $\mathcal{SF}_7$ | $\mathcal{SF}_8$ | $\mathcal{SF}_9$ | $\mathcal{SF}_{10}$ | $\mathcal{SF}_{11}$ | $\mathcal{SF}_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $Alt_1$ | 13.6155 | 29.016 | 24.446 | 20.472 | 19.0848 | 14.403 | 20.771 | 33.629 | 22.3045 | 16.175 | 27.377 | 14.236 |
| $Alt_2$ | 29.855 | 41.805 | 41.742 | 20.5058 | 21.48 | 40.68 | 12.077 | 16.4799 | 15.7316 | 24.56 | 31.0284 | 27.511 |
| $Alt_3$ | 12.9506 | 18.1925 | 16.8205 | 18.211 | 24.9343 | 27.742 | 14.2492 | 22.564 | 15.7259 | 15.19 | 29.436 | 17.198 |

**Table 22.**

| $\mathcal{P}(\mathcal{S}_{ji})^-$ | $\mathcal{SF}_1$ | $\mathcal{SF}_2$ | $\mathcal{SF}_3$ | $\mathcal{SF}_4$ | $\mathcal{SF}_5$ | $\mathcal{SF}_6$ | $\mathcal{SF}_7$ | $\mathcal{SF}_8$ | $\mathcal{SF}_9$ | $\mathcal{SF}_{10}$ | $\mathcal{SF}_{11}$ | $\mathcal{SF}_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $Alt_1$ | 9.077 | 9.672 | 9.1669 | 6.1105 | 7.1568 | 8.4036 | 6.923 | 7.472 | 10.4504 | 9.252 | 9.1258 | 11.023 |
| $Alt_2$ | 6.634 | 9.289 | 9.2809 | 6.8353 | 7.16 | 9.0448 | 9.393 | 9.6131 | 9.1769 | 8.186 | 6.844 | 9.1717 |
| $Alt_3$ | 4.8869 | 10.6165 | 9.8119 | 10.5188 | 6.233 | 9.2445 | 8.3907 | 8.462 | 9.1726 | 10.0629 | 7.3587 | 10.031 |

**Step 4:** Calculate the consensus performance index by Eq. (14) as in Table 23 then aggregated the consensus performance index to get consensus performance for each criteria as shown in Table 24.

**Table 23.**

| $\mathcal{P}(\mathcal{C}i_{ji})$ | $\mathcal{SF}_1$ | $\mathcal{SF}_2$ | $\mathcal{SF}_3$ | $\mathcal{SF}_4$ | $\mathcal{SF}_5$ | $\mathcal{SF}_6$ | $\mathcal{SF}_7$ | $\mathcal{SF}_8$ | $\mathcal{SF}_9$ | $\mathcal{SF}_{10}$ | $\mathcal{SF}_{11}$ | $\mathcal{SF}_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $Alt_1$ | 0.6 | 0.75 | 0.727 | 0.770 | 0.727 | 0.632 | 0.75 | 0.818 | 0.681 | 0.629 | 0.75 | 0.563 |
| $Alt_2$ | 0.818 | 0.818 | 0.818 | 0.7499 | 0.75 | 0.818 | 0.562 | 0.632 | 0.632 | 0.75 | 0.819 | 0.749 |
| $Alt_3$ | 0.726 | 0.631 | 0.631 | 0.634 | 0.800 | 0.75 | 0.629 | 0.727 | 0.632 | 0.601 | 0.800 | 0.632 |

**Table 24.**

| $\mathcal{P}(\mathcal{C}i_{ji})$ | $\mathcal{C}_1$ | $\mathcal{C}_2$ | $\mathcal{C}_3$ | $\mathcal{C}_4$ |
|---|---|---|---|---|
| $Alt_1$ | 0.692 | 0.7097 | 0.7497 | 0.647 |
| $Alt_2$ | 0.818 | 0.773 | 0.609 | 0.773 |
| $Alt_3$ | 0.663 | 0.728 | 0.663 | 0.678 |

**Stage 3 Satisfaction index and ranking**

This is the final stage for the APPRESAL method to get the user preferred and best city performance.

**Step 1:** ranking the sub-factors preference for each criterion as it exists in Table 25 and also rank the sub-factor performance for each alternative as shown in Table 26.

Note that we have ordered pairs from x and y where x represent the preference rank and y represent performance rank after applying these ordered pairs, we found Satisfaction level for each alternative represented in Figure 4.

**Table 25.** Preference ranking values x of all sub-factors.

| | $\mathcal{SF}_1$ | $\mathcal{SF}_2$ | $\mathcal{SF}_3$ | $\mathcal{SF}_4$ | $\mathcal{SF}_5$ | $\mathcal{SF}_6$ | $\mathcal{SF}_7$ | $\mathcal{SF}_8$ | $\mathcal{SF}_9$ | $\mathcal{SF}_{10}$ | $\mathcal{SF}_{11}$ | $\mathcal{SF}_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x$ | 10 | 3 | 4 | 6 | 12 | 11 | 9 | 5 | 7 | 1 | 8 | 2 |

**Table 26.** Preference ranking values y of all sub-factors.

| y | $\mathcal{SF}_1$ | $\mathcal{SF}_2$ | $\mathcal{SF}_3$ | $\mathcal{SF}_4$ | $\mathcal{SF}_5$ | $\mathcal{SF}_6$ | $\mathcal{SF}_7$ | $\mathcal{SF}_8$ | $\mathcal{SF}_9$ | $\mathcal{SF}_{10}$ | $\mathcal{SF}_{11}$ | $\mathcal{SF}_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $Alt_1$ | 11 | 3 | 6 | 2 | 7 | 9 | 4 | 1 | 8 | 10 | 5 | 12 |
| $Alt_2$ | 2 | 3 | 4 | 8 | 6 | 5 | 12 | 10 | 11 | 7 | 1 | 9 |
| $Alt_3$ | 5 | 9 | 10 | 6 | 1 | 3 | 11 | 4 | 7 | 12 | 2 | 8 |

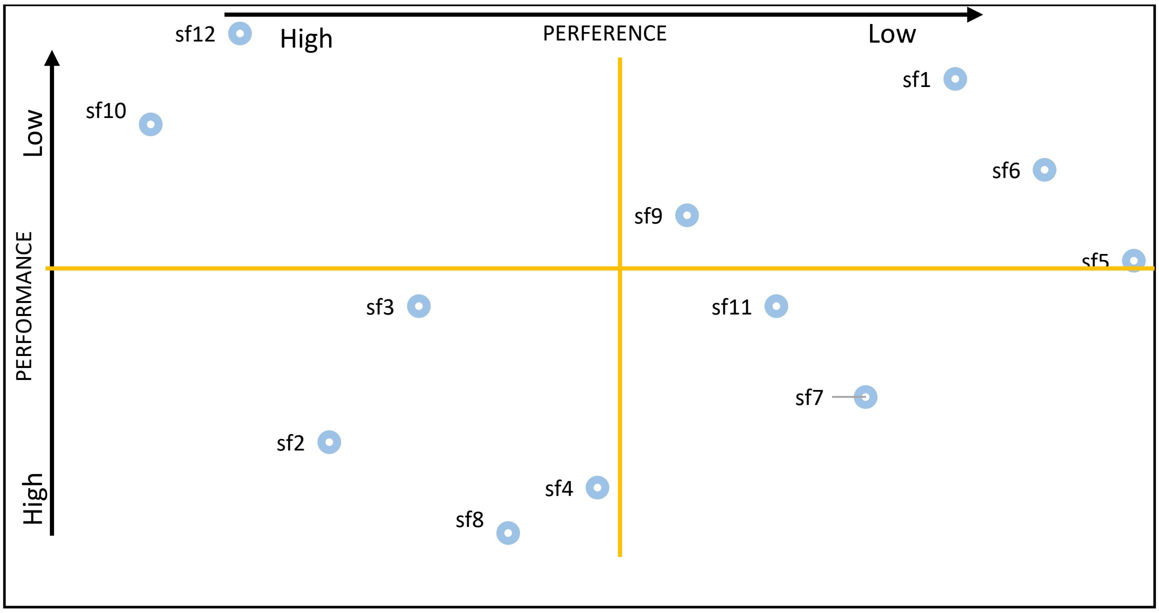


**Figure 4a.**

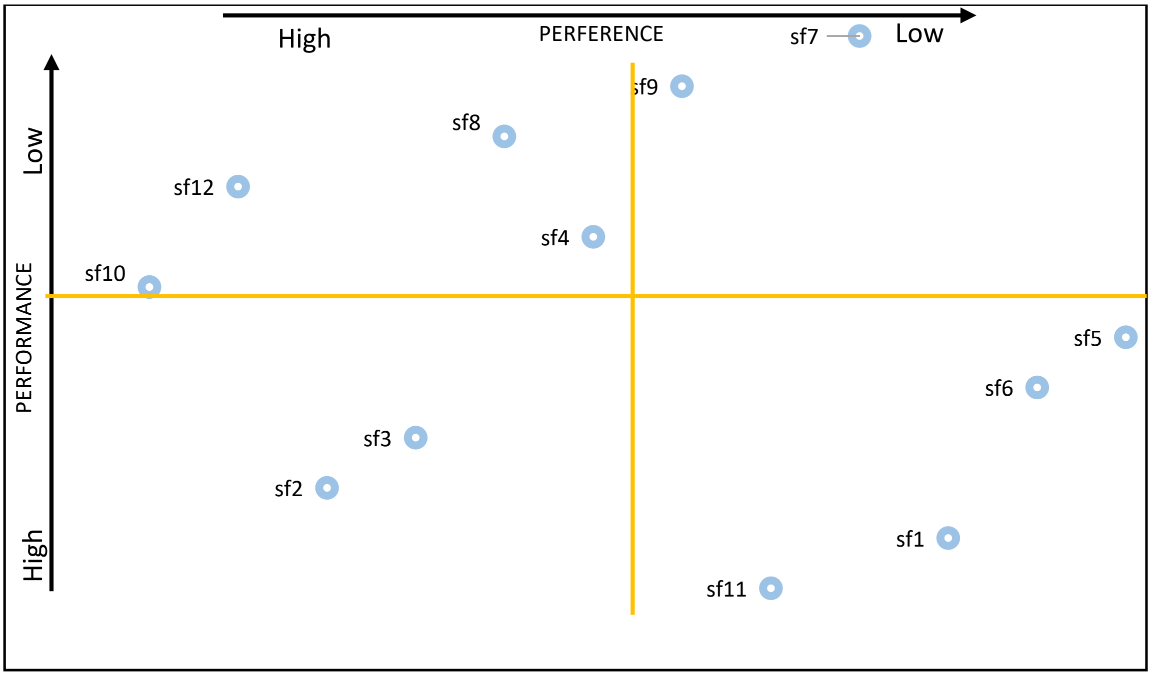


**Figure 4b.**

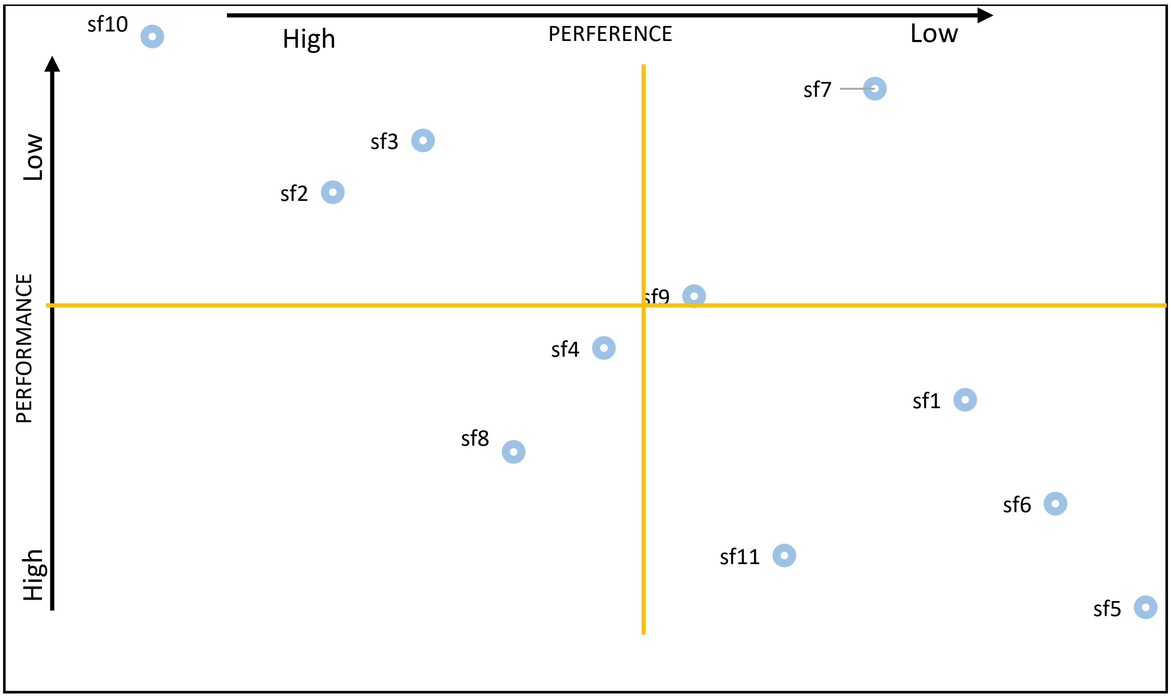


**Figure 4c.**

**Step 2 :** use Eq. (15) to rank alternatives after applying equation we found that second alternative is best one as it has the heights value for $\mathcal{R}_i$

**Table 27.**

| $\mathcal{W}(\mathcal{C}i)_j * \mathcal{P}(\mathcal{C}i_{ji})$ | $\mathcal{C}_1$ | $\mathcal{C}_2$ | $\mathcal{C}_3$ | $\mathcal{C}_4$ | Ranking $\mathcal{R}_i$ | Rank |
|---|---|---|---|---|---|---|
| $Alt_1$ | 0.446 | 0.44157 | 0.4833 | 0.4548 | 1.8257 | 2 |
| $Alt_2$ | 0.5275 | 0.4806 | 0.3926 | 0.5434 | 1.9444 | 1 |
| $Alt_3$ | 0.4276 | 0.4529 | 0.4274 | 0.4766 | 1.7845 | 3 |

Results:

Table 15 shown that $\mathcal{C}_4$ (Road) of BRT system is most important criteria followed by $\mathcal{C}_1$ $(Bus\ way)$ then $\mathcal{C}_3(Bus)$ and finally $\mathcal{C}_2$ $(station)$. Form Figure 5 and Table 27 we found that $\boldsymbol{Alt_2} > \boldsymbol{Alt_1} > \boldsymbol{Alt_3}$ shoen that Bogotá, Colombia is the best is the best of all because it provides engagement services with a high performance rate.

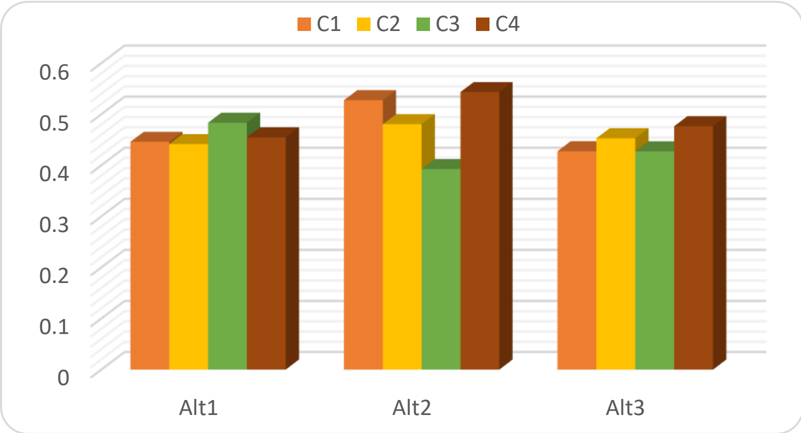


**Figure 5.**

## 5 | Conclusion

BRT is a public transportation system that uses dedicated lanes to provide fast, efficient bus service. BRT systems often feature stations with elevated platforms for easy boarding, prepayment systems to speed up boarding, and traffic signal priority to keep buses moving smoothly. BRT is designed to offer many of the benefits of light rail or subway systems at a lower cost and with more flexibility. It's commonly implemented in cities around the world as a way to improve public transportation and reduce congestion. The APPRISAL method typically refers to a structured approach used in performance appraisal or evaluation systems within organizations. It stands for: Assessment: This involves evaluating against predetermined criteria or standards. Performance: Monitoring and tracking performance over time is essential.

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All authors contributed equally to this work.

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## Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

## Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

## Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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