The Use of Neutrosophic Methods of Operation Research in the Management of Corporate Work

Maissam Jdid [1],* ID **and Florentin Smarandache** [2] ID

[1] Faculty member, Damascus University, Faculty of Science, Syria; maissam.jdid66@damascusuniversity.edu.sy.

[2] University of New Mexico, Mathematics, Physics and Natural Sciences Division 705 Gurley Ave., Gallup, NM 87301, USA; smarand@unm.edu.

* Correspondence: maissam.jdid66@damascusuniversity.edu.sy.

Abstract: The science of operations research is one of the modern sciences that have made a great revolution in all areas of life through the methods provided by it, suitable and appropriate to solve most of the problems that were facing researchers, scholars and those interested in the development of societies, and the most beneficiaries of this science were companies and institutions that are looking for scientific methods that help them manage their work so that they achieve the greatest profit and the lowest cost, and one of the important methods that have been used in the management of companies we offer in this research two methods, Dynamic programming method. This method has been used in many practical matters and helped decision-makers in companies to achieve a maximum profit and less cost by formulating the reality of the state of the company and the data provided by decision-makers with a dynamic mathematical model that is solved using methods of solving dynamic models and we will provide in this research an example of this through the issue of choosing the optimal investment for the budget of a company so that it achieves a maximum profit, and the method of programming with integers: the method that provided these companies with solutions with integer values suitable for the nature of its work, through the use of the binary integer in the formulation of the appropriate mathematical model on the one hand, and on the other hand, the use of the binary integer variable helped to convert some nonlinear models that lead to some practical problems into linear models, and it should be noted here that in the previous two methods there is something indeterminable because we must make a decision in choosing or not choosing something, but the optimal solution that we will get remains A specific value because we are building the mathematical model for any realistic issue through the data provided by those responsible for the work and these data are calculated quantities and therefore they are uncertain values because their validity depends on the circumstances surrounding the work environment, they may be exposed to increase or decrease, and therefore the optimal solution on which the company will base its decision is suitable for specific values and any change in them can cause the company an uncalculated loss, so in this research we will use the concepts of neutrosophic science,   the branch of science founded by the American scientist Florentin Smarandache in 1995 based on his belief that there is no absolute truth, a science that is interested in the study of ideas and concepts that are neither true nor false, but just   in-between, and we will take the data (calculated quantities) neutrosophic values that are specified or unspecified values are any set close to the calculated quantities, then the resulting mathematical model is a neutrosophic model and the optimal solution has neutrosophic values and thanks to the indefinite uncertainty that these values have, companies from the development of appropriate plans for all circumstances and thus achieve the greatest profit and the lowest cost, and we will clarify the above through two issues, the issue of optimal designation of a warehouse site, which we will formulate the mathematical model of using the neutrosophic integer programming method - and the issue of capital budget, which we will present in two different forms, we use in the first form the neutrosophic integer programming method and in the second the neutrosophic dynamic programming method.

Keywords: Operations Research; Neutrosophic Integer Programming; Binary Integer Variable; Neutrosophic Dynamic Programming; Neutrosophic Science; Neutrosophic Linear Mathematical Model; Neutrosophic Nonlinear Mathematical Model; The Problem of Locating a Repository; The Problem of Capital Budget.

1. Introduction

The splendor of any science is completed through its scientific methods that help solve practical issues facing us in our daily lives and work on the development of societies, and one of the important methods presented by the science of operations research is the method of programming with integers, which depends on building mathematical models using the binary correct variable, which enabled researchers to convert some nonlinear mathematical models into linear mathematical models, in addition to that when solving these models, beam compounds were the optimal solution correct values that meet the nature of the issue under study and do not need to rotate fractional values for the optimal solution to obtain correct values as we used to do previously, the greatness of the science of operations research is when it meets with the concepts of neutrosophic science, the latest and most important thing that science has offered in our time, the science that brought values out of determination to indeterminacy, this margin of freedom can be greatly benefited from when the methods of operations research are reformulated using its concepts, a complement to what we have presented from research in which we formulated some methods of process research according to the concepts of neutrosophic [1–21]. Due to the importance of the dynamic programming method, we also presented a paper entitled Neutrosophic Dynamic Programming, through which we explained how to formulate dynamic models using the concepts of neutrosophic science [22]. In another paper, we used the neutrosophic integer programming method entitled Neutrosophic Mathematical Model of the product mixture problem using the binary integer variable [23]. In this research, we will use the method of dynamic neutrosophic programming and the method of neutrosophic integer programming to help companies manage their work in a scientific manner that enables them to exploit the resources available to them in an ideal way that achieves the greatest profit and the least loss by formulating two issues: the question of optimal designation of a warehouse site and the question of capital budget using the concepts of neutrosophic science It should be noted that these two issues were studied using classical values, see reference [24, 25].

2. Discussion

Since the aim of this research is to help decision-makers in companies and institutions to make the optimal decision to ensure the greatest profit and the lowest cost, we must study the issue at hand a good study through which we can determine the data that are affected by the surrounding conditions, and then we take these neutrosophic data values any unspecified values take the form

$Na_i = a_i + \varepsilon_i$ where ε_i is the indeterminacy on the data can take one of the forms $[\lambda_{i1}, \lambda_{i2}]$ أو

$\{\lambda_{i1}, \lambda_{i2}\}$.. Or----- otherwise, which is any neighborhood containing the value a_i.

We clarify the above through the following problems:

2.1 The first issue

An executive in one of the companies asked an expert in the science of operations research to help him obtain an optimal solution through which to achieve the lowest cost of transportation and operation of warehouses he wants to establish in order to expand the company's work and provided him with information through which the expert formulated the following issue: The text of the problem according to the concepts of neutrosophic science: A retail company plans to expand its activities in a specific area by establishing two new warehouses, the following Table 1 shows the potential locations, the number of customers and the possibility of meeting the demand for the sites where (∗) has been placed in the event that the site can meet the customer's request and put (×) the opposite and code Nc_{ij} For the cost of transferring one unit from site i to customer j he got the following Table:

Table 1. Transportation cost in case of location selection.

Customer site	B_1	B_2	B_3	B_4
A_1	* Nc_{11}	* Nc_{12}	×	* Nc_{14}
A_2	* Nc_{21}	* Nc_{22}	* Nc_{23}	* Nc_{24}
A_3	×	* Nc_{32}	* Nc_{33}	* Nc_{34}
Customer orders	D_1	D_2	D_3	D_4

Table 2 shows the following information available for each of the candidate locations for warehouses.

Table 2. Operation information.

Information site	Operating cost per unit (monetary unit)	Initial Invested Capital (Monetary Unit)	Site Capacity
first	Np_1	k_1	A_1
second	Np_2	k_2	A_2
third	Np_3	k_3	A_3

It is required to choose suitable locations for warehouses that make the total costs of investment, operation and transportation as small as possible.

Building the mathematical model: Each site has a fixed capital cost independent of the quantity stored in the warehouse referred to that site and also has a variable cost proportional to the quantity transported, and therefore the total cost of establishing and operating the warehouse is a non-linear function of the stored quantity and using binary integer variables can be formulated the issue of determining the location of the warehouse in a program with integers where we assume that the binary integer variableδ_i Symbolizes the decision to choose the site or not to choose it in other words

$$\delta_i = \begin{cases} 1 & \text{if we chose the site } i \\ 0 & \text{otherwise} \end{cases}$$

Suppose that x_{ij} is the quantity transferred from site i to customer j, so the constraint expressing the ability of the first site to meet the requests is as follows:

$$x_{11} + x_{12} + x_{14} \leq A_1 \delta_1$$

When $\delta_1 = 1$, the first location with capacityA_1 is chosen. The quantity transported from the first site cannot exceed the capacity of that siteA_1 When $\delta_1 = 0$ the non-negative variables

$x_{11}, x_{12}, x_{14} = 0$ directly, indicating that it is not possible to ship from the first location

In a similar way, we obtain the following two constraints for the second and third signatories.

$$x_{21} + x_{22} + x_{23} + x_{24} \leq A_2 \delta_2$$

$$x_{31} + x_{33} + x_{34} \leq A_3 \delta_3$$

To choose exactly two locations, we need the following restriction:

$$\delta_1 + \delta_2 + \delta_3 = 2$$

As δ_1 can take one of the values of 0 or 1 only, the new constraint will force two variables from among the three variables, δ_i to be equal to one

The restrictions for customer requests can be written as follows:

first customer $x_{11} + x_{21} = D_1$

Second customer $x_{12} + x_{22} + x_{32} = D_2$

Third customer $x_{23} + x_{33} = D_3$

Forth customer $x_{14} + x_{24} + x_{34} = D_4$

To write the objective function, we note that the total cost of investment, operation and transportation for the first site is as follows:

$$k_1\delta_1 + Np_1(x_{11} + x_{12} + x_{14}) + Nc_{11}x_{11} + Nc_{12}x_{12} + Nc_{14}x_{14}$$

When we do not choose the first site, variable $\delta_1 = 0$ And that forces the variables

$$x_{11}, x_{12}, x_{14} = 0$$

In a similar way, the cost functions of the second and third sites can be written, and thus the full formulation of the issue of assigning the location of the warehouse is reduced to the following correct mixed program: Z is meant to be made minimal

$$Z = k_1\delta_1 + Np_1(x_{11} + x_{12} + x_{14}) + Nc_{11}x_{11} + Nc_{12}x_{12} + Nc_{14}x_{14} + k_2\delta_2 + Np_2(x_{21} + x_{22} + x_{23} + x_{24}) + Nc_{21}x_{21} + Nc_{22}x_{22} + Nc_{23}x_{23} + Nc_{24}x_{24} + k_3\delta_3 + Np_3(x_{32} + x_{33} + x_{34}) + Nc_{32}x_{32} + Nc_{33}x_{33} + Nc_{34}x_{34}$$

considering the following restrictions:

$$x_{11} + x_{12} + x_{14} \leq A_1\delta_1$$
$$x_{21} + x_{22} + x_{23} + x_{24} \leq A_2\delta_2$$
$$x_{31} + x_{33} + x_{34} \leq A_3\delta_3$$
$$\delta_1 + \delta_2 + \delta_3 = 2$$
$$x_{11} + x_{21} = D_1$$
$$x_{12} + x_{22} + x_{32} = D_2$$
$$x_{23} + x_{33} = D_3$$
$$\delta_i \text{ true variable for } i = 1, 2, 3$$
$$x_{ij} \geq 0 \; ; i = 1, 2, 3 \; and \; j = 1, 2, 3, 4$$

2.2 The second issue

The second request addressed by the executive was how I can choose the appropriate projects to operate a limited capital available in the company through a number of projects presented, through the information provided by the official in charge of managing the company, the expert formulated the following issue:

The issue of the capital budget: A company plans to disburse its capital during the T_j periods. where $j = 1, 2, − − −, n$, and there is A_i A proposed project where $i = 1, 2, − − −, m$ versus a limited capital B_j Available for investment in period j and when choosing any project i becomes in need of a certain capital in each period j we denote it Na_{ij}. It is a neutrosophic value, the value of each project is measured in terms of the liquidity flow corresponding to the project in each period minus the value

of inflation, and this is called net present value (NPV), we denote it Nv_i Accordingly, the following Table 3 can be organized:

Table 3. Return on Investment during periods.

period ⟍ project	T_1	T_2	-----------	T_n
A_1	Na_{11}	Na_{12}	----------	Na_{11}
A_2	Na_{21}	Na_{22}		Na_{2n}
--------	----------	----------	--------	----------
A_m	Na_{m1}	Na_{m2}	----------	Na_{mn}
Limited capital	B_1	B_2	-----------	B_n

What is required in this problem is to select the right projects that maximize the total value (NPV) of all selected projects. Formulation of the mathematical model:

Here we assume a binary integer variable x_j It takes the value one if the project j is selected and takes the value zero if the project j is not selected

$$x_i = \begin{cases} 1 & \text{if we chose project } i \\ 0 & \text{otherwise} \end{cases}$$

Then the objective function is given by the following relationship:

$$Z = \sum_{i=1}^{m} Nv_i\, x_i$$

Then the objective function is given by the following relationship:

$$\sum_{i=1}^{m} Na_{ij}\, x_i \leq B_j \quad ; j = 1, ---, n$$

Accordingly, we get the following mathematical model:

Find the maximum value of the function:

$$Z = \sum_{i=1}^{m} Nv_i\, x_i$$

Considering the following restrictions:

$$\sum_{i=1}^{m} Na_{ij}\, x_i \leq B_j \quad ; j = 1, ---, n$$

x_i A binary variable takes one of the values 0 or 1 for all values of $i = 1, ---, m$ in the previous two issues, we got models with integers that have special methods of solution. This research cannot be presented and we will present them in later research using the concepts of neutrosophic science.

2.3 The third issue

in the reference [25] an example of a dynamic programming problem, which is the capital budget problem using classical values, is presented. In this paper, we will reformulate the general model of this problem using neutrosophic values, and we will apply this to the example given in the reference so that we can compare the use of classical values and neutrosophic values using the optimal solution that we will get.

Another presentation of the problem of the neutrosophic capital budget in reference [25] the following question was raised: the budget of a company (5 million monetary units) wants to be distributed to four different investment projects, if you know that each investment has a certain return corresponding to each amount of one million and is shown in the attached table, it is required to find the optimal distribution of the budget if (5 million and 4 million) between the four projects in a way that obtains the greatest profit

Table 5. Periodic amounts classic values.

Invesment projects / Invested amounts	$F_1(x_1)$	$F_2(x_2)$	$F_3(x_3)$	$F_4(x_4)$
0	0	0	0	0
1	0.56	0.5	0.3	0.4
2	0.9	0.82	0.5	0.66
3	1.3	1.1	0.8	0.84
4	1.56	1.3	1.00	0.96
5	1.8	1.5	1.24	1.06

This issue is a dynamic programming issue that was addressed using the tables method and the summary table for the final solution was as follows:

Table 6. Final table of solution and classic values.

Z_4	x_4	$F_4(x_4)$	Z_3	$G_3^*(Z_3)$	$G_4(Z_4)$
5	0	0	5	2.12	2.12
	1	0.40	4	1.80	2.20
	2	0.66	3	1.40	2.06
	3	0.84	2	1.06	1.90
	4	0.96	1	0.56	1.52
	5	1.06	0	0	1.06
4	0	0	4	1.80	1.80
	1	0.40	3	1.40	1.80
	2	0.66	2	1.06	1.72
	3	0.84	1	0.56	1.40
	4	0.96	0	0	0.96

From the table, we find that the maximum return if (5) million is invested equals to $G_4^*(Z_4) = \mathbf{2.20}$

This value corresponds to $x_4 = 1$, $x_3 = 0$, $Z_3 = 4$ and therefore from the relations $Z_i = Z_{i-1} + x_i$

We find the following distribution:

$$G_4^*(Z_4) = 2.20 \Rightarrow Z_4 = 5 \Rightarrow \begin{cases} x_4 = 1 \\ x_3 = 0 \\ x_2 = 1 \\ x_1 = 3 \end{cases}$$

$x_4 = 1$ أو $x_4 = 0$:

From the table, we find that the maximum return if (4) million is invested equals $G_4^*(Z_4) = 1.80$

This value corresponds to , $x_4 = 1$ or $x_4 = 0$ and the distribution is as follows:

$$G_4^*(Z_4) = 1.80 \Rightarrow Z_4 = 4 \Rightarrow \begin{cases} x_4 = 0 \\ or \\ x_4 = 1 \end{cases} \Rightarrow \begin{cases} \begin{cases} x_4 = 0 \\ x_3 = 0 \\ x_2 = 1 \\ x_1 = 3 \end{cases} \\ 0r \\ \begin{cases} x_4 = 1 \\ x_3 = 0 \\ x_2 = 1 \\ x_1 = 2 \end{cases} \end{cases}$$

To clarify the extent to which neutrosophic values affect the final result of the solution, that is, on the maximum profit of the company In the beginning, we formulate the general model for this problem, neutrosophic science concepts, and accordingly we determine the data that are affected by the surrounding conditions of the work environment and here we take the values that express the profit from investing an amount of amounts in one of the projects neutrosophic values because they are the values that are affected by the surrounding conditions

We get the following problem:

The problem of the distribution of the neutrosophic capital budget: the budget of the company is M monetary unit that you want to distribute to n of different investment projects, $B_1, B_2, ---, B_n$ if you know that for every amount invested in a project, there is a profit (i.e. the profit is related to the amount invested into the project), $NF_1(x_1), NF_2(x_2), ---, NF_n(x_n)$، shown in the attached table, it

is required to find the optimal distribution of the budget between n projects so that the company gets the greatest profit

Table 4. Return on Investment by amount used.

Ivestment projects / Invested amounts	B_1	B_2	---------	B_n
1	Nc_{11}	Nc_{12}	---------	Nc_{1n}
2	Nc_{21}	Nc_{22}	---------	Nc_{2n}
----------	------	--------	--------	--------
m	Nc_{m1}	Nc_{m2}	--------	Nc_{mn}

After studying the data of this issue, we note that it can be formulated in the form of a dynamic programming problem corresponding to the case of one dimension and we mean the criterion that affects the decision-making process at a certain stage of obtaining the optimal solution and one

influencer and we mean by it the goal follower and here we note that the goal is the only one which is to achieve the greatest profit so we can represent it with the following relationships:

$$Z_i = NF_i(x_i) \; ; i = 1, 2, - - -, n$$

Z_i is the function that expresses the state reached by the problem at a certain partial point x_i is secision variable in stage i

The relationship between the state in a given phase i−1 and the next phase i is written as follows:

$$Z_i = g_i(x_i, Z_{i-1})$$

These relationships represent constraints

The goal function is written according to the partial goal dependencies in the previous stages with the following relationship:

$$NG_i(Z_i) = NF_i(x_i, Z_{i-1}, Z_i) \; ; i = 1, 2, - - -, n$$

This function takes an optimal value at each stage and this value is given by the following relationship: $NG_n^*(Z_n) = \underset{x_1 \to x_n}{opt} [NF_1(x_i, Z_{i-1}, Z_i) + - - - + NF_n(x_n, Z_{n-1}, Z_n)]$

Which can be expressed as follows:

$$NG_n^*(Z_n) = \underset{x_n}{opt} [NF_n(x_n, Z_{n-1}, Z_n) + NG_{n-1}^*(Z_{n-1})]$$

whereas

$NG_n^*(Z_n)$ – **optimal goal function   at stage** n

$NG_{n-1}^*(Z_{n-1})$ – **optimal goal function   at stage** $n - 1$

they be solved as follows:

$$NG_i^*(Z_i) = \underset{x_i}{0pt} NF_i[(x_i, Z_{i-1}, Z_i)] \; ; i = 1, 2, - - -, n \quad (1)$$

Restrictions:

$$Z_i = g_i(x_i, Z_{i-1}) \; ; i = 1, 2, - - -, n \quad (2)$$

The relationships,1. and restrictions,2. It includes all one-dimensional, one-effect neutrosophic dynamic programming problems and is solved in the form of tables that we will illustrate by solving the following example: Example: (We will take some neutrosophic values) The budget of the company M=\$ 5 million wants to spread it among four projects B_1, B_2, B_3, B_4 of the various investment projects, if you know that for every million investment in a project, there is a profit (that is, the profit is related to the amount that is invested into the project), shown in the attached table, it is required to find the optimal distribution of the budget between the four projects so that the company achieves the greatest profit

Table 7. Return on Investment with neutrosophic values.

Investment projects / Invested amounts	B_1	B_2	B_3	B_4
0	0	0	0	0
1	[0.52 , 0.57]	0.5	0.3	0.4
2	0.9	[0.8 , 0.85]	0.5	0.66
3	1.3	1.1	[0.79 , 0.82]	0.84
4	1.56	1.3	1.00	[0.9 , 1.3]
5	1.8	1.5	1.24	[1.05 , 1.2]

This problem is formulated with a dynamic programming program and the optimal solution is found using the following relationships: We denote $NF_i(x_i)$. -function at each stage (the value of the profit returned from the use of the amount x_i in project , x_1, x_2, x_3 , x_4 is the amount of money distributed over the four projects

$$Z_i = Z_{i-1} + x_i$$

Solution equations:

$$NG_1^*(Z_1) = NF_1(x_1)$$

$$NG_2^*(Z_2) = \underset{x_2}{Max}\,[NF_2(x_2) + NG_1^*(Z_1)]$$

$$NG_3^*(Z_3) = \underset{x_3}{Max}\,[NF_3(x_3) + NG_2^*(Z_2)]$$

$$NG_4^*(Z_4) = \underset{x_4}{Max}\,[NF_4(x_4) + NG_3^*(Z_3)]$$

1- Solution using tables

2- Table Z_1

Table 8. Z_1 table.

Z_1	x_1	$G_1^*(Z_1)$	
0	0	0	
1	1	[0.52 , 0.57]	
2	2	0.9	$Z_0 = 0$
3	3	1.3	$Z_1 = Z_0 + x_1$
4	4	1.56	
5	5	1.8	

1- Table Z_2 where $Z_2 = Z_1 + x_2$

Table 9. Z_2 table.

Z_2	x_2	$NF_2(x_2)$	Z_1	$NG_1^*(Z_1)$	$NG_2(Z_2)$
0	0	0	0	0	0
1	0	0	1	$[0.52\,,0.57]$	$[0.52\,,0.57]$
	1	0.5	0	0	0.5
2	0	0	2	0.9	0.9
	1	0.5	1	$[0.52\,,0.57]$	$[1.02\,,1.07]$
	2	$[0.8\,,0.85]$	0	0	$[0.8\,,0.85]$
3	0	0	3	1.3	1.3
	1	0.5	2	0.9	1.4
	2	$[0.8\,,0.85]$	1	$[0.52\,,0.57]$	$[1.32\,,1.42]$
	3	1.1	0	0	1.1
4	0	0	4	1.56	1.56
	1	0.5	3	1.3	1.8
	2	$[0.8\,,0.85]$	2	0.9	$[1.7\,,1.75]$
	3	1.1	1	$[0.52\,,0.57]$	$[1.62\,,1.67]$
	4	1.3	0	0	1.3
5	0	0	5	1.8	1.8
	1	0.5	4	1.56	2.06
	2	$[0.8\,,0.85]$	3	1.3	$[2.1\,,2.15]$
	3	1.1	2	0.9	2.0
	4	1.3	1	$[0.52\,,0.57]$	$[1.82\,,1.87]$
	5	1.5	0	0	1.5

The values colored yellow represent the maximum values of profit returned for each amount in which the second project is supplied, so we write a table, **Z_2** the abbreviation that shows the optimal values.

Table 10. Optimal values of Z_2.

Z_2	x_2	$NG_2^*(Z_2)$	
0	0	0	
1	0	$[0.52\,,0.57]$	
2	1	$[1.02\,,1.07]$	
3	2	$[1.4\,,1.42]$	$Z_2 = Z_1 + x_2$
4	2	$[1.7\,,1.75]$	
5	2	$[2.1\,,2.15]$	

2- Z_3 table   where $Z_3 = Z_2 + x_3$

Table 11. Z_3 table.

Z_3	x_3	$NF_3(x_3)$	Z_2	$NG_2^*(Z_2)$	$NG_3(Z_3)$
0	0	0	0	0	0
1	0	0	1	[0.52 , 0.57]	[0.52 , 0.57]
	1	0.3	0	0	0.3
2	0	0	2	[1.02 , 1.07]	[1.02 , 1.07]
	1	0.3	1	[0.52 , 0.57]	[0.82 , 0.87]
	2	0.5	0	0	0.5
3	0	0	3	[1.4 , 1.42]	[1.4 , 1.42]
	1	0.3	2	[1.02 , 1.07]	[1.32 , 1.37]
	2	0.5	1	[0.52 , 0.57]	[1.02 , 1.07]
	3	[0.79 , 0.82]	0	0	[0.79 , 0.82]
4	0	0	4	[1.7 , 1.75]	[1.7 , 1.75]
	1	0.3	3	[1.4 , 1.42]	[1.7 , 1.72]
	2	0.5	2	[1.02 , 1.07]	[1.52 , 1.57]
	3	[0.79 , 0.82]	1	[0.52 , 0.57]	[1.31 , 1.39]
	4	1.0	0	0	1.0
5	0	0	5	[2.1 , 2.15]	[2.1 , 2.15]
	1	0.3	4	[1.7 , 1.75]	[2 , 2.05]
	2	0.5	3	[1.4 , 1.42]	[1.9 , 1.92]
	3	[0.79 , 0.82]	2	[1.02 , 1.07]	[1.81 , 1.89]
	4	1.0	1	[0.52 , 0.57]	[1.52 , 1.57]
	5	1.24	0	0	1.24

The values colored yellow represent the maximum values of profit payable for each amount in which the second project is supplied, so we write a table Z_3 The abbreviation that shows the optimal values.

Table 12. Optimal values.

Z_3	x_3	$NG_3^*(Z_3)$	
0	0	0	
1	0	[0.52 , 0.57]	
2	0	[1.02 , 1.07]	
3	0	[1.4 , 1.42]	$Z_3 = Z_2 + x_3$
4	0	[1.7 , 1.75]	
5	0	[2.1 , 2.15]	

3- Z_3 table where $Z_4 = Z_3 + x_4$

Table 13. Z_4 table.

Z_4	x_4	$NF_4(x_4)$	Z_3	$NG_3^*(Z_3)$	$NG_4(Z_4)$
0	0	0	0	0	0
1	0	0	1	[0.52 , 0.57]	[0.52 , 0.57]
	1	0.4	0	0	0.4
2	0	0	2	[1.02 , 1.07]	[1.02 , 1.07]
	1	0.4	1	[0.52 , 0.57]	[0.92 , 0.97]
	2	0.66	0	0	0.66
3	0	0	3	[1.4 , 1.42]	[1.4 , 1.42]
	1	0.4	2	[1.02 , 1.07]	[1.42 , 1.47]
	2	0.66	1	[0.52 , 0.57]	[1.18 , 1.23]
	3	0.84	0	0	0.84
4	0	0	4	[1.7 , 1.75]	[1.7 , 1.75]
	1	0.4	3	[1.4 , 1.42]	[1.8 , 1.82]
	2	0.66	2	[1.02 , 1.07]	[1.68 , 1.73]
	3	0.84	1	[0.52 , 0.57]	[1.36 , 1.41]
	4	[0.9 , 1.3]	0	0	[0.9 , 1.3]
5	0	0	5	[2.1 , 2.15]	[2.1 , 2.15]
	1	0.4	4	[1.7 , 1.75]	[2.1 , 2.15]
	2	0.66	3	[1.4 , 1.42]	[2.06 , 2.08]
	3	0.84	2	[1.02 , 1.07]	[1.86 , 1.91]
	4	[0.9 , 1.3]	1	[0.52 , 0.57]	[1.42 , 1.87]
	5	[1.05 , 1.2]	0	0	[1.05 , 1.2]

The values colored yellow represent the maximum values of profit returned for each amount in which the second project is supplied, so we write a table,Z-4. The abbreviation that shows the optimal values

Table 14. the optimal values of Z_4.

Z_4	x_4	$NG_4^*(Z_4)$	
0	0	0	
1	0	[0.52 , 0.57]	
2	0	[1.02 , 1.07]	
3	1	[1.42 , 1.47]	$Z_4 = Z_3 + x_4$
4	1	[1.8 , 1.82]	
5	0 *or* 1	[2.1 , 2.15]	

Conclusion of the optimal solution: Note that the maximum profit value when investing the amount (5) available in the company is achieved at two values of x_4 Therefore, the optimal distribution is as follows:

$$NG_4^*(Z_4) = [2.1 \,,\, 2.15] \Rightarrow Z_4 = 5 \Rightarrow \begin{cases} x_4 = 0 \\ or \\ x_4 = 1 \end{cases} \Rightarrow \begin{cases} \begin{cases} x_4 = 0 \\ x_3 = 0 \\ x_2 = 2 \\ x_1 = 3 \end{cases} \\ 0r \\ \begin{cases} x_4 = 1 \\ x_3 = 0 \\ x_2 = 2 \\ x_1 = 2 \end{cases} \end{cases}$$

To obtain a maximum profit of a value belonging to the range, $[\mathbf{2.1} \,,\, \mathbf{2.15}]$ an amount of (5) million can be distributed in one of the two forms:

Figure 1: We provide the first project with (3) million dollars and the second project with (2) million dollars and cancel investment in the third and fourth projects Figure 2: We provide the first project (2) million dollars, the second project (2) million dollars, the fourth project (1) million dollars, and cancel the investment in the third project to obtain a maximum profit of a value belonging to the range$[\mathbf{1.8} \,,\, \mathbf{1.82}]$ The amount of (4) million can be distributed as follows: We provide the first project with (1) million dollars, the second project with (2) million dollars, and the fourth project with (1) million dollars, and cancel the investment in the third project

$$NG_4^*(Z_4) = [\mathbf{1.8} \,,\, \mathbf{1.82}] \Longrightarrow Z_4 = 4 \Longrightarrow \begin{cases} x_4 = 1 \\ x_3 = 0 \\ x_2 = 2 \\ x_1 = 1 \end{cases}$$

Note: When calculating $\mathbf{Z_4}$ We can only calculate for (4) and (5) million, i.e. sufficiency in the following table:

Table 15. Table of optimal values for Z_4 with the short solution.

Z_4	x_4	$NF_4(x_4)$	Z_3	$NG_3^*(Z_3)$	$NG_4(Z_4)$
4	0	0	4	[1.7 , 1.75]	[1.7 , 1.75]
	1	0.4	3	[1.4 , 1.42]	[1.8 , 1.82]
	2	0.66	2	[1.02 , 1.07]	[1.68 , 1.73]
	3	0.84	1	[0.52 , 0.57]	[1.36 , 1.41]
	4	[0.9 , 1.3]	0	0	[0.9 , 1.3]
5	0	0	5	[2.1 , 2.15]	[2.1 , 2.15]
	1	0.4	4	[1.7 , 1.75]	[2.1 , 2.15]
	2	0.66	3	[1.4 , 1.42]	[2.06 2.08]
	3	0.84	2	[1.02 , 1.07]	[1.86 , 1.91]
	4	[0.9 , 1.3]	1	[0.52 , 0.57]	[1.42 , 1.87]
	5	[1.05 , 1.2]	0	0	[1.05 , 1.2]

Comparison for the investment of (5) million:

Table 16. comparison in case the amount is (5) millions.

Type of profit values / Projects	Optimal distribution of amount in case that some profit values are neutrosophic values		Optimal distribution of amount in case that some profit values are classical values
Forth project	$x_4 = 1$ 0r $x_4 = 0$		$x_4 = 1$
Third project	$x_3 = 0$	$x_3 = 0$	$x_3 = 0$
Second project	$x_2 = 2$	$x_2 = 2$	$x_2 = 1$
First project	$x_1 = 3$	$x_1 = 1$	$x_1 = 3$
Maximal general profit	$NG_4^*(Z_4) = [2.1 , 2.15]$		$G_4^*(Z_4) = 2.20$

Comparison for the investment of (4) million:

Table 17. comparison in case the amount is (4) millions.

Type of profit values / Projects	Optimal distribution of amount in case that some profit values are neutrosophic values	Optimal distribution of amount in case that some profit values are classical values	
Forth project	$x_4 = 1$	$x_4 = 0$ or $x_4 = 1$	
Third project	$x_3 = 0$	$x_3 = 0$	$x_3 = 0$
Second project	$x_2 = 2$	$x_2 = 1$	$x_2 = 1$
First project	$x_1 = 1$	$x_1 = 3$	$x_1 = 2$
Maximal general profit	$NG_4^*(Z_4) = [1.8 , 1.82]$	$G_4^*(Z_4) = 1.80$	

3. Conclusion and Results

In this research, we presented a study whose importance lies in the following points:

1. Obtaining a general formula for the issue of establishing new warehouses that help companies and institutions expand their work at the lowest cost by choosing the ideal locations for expansion points

2. Obtaining a general formula for the issue of distributing the companies' capital budget through the ideal investment for them in selected projects within a group of proposed projects so that the company achieves the maximum profit.

3. Clarifying the role of the binary integer in converting some nonlinear models into linear models and also obtain solutions with integers appropriate to the nature of the problems under study.

4. Focusing on the need to use the concepts of neutrosophic science to obtain optimal solutions suitable for all circumstances surrounding the work environment.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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