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Separation Axioms in Neutrosophic Topological Spaces

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Abstract: In this article, we first establish some results based on single-valued neutrosophic sets. Next, we define a subspace topology in a neutrosophic topological space and investigate some properties. We then define the neutrosophic T_0, T_1, T_2-spaces and study their various properties offering adequate examples.

Keywords: Neutrosophic subspace; Neutrosophic T_0-space; Neutrosophic T_1-space; Neutrosophic T_2 -space; Neutrosophic Compact set; Neutrosophic continuous function; Neutrosophic homeomorphism.

1. Introduction

Zadeh [36] uncovered the concept of a fuzzy set in 1965, and Atanassov [1] introduced the intuitionistic fuzzy set, a generalized version of a fuzzy set, in 1986. After a decade, Florentin Smarandache [26-28] developed and studied a new branch of philosophy called "Neutrosophy". Smarandache [28] demonstrated that a neutrosophic set is a generalization of an intuitionistic fuzzy set. Just like an intuitionistic fuzzy set, a neutrosophic set assigns degrees of membership and non-membership to its elements. However, it incorporates an additional measure called the degree of indeterminacy to determine the level of membership. In a neutrosophic set, all three neutrosophic components are independent of one another, which is an important characteristic of the neutrosophic set.

After Smarandache had brought the thought of neutrosophy, it was studied and taken ahead by many researchers [6, 30, 34, 35]. Due to its flexibility and effectiveness, neutrosophy is attracting researchers from various fields around the world, and it has proven to be useful not only in the development of science and technology but also in other areas. For example, Abdel-Basset et al. [3, 4] studied the applications of neutrosophic theory in several scientific fields, while Pramanik and Roy [23] analyzed the conflict between India and Pakistan over Jammu-Kashmir using neutrosophic game theory. Furthermore, researchers have applied neutrosophic theory to medical diagnosis [5, 15], decision-making problems [13, 22], image processing [16], and many other fields.

In 2002, Smarandache [27] introduced the concept of neutrosophic topology on the non-standard interval, and Lupiáñez [18-20] subsequently investigated many properties of neutrosophic topological spaces. In 2012, Salama & Alblowi[29] revealed the idea of neutrosophic topological space as an extension of intuitionistic fuzzy topological space developed by D.Coker [10] in 1997. Salama et al.[32] later introduced the concept of neutrosophic continuous functions. In 2016, Karatas and Kuru[17] redefined single-valued neutrosophic set operations and examined important properties associated with neutrosophic topological spaces. Subsequently, various notions related to neutrosophic topological spaces were developed by numerous researchers [2, 11, 12, 14, 24, 25, 30, 31, 33]. For instance, Al-Nafee et al. [8] utilized neutrosophic crisp points to construct separation axioms in neutrosophic crisp topological spaces and examined the relationships between them. In 2020, Ahu and Ferhat [7] introduced the concept of neutrosophic pre-separation axioms in neutrosophic soft topological spaces and explored the connections among these separation axioms. Additionally, A. Mehmood et al. [21] developed and studied the neutrosophic soft p-separation axioms in neutrosophic soft topological spaces, while V. Amarendra Babu and J. Aswini [9] investigated separation axioms in supra neutrosophic crisp topological spaces in 2021.

The primary objective of this article is to define and explore the separation axioms in neutrosophic topological spaces. Prior to that, we shall first investigate some properties of single-valued neutrosophic sets. Additionally, we shall define the subspace topology (relative topology) in a neutrosophic topological space and examine a few properties.

The article is organized by conferring some basic notions in section 2. In section 3, we establish some results in connection with single-valued neutrosophic sets. We then define neutrosophic subspace with example and investigate some properties. In section 4, we define neutrosophic T_0, T_1, T_2-spaces and study various properties. In section 5, we confer a conclusion.

2. Preliminaries

2.1. Definition: [26] Let X be the universe of discourse. A neutrosophic set A over X is defined as $A = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in X\}$, where the functions $\mathcal{T}_A, \mathcal{I}_A, \mathcal{F}_A$ are real standard or non-standard subsets of $]^-0, 1^+[$, i.e., $\mathcal{T}_A: X \to\,]^-0, 1^+[$, $\mathcal{I}_A: X \to\,]^-0, 1^+[$, $\mathcal{F}_A: X \to\,]^-0, 1^+[$ and $^-0 \leq \mathcal{T}_A(x) + \mathcal{I}_A(x) + \mathcal{F}_A(x) \leq 3^+$.

The neutrosophic set A is characterized by the truth-membership function $\mathcal{T}_A$, indeterminacy-membership function $\mathcal{I}_A$, falsehood-membership function $\mathcal{F}_A$.

2.2. Definition: [35] Let X be the universe of discourse. A single-valued neutrosophic set A over X is defined as $A = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in X\}$, where $\mathcal{T}_A, \mathcal{I}_A, \mathcal{F}_A$ are functions from X to $[0,1]$ and $0 \leq \mathcal{T}_A(x) + \mathcal{I}_A(x) + \mathcal{F}_A(x) \leq 3$.

The set of all single-valued neutrosophic sets over X is denoted by $\mathcal{N}(X)$.

Throughout this article, a neutrosophic set (NS, for short) will mean a single-valued neutrosophic set.

2.3. Definition: [17] Let $A, B \in \mathcal{N}(X)$. Then

(i) (Inclusion): If $\mathcal{T}_A(x) \leq \mathcal{T}_B(x), \mathcal{I}_A(x) \geq \mathcal{I}_B(x), \mathcal{F}_A(x) \geq \mathcal{F}_B(x)$ for all $x \in X$ then A is said to be a neutrosophic subset of B and which is denoted by $A \subseteq B$.

(ii) (Equality): If $A \subseteq B$ and $B \subseteq A$ then $A = B$.

(iii) (Intersection): The intersection of A and B, denoted by $A \cap B$, is defined as $A \cap B = \{\langle x, \mathcal{T}_A(x) \wedge \mathcal{T}_B(x), \mathcal{I}_A(x) \vee \mathcal{I}_B(x), \mathcal{F}_A(x) \vee \mathcal{F}_B(x)\rangle : x \in X\}$.

(iv) (Union): The union of A and B, denoted by $A \cup B$, is defined as $A \cup B = \{\langle x, \mathcal{T}_A(x) \vee \mathcal{T}_B(x), \mathcal{I}_A(x) \wedge \mathcal{I}_B(x), \mathcal{F}_A(x) \wedge \mathcal{F}_B(x)\rangle : x \in X\}$.

(v) (Complement): The complement of the NS A, denoted by A^c, is defined as $A^c = \{\langle x, \mathcal{F}_A(x), 1 - \mathcal{I}_A(x), \mathcal{T}_A(x)\rangle : x \in X\}$.

(vi) (Universal Set): If $\mathcal{T}_A(x) = 1, \mathcal{I}_A(x) = 0, \mathcal{F}_A(x) = 0$ for all $x \in X$ then A is said to be neutrosophic universal set and which is denoted by $\tilde{X}$.

(vii) (Empty Set): If $\mathcal{T}_A(x) = 0, \mathcal{I}_A(x) = 1, \mathcal{F}_A(x) = 1$ for all $x \in X$ then A is said to be neutrosophic empty set and which is denoted by $\tilde{\emptyset}$.

2.4. Definition: [29] Let $\{A_i: i \in \triangle\} \subseteq \mathcal{N}(X)$, where $\triangle$ is an index set. Then

(i) $\cup_{i\in\triangle} A_i = \{\langle x, \vee_{i\in\triangle} \mathcal{T}_{A_i}(x), \wedge_{i\in\triangle} \mathcal{I}_{A_i}(x), \wedge_{i\in\triangle} \mathcal{F}_{A_i}(x)\rangle: x \in X\}$.

(ii) $\cap_{i\in\triangle} A_i = \{\langle x, \wedge_{i\in\triangle} \mathcal{T}_{A_i}(x), \vee_{i\in\triangle} \mathcal{I}_{A_i}(x), \vee_{i\in\triangle} \mathcal{F}_{A_i}(x)\rangle: x \in X\}$.

2.5. Definition:[17] Let $\tau \subseteq \mathcal{N}(X)$. Then τ is called a neutrosophic topology on X if

(i) $\widetilde{\emptyset}$ and $\tilde{X}$ belong to τ.

(ii) Arbitrary union of neutrosophic sets in τ is in τ.

(iii) Intersection of any two neutrosophic sets in τ is in τ.

If τ is a neutrosophic topology on X then the pair (X,τ) is called a neutrosophic topological space (NTS, for short) over X. The members of τ are called neutrosophic τ-open sets (neutrosophic open sets or open sets, for short) in X. If for an NS A, $A^c \in \tau$ then A is said to be a neutrosophic τ-closed set (neutrosophic closed set or closed set, for short) in X.

2.6. Definition: [24] Let $\mathcal{N}(X)$ be the set of all neutrosophic sets over X. An NS $P = \{\langle x, \mathcal{T}_P(x), \mathcal{I}_P(x), \mathcal{F}_P(x)\rangle: x \in X\}$ is called a neutrosophic point (NP, for short) iff for any element $y \in X$, $\mathcal{T}_P(y) = \alpha, \mathcal{I}_P(y) = \beta, \mathcal{F}_P(y) = \gamma$ for $y = x$ and $\mathcal{T}_P(y) = 0, \mathcal{I}_P(y) = 1, \mathcal{F}_P(y) = 1$ for $y \neq x$, where $0 < \alpha \leq 1, 0 \leq \beta < 1, 0 \leq \gamma < 1$. A neutrosophic point $P = \{\langle x, \mathcal{T}_P(x), \mathcal{I}_P(x), \mathcal{F}_P(x)\rangle: x \in X\}$ will be denoted by $x_{\alpha,\beta,\gamma}$. For the NP $x_{\alpha,\beta,\gamma}$, x will be called its support. The complement of the NP $x_{\alpha,\beta,\gamma}$ will be denoted by $(x_{\alpha,\beta,\gamma})^c$. An NS $P = \{\langle x, \mathcal{T}_P(x), \mathcal{I}_P(x), \mathcal{F}_P(x)\rangle: x \in X\}$ is called a neutrosophic crisp point (NCP, for short) iff for any element $y \in X$, $\mathcal{T}_P(y) = 1, \mathcal{I}_P(y) = 0, \mathcal{F}_P(y) = 0$ for $y = x$ and $\mathcal{T}_P(y) = 0, \mathcal{I}_P(y) = 1, \mathcal{F}_P(y) = 1$ for $y \neq x$.

2.7. Definition: [32] Let X and Y be two non-empty sets and $f: X \to Y$ be a function. Also let $A \in \mathcal{N}(X)$ and $B \in \mathcal{N}(Y)$. Then

(i) Image of A under f is defined by $f(A) = \{\langle y, f(\mathcal{T}_A)(y), f(\mathcal{I}_A)(y), (1 - f(1 - \mathcal{F}_A))(y)\rangle: y \in Y\}$,

where $f(\mathcal{T}_A)(y) = \begin{cases} \sup\{\mathcal{T}_A(x): x \in f^{-1}(y)\} & if \ \ f^{-1}(y) \neq \emptyset \\ 0 & if \ \ f^{-1}(y) = \emptyset \end{cases}$

$$f(\mathcal{I}_A)(y) = \begin{cases} \inf\{\mathcal{I}_A(x): x \in f^{-1}(y)\} & if \ \ f^{-1}(y) \neq \emptyset \\ 1 & if \ \ f^{-1}(y) = \emptyset \end{cases}$$

$$(1 - f(1 - \mathcal{F}_A))(y) = \begin{cases} \inf\{\mathcal{F}_A(x): x \in f^{-1}(y)\} & if \ \ f^{-1}(y) \neq \emptyset \\ 1 & if \ \ f^{-1}(y) = \emptyset \end{cases}$$

(ii) Pre-image of B under f is defined by $f^{-1}(B) = \{\langle x, f^{-1}(\mathcal{T}_B)(x), f^{-1}(\mathcal{I}_B)(x), f^{-1}(\mathcal{F}_B)(x)\rangle: x \in X\}$

2.8. Theorem: [32] Let $f: X \to Y$ be a function. Also let $A, A_i \in \mathcal{N}(X), i \in I$ and $B, B_j \in \mathcal{N}(Y), j \in J$. Then the following hold.

(i) $A_1 \subseteq A_2 \Leftrightarrow f(A_1) \subseteq f(A_2)$, $B_1 \subseteq B_2 \Leftrightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$.

(ii) $A \subseteq f^{-1}(f(A))$ and if f is injective then $A = f^{-1}(f(A))$.

(iii) $f^{-1}(f(B)) \subseteq B$ and if f is surjective then $f^{-1}(f(B)) = B$.

(iv) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$ and $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$.

(v) $f(\cup A_i) = \cup f(A_i)$, $f(\cap A_i) \subseteq \cap f(A_i)$ and if f is injective then $f(\cap A_i) = \cap f(A_i)$.

(vi) $f^{-1}(\widetilde{\emptyset}_Y) = \widetilde{\emptyset}_X$, $f^{-1}(\tilde{Y}) = \tilde{X}$.

(vii) $f(\widetilde{\emptyset}_X) = \widetilde{\emptyset}_Y$, $f(\tilde{X}) = \tilde{Y}$ if f is surjective.

2.9. Definition: [33] Let f be a function from an NTS (X, τ) to another NTS (Y, σ). Then

(i) f is called a neutrosophic continuous function if $f^{-1}(G) \in \tau$ for all $G \in \sigma$ then

(ii) f is called a neutrosophic open function if $f(G) \in \sigma$ for all $G \in \tau$.

(iii) f is called a neutrosophic closed function if $f(G)$ is a neutrosophic closed set in Y for every neutrosophic closed set G in X.

(iv) f is called a neutrosophic homeomorphism if the following three conditions hold:

a. f is a bijective function.

b. f is a neutrosophic continuous function.

c. f^{-1} is a neutrosophic continuous function.

2.10. Definition:[24] Let (X, τ) be a neutrosophic topological space. An NS $A \in \mathcal{N}(X)$ is called a neutrosophic neighbourhood or simply neighbourhood (nhbd for short) of an NP $x_{\alpha,\beta,\gamma}$ iff there exists an NS $B \in \tau$ such that $x_{\alpha,\beta,\gamma} \in B \subseteq A$.

2.11. Definition: [14] Let (X, τ) be a neutrosophic topological space. A subcollection $\mathcal{B}$ of τ is called a base for τ iff for each $A \in \tau$, there exists a subcollection $\{A_i : i \in \Delta\} \subseteq \mathcal{B}$ such that $A = \cup \{A_i : i \in \Delta\}$, where Δ is an index set.

2.12. Definition:[14] Let (X, τ) be a neutrosophic topological space and $A \in \mathcal{N}(X)$. A collection $C = \{G_\lambda : \lambda \in \Delta\}$ of neutrosophic open sets of X is called a neutrosophic open cover (NOC, in short) of A if $A \subseteq \cup_{\lambda \in \Delta} G_\lambda$. We then say C covers A. In particular, C is said to be a NOC of X iff $\tilde{X} = \cup_{\lambda \in \Delta} G_\lambda$.

Let C be a NOC of the NS A and $C' \subseteq C$. Then C' is called a neutrosophic open subcover (NOSC, in short) of C if C' covers A.

2.13. Definition:[14] An NS A in an NTS (X, τ) is said to be neutrosophic compact set iff every NOC of A has a finite NOSC. In particular, the space X is said to be neutrosophic compact space iff every NOC of X has a finite NOSC.

3. Neutrosophic Subspaces

In this section we try to establish some results related to single-valued neutrosophic sets. After that, we define neutrosophic subspace with example and then investigate some properties.

3.1. Definition: Let X, Y be two crisp sets such that $Y \neq \emptyset$ and $Y \subseteq X$. We define $\tilde{Y} = \{\langle x, \alpha, \beta, \gamma \rangle : x \in X\}$, where $\alpha = 1, \beta = 0, \gamma = 0$ if $x \in Y$ and $\alpha = 0, \beta = 1, \gamma = 1$ if $x \in X \backslash Y$. The set of all single-valued neutrosophic sets over Y will be denoted by $\mathcal{N}(Y)$.

3.2. Definition: Let X, Y be two crisp sets such that $Y \neq \emptyset$ and $Y \subseteq X$. Then for an NS $A \in \mathcal{N}(X)$, we define $A|_Y = \{\langle x, \mathcal{T}_{A|_Y}(x), \mathcal{I}_{A|_Y}(x), \mathcal{F}_{A|_Y}(x) \rangle : x \in X\}$, where $\mathcal{T}_{A|_Y}(x) = \mathcal{T}_A(x)$, $\mathcal{I}_{A|_Y}(x) = \mathcal{I}_A(x)$, $\mathcal{F}_{A|_Y}(x) = \mathcal{F}_A(x)$ if $x \in Y$ and $\mathcal{T}_{A|_Y}(x) = 0$, $\mathcal{I}_{A|_Y}(x) = 1$, $\mathcal{F}_{A|_Y}(x) = 1$ if $x \in X \backslash Y$.

3.3. Remark: From the definitions 3.1 and 3.2, it is clear that

1. $A|_Y \in \mathcal{N}(Y)$ for every $A \in \mathcal{N}(X)$.
2. Every NS A over Y can be considered as an NS over X by taking $\mathcal{T}_A(x) = 0, \mathcal{I}_A(x) = 1, \mathcal{F}_A(x) = 1$ for all $x \in X \backslash Y$.
3. $\tilde{X}|_Y = \tilde{Y}$ and $\tilde{\emptyset}|_Y = \tilde{\emptyset}$.

3.4. Proposition: Let X, Y, Z be three sets such that $\emptyset \neq Z \subseteq Y \subseteq X$. Let $A \in \mathcal{N}(X)$ and $\{A_\lambda : \lambda \in \Delta\} \subseteq \mathcal{N}(X)$, where Δ is an index set. Then

(i) $(\bigcup_{\lambda\in\Delta} A_\lambda)|_Y = \bigcup_{\lambda\in\Delta} (A_\lambda|_Y)$.

(ii) $(\bigcap_{\lambda\in\Delta} A_\lambda)|_Y = \bigcap_{\lambda\in\Delta} (A_\lambda|_Y)$.

(iii) $A^c|_Y = (A|_Y)^c$.

(iv) $(A|_Y)|_Z = A|_Z$.

Proofs:

(i)
$$\begin{aligned}(\textstyle\bigcup_{\lambda\in\Delta} A_\lambda)|_Y &= \{\langle x, \mathcal{T}_{(\cup_{\lambda\in\Delta}A_\lambda)|_Y}(x), \mathcal{I}_{(\cup_{\lambda\in\Delta}A_\lambda)|_Y}(x), \mathcal{F}_{(\cup_{\lambda\in\Delta}A_\lambda)|_Y}(x)\rangle : x \in X\}\\
&= \{\langle x, \mathcal{T}_{(\cup_{\lambda\in\Delta}A_\lambda)|_Y}(x), \mathcal{I}_{(\cup_{\lambda\in\Delta}A_\lambda)|_Y}(x), \mathcal{F}_{(\cup_{\lambda\in\Delta}A_\lambda)|_Y}(x)\rangle : x \in Y\} \cup\\
&\quad \{\langle x, \mathcal{T}_{(\cup_{\lambda\in\Delta}A_\lambda)|_Y}(x), \mathcal{I}_{(\cup_{\lambda\in\Delta}A_\lambda)|_Y}(x), \mathcal{F}_{(\cup_{\lambda\in\Delta}A_\lambda)|_Y}(x)\rangle : x \in X\setminus Y\}\\
&= \{\langle x, \mathcal{T}_{\cup_{\lambda\in\Delta}A_\lambda}(x), \mathcal{I}_{\cup_{\lambda\in\Delta}A_\lambda}(x), \mathcal{F}_{\cup_{\lambda\in\Delta}A_\lambda}(x)\rangle : x \in Y\} \cup\\
&\quad \{\langle x, 0,1,1\rangle : x \in X\setminus Y\}\\
&= \{\langle x, \vee_{\lambda\in\Delta} \mathcal{T}_{A_\lambda}(x), \wedge_{\lambda\in\Delta} \mathcal{I}_{A_\lambda}(x), \wedge_{\lambda\in\Delta} \mathcal{F}_{A_\lambda}(x)\rangle : x \in Y\}\\
&= \{\langle x, \vee_{\lambda\in\Delta} \mathcal{T}_{A_\lambda|_Y}(x), \wedge_{\lambda\in\Delta} \mathcal{I}_{A_\lambda|_Y}(x), \wedge_{\lambda\in\Delta} \mathcal{F}_{A_\lambda|_Y}(x)\rangle : x \in Y\}\\
&= \textstyle\bigcup_{\lambda\in\Delta} [\{\langle x, \mathcal{T}_{A_\lambda|_Y}(x), \mathcal{I}_{A_\lambda|_Y}(x), \mathcal{F}_{A_\lambda|_Y}(x)\rangle : x \in Y\} \cup \{\langle x, 0,1,1\rangle : x \in X\setminus Y\}]\\
&= \textstyle\bigcup_{\lambda\in\Delta} (A_\lambda|_Y)\end{aligned}$$

(ii)
$$\begin{aligned}(\textstyle\bigcap_{\lambda\in\Delta} A_\lambda)|_Y &= \{\langle x, \mathcal{T}_{(\cap_{\lambda\in\Delta}A_\lambda)|_Y}(x), \mathcal{I}_{(\cap_{\lambda\in\Delta}A_\lambda)|_Y}(x), \mathcal{F}_{(\cap_{\lambda\in\Delta}A_\lambda)|_Y}(x)\rangle : x \in X\}\\
&= \{\langle x, \mathcal{T}_{(\cap_{\lambda\in\Delta}A_\lambda)|_Y}(x), \mathcal{I}_{(\cap_{\lambda\in\Delta}A_\lambda)|_Y}(x), \mathcal{F}_{(\cap_{\lambda\in\Delta}A_\lambda)|_Y}(x)\rangle : x \in Y\} \cup\\
&\quad \{\langle x, \mathcal{T}_{(\cap_{\lambda\in\Delta}A_\lambda)|_Y}(x), \mathcal{I}_{(\cap_{\lambda\in\Delta}A_\lambda)|_Y}(x), \mathcal{F}_{(\cap_{\lambda\in\Delta}A_\lambda)|_Y}(x)\rangle : x \in X\setminus Y\}\\
&= \{\langle x, \mathcal{T}_{\cap_{\lambda\in\Delta}A_\lambda}(x), \mathcal{I}_{\cap_{\lambda\in\Delta}A_\lambda}(x), \mathcal{F}_{\cap_{\lambda\in\Delta}A_\lambda}(x)\rangle : x \in Y\} \cup\\
&\quad \{\langle x, 0,1,1\rangle : x \in X\setminus Y\}\\
&= \{\langle x, \wedge_{\lambda\in\Delta} \mathcal{T}_{A_\lambda}(x), \vee_{\lambda\in\Delta} \mathcal{I}_{A_\lambda}(x), \vee_{\lambda\in\Delta} \mathcal{F}_{A_\lambda}(x)\rangle : x \in Y\}\\
&= \{\langle x, \wedge_{\lambda\in\Delta} \mathcal{T}_{A_\lambda|_Y}(x), \vee_{\lambda\in\Delta} \mathcal{I}_{A_\lambda|_Y}(x), \vee_{\lambda\in\Delta} \mathcal{F}_{A_\lambda|_Y}(x)\rangle : x \in Y\}\\
&= \textstyle\bigcap_{\lambda\in\Delta} [\{\langle x, \mathcal{T}_{A_\lambda|_Y}(x), \mathcal{I}_{A_\lambda|_Y}(x), \mathcal{F}_{A_\lambda|_Y}(x)\rangle : x \in Y\} \cup \{\langle x, 0,1,1\rangle : x \in X\setminus Y\}]\\
&= \textstyle\bigcap_{\lambda\in\Delta} (A_\lambda|_Y)\end{aligned}$$

(iii)
$$\begin{aligned}A^c|_Y &= \{\langle x, \mathcal{T}_{A^c|_Y}(x), \mathcal{I}_{A^c|_Y}(x), \mathcal{F}_{A^c|_Y}(x)\rangle : x \in X\}\\
&= \{\langle x, \mathcal{T}_{A^c}(x), \mathcal{I}_{A^c}(x), \mathcal{F}_{A^c}(x)\rangle : x \in Y\} \cup \{\langle x, 0,1,1\rangle : x \in X\setminus Y\}\\
&= \{\langle x, \mathcal{T}_{A^c}(x), \mathcal{I}_{A^c}(x), \mathcal{F}_{A^c}(x)\rangle : x \in Y\}\\
&= \{\langle x, \mathcal{T}_{A}(x), \mathcal{I}_{A}(x), \mathcal{F}_{A}(x)\rangle : x \in Y\}^c\\
&= \{\langle x, \mathcal{T}_{A|_Y}(x), \mathcal{I}_{A|_Y}(x), \mathcal{F}_{A|_Y}(x)\rangle : x \in Y\}^c\\
&= (\{\langle x, \mathcal{T}_{A|_Y}(x), \mathcal{I}_{A|_Y}(x), \mathcal{F}_{A|_Y}(x)\rangle : x \in Y\} \cup \{\langle x, 0,1,1\rangle : x \in Y\})^c\\
&= \{\langle x, \mathcal{T}_{(A|_Y)^c}(x), \mathcal{I}_{(A|_Y)^c}(x), \mathcal{F}_{(A|_Y)^c}(x)\rangle : x \in X\}\\
&= (A|_Y)^c\end{aligned}$$

(iv)
$$\begin{aligned}(A|_Y)|_Z &= \{\langle x, \mathcal{T}_{(A|_Y)|_Z}(x), \mathcal{I}_{(A|_Y)|_Z}(x), \mathcal{F}_{(A|_Y)|_Z}(x)\rangle : x \in X\}\\
&= \{\langle x, \mathcal{T}_{A|_Y}(x), \mathcal{I}_{A|_Y}(x), \mathcal{F}_{A|_Y}(x)\rangle : x \in Z\} \cup \{\langle x, 0,1,1\rangle : x \notin Z\}\\
&= \{\langle x, \mathcal{T}_{A}(x), \mathcal{I}_{A}(x), \mathcal{F}_{A}(x)\rangle : x \in Y\cap Z\} \cup \{\langle x, 0,1,1\rangle : x \notin Y\cap Z\}\end{aligned}$$

$$= \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in Z\} \cup \{\langle x, 0,1,1\rangle : x \notin Z\}$$
$$= \{\langle x, \mathcal{T}_{A|_Z}(x), \mathcal{I}_{A|_Z}(x), \mathcal{F}_{A|_Z}(x)\rangle : x \in Z\} \cup \{\langle x, 0,1,1\rangle : x \notin Z\}$$
$$= \{\langle x, \mathcal{T}_{A|_Z}(x), \mathcal{I}_{A|_Z}(x), \mathcal{F}_{A|_Z}(x)\rangle : x \in X\}$$
$$= A|_Z$$

***3.5. Proposition*:** Let Y, Z be two non-empty subsets of X and let $A \in \mathcal{N}(X)$. Then $A|_{(Y\cap Z)} = (A|_Y) \cap (A|_Z)$.

Proof:

$$A|_{(Y\cap Z)} = \{\langle x, \mathcal{T}_{A|_{(Y\cap Z)}}(x), \mathcal{I}_{A|_{(Y\cap Z)}}(x), \mathcal{F}_{A|_{(Y\cap Z)}}(x)\rangle : x \in X\}$$
$$= \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in Y \cap Z\} \cup \{\langle x, 0,1,1\rangle : x \notin Y \cap Z\}$$
$$= \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in Y \cap Z\}$$
$$= \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in Y\} \cap \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in Z\}$$
$$= [\{\langle x, \mathcal{T}_{A|_Y}(x), \mathcal{I}_{A|_Y}(x), \mathcal{F}_{A|_Y}(x)\rangle : x \in Y\} \cup \{\langle x, 0,1,1\rangle : x \notin Y\}] \cap$$
$$[\{\langle x, \mathcal{T}_{A|_Z}(x), \mathcal{I}_{A|_Z}(x), \mathcal{F}_{A|_Z}(x)\rangle : x \in Z\} \cup \{\langle x, 0,1,1\rangle : x \notin Z\}]$$
$$= \{\langle x, \mathcal{T}_{A|_Y}(x), \mathcal{I}_{A|_Y}(x), \mathcal{F}_{A|_Y}(x)\rangle : x \in X\} \cap \{\langle x, \mathcal{T}_{A|_Z}(x), \mathcal{I}_{A|_Z}(x), \mathcal{F}_{A|_Z}(x)\rangle : x \in X\}$$
$$= (A|_Y) \cap (A|_Z)$$

3.6. Proposition: Let (X, τ) be an NTS. Let $\emptyset \neq Y \subseteq X$ and $\tau|_Y = \{G|_Y : G \in \tau\}$. Then $(Y, \tau|_Y)$ is an NTS.

Proof:

1. $\tilde{X}, \tilde{\emptyset} \in \tau \Rightarrow \tilde{X}|_Y, \tilde{\emptyset}|_Y \in \tau|_Y$. As $\tilde{Y} = \tilde{X}|_Y$ and $\tilde{\emptyset} = \tilde{\emptyset}|_Y$, so $\tilde{Y}, \tilde{\emptyset} \in \tau|_Y$.
2. Let $\{G_i : i \in \Delta\} \subseteq \tau|_Y$ Then for each $i \in \Delta$, $G_i = G_i'|_Y$ for some $G_i' \in \tau$. Now $\cup_{i\in\Delta} G_i = \cup_{i\in\Delta} (G_i'|_Y) == (\cup_{i\in\Delta} G_i')|_Y \in \tau|_Y$ [$\because \cup_{i\in\Delta} G_i' \in \tau$ and by 3.4(i)].
3. Let $G, H \in \tau|_Y$. Then $G = G'|_Y$ and $H = H'|_Y$ for some $G', H' \in \tau$. Now $G \cap H = (G'|_Y) \cap (H'|_Y) = (G' \cap H')|_Y \in \tau|_Y$ [$\because G' \cap H' \in \tau$ and by 3.4(ii)]

   Hence $(Y, \tau|_Y)$ is an NTS.

3.7. Definition: Let (X, τ) be an NTS. Let $\emptyset \neq Y \subseteq X$ and $\tau|_Y = \{G|_Y : G \in \tau\}$. Then $(Y, \tau|_Y)$ [by 3.6] is an NTS. The topology $\tau|_Y$ is called the neutrosophic relative topology of τ on Y or the neutrosophic subspace topology of Y and the NTS $(Y, \tau|_Y)$ is called a neutrosophic subspace (or a subspace, for short) of the NTS (X, τ).

Members of $\tau|_Y$ are called $\tau|_Y$-open sets in Y. An NS $A \in \mathcal{N}(Y)$ such that $A^c \in \tau|_Y$ is called a $\tau|_Y$-closed set in Y.

$(Y, \tau|_Y)$ is called a neutrosophic open subspace or neutrosophic closed subspace of (X, τ) according as $\tilde{Y} \in \tau$ or $\tilde{Y} \in \tau^c$.

3.8. Example: Let $X = \{a, b\}$ and $\tau = \{\tilde{\emptyset}, \tilde{X}, A, B, A \cap B, A \cup B\}$, where $A = \{\langle a, 0.5,0.4,0.2\rangle, \langle b, 0.6,0.3,0.5\rangle\}$ and $B = \{\langle a, 0.3,0.4,0.6\rangle, \langle b, 0.4,0.7,0.3\rangle\}$. Clearly (X, τ) is an NTS. Let $Y = \{a\}$. Then $\tilde{X}|_Y = \{\langle a, 1,0,0\rangle, \langle b, 0,1,1\rangle\} = \tilde{Y}$, $\tilde{\emptyset}|_Y = \{\langle a, 0,1,1\rangle, \langle b, 0,1,1\rangle\} = \tilde{\emptyset}$, $A|_Y =$

$\{\langle a, 0.5,0.4,0.2\rangle, \langle b, 0,1,1\rangle\}$, $B|_Y = \{\langle a, 0.3,0.4,0.6\rangle, \langle b, 0,1,1\rangle\}$, $(A \cap B)|_Y = \{\langle a, 0.3,0.4,0.6\rangle, \langle b, 0,1,1\rangle\}$, $(A \cup B)|_Y = \{\langle a, 0.5,0.4,0.2\rangle, \langle b, 0,1,1\rangle\}$.

Clearly $\tau|_Y = \{\widetilde{\emptyset}, \tilde{Y}, A|_Y, B|_Y, (A \cap B)|_Y, (A \cup B)|_Y\}$ is a neutrosophic subspace topology of Y, i.e., $(Y, \tau|_Y)$ is a neutrosophic subspace of (X, τ).

3.9. Proposition: Let (Y, σ) be a subspace of an NTS (X, τ) and (Z, μ) be a subspace of (Y, σ). Then (Z, μ) is a subspace of (X, τ).

Proof: Since $Z \subseteq Y \subseteq X$, so $Z \subseteq X$. We need to show that $\tau|_Z = \mu$. Let $G \in \mu$. Since (Z, μ) is a subspace of (Y, σ), so there exists $H \in \sigma$ such that $G = H|_Z$. Again since (Y, σ) is a subspace of (X, τ), so there exists $K \in \tau$ such that $H = K|_Y$. Then $G = H|_Z = (K|_Y)|_Z = K|_Z$ [by 3.4(iv)]. Since $K|_Z \in \tau|_Z$, so $G \in \tau|_Z$. Therefore $\mu \subseteq \tau|_Z$. Next suppose that $U \in \tau|_Z$. Then there exists $V \in \tau$ such that $U = V|_Z$. Since (Y, σ) is a subspace of (X, τ), so $V|_Y \in \sigma$. Again since (Z, μ) is a subspace of (Y, σ), so $(V|_Y)|_Z \in \mu \Rightarrow V|_Z \in \mu \Rightarrow U \in \mu$. Therefore $\tau|_Z \subseteq \mu$. Hence $\tau|_Z = \mu$, i.e., (Z, μ) is a subspace of (X, τ).

3.10. Proposition: Let Y and Z be two subspaces of an NTS (X, τ). If $Y \subseteq Z$ then Y is a subspace of Z.

Proof: Let (Y, σ) and (Z, μ) be the subspaces of the NTS (X, τ). Then $\tau|_Y = \sigma$ and $\tau|_Z = \mu$. Now $\mu|_Y = \{A|_Y : A \in \mu\} = \{(B|_Z)|_Y : B \in \tau \text{ and } B|_Z = A \in \mu\} = \{B|_Y : B \in \tau\} = \tau|_Y = \sigma$. Since $\mu|_Y = \sigma$, so Y is a subspace of Z.

***3.11. Proposition*:** Let $(Y, \tau|_Y)$ be a subspace of an NTS (X, τ) and $A \in \mathcal{N}(Y)$. Then A is $\tau|_Y$-closed iff $A = F|_Y$ for some τ-closed set F in X.

Proof: A is $\tau|_Y$-closed in $Y \Leftrightarrow A^c$ is $\tau|_Y$-open in $Y \Leftrightarrow A^c = G|_Y$ for some $G \in \tau \Leftrightarrow A = (G|_Y)^c \Leftrightarrow A = G^c|_Y$ [3.4(iii)] $\Leftrightarrow A = F|_Y$, where $F = G^c$ is a τ-closed set in X.

***3.12. Remark*:** From 3.11, it is easy to conclude that if $(Y, \tau|_Y)$ is a subspace of an NTS (X, τ) then $(\tau|_Y)^c = \tau^c|_Y$.

***3.13. Proposition*:** Let $(Y, \tau|_Y)$ be a subspace of an NTS (X, τ) and let $\mathcal{B}$ be a base for τ. Then $\mathcal{B}|_Y = \{B|_Y : B \in \mathcal{B}\}$ is a base for $\tau|_Y$.

Proof: Let H be a $\tau|_Y$-open set in Y. Also let $x_{\alpha,\beta,\gamma} \in H$ be an arbitrary NP. Then there exists a τ-open set G such that $H = G|_Y$. Since $\mathcal{B}$ is a base for τ, so there exists a $B \in \mathcal{B}$ such that $x_{\alpha,\beta,\gamma} \in B \subseteq G$. Therefore $x_{\alpha,\beta,\gamma} \in B|_Y \subseteq G|_Y = H$ as $x_{\alpha,\beta,\gamma} \in \mathcal{N}(Y)$. Thus for any $x_{\alpha,\beta,\gamma} \in H$, there exists a member $B|_Y$ of $\mathcal{B}|_Y$ such that $x_{\alpha,\beta,\gamma} \in B|_Y \subseteq H$. Therefore $H = \cup \{B|_Y : B|_Y \in \mathcal{B}|_Y \text{ and } B|_Y \subseteq H\}$. Hence $\mathcal{B}|_Y$ is a base for $\tau|_Y$.

4. Neutrosophic Separation Axioms

Here we study the separation axioms in neutrosophic topological spaces. But, before that, we put forward two definitions.

***4.1. Definition*:** A property of an NTS (X, τ) is said to be hereditary if whenever the space X has that property, then so does every subspace of it.

4.2. Definition: A property of an NTS (X,τ) is said to be a topological property or topological invariant if each space homeomorphic to X has that property whenever the space X has that property. In other words, a property of an NTS is said to be a topological property iff it is preserved under homeomorphism.

4.3. Definition: An NTS (X,τ) is called a neutrosophic T_0-space or (NT_0-space, for short) iff for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x\neq y$, there exists a $U\in\tau$ such that $x_{\alpha,\beta,\gamma}\in U$, $y_{\alpha',\beta',\gamma'}\notin U$ or there exists a $V\in\tau$ such that $x_{\alpha,\beta,\gamma}\notin V$, $y_{\alpha',\beta',\gamma'}\in V$.

4.4. Example: Let $X=\{a,b\}$ and $\tau=\{\widetilde{\emptyset},\tilde{X},A,B\}$, where $A=\{\langle a,1,0,0\rangle,\langle b,0,1,1\rangle\}$ and $B=\{\langle a,0,1,1\rangle,\langle b,1,0,0\rangle\}$. Clearly (X,τ) is an NTS and it is a NT_0-space.

4.5. Example: Let $X=\{a,b\}$ and $\tau=\{\widetilde{\emptyset},\tilde{X}\}$. Clearly (X,τ) is an NTS but it is not a NT_0-space.

4.6. Proposition: Let τ and τ^* be two neutrosophic topologies on a set X such that τ^* is finer than τ. If (X,τ) is a NT_0-space then (X,τ^*) is also a NT_0-space.

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x\neq y$, be two NPs in X. Since (X,τ) is a NT_0-space, so there exists a $G\in\tau$ such that $x_{\alpha,\beta,\gamma}\in G$, $y_{\alpha',\beta',\gamma'}\notin G$ or there exists a $H\in\tau$ such that $x_{\alpha,\beta,\gamma}\notin H$, $y_{\alpha',\beta',\gamma'}\in H$. Since τ^* is finer than τ, so $G,H\in\tau\Rightarrow G,H\in\tau^*$. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x\neq y$, there exists a $G\in\tau^*$ such that $x_{\alpha,\beta,\gamma}\in G$, $y_{\alpha',\beta',\gamma'}\notin G$ or there exists a $H\in\tau^*$ such that $x_{\alpha,\beta,\gamma}\notin H$, $y_{\alpha',\beta',\gamma'}\in H$. Hence (X,τ^*) is also a NT_0-space.

4.7. Proposition: Let (X,τ) be a NT_0-space. Then every neutrosophic subspace of X is a NT_0-space and hence the property is hereditary.

Proof: Let $(Y,\tau|_Y)$ be a neutrosophic subspace of (X,τ), where $\tau|_Y=\{G|_Y: G\in\tau\}$. We want to show $(Y,\tau|_Y)$ is a NT_0-space. Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be two NPs in Y such that $x\neq y$. Then $x_{\alpha,\beta,\gamma}, y_{\alpha',\beta',\gamma'}\in X$, $x\neq y$. Since (X,τ) is a NT_0-space, so there exists a τ-open NS U such that $x_{\alpha,\beta,\gamma}\in U$, $y_{\alpha',\beta',\gamma'}\notin U$ or there exists a τ-open NS V such that $x_{\alpha,\beta,\gamma}\notin V$, $y_{\alpha',\beta',\gamma'}\in V$. Then $(x_{\alpha,\beta,\gamma}\in U|_Y,\ y_{\alpha',\beta',\gamma'}\notin U|_Y)$ or $(x_{\alpha,\beta,\gamma}\notin V|_Y,\ y_{\alpha',\beta',\gamma'}\in V|_Y)$. Also $U|_Y, V|_Y\in\tau|_Y$. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in Y such that $x\neq y$, there exists a $\tau|_Y$-open NS $U|_Y$ such that $x_{\alpha,\beta,\gamma}\in U|_Y$, $y_{\alpha',\beta',\gamma'}\notin U|_Y$ or there exists a $\tau|_Y$-open NS $V|_Y$ such that $x_{\alpha,\beta,\gamma}\notin V|_Y$, $y_{\alpha',\beta',\gamma'}\in V|_Y$ Therefore $(Y,\tau|_Y)$ is a NT_0-space and hence the property is hereditary.

4.8. Proposition: Let (X,τ) be an NTS. Then X is a NT_0-space iff for any two distinct neutrosophic crisp points $x_{1,0,0}$ and $y_{1,0,0}$ in X, $(x_{1,0,0})\hat{q}[cl(y_{1,0,0})]$ or $(y_{1,0,0})\hat{q}[cl(x_{1,0,0})]$.

Proof: Necessary part: Suppose that both $(x_{1,0,0})\hat{q}[cl(y_{1,0,0})]$ and $(y_{1,0,0})\hat{q}[cl(x_{1,0,0})]$ are false. Then $(x_{1,0,0})q[cl(y_{1,0,0})]$ and $(y_{1,0,0})q[cl(x_{1,0,0})]$ are true. Now $(x_{1,0,0})q[cl(y_{1,0,0})]\Rightarrow x_{1,0,0}\notin[cl(y_{1,0,0})]^c\Rightarrow x_{1,0,0}\notin[\cap\{G: G$ is a τ-closed NS and $y_{1,0,0}\in G\}]^c\Rightarrow x_{1,0,0}\notin\cup\{G^c: G^c$ is a τ-open NS and $y_{1,0,0}\notin G^c\}\Rightarrow x_{1,0,0}\notin G^c$ for all τ-open NSs G^c such that $y_{1,0,0}\notin G^c$. This ensures that if H is a τ-open NS such that $y_{1,0,0}\in H$ then $x_{1,0,0}\in H$. Similarly $(y_{1,0,0})q[cl(x_{1,0,0})]$ implies that if K is a τ-open NS such that $x_{1,0,0}\in K$ then $y_{1,0,0}\in K$. Thus every τ-open NS containing one of $x_{1,0,0}$ and $y_{1,0,0}$ must contain the other. But this is a contradiction to our assumption that X is a NT_0-space. Therefore $(x_{1,0,0})\hat{q}[cl(y_{1,0,0})]$ or $(y_{1,0,0})\hat{q}[cl(x_{1,0,0})]$.

Converse part: $x_{\alpha,\beta,\gamma}$ and $y_{p,q,r}$ be any two NPs in X such that $x \neq y$. Now by hypothesis, $(x_{1,0,0})\hat{q}[cl(y_{1,0,0})]$ or $(y_{1,0,0})\hat{q}[cl(x_{1,0,0})]$. If $(x_{1,0,0})\hat{q}[cl(y_{1,0,0})]$ then $x_{1,0,0} \in [cl(y_{1,0,0})]^c$, which gives $x_{\alpha,\beta,\gamma} \in [cl(y_{1,0,0})]^c$. Obviously $y_{p,q,r} \notin [cl(y_{1,0,0})]^c$. Since $cl(y_{1,0,0})$ is a τ-closed NS, so $[cl(y_{1,0,0})]^c$ is a τ-open NS. Thus there exists a τ-open NS $[cl(y_{1,0,0})]^c$ in X such that $x_{\alpha,\beta,\gamma} \in [cl(y_{1,0,0})]^c$ but $y_{p,q,r} \notin [cl(y_{1,0,0})]^c$. Similarly if $(y_{1,0,0})\hat{q}[cl(x_{1,0,0})]$ then there exists a τ-open NS $[cl(x_{1,0,0})]^c$ in X such that $x_{\alpha,\beta,\gamma} \notin [cl(x_{1,0,0})]^c$ but $y_{p,q,r} \in [cl(x_{1,0,0})]^c$. Therefore (X,τ) is a NT_0-space.
Hence proved.

***4.9. Proposition*:** Let f be a one-one neutrosophic continuous function from an NTS (X,τ) to the NTS (Y,σ). If (Y,σ) is NT_0 then (X,τ) is also a NT_0-space.
Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since Y is NT_0, so there exists a σ-open NS G such that $y^1_{p,q,r} \in G$, $y^2_{p',q',r'} \notin G$ or there exists a σ-open NS H such that $y^1_{p,q,r} \notin H$, $y^2_{p',q',r'} \in H$. Again, since f is neutrosophic continuous, so $f^{-1}(G)$ is a τ-open NS. Also $y^1_{p,q,r} \in G \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(G) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$ and $y^2_{p',q',r'} \notin G \Rightarrow f^{-1}(y^2_{p',q',r'}) \notin f^{-1}(G) \Rightarrow x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$. Similarly $f^{-1}(H)$ is a τ-open NS such that $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$, $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exists a τ-open NS $f^{-1}(G)$ such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(G), x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$ or there exists a τ-open NS $f^{-1}(H)$ such that $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$. Therefore (X,τ) is a NT_0-space. Hence proved.

***4.10. Proposition*:** The property of being NT_0-space is preserved under a bijective neutrosophic open function.
Proof: Let (X,τ) and (Y,σ) be two NTSs. Also let (X,τ) be a NT_0-space and $f: X \to Y$ be a bijective neutrosophic open function. We show that (Y,σ) ia a NT_0-space. Let $y^1_{p,q,r}$ and $y^2_{p',q',r'}$ be two NPs in Y such that $y^1 \neq y^2$. Since f is bijective, so there exist two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$, $x^1 \neq x^2$, in X such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$. Since X is NT_0, so there exists a τ-open NS G such that $x^1_{\alpha,\beta,\gamma} \in G$, $x^2_{\alpha',\beta',\gamma'} \notin G$ or there exists a τ-open NS H such that $x^1_{\alpha,\beta,\gamma} \notin H$, $x^2_{\alpha',\beta',\gamma'} \in H$. Suppose G exists such that $x^1_{\alpha,\beta,\gamma} \in G$ and $x^2_{\alpha',\beta',\gamma'} \notin G$. Since f is a neutrosophic open function, so $f(G)$ is a σ-open NS such that $y^1_{p,q,r} = f(x^1_{\alpha,\beta,\gamma}) \in f(G)$ and $y^2_{p',q',r'} = f(x^2_{\alpha',\beta',\gamma'}) \notin f(G)$. Similarly if H exists such that $x^1_{\alpha,\beta,\gamma} \notin H$ and $x^2_{\alpha',\beta',\gamma'} \in H$ then $f(H)$ is a σ-open NS such that $y^1_{p,q,r} = f(x^1_{\alpha,\beta,\gamma}) \notin f(H)$ and $y^2_{p',q',r'} = f(x^2_{\alpha',\beta',\gamma'}) \in f(H)$. Thus for any two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$ in Y such that $y^1 \neq y^2$, there exists a σ-open NS $f(G)$ such that $y^1_{p,q,r} \in f(G)$, $y^2_{p',q',r'} \notin f(G)$ or there exists a σ-open NS $f(H)$ such that $y^1_{p,q,r} \notin f(H)$, $y^2_{p',q',r'} \in f(H)$. Therefore (Y,σ) is a NT_0-space. Hence proved.

***4.11. Proposition*:** The property of being NT_0-space is a topological property.

Proof: Let (X,τ) and (Y,σ) be two NTSs. Also let (X,τ) be a NT_0-space and $f: X \to Y$ be a neutrosophic homeomorphism. Since f is a neutrosophic homeomorphism, so f is a bijective neutrosophic open function. Therefore by the proposition 4.10, (Y,σ) is a NT_0-space. Hence proved.

***4.12. Definition*:** An NTS (X,τ) is called a neutrosophic T_1-space (NT_1-space, for short) iff for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists a $U \in \tau$ such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ and there exists a $V \in \tau$ such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$.

***4.13. Example*:** Let $X = \{a,b\}$ and $\tau = \{\widetilde{\emptyset}, \widetilde{X}, A, B\}$, where $A = \{\langle a,1,0,0\rangle, \langle b,0,1,1\rangle\}$ and $B = \{\langle a,0,1,1\rangle, \langle b,1,0,0\rangle\}$. Clearly (X,τ) is an NTS and it is a NT_1-space.

***4.14. Example*:** Let $X = \{a,b\}$ and $\tau = \{\widetilde{\emptyset}, \widetilde{X}\}$. Clearly (X,τ) is an NTS but it is not a NT_1-space.

***4.15. Proposition*:** Let τ and τ^* be two neutrosophic topologies on a set X such that τ^* is finer than τ. If (X,τ) is a NT_1-space then (X,τ^*) is also a NT_1-space.

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, be two NPs in X. Since (X,τ) is a NT_1-space, so there exists a $G \in \tau$ such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ and there exists a $H \in \tau$ such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Since τ^* is finer than τ, so $G, H \in \tau \Rightarrow G, H \in \tau^*$. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in X such that $x \neq y$, there exists a $G \in \tau^*$ such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ and there exists a $H \in \tau^*$ such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Hence (X,τ^*) is a NT_1-space.

***4.16. Proposition*:** Let (X,τ) be an NTS. If (X,τ) is a NT_1-space then it is a NT_0-space.

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, be two NPs in X. Since X is NT_1-space, so there exists a $U \in \tau$ such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ and there exists a $V \in \tau$ such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$. Hence (X,τ) is a NT_0-space.

***4.17. Remark*:** Converse of the proposition 4.16 is not true. We establish it by the following counter example.

Let $X = \{a,b\}$ and $\tau = \{\widetilde{\emptyset}, \widetilde{X}, A\}$, where $A = \{\langle a,1,0,0\rangle, \langle b,0,1,1\rangle\}$. Clearly (X,τ) is a NT_0-space but not a NT_1-space.

***4.18. Proposition*:** Let (X,τ) be a NT_1-space. Then every neutrosophic subspace of X is a NT_1-space and hence the property is hereditary.

Proof: Let $(Y,\tau|_Y)$ be a neutrosophic subspace of (X,τ), where $\tau|_Y = \{G|_Y : G \in \tau\}$. We want to show $(Y,\tau|_Y)$ is a NT_1-space. Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be two NPs in Y such that $x \neq y$. Then $x_{\alpha,\beta,\gamma}, y_{\alpha',\beta',\gamma'} \in X$, $x \neq y$. Since (X,τ) is NT_1-space, so there exists a τ-open NS U such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ and there exists a τ-open NS V such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$. Then $(x_{\alpha,\beta,\gamma} \in U|_Y$, $y_{\alpha',\beta',\gamma'} \notin U|_Y)$ and $(x_{\alpha,\beta,\gamma} \notin V|_Y$, $y_{\alpha',\beta',\gamma'} \in V|_Y)$. Also $U|_Y, V|_Y \in \tau|_Y$. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in Y such that $x \neq y$, there exists a $\tau|_Y$-open NS $U|_Y$ such that $x_{\alpha,\beta,\gamma} \in U|_Y$,

$y_{\alpha',\beta',\gamma'} \notin U|_Y$ and there exists a $\tau|_Y$-open NS $V|_Y$ such that $x_{\alpha,\beta,\gamma} \notin V|_Y$, $y_{\alpha',\beta',\gamma'} \in V|_Y$ Therefore $(Y, \tau|_Y)$ is a NT_1-space and hence the property is hereditary.

***4.19. Proposition*:** Let (X, τ) be an NTS. Then every NCP in X is a τ-closed NS iff X is a NT_1-space.

Proof: Necessary part: Let $x_{\alpha,\beta,\gamma}$ and $y_{p,q,r}$ be two NPs in X such that $x \neq y$. Since $x \neq y$, so $x_{\alpha,\beta,\gamma} \in (y_{1,0,0})^c$. By hypothesis, $y_{1,0,0}$ is a τ-closed NS. Therefore $(y_{1,0,0})^c$ is a τ-open NS. Thus there exists a τ-open NS $(y_{1,0,0})^c$ such that $x_{\alpha,\beta,\gamma} \in (y_{1,0,0})^c$ but $y_{p,q,r} \notin (y_{1,0,0})^c$. Similarly $(x_{1,0,0})^c$ is a τ-open NS such that $y_{p,q,r} \in (x_{1,0,0})^c$ but $x_{\alpha,\beta,\gamma} \notin (x_{1,0,0})^c$. Therefore X is a NT_1-space.

Sufficient part: Let $x_{1,0,0}$ be an NCP in X. Also let $y_{p,q,r}$ be an NP in X such that $x \neq y$. Then $y_{p,q,r} \in (x_{1,0,0})^c$. Let us consider an NP $x_{\alpha,\beta,\gamma}$ with support x. Since X is a NT_1-space, so for $y_{p,q,r}$ and $x_{\alpha,\beta,\gamma}$, there exists a τ-open NS G such that $y_{p,q,r} \in G$ and $x_{\alpha,\beta,\gamma} \notin G$. Since for all α, β, γ with $0 < \alpha \leq 1, 0 \leq \beta < 1, 0 \leq \gamma < 1$, one such G exists, therefore we must have a τ-open NS H such that $y_{p,q,r} \in H$ and $x_{1,0,0} \cap H = \widetilde{\emptyset}$, i.e., $y_{p,q,r} \in H \subseteq (x_{1,0,0})^c$. Therefore $(x_{1,0,0})^c$ is a τ-open NS and hence $x_{1,0,0}$ is a τ-closed NS.

Hence proved.

***4.20. Proposition*:** Let f be a one-one neutrosophic continuous function from an NTS (X, τ) to the NTS (Y, σ). If (Y, σ) is NT_1 then (X, τ) is also a NT_1-space.

Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since Y is NT_1, so there exists a σ-open NS G such that $y^1_{p,q,r} \in G$, $y^2_{p',q',r'} \notin G$ and there exists a σ-open NS H such that $y^1_{p,q,r} \notin H$, $y^2_{p',q',r'} \in H$. Since f is neutrosophic continuous, so $f^{-1}(G)$ and $f^{-1}(H)$ are τ-open NSs. Also $y^1_{p,q,r} \in G \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(G) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$ and $y^2_{p',q',r'} \notin G \Rightarrow f^{-1}(y^2_{p',q',r'}) \notin f^{-1}(G) \Rightarrow x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$. Similarly $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$ and $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exists a τ-open NS $f^{-1}(G)$ such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(G), x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$ and there exists a τ-open NS $f^{-1}(H)$ such that $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$. Therefore (X, τ) is a NT_1-space. Hence proved.

***4.21. Proposition*:** The property of being NT_1-space is preserved under a bijective neutrosophic open function.

Proof: Let (X, τ) and (Y, σ) be two NTSs. Also let (X, τ) be a NT_1-space and $f: X \to Y$ be a bijective neutrosophic open function. We show that (Y, σ) ia a NT_1-space. Let $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, be two NPs in Y. Since f is bijective, so there exist two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$, $x^1 \neq x^2$, in X such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$. Since X is NT_1, so there exists a τ-open NS G such that $x^1_{\alpha,\beta,\gamma} \in G$, $x^2_{\alpha',\beta',\gamma'} \notin G$ and there exists a τ-open NS H such that $x^1_{\alpha,\beta,\gamma} \notin H$, $x^2_{\alpha',\beta',\gamma'} \in H$. Since f is a neutrosophic open function, so $f(G)$ is a σ-open NS such that $y^1_{p,q,r} = f(x^1_{\alpha,\beta,\gamma}) \in f(G)$ and $y^2_{p',q',r'} = f(x^2_{\alpha',\beta',\gamma'}) \notin f(G)$. Similarly $f(H)$ is a σ-open NS such that $y^1_{p,q,r} = f(x^1_{\alpha,\beta,\gamma}) \notin f(H)$ and $y^2_{p',q',r'} = f(x^2_{\alpha',\beta',\gamma'}) \in f(H)$. Thus for any two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$ in Y such that $y^1 \neq y^2$, there exists a σ-open NS $f(G)$ such that $y^1_{p,q,r} \in f(G)$, $y^2_{p',q',r'} \notin f(G)$ and there exists a σ-open NS $f(H)$ such that $y^1_{p,q,r} \notin f(H)$, $y^2_{p',q',r'} \in f(H)$. Therefore (Y,σ) is a NT_1-space. Hence proved.

4.22. Proposition: The property of being NT_1-space is a topological property.

Proof: Let (X,τ) and (Y,σ) be two NTSs. Also let (X,τ) be a NT_1-space and $f: X \to Y$ be a neutrosophic homeomorphism. Since f is a neutrosophic homeomorphism, so f is a bijective neutrosophic open function. Therefore by the proposition 4.21, (Y,σ) is a NT_1-space. Hence proved.

4.23. Proposition: Let (X,τ) be an NTS. Then X is NT_1 iff the intersection of all the neutrosophic neighbourhoods of an arbitrary NP of X is an NP.

Proof: Necessary part: Let $x_{\alpha,\beta,\gamma}$ be an arbitrary NP in X and N be the intersection of all the neutrosophic neighbourhoods of $x_{\alpha,\beta,\gamma}$. Also let $y_{p,q,r}$ be any NP in X such that $x \neq y$. Since X is NT_1, so there exists a neutrosophic neighbourhood G of $x_{\alpha,\beta,\gamma}$ such that $y_{p,q,r} \notin G$ and consequently $y_{p,q,r} \notin N$. Since $y_{p,q,r}$ is arbitrary, so $N = x_{\alpha,\beta,\gamma}$.

Sufficient part: Let $x_{\alpha,\beta,\gamma}$ and $y_{p,q,r}$ be any two NPs in X such that $x \neq y$. By hypothesis, the intersection of all the neutrosophic neighbourhoods of $x_{\alpha,\beta,\gamma}$ is $x_{\alpha,\beta,\gamma}$. So, there must exist a neutrosophic neighbourhood of $x_{\alpha,\beta,\gamma}$ which does not contain $y_{p,q,r}$. Similarly, there must exist a neutrosophic neighbourhood of $y_{p,q,r}$ which does not contain $x_{\alpha,\beta,\gamma}$. Therefore X is a NT_1-space. Hence proved.

4.24 Definition: An NTS (X,τ) is called a neutrosophic T_2-space or neutrosophic Hausdorff space (NT_2-space or N-Hausdorff space, for short) iff for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exist $U, V \in \tau$ such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \in V$ and $U \cap V = \tilde{\emptyset}$.

4.25. Example: Let $X = \{a, b\}$ and $\tau = \{\tilde{\emptyset}, \tilde{X}, A, B\}$, where $A = \{\langle a, 1,0,0\rangle, \langle b, 0,1,1\rangle\}$ and $B = \{\langle a, 0,1,1\rangle, \langle b, 1,0,0\rangle\}$. Clearly (X,τ) is an NTS and it is a NT_2-space.

4.26. Example: Let $X = \{a, b\}$ and $\tau = \{\tilde{\emptyset}, \tilde{X}\}$. Clearly (X,τ) is an NTS but it is not a NT_2-space.

4.27. Proposition: Let τ and τ^* be two neutrosophic topologies on a set X such that τ^* is finer than τ. If (X,τ) is a NT_2-space then (X,τ^*) is also a NT_2-space.

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, be two NPs in X. Since (X,τ) is a NT_2-space, so there exist $G, H \in \tau$ such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \in H$ and $G \cap H = \tilde{\emptyset}$. Since τ^* is finer than τ, so $G, H \in \tau \Rightarrow G, H \in \tau^*$. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in X such that $x \neq y$, there exist $G, H \in \tau^*$ such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \in H$ and $G \cap H = \tilde{\emptyset}$.. Hence (X,τ^*) is a NT_2-space.

4.28. Proposition: Let (X,τ) be an NTS. If (X,τ) is a NT_2-space then it is a NT_1-space.

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x \neq y$. Since (X,τ) is a NT_2-space, so there exist τ-open NSs H and K such that $x_{\alpha,\beta,\gamma} \in H$, $y_{\alpha',\beta',\gamma'} \in K$ and $H \cap K = \tilde{\emptyset}$. Since $x_{\alpha,\beta,\gamma} \in H$ and $H \cap K = \tilde{\emptyset}$, so $x_{\alpha,\beta,\gamma} \notin K$. Similarly, $y_{\alpha',\beta',\gamma'} \notin H$. Thus there exists a $H \in \tau$ such that $x_{\alpha,\beta,\gamma} \in H$, $y_{\alpha',\beta',\gamma'} \notin H$ and there exists a $K \in \tau$ such that $x_{\alpha,\beta,\gamma} \notin K$, $y_{\alpha',\beta',\gamma'} \in K$. Hence (X,τ) is a NT_1-space.

4.29. Lemma: The co-finite NTS $(\mathbb{N},\tau)$ is not a NT_2-space, where $\mathbb{N}$ is the set of all natural numbers.

Proof: Let $\widetilde{\mathbb{N}} = \{\langle x, 1,0,0\rangle: x \in \mathbb{N}\}$ and $\widetilde{\emptyset} = \{\langle x, 0,1,1\rangle: x \in \mathbb{N}\}$. Given that τ is a co-finite topology on $\mathbb{N}$, so τ is the set containing $\widetilde{\emptyset}$ and all those neutrosophic sets over $\mathbb{N}$ whose complements are finite. We show that the co-finite NTS $(\mathbb{N}, \tau)$ is not a NT_2-space.
Suppose, on the contrary, that $(\mathbb{N}, \tau)$ is a NT_2-space. Then for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in $\mathbb{N}$ such that $x \neq y$, there exist τ-open NSs G, H such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \in H$ and $G \cap H = \widetilde{\emptyset}$. Now $G \cap H = \widetilde{\emptyset} \Rightarrow (G \cap H)^c = (\widetilde{\emptyset})^c \Rightarrow G^c \cup H^c = \widetilde{\mathbb{N}}$, which is not possible as $\widetilde{\mathbb{N}}$ is an infinite neutrosophic set and $G^c \cup H^c$ is a finite neutrosophic set being the union of two finite neutrosophic sets G^c and H^c.
Therefore the co-finite NTS $(\mathbb{N}, \tau)$ is a not a NT_2-space.

***4.30. Remark*:** Converse of the proposition 4.28 is not true. We establish it by the following counter example.
We consider the co-finite NTS $(\mathbb{N}, \tau)$, where $\mathbb{N}$ is the set of all natural numbers. In the lemma 4.29, we have shown that $(\mathbb{N}, \tau)$ is not a NT_2-space.
We now show that $(\mathbb{N}, \tau)$ is a NT_1-space. Let $\widetilde{\mathbb{N}} = \{\langle x, 1,0,0\rangle: x \in \mathbb{N}\}$ and $\widetilde{\emptyset} = \{\langle x, 0,1,1\rangle: x \in \mathbb{N}\}$. Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be two NPs in $\mathbb{N}$ such that $x \neq y$. Now $(\widetilde{\mathbb{N}} \backslash x_{1,0,0})^c = x_{1,0,0}$, a finite NS. Therefore $\widetilde{\mathbb{N}} \backslash x_{1,0,0}$ is a τ-open NS. Obviously $y_{\alpha',\beta',\gamma'} \in \widetilde{\mathbb{N}} \backslash x_{1,0,0}$ but $x_{\alpha,\beta,\gamma} \notin \widetilde{\mathbb{N}} \backslash x_{1,0,0}$. Similarly $\widetilde{\mathbb{N}} \backslash y_{1,0,0}$ is a τ-open NS such that $x_{\alpha,\beta,\gamma} \in \widetilde{\mathbb{N}} \backslash y_{1,0,0}$ but $y_{\alpha',\beta',\gamma'} \notin \widetilde{\mathbb{N}} \backslash y_{1,0,0}$. Therefore $(\mathbb{N}, \tau)$ is a NT_1-space.
Thus the co-finite NTS $(\mathbb{N}, \tau)$ is a NT_1-space but not a NT_2-space.

***4.31. Proposition*:** Let (X, τ) be a NT_2-space. Then every neutrosophic subspace of X is a NT_2-space and hence the property is hereditary.
Proof: Let $(Y, \tau|_Y)$ be a neutrosophic subspace of (X, τ), where $\tau|_Y = \{G|_Y: G \in \tau\}$. We want to show $(Y, \tau|_Y)$ is a NT_2-space. Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be two NPs in Y such that $x \neq y$. Then $x_{\alpha,\beta,\gamma}, y_{\alpha',\beta',\gamma'} \in X$, $x \neq y$. Since (X, τ) is NT_2-space, so there exist τ-open NSs U, V such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \in V$ and $U \cap V = \widetilde{\emptyset}$. Then $x_{\alpha,\beta,\gamma} \in U|_Y$, $y_{\alpha',\beta',\gamma'} \in V|_Y$ and $(U|_Y) \cap (V|_Y) = (U \cap V)|_Y = \widetilde{\emptyset}|_Y = \widetilde{\emptyset}$. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in Y such that $x \neq y$, there exist $\tau|_Y$-open NSs $U|_Y$, $V|_Y$ such that $x_{\alpha,\beta,\gamma} \in U|_Y$, $y_{\alpha',\beta',\gamma'} \in V|_Y$ and $(U|_Y) \cap (V|_Y) = \widetilde{\emptyset}$. Therefore $(Y, \tau|_Y)$ is a NT_2-space and hence the property is hereditary.

4.32. Proposition: If f is a one-one neutrosophic continuous function from an NTS (X, τ) to a neutrosophic Hausdorff space (Y, σ) then (X, τ) is also a neutrosophic Hausdorff space.
Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since (Y, σ) is neutrosophic Hausdorff, so there exist σ-open NSs H_1, H_2 such that $y^1_{p,q,r} \in H_1$, $y^2_{p',q',r'} \in H_2$ and $H_1 \cap H_2 = \widetilde{\emptyset}$. Since f is neutrosophic continuous, so $f^{-1}(H_1)$ and $f^{-1}(H_2)$ are τ-open NSs. Now $f^{-1}(H_1) \cap f^{-1}(H_2) = f^{-1}(H_1 \cap H_2) = f^{-1}(\widetilde{\emptyset}) = \widetilde{\emptyset}$. Also $y^1_{p,q,r} \in H_1 \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(H_1) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(H_1)$. Similarly $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H_2)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exist τ-

open NSs $f^{-1}(H_1)$, $f^{-1}(H_2)$ such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(H_1)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H_2)$ and $f^{-1}(H_1) \cap f^{-1}(H_2) = \widetilde{\emptyset}$. Therefore (X,τ) is a neutrosophic Hausdorff space. Hence proved.

4.33. Proposition: The property of being NT_2-space is preserved under a bijective neutrosophic open function.

Proof: Let (X,τ) and (Y,σ) be two NTSs. Also let (X,τ) be a NT_2-space and $f: X \to Y$ be a bijective neutrosophic open function. We show that (Y,σ) ia a NT_2-space. Let $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, be two NPs in Y. Since f is bijective, so there exist two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$, $x^1 \neq x^2$, in X such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$. Since X is NT_2, so there exist τ-open NSs G, H such that $x^1_{\alpha,\beta,\gamma} \in G$, $x^2_{\alpha',\beta',\gamma'} \in H$ and $G \cap H = \widetilde{\emptyset}$. Since f is a neutrosophic open function, so $f(G), f(H)$ are σ-open NSs such that $y^1_{p,q,r} = f(x^1_{\alpha,\beta,\gamma}) \in f(G)$, $y^2_{p',q',r'} = f(x^2_{\alpha',\beta',\gamma'}) \in f(H)$. Again since f is bijective, so $f(G) \cap f(H) = f(G \cap H) = f(\widetilde{\emptyset}) = \widetilde{\emptyset}$. Thus for any two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$ in Y such that $y^1 \neq y^2$, there exist σ-open NSs $f(G), f(H)$ such that $y^1_{p,q,r} \in f(G)$, $y^2_{p',q',r'} \in f(H)$ and $f(G) \cap f(H) = \widetilde{\emptyset}$. Therefore (Y,σ) is a NT_2-space. Hence proved.

4.34. Proposition: The property of being NT_2-space is a topological property.

Proof: Let (X,τ) and (Y,σ) be two NTSs. Also let (X,τ) be a NT_2-space and $f: X \to Y$ be a neutrosophic homeomorphism. Since f is a neutrosophic homeomorphism, so f is a bijective neutrosophic open function. Therefore by the proposition 4.33, (Y,σ) is a NT_2-space. Hence proved.

4.35. Proposition: Let A be a neutrosophic compact subset of a neutrosophic Hausdorff space (X,τ) such that $A \cap A^c = \widetilde{\emptyset}$. Then A is a neutrosophic closed set.

Proof: We want to show that A is τ-closed, i.e. A^c is τ-open. Let $x_{\alpha,\beta,\gamma}$ be an NP in A^c. Since X is neutrosophic Hausdorff, so for any NP $y^i_{p,q,r} \in A$ (Obviously $x \neq y$ as $A \cap A^c = \widetilde{\emptyset}$), there exist τ-open NSs $G_i(x_{\alpha,\beta,\gamma})$, $H_i(y^i_{p,q,r})$ such that $x_{\alpha,\beta,\gamma} \in G_i(x_{\alpha,\beta,\gamma})$, $y^i_{p,q,r} \in H_i(y^i_{p,q,r})$ and $G_i(x_{\alpha,\beta,\gamma}) \cap H_i(y^i_{p,q,r}) = \widetilde{\emptyset}$ for each $i \in \Delta$, where Δ is an index set. Clearly $\{H_i(y^i_{p,q,r}): i \in \Delta\}$ is a NOC of A. Since A is neutrosophic compact, so A has a finite NOSC, i.e., $A \subseteq \bigcup_{k=1}^{n} H_{i_k}(y^{i_k}_{p,q,r})$. Let $G_{i_k}(x_{\alpha,\beta,\gamma})$ be the neutrosophic open sets corresponding to the neutrosophic open sets $H_{i_k}(y^{i_k}_{p,q,r})$, $k = 1,2,3,\cdots,n$. Let $M = \bigcap_{k=1}^{n} G_{i_k}(x_{\alpha,\beta,\gamma})$ and $N = \bigcup_{k=1}^{n} H_{i_k}(y^{i_k}_{p,q,r})$. Obviously M is a τ-open set. We claim that $M \cap N = \widetilde{\emptyset}$. Let $z_{\alpha',\beta',\gamma'}$ be an arbitrary NP in N. Then $z_{\alpha',\beta',\gamma'} \in H_{i_k}(y^{i_k}_{p,q,r})$ for some $k, 1 \leq k \leq n \Rightarrow z_{\alpha',\beta',\gamma'} \notin G_{i_k}(x_{\alpha,\beta,\gamma})$ for some $k, 1 \leq k \leq n \Rightarrow z_{\alpha',\beta',\gamma'} \notin M$. Again if $u_{r,s,t} \in M$ be an arbitrary NP then $u_{r,s,t} \in G_{i_k}(x_{\alpha,\beta,\gamma})$ for all $k, 1 \leq k \leq n \Rightarrow u_{r,s,t} \notin H_{i_k}(y^{i_k}_{p,q,r})$ for all $k, 1 \leq k \leq n \Rightarrow u_{r,s,t} \notin N$. Thus $M \cap N = \widetilde{\emptyset}$. Since $A \subseteq N$ and since $M \cap N = \widetilde{\emptyset}$, so $A \cap M = \widetilde{\emptyset}$ and therefore $M \subseteq A^c$. Since M is a τ-open set and since $x_{\alpha,\beta,\gamma} \in M$, so M is a τ-neighbourhood of $x_{\alpha,\beta,\gamma}$. Since $M \subseteq A^c$, so A^c is also a τ-neighbourhood of $x_{\alpha,\beta,\gamma}$. Since $x_{\alpha,\beta,\gamma}$ is an arbitrary NP in A^c, so A^c is a τ-neighbourhood of each of its NPs. Therefore A^c is τ-open, i.e., A is a τ-closed NS. Hence proved.

4.36. Proposition: Let (X,τ) be a neutrosophic Hausdorff space. If $x_{\alpha,\beta,\gamma}$ is an NP in X and A is a neutrosophic compact subset of X such that $x_{\alpha,\beta,\gamma} \cap A = \widetilde{\emptyset}$ then $x_{\alpha,\beta,\gamma}$ and A can be separated by two disjoint neutrosophic open sets.

Proof: Since $x_{\alpha,\beta,\gamma} \cap A = \widetilde{\emptyset}$, so $x_{\alpha,\beta,\gamma} \in A^c$. Since X is neutrosophic Hausdorff, so for any NP $y^i_{p,q,r} \in A$, $x \neq y$, there exist τ-open NSs $G_i(x_{\alpha,\beta,\gamma})$, $H_i(y^i_{p,q,r})$ such that $x_{\alpha,\beta,\gamma} \in G_i(x_{\alpha,\beta,\gamma})$, $y^i_{p,q,r} \in H_i(y^i_{p,q,r})$ and $G_i(x_{\alpha,\beta,\gamma}) \cap H_i(y^i_{p,q,r}) = \widetilde{\emptyset}$ for each $i \in \Delta$, where Δ is an index set. Clearly $\{H_i(y^i_{p,q,r}): i \in \Delta\}$ is a NOC of A. Since A is neutrosophic compact, so A has a finite NOSC, i.e., $A \subseteq \bigcup_{k=1}^{n} H_{i_k}(y^{i_k}_{p,q,r})$. Let $G_{i_k}(x_{\alpha,\beta,\gamma})$ be the τ-open NSs corresponding to the τ-open NSs $H_{i_k}(y^{i_k}_{p,q,r})$, $k = 1,2,3,\cdots,n$. Let $M = \bigcap_{k=1}^{n} G_{i_k}(x_{\alpha,\beta,\gamma})$ and $N = \bigcup_{k=1}^{n} H_{i_k}(y^{i_k}_{p,q,r})$. Obviously M and N are neutrosophic open sets such that $x_{\alpha,\beta,\gamma} \in M$ and $A \subseteq N$. We claim that $M \cap N = \widetilde{\emptyset}$. Let $z_{\alpha',\beta',\gamma'}$ be an arbitrary NP in N. Then $z_{\alpha',\beta',\gamma'} \in H_{i_k}(y^{i_k}_{p,q,r})$ for some $k, 1 \leq k \leq n \Rightarrow z_{\alpha',\beta',\gamma'} \notin G_{i_k}(x_{\alpha,\beta,\gamma})$ for some $k, 1 \leq k \leq n \Rightarrow z_{\alpha',\beta',\gamma'} \notin M$. Again if $u_{r,s,t} \in M$ be an arbitrary NP then $u_{r,s,t} \in G_{i_k}(x_{\alpha,\beta,\gamma})$ for all $k, 1 \leq k \leq n \Rightarrow u_{r,s,t} \notin H_{i_k}(y^{i_k}_{p,q,r})$ for all $k, 1 \leq k \leq n \Rightarrow u_{r,s,t} \notin N$. Therefore $M \cap N = \widetilde{\emptyset}$. Hence proved.

4.37. Proposition: Let A be a neutrosophic compact subset of a neutrosophic Hausdorff space (X, τ). If $x_{\alpha,\beta,\gamma}$ is an NP in X such that $x_{\alpha,\beta,\gamma} \cap A = \widetilde{\emptyset}$ then there exists a neutrosophic open set G such that $x_{\alpha,\beta,\gamma} \in G \subseteq A^c$.

Proof: Immediately from 4.36.

4.38. Proposition: Let A and B be disjoint neutrosophic compact subsets of a neutrosophic Hausdorff space (X, τ). Then there exist disjoint neutrosophic open sets G and H such that $A \subseteq G$ and $B \subseteq H$.

Proof: Let $x_{\alpha,\beta,\gamma} \in A$. Then $x_{\alpha,\beta,\gamma} \notin B$ as $A \cap B = \widetilde{\emptyset}$. Since X is neutrosophic Hausdorff, so for any $y_{\alpha',\beta',\gamma'} \in B$, there exist disjoint τ-open NSs $G(y_{\alpha',\beta',\gamma'})$ and $H(y_{\alpha',\beta',\gamma'})$ such that $x_{\alpha,\beta,\gamma} \in G(y_{\alpha',\beta',\gamma'})$ and $y_{\alpha',\beta',\gamma'} \in H(y_{\alpha',\beta',\gamma'})$. The collection $\{H(y_{\alpha',\beta',\gamma'}): y_{\alpha',\beta',\gamma'} \in B\}$ is evidently a NOC of B. Since B is neutrosophic compact, so there exist finitely many NPs $y^i_{p,q,r}, i = 1,2,3,\ldots,n$ of B such that $B \subseteq \bigcup_{i=1}^{n} H(y^i_{p,q,r})$. Let $H(x_{\alpha,\beta,\gamma}) = \bigcup_{i=1}^{n} H(y^i_{p,q,r})$ and $G(x_{\alpha,\beta,\gamma}) = \bigcap_{i=1}^{n} G(y^i_{p,q,r})$, where $G(y^i_{p,q,r})$ are the τ-open NSs corresponding to the τ-open NSs $H(y^i_{p,q,r})$. Then clearly $H(x_{\alpha,\beta,\gamma})$ and $G(x_{\alpha,\beta,\gamma})$ are τ-open NSs such that $x_{\alpha,\beta,\gamma} \in G(x_{\alpha,\beta,\gamma}), B \subseteq H(x_{\alpha,\beta,\gamma})$ and $G(x_{\alpha,\beta,\gamma}) \cap H(x_{\alpha,\beta,\gamma}) = \widetilde{\emptyset}$. Now suppose that $x_{\alpha,\beta,\gamma}$ is an arbitrary NP in A. We construct $G(x_{\alpha,\beta,\gamma})$ and $H(x_{\alpha,\beta,\gamma})$ as above. Evidently $\{G(x_{\alpha,\beta,\gamma}): x_{\alpha,\beta,\gamma} \in A\}$ is a NOC of A. Since A is neutrosophic compact, so there exist finitely many NPs $x^j_{r,s,t}, j = 1,2,3,\ldots,m$ of A such that $A \subseteq \bigcup_{j=1}^{m} G(x^j_{r,s,t})$. Let $G = \bigcup_{j=1}^{m} G(x^j_{r,s,t})$ and $H = \bigcap_{j=1}^{m} H(x^j_{r,s,t})$, where $H(x^j_{r,s,t})$ are the τ-open NSs corresponding to the τ-open NSs $G(x^j_{r,s,t})$. Clearly G and H are neutrosophic open sets such that $A \subseteq G$, $B \subseteq H$ and $G \cap H = \widetilde{\emptyset}$. Hence proved.

5. Conclusion

In this article, our primary objective was to explore the separation axioms in neutrosophic topological spaces. Just like in the study of topological spaces in classical, fuzzy or other settings, the significance of subspace topology and subspaces can not be overlooked, as many properties of topological spaces are interconnected with subspaces. Therefore, in section 3, we have introduced the concept of neutrosophic subspace, and investigated a few properties of it. Before delving into neutrosophic subspaces, we have laid the groundwork by establishing some results based on single-valued neutrosophic sets which have played a crucial role in the study of neutrosophic subspaces.

Moving forward in section 4, we have defined neutrosophic T_0, T_1 and T_2-spaces in relation to neutrosophic topological spaces and examined various properties associated with these separation axioms. Our future research will aim to explore other notions associated with neutrosophic topological spaces. We hope that the findings presented in this article will prove beneficial to the research community and contribute to the advancement of various aspects of neutrosophic topology.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

References

1. Atanassov, K. (1986). Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20, 87-96.
2. Arar, M. (2020). About Neutrosophic Countably Compactness. Neutrosophic Sets and Systems, 36(1), 246-255.
3. Abdel-Basset, M. ; Gamal, A. ; Chakrabortty, R.K. ; Ryan, M.J. (2021). A new hybrid multi-criteria decision-making approach for location selection of sustainable offshore wind energy stations : A case study. Journal of Cleaner Production, 280, DOI : 10.1016/j.jclepro.2020.124462
4. Abdel-Basset, M. ; Mohamed, R. ; Smarandache, F. ; Elhoseny, M. (2021). A New Decision-Making Model Based on Plithogenic Set for Supplier Selection. Computers, Materials & Continua, 66(3), 2751-2769.
5. Abdel-Basset, M. ; Gamal, A. ; Manogaran, G. ; Long, H.V. (2020). A novel group decision making model based on neutrosophic sets for heart disease diagnosis. Multimedia Tools and Applications, 79, 9977-10002.
6. Alblowi, S.A. ; Salma, A.A. ; Eisa, M. (2014). New concepts of neutrosophic sets. Int.J. of Math and Comp. Appl. Research, 4(1), 59-66.
7. Açikgöz, A. ; Esenbel, F. (2020). An Approach to pre-separation axioms in neutrosophic soft topological spaces. Commun.Fac.Sci.Univ.Ank.Ser. A1 Math. Stat., 69(2), 1389-1404.
8. Al-Nafee, A.B. ; Al-Hamido,R.K. ; Smarandache, F. (2019). Separation Axioms in Neutrosophic Crisp Topological Spaces. Neutrosophic Sets and Systems, 25(1), 25-32.
9. Babu, V. A. ; Aswini, J. (2021). Separation axioms in supra neutrosophic crisp topological spaces. Advances and Applications in Mathematical Sciences, 20(6), 1115-1128.
10. Coker, D. (1997). An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems, 88, 81-89.
11. Das, S. ; Pramanik, S. (2020). Generalized neutrosophic b-open sets in neutrosophic topological space. Neutrosophic Sets and Systems, 38, 235-243.
12. Das, R. and Tripathy, B.C. (2020). Neutrosophic multiset topological space. Neutrosophic Sets and Systems, 35, 142-152.
13. Das, S. ; Das, R. & Tripathy, B. C. (2020). Multi-criteria group decision making model using single-valued neutrosophic set. LogForum, 16(3), 421-429.
14. Dey. S ; Ray, G.C. Covering properties in Neutrosophic Topological Spaces. Neutrosophic Sets and Systems, 51, 525-537.
15. Dey. S ; Ray, G.C. (2022). Redefined neutrosophic composite relation and its application in medical diagnosis. Int. J. Nonlinear Anal. Appl., 13(Special issue), 43-52.
16. Guo, Y. ; Cheng, H.D. (2009). New neutrosophic approach to image segmentation. Pattern Recognition, 42, 587-595.
17. Karatas, S. ; Kuru, C. (2016). Neutrosophic Topology. Neutrosophic Sets and Systems, 13(1), 90-95.

18. Lupiáñez, F.G. (2008). On neutrosophic topology. The International Journal of Systems and Cybernetics, 37(6), 797-800.
19. Lupiáñez, F.G. (2009). Interval neutrosophic sets and topology. The International Journal of Systems and Cybernetics, 38(3/4), 621-624.
20. Lupiáñez, F.G. (2009). On various neutrosophic topologies. The International Journal of Systems and Cybernetics, 38(6), 1009-1013.
21. Mehmood, A. ; Nadeem, F. ; Nordo, G. ; Zamir, M. ; Park, C. ; Kalsoom, H. ; Jabeen, S. ; KHAN, M.I. (2020). Generalized Neutrosophic Separation Axioms in Neutrosophic Soft Topological Spaces. Neutrosophic Sets and Systems, 32(1), 38-51.
22. Majumder, P. ; Das, S. ; Das, R. and Tripathy, B.C. (2021). Identification of the Most Significant Risk Factor of COVID-19 in Economy Using Cosine Similarity Measure under SVPNS-Environment. Neutrosophic Sets and Systems, 46, 112-127. DOI: 10.5281/zenodo.5553497.
23. Pramanik, S. ; Roy, T. (2014). Neutrosophic Game Theoretic Approach to Indo-Pak Conflict over Jammu-Kashmir. Neutrosophic Sets and Systems, 2(1), 82-101.
24. Ray, G.C. ; Dey, S. (2021). Neutrosophic point and its neighbourhood structure. Neutrosophic Sets and Systems, 43, 156-168.
25. Ray, G.C. ; Dey, S. (2021). Relation of Quasi-coincidence for Neutrosophic Sets. Neutrosophic Sets and Systems, 46, 402-415.
26. Smarandache, F. (1999). A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability. American Research Press, Rehoboth, NM.
27. Smarandache, F. (2002). Neutrosophy and neutrosophic logic. First international conference on neutrosophy, neutrosophic logic, set, probability, and statistics, University of New Mexico, Gallup, NM 87301, USA .
28. Smarandache, F. (2005). Neutrosophic set - a generalization of the intuitionistic fuzzy set. International Journal of Pure and Applied Mathematics, 24(3), 287-297.
29. Salama, A.A. ; Alblowi, S. (2012). Neutrosophic set and Neutrosophic Topological Spaces. IOSR Journal of Mathematics, 3(4), 31-35.
30. Salama, A.A. ; Alblowi, S. (2012). Generalized neutrosophic set and generalized neutrosophic topological spaces. Computer Science and Engineering, 2(7), 129-132.
31. Salama, A.A. ; Alagamy, H. (2013). Neutrosophic filters. Int. J.of Comp. Sc. Eng. and I.T. Research, 3(1), 307-312.
32. Salama, A. A. ; Smarandache, F. ; Kroumov, V. (2014). Closed sets and Neutrosophic Continuous Functions. Neutrosophic Sets and Systems, 4, 4-8.
33. *S* enyurt, S. ; Kaya, G. (2017). On Neutrosophic Continuity. Ordu University Journal of Science and Technology, 7(2), 330-339.
34. Salma, A.A. ; Smarandache, F. (2015). Neutrosophic Set Theory. The Educational Publisher 415 Columbus, Ohio.
35. Wang, H. ; Smarandache, F. ; Zhang, Y.Q. ; Sunderraman, R. (2010). Single valued neutrosophic sets. Multispace Multistruct, 4, 410-413.
36. Zadeh, L.A. (1965). Fuzzy sets. Inform. and Control, 8, 338-353.

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