Neutrosophic Systems with Applications, Vol. 7, 2023
https://doi.org/10.61356/j.nswa.2023.27

# Properties of Redefined Neutrosophic Composite Relation

**Sudeep Dey [1],*** and **Gautam Chandra Ray [2]**

[1] Department of Mathematics, Science College, Kokrajhar, Assam, India; sudeep.dey.1976@gmail.com.
[2] Department of Mathematics, Central Institute of Technology, Kokrajhar, Assam, India; gautomofcit@gmail.com.

* Correspondence: sudeep.dey.1976@gmail.com.

**Abstract:** The notion of single-valued neutrosophic composite relation was redefined by S.Dey and G.C.Ray in the year 2022. In this article, we investigate some basic properties of the redefined neutrosophic composite relation.

**Keywords:** Neutrosophic set; Single-valued neutrosophic set; Single-valued neutrosophic relation; Redefined neutrosophic composite relation.

## 1. Introduction

The concept of neutrosophic set was introduced by Smarandache [13,14] in the 1990's. Afterwards many researchers [7,8,11,12,15] studied and developed it. Since its inception, the neutrosophic set has garnered significant interest from researchers worldwide due to its flexibility and effectiveness. It has proven to be not only valuable in the advancement of science and technology but also applicable in various other fields. For instance, works[1,2,6,18,19] on medical diagnosis, decision-making problems, image processing, social issues etc. had also been done in a neutrosophic environment.

In 2010, Wang et al.[16] further developed the notion of a single-valued neutrosophic set. Salma et.al. [9,10] added the thinking of neutrosophic relation and studied some of its properties. Building upon these concepts, Yang et al.[17] in 2016 introduced single-valued neutrosophic relation and investigated some properties. Taking the concept forward, Kim et al.[5] generalized the notion of a of single-valued neutrosophic relation from a set X to a set Y. The authors also introduced the composition of two neutrosophic relations and throughly examined various properties associated with it.

More recently, in 2022, S.Dey and G.C.Ray [3] introduced a novel definition for the neutrosophic composite relation of two single-valued neutrosophic relations. In this article, we aim to explore and investigate some properties related to the redefined neutrosophic composite relation.

## 2. Preliminaries

In this section we confer some basic concepts which will be helpful in the later sections.

***2.1. Definition: [13]*** Let $X$ be the universe of discourse. A neutrosophic set $A$ over $X$ is defined as $A=\{\langle x,\mathcal{T}_A(x),\mathcal{I}_A(x),\mathcal{F}_A(x)\rangle : x\in X\}$, where the functions $\mathcal{T}_A,\mathcal{I}_A,\mathcal{F}_A$ are real standard or non-standard subsets of $]^-0,1^+[$, i.e., $\mathcal{T}_A: X\to\ ]^-0,1^+[, \mathcal{I}_A: X\to\ ]^-0,1^+[, \mathcal{F}_A: X\to\ ]^-0,1^+[$ and $^-0\le\mathcal{T}_A(x)+\mathcal{I}_A(x)+\mathcal{F}_A(x)\le 3^+$.

The neutrosophic set $A$ is characterized by the truth-membership function $\mathcal{T}_A$, indeterminacy-membership function $\mathcal{I}_A$, falsehood-membership function $\mathcal{F}_A$.

***2.2. Definition:[16]*** Let $X$ be the universe of discourse. A single-valued neutrosophic set (SVNS, for short) $A$ over $X$ is defined as $A=\{\langle x,\mathcal{T}_A(x),\mathcal{I}_A(x),\mathcal{F}_A(x)\rangle : x\in X\}$, where $\mathcal{T}_A,\mathcal{I}_A,\mathcal{F}_A$ are functions from $X$ to $[0,1]$ and $0\le\mathcal{T}_A(x)+\mathcal{I}_A(x)+\mathcal{F}_A(x)\le 3$.

The functions $\mathcal{T}_A, \mathcal{I}_A, \mathcal{F}_A$ denote respectively the degrees of truth-membership, indeterminacy-membership, falsehood-membership of the element $x \in X$ in $A$.
The set of all single-valued neutrosophic sets over $X$ is denoted by $\mathcal{N}(X)$.
***2.3. Definition:*** **[4]** Let $A, B \in \mathcal{N}(X)$. Then

i. (Inclusion): If $\mathcal{T}_A(x) \leq \mathcal{T}_B(x), \mathcal{I}_A(x) \geq \mathcal{I}_B(x), \mathcal{F}_A(x) \geq \mathcal{F}_B(x)$ for all $x \in X$ then $A$ is said to be a neutrosophic subset of $B$ and which is denoted by $A \subseteq B$.

ii. (Equality): If $A \subseteq B$ and $B \subseteq A$ then $A = B$.

iii. (Intersection): The intersection of $A$ and $B$, denoted by $A \cap B$, is defined as $A \cap B = \{\langle x, \mathcal{T}_A(x) \wedge \mathcal{T}_B(x), \mathcal{I}_A(x) \vee \mathcal{I}_B(x), \mathcal{F}_A(x) \vee \mathcal{F}_B(x)\rangle : x \in X\}$.

iv. (Union): The union of $A$ and $B$, denoted by $A \cup B$, is defined as $A \cup B = \{\langle x, \mathcal{T}_A(x) \vee \mathcal{T}_B(x), \mathcal{I}_A(x) \wedge \mathcal{I}_B(x), \mathcal{F}_A(x) \wedge \mathcal{F}_B(x)\rangle : x \in X\}$.

v. (Complement): The complement of the NS $A$, denoted by $A^c$, is defined as $A^c = \{\langle x, \mathcal{F}_A(x), 1 - \mathcal{I}_A(x), \mathcal{T}_A(x)\rangle : x \in X\}$

vi. (Universal Set): If $\mathcal{T}_A(x) = 1, \mathcal{I}_A(x) = 0, \mathcal{F}_A(x) = 0$ for all $x \in X$ then $A$ is said to be neutrosophic universal set and which is denoted by $\tilde{X}$.

vii. (Empty Set): If $\mathcal{T}_A(x) = 0, \mathcal{I}_A(x) = 1, \mathcal{F}_A(x) = 1$ for all $x \in X$ then $A$ is said to be neutrosophic empty set and which is denoted by $\widetilde{\emptyset}$.

***2.4. Definition:*** **[4]** Let $A, B \in \mathcal{N}(X)$ and $\{A_i : i \in \Delta\} \subseteq \mathcal{N}(X)$, $\Delta$ is an index set. Then the following hold.

i. $A \cup A = A$ and $A \cap A = A$

ii. $A \cup B = B \cup A$ and $A \cap B = B \cap A$

iii. $A \cup \widetilde{\emptyset} = A$ and $A \cup \tilde{X} = \tilde{X}$

iv. $A \cap \widetilde{\emptyset} = \widetilde{\emptyset}$ and $A \cap \tilde{X} = A$

v. $A \cap (B \cap C) = (A \cap B) \cap C$ and $A \cup (B \cup C) = (A \cup B) \cup C$

vi. $(A^c)^c = A$

vii. $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$

viii. $(\cup_{i\in\Delta} A_i)^c = \cap_{i\in\Delta} A_i^c$ and $(\cap_{i\in\Delta} A_i)^c = \cup_{i\in\Delta} A_i^c$

ix. $B \cup (\cap_{i\in\Delta} A_i) = \cap_{i\in\Delta} (B \cup A_i)$

x. $B \cap (\cup_{i\in\Delta} A_i) = \cup_{i\in\Delta} (B \cap A_i)$

***2.5. Definition:*** **[5]** Let $X, Y, Z$ be three ordinary sets. Then $R$ is called a single-valued neutrosophic relation (SVNR, for short) from $X$ to $Y$ if it is a SVNS in $X \times Y$ having the form $R = \{\langle (x, y), \mathcal{T}_R(x, y), \mathcal{I}_R(x, y), \mathcal{F}_R(x, y)\rangle : (x, y) \in X \times Y\}$, where $\mathcal{T}_R : X \times Y \to [0,1], \mathcal{I}_R : X \times Y \to [0,1], \mathcal{F}_R : X \times Y \to [0,1]$ denote respectively the truth-membership function, indeterminacy-membership function, falsity-membership function.
In particular, a SVNR from from $X$ to $X$ is called a SVNR in $X$.
The empty SVNR and the whole SVNR in $X$, denoted by $\widetilde{\emptyset}_N$ and $\tilde{X}_N$ respectively, are defined as $\widetilde{\emptyset}_N = \{\langle (x, y), 0,1,1\rangle : (x, y) \in X \times X\}$ and $\tilde{X}_N = \{\langle (x, y), 1,0,0\rangle : (x, y) \in X \times X\}$.

The set of all SVNRs from $X$ to $Y$ is denoted by $SVNR(X \times Y)$ and the set of all SVNRs in $X$ is denoted by $SVNR(X)$.

**2.6. Definition: [5]** Let $R \in SVNR(X \times Y)$. Then

i. The inverse of $R$, denoted by $R^{-1}$, is a SVNR from $Y$ to $X$ defined as $R^{-1}(y,x) = R(x,y)$ for each $(y,x) \in Y \times X$.

ii. The complement of $R$, denoted by $R^c$, is a SVNR from $X$ to $Y$ defined as $\mathcal{T}_R^c(x,y) = \mathcal{F}_R(x,y), \mathcal{I}_R^c(x,y) = 1 - \mathcal{I}_R(x,y), \mathcal{F}_R^c(x,y) = \mathcal{T}_R(x,y)$ for each $(x,y) \in X \times Y$.

***2.7. Definition:*** **[5]** Let $R, S \in SVNR(X \times Y)$. Then

i. $R$ is said to be contained in $S$, denoted by $R \subseteq S$, if $\mathcal{T}_R(x,y) \leq \mathcal{T}_S(x,y), \mathcal{I}_R(x,y) \geq \mathcal{I}_S(x,y)$, $\mathcal{F}_R(x,y) \geq \mathcal{I}_S(x,y)$ for each $(x,y) \in X \times Y$.

ii. $R$ is said to be equal to $S$, denoted by $R = S$, if $R \subseteq S$ and $S \subseteq R$.

iii. The intersection of $R$ and $S$, denoted by $R \cap S$, is defined as $R \cap S = \{\langle (x,y), \mathcal{T}_R(x,y) \wedge \mathcal{T}_S(x,y), \mathcal{I}_R(x,y) \vee \mathcal{I}_S(x,y), \mathcal{F}_R(x,y) \vee \mathcal{F}_S(x,y) \rangle : (x,y) \in X \times Y\}$.

iv. The union of $R$ and $S$, denoted by $R \cup S$, is defined as $R \cup S = \{\langle (x,y), \mathcal{T}_R(x,y) \vee \mathcal{T}_S(x,y), \mathcal{I}_R(x,y) \wedge \mathcal{I}_S(x,y), \mathcal{F}_R(x,y) \wedge \mathcal{F}_S(x,y) \rangle : (x,y) \in X \times Y\}$.

***2.8. Definition:*** **[5]** Let $X, Y, Z$ be three ordinary sets. Also let $R \in SVNR(X \times Y)$ and $S \in SVNR(Y \times Z)$. Then the composition(max-min-max composition) of $R$ and $S$, denoted by $S \circ R$, is a SVNR from $X$ to $Z$ defined as

$$S \circ R = \{\langle (x,z), \mathcal{T}_{S \circ R}(x,z), \mathcal{I}_{S \circ R}(x,z), \mathcal{F}_{S \circ R}(x,z) \rangle : (x,z) \in X \times Z\},$$

where

$$\mathcal{T}_{S \circ R}(x,z) = \vee_{y \in Y} (\mathcal{T}_R(x,y) \wedge \mathcal{T}_S(y,z)),$$
$$\mathcal{I}_{S \circ R}(x,z) = \wedge_{y \in Y} (\mathcal{I}_R(x,y) \vee \mathcal{I}_S(y,z)),$$
$$\mathcal{F}_{S \circ R}(x,z) = \wedge_{y \in Y} (\mathcal{F}_R(x,y) \vee \mathcal{F}_S(y,z)).$$

***2.9. Definition:*****[5]**

i. The single-valued neutrosophic identity relation in $X$, denoted by $I_X$, is defined as : for each $(x,y) \in X \times X$ , $\mathcal{T}_{I_X}(x,y) = 1, \mathcal{I}_{I_X}(x,y) = 0, \mathcal{F}_{I_X}(x,y) = 0$ if $x = y$ and $\mathcal{T}_{I_X}(x,y) = 0, \mathcal{I}_{I_X}(x,y) = 1, \mathcal{F}_{I_X}(x,y) = 1$ if $x \neq y$.

ii. A SVNR $R$ in $X$ is said to be reflexive if for each $x \in X$, $\mathcal{T}_R(x,x) = 1, \mathcal{I}_R(x,x) = 0, \mathcal{F}_R(x,x) = 0$.

iii. A SVNR $R$ in $X$ is said to be symmetric if for each $(x,y) \in X \times X$, $\mathcal{T}_R(x,y) = \mathcal{T}_R(y,x), \mathcal{I}_R(x,y) = \mathcal{I}_R(y,x), \mathcal{F}_R(x,y) = \mathcal{F}_R(y,x)$.

iv. A SVNR $R$ in $X$ is said to be transitive if $R \circ R \subseteq R$, i.e., $R^2 \subseteq R$.

***2.10. Proposition:*****[5]** Let $X$ be an ordinary set and $R \in SVNR(X)$. Then $R$ is symmetric iff $R^{-1} = R$.

***2.11. Definition:*[3]** Let $X, Y, Z$ be three ordinary sets. Also let $R \in SVNR(X \times Y)$ and $S \in SVNR(Y \times Z)$. Then the redefined neutrosophic composite relation of the SVNRs $R$ and $S$, denoted by $S \circ R$, is a SVNR from $X$ to $Z$ defined as

$$S \circ R = \{\langle (x,z), \mathcal{T}_{S\circ R}(x,z), \mathcal{I}_{S\circ R}(x,z), \mathcal{F}_{S\circ R}(x,z)\rangle : (x,z) \in X \times Z\},$$

where

$$\mathcal{T}_{S\circ R}(x,z) = \vee_{y\in Y} \frac{\mathcal{T}_R(x,y)+\mathcal{T}_S(y,z)}{2},$$

$$\mathcal{I}_{S\circ R}(x,z) = \wedge_{y\in Y} \frac{\mathcal{I}_R(x,y)+\mathcal{I}_S(y,z)}{2},$$

$$\mathcal{F}_{S\circ R}(x,z) = \wedge_{y\in Y} \frac{\mathcal{F}_R(x,y)+\mathcal{F}_S(y,z)}{2}.$$

***2.12 Example*:** Let $X = \{a, b\}, Y = \{p, q\}, Z = \{u, v\}$. Also let $R \in SVNR(X \times Y)$ and $S \in SVNR(Y \times Z)$ be given by the Table-1, Table-2.

Table-1

| $R$ | $p$ | $q$ |
|---|---|---|
| $a$ | (0.6, 0.1, 0.2) | (0.1, 0.2, 0.7) |
| $b$ | (0.5, 0.6, 0.7) | (0.3, 0.2, 0.1) |

Table-2

| $S$ | $u$ | $v$ |
|---|---|---|
| $p$ | (0.5, 0.3, 0.2) | (0.6, 0.4, 0.3) |
| $q$ | (0.9, 0.1, 0.2) | (0.2, 0.5, 0.4) |

Then by using the definition 2.11, we have

$$\mathcal{T}_{S\circ R}(a,u) = \vee_{y\in Y} \frac{\mathcal{T}_R(a,y)+\mathcal{T}_S(y,u)}{2} = \vee\{\frac{0.6+0.5}{2}, \frac{0.1+0.9}{2}\} = 0.55.$$

$$\mathcal{I}_{S\circ R}(a,u) = \wedge_{y\in Y} \frac{\mathcal{I}_R(a,y)+\mathcal{I}_S(y,u)}{2} = \wedge\{\frac{0.1+0.3}{2}, \frac{0.2+0.1}{2}\} = 0.15.$$

$$\mathcal{F}_{S\circ R}(a,u) = \wedge_{y\in Y} \frac{\mathcal{F}_R(a,y)+\mathcal{F}_S(y,u)}{2} = \wedge\{\frac{0.2+0.2}{2}, \frac{0.7+0.2}{2}\} = 0.20.$$

Similarly proceeding for the pairs $(a, v), (b, u), (b, v)$, we get the redefined neutrosophic composite relation $S \circ R \in SVNR(X \times Z)$ as shown in the following Table-3.

Table-3

| $S \circ R$ | $u$ | $v$ |
|---|---|---|
| $a$ | (0.55, 0.15, 0.20) | (0.60, 0.25, 0.25) |
| $b$ | (0.60, 0.15, 0.15) | (0.55, 0.35, 0.25) |

**Main Results:** In this section we study the properties of redefined neutrosophic composite relation.

***3.1. Proposition*:** Let $X, Y, Z$ be three ordinary sets. Also let $R, S \in SVNR(X \times Y)$ and $P \in SVNR(Y \times Z)$. Then

i. $P \circ (R \cup S) = (P \circ R) \cup (P \circ S)$.

ii. $R \subseteq S \Rightarrow P \circ R \subseteq P \circ S$.

iii. $(P \circ R)^{-1} = R^{-1} \circ P^{-1}$.

**Proof:**

i. Clearly $P \circ (R \cup S), (P \circ R) \cup (P \circ S) \in SVNR(X \times Z)$. Let $(x, z) \in X \times Z$. Then

$$\mathcal{T}_{P\circ(R\cup S)}(x,z) = \bigvee_{y\in Y} \frac{\mathcal{T}_{R\cup S}(x,y)+\mathcal{T}_P(y,z)}{2}$$

$$= \bigvee_{y\in Y} \frac{(\mathcal{T}_R(x,y)\vee\mathcal{T}_S(x,y))+\mathcal{T}_P(y,z)}{2}$$

$$= \bigvee_{y\in Y} [\frac{\mathcal{T}_R(x,y)+\mathcal{T}_P(y,z)}{2} \vee \frac{\mathcal{T}_S(x,y)+\mathcal{T}_P(y,z)}{2}]$$

$$= [\bigvee_{y\in Y} \frac{\mathcal{T}_R(x,y)+\mathcal{T}_P(y,z)}{2}] \vee [\bigvee_{y\in Y} \frac{\mathcal{T}_S(x,y)+\mathcal{T}_P(y,z)}{2}]$$

$$= \mathcal{T}_{P\circ R}(x,z) \vee \mathcal{T}_{P\circ S}(x,z)$$
$$= \mathcal{T}_{(P\circ R)\cup(P\circ S)}(x,z)$$

Similarly we can show that $\mathcal{I}_{P\circ(R\cup S)}(x,z) = \mathcal{I}_{(P\circ R)\cup(P\circ S)}(x,z)$ and $\mathcal{F}_{P\circ(R\cup S)}(x,z) = \mathcal{F}_{(P\circ R)\cup(P\circ S)}(x,z)$. Therefore $P \circ (R \cup S) = (P \circ R) \cup (P \circ S)$.

ii. Clearly $P \circ R, P \circ S \in SVNR(X \times Z)$. Let $(x, z) \in X \times Z$. Then

$$\mathcal{T}_{P\circ R}(x,z) = \bigvee_{y\in Y} \frac{\mathcal{T}_R(x,y)+\mathcal{T}_P(y,z)}{2} \leq \bigvee_{y\in Y} \frac{\mathcal{T}_S(x,y)+\mathcal{T}_P(y,z)}{2} [\because \mathrm{R} \subseteq \mathrm{S}] = \mathcal{T}_{P\circ S}(x,z).$$

Therefore, $\mathcal{T}_{P\circ R}(x,z) \leq \mathcal{T}_{P\circ S}(x,z)$.
Similarly we can show that $\mathcal{I}_{P\circ R}(x,z) \geq \mathcal{I}_{P\circ S}(x,z)$ and $\mathcal{F}_{P\circ R}(x,w) \geq \mathcal{F}_{P\circ S}(x,z)$.
Hence $P \circ R \subseteq P \circ S$.

iii. Clearly $P \circ R \in SVNR(X \times Z)$ and $(P \circ R)^{-1}, R^{-1} \circ P^{-1} \in SVNR(Z \times X)$. Let $(z, x) \in Z \times X$. Then

$$\mathcal{T}_{(P\circ R)^{-1}}(z,x) = \mathcal{T}_{P\circ R}(x,z)$$
$$= \bigvee_{y\in Y} \frac{\mathcal{T}_R(x,y)+\mathcal{T}_P(y,z)}{2}$$

$$= \bigvee_{y\in Y} \frac{\mathcal{T}_{R^{-1}}(y,x)+\mathcal{T}_{P^{-1}}(z,y)}{2}$$

$$= \mathcal{T}_{R^{-1}\circ P^{-1}}(z,x)$$

Similarly we can show that $\mathcal{I}_{(P\circ R)^{-1}}(z,x) = \mathcal{I}_{R^{-1}\circ P^{-1}}(z,x)$ and $\mathcal{F}_{(P\circ R)^{-1}}(z,x) = \mathcal{F}_{R^{-1}\circ P^{-1}}(z,x)$. Therefore $(P \circ R)^{-1} = R^{-1} \circ P^{-1}$.

***3.2. Remark*:** Redefined neutrosophic composite relation is not commutative. We shall establish by the following counter example.
Let $X = \{a,b\}, Y = \{p,q\}, Z = \{u,v\}$. Also let $R \in SVNR(X \times Y), P \in SVNR(Y \times Z)$. Obviously $P \circ R \in SVNR(X \times Z)$ and $R \circ P \in SVNR(Y \times Y)$. Therefore $P \circ R \neq R \circ P$.

***3.3. Remark:*** Redefined neutrosophic composite relation is not associative. We shall establish by the following counter example.
Let $X = \{a,b\}, Y = \{p,q\}, Z = \{u,v\}, W = \{x,y\}$. Also let $R \in SVNR(X \times Y), P \in SVNR(Y \times Z)$ and $Q \in SVNR(Z \times W)$ be given by the following Table-4, Table-5, Table-6.

Table-4

| $R$ | $p$ | $q$ |
|---|---|---|
| $a$ | (.6,.1,.2) | (.1,.2,.7) |
| $b$ | (.5,.6,.7) | (.3,.2,.1) |

Table-5

| $P$ | $u$ | $v$ |
|---|---|---|
| $p$ | (.5,.3,.2) | (.6,.4,.3) |
| $q$ | (.9,.1,.2) | (.3,.2,.1) |

Table-6

| $Q$ | $x$ | $y$ |
|---|---|---|
| $u$ | (.5,.4,.2) | (.5,.3,.1) |
| $v$ | (.8,.2,.1) | (.3,.6,.4) |

Then by using the definition 2.11, we find the redefined neutrosophic composite relations $P \circ R \in SVNR(X \times Z), Q \circ P \in SVNR(Y \times W), Q \circ (P \circ R) \in SVNR(X \times W), (Q \circ P) \circ R \in SVNR(X \times W)$ as shown in the following Table-7, Table-8, Table-9, Table-10.

Table-7

| $P \circ R$ | $u$ | $v$ |
|---|---|---|
| $a$ | (.55,.15,.20) | (.60,.25,.25) |
| $b$ | (.60,.15,.15) | (.55,.35,.25) |

Table-8

| $Q \circ P$ | $x$ | $y$ |
|---|---|---|
| $p$ | (.70,.20,.15) | (.50,.40,.25) |
| $q$ | (.70,.25,.15) | (.70,.30,.20) |

Table-9

| $Q \circ (P \circ R)$ | $x$ | $y$ |
|---|---|---|
| $a$ | (.475,.225,.225) | (.475,.225,.225) |
| $b$ | (.475,.225,.225) | (.475,.225,.225) |

Table-10

| $(Q \circ P) \circ R$ | $x$ | $y$ |
|---|---|---|
| $a$ | (.65,.15,.175) | (.55,.25,.225) |
| $b$ | (.60,.225,.125) | (.50,.25,.15) |

We see that $\mathcal{T}_{Q\circ(P\circ R)}(a,x) = 0.475$ and $\mathcal{T}_{(Q\circ P)\circ R}(a,x) = 0.65$ . Since $\mathcal{T}_{Q\circ(P\circ R)}(a,x) \neq \mathcal{T}_{(Q\circ P)\circ R}(a,x)$, so $Q \circ (P \circ R) \neq (Q \circ P) \circ R$.

***3.4. Remark*****:** Redefined neutrosophic composite relation is not distributive over intersection. We shall establish by the following counter example.

Let $X = \{a,b\}, Y = \{p,q\}, Z = \{u,v\}, W = \{x,y\}$ . Also let $R,S \in SVNR(X \times Y), P \in SVNR(Y \times Z)$ be given by the Table-11, Table-12, Table-13.

Table-11

| $R$ | $p$ | $q$ |
|---|---|---|
| $a$ | (.6,.1,.2) | (.1,.2,.7) |
| $b$ | (.5,.6,.7) | (.3,.2,.1) |

Table-12

| $S$ | $p$ | $q$ |
|---|---|---|
| $a$ | (.8,.7,.3) | (.2,.0,.7) |
| $b$ | (.7,.2,.3) | (.5,.6,.4) |

Table-13

| $P$ | $u$ | $v$ |
|---|---|---|
| $p$ | (.5,.3,.2) | (.6,.4,.3) |
| $q$ | (.9,.1,.2) | (.3,.2,.1) |

Then by using the definition 4(`)@, we find the SVNRs $R \cap S \in SVNR(X \times Y), P \circ (R \cap S) \in SVNR(X \times Z), P \circ R \in SVNR(X \times Z), P \circ S \in SVNR(X \times Z)$ and $(P \circ R) \cap (P \circ S) \in SVNR(X \times Y)$ as shown in the following Table-14, Table-15, Table-16, Table-17, Table-18.

Table-14

| $P \circ R$ | $u$ | $v$ |
|---|---|---|
| $a$ | (.55,.15,.20) | (.60,.25,.25) |
| $b$ | (.60,.15,.15) | (.55,.35,.25) |

Table-15

| $P \circ S$ | $u$ | $v$ |
|---|---|---|
| $a$ | (.65,.05,.25) | (.70,.25,.30) |
| $b$ | (.70,.25,.25) | (.65,.30,.30) |

Table-16

| $R \cap S$ | $p$ | $q$ |
|---|---|---|
| $a$ | (.6,.7,.3) | (.1,.2,.7) |
| $b$ | (.5,.6,.7) | (.3,.6,.4) |

Table-17

| $P \circ (R \cap S)$ | $u$ | $v$ |
|---|---|---|
| $a$ | (.55,.15,.25) | (.60,.35,.30) |
| $b$ | (.60,.35,.30) | (.55,.50,.40) |

Table-18

| $(P \circ R) \cap (P \circ S)$ | $u$ | $v$ |
|---|---|---|
| $a$ | (.55,.05,.20) | (.60,.25,.25) |
| $b$ | (.60,.15,.15) | (.55,.30,.25) |

From the Table-17 and Table-18, it is easy to see that

$$\mathcal{I}_{P\circ(R\cap S)}(a,u) = .15 \quad \text{and} \quad \mathcal{I}_{(P\circ R)\cap(P\circ S)}(a,u) = .05.$$

Therefore $P \circ (R \cap S) \neq (P \circ R) \cap (P \circ S)$.

***3.5. Proposition*****:** Let $X$ be an ordinary set and $R, S \in SVNR(X \times X)$. If $R, S$ are reflexive then $S \circ R$ is reflexive.

**Proof:** For any two elements $x, y \in X$, we have

$$\mathcal{T}_{S\circ R}(x,x) = \bigvee_{y\in X} \frac{\mathcal{T}_R(x,y)+\mathcal{T}_S(y,x)}{2}$$

$$= [\bigvee_{y\neq x} \frac{\mathcal{T}_R(x,y)+\mathcal{T}_S(y,x)}{2}] \vee [\frac{\mathcal{T}_R(x,x)+\mathcal{T}_S(x,x)}{2}]$$

$$= [\bigvee_{y\neq x} \frac{\mathcal{T}_R(x,y)+\mathcal{T}_S(y,x)}{2}] \vee [\frac{1+1}{2}] (\because R \text{ and } S \text{ are reflexive})$$

$$= [\bigvee_{y\neq x} \frac{\mathcal{T}_R(x,y)+\mathcal{T}_S(y,x)}{2}] \vee 1 = 1$$

Again

$$\mathcal{I}_{S\circ R}(x,x) = \underset{y\in X}{\wedge} \frac{\mathcal{I}_R(x,y)+\mathcal{I}_S(y,x)}{2}$$

$$= [\underset{y\neq x}{\wedge} \frac{\mathcal{X}_R(x,y)+\mathcal{I}_S(y,x)}{2}]\wedge[\frac{\mathcal{I}_R(x,x)+\mathcal{I}_S(x,x)}{2}]$$

$$= [\underset{y\neq x}{\wedge} \frac{\mathcal{I}_R(x,y)+\mathcal{I}_S(y,x)}{2}]\wedge [\frac{0+0}{2}](\because R \text{ and } S \text{ are reflexive})$$

$$= [\underset{y\neq x}{\wedge} \frac{\mathcal{I}_R(x,y)+\mathcal{I}_S(y,x)}{2}]\wedge 0$$

$$= 0$$

Similarly we can show that $\mathcal{F}_{S\circ R}(x,x) = 0$.

Therefore, $S\circ R$ is reflexive.

***3.6. Remark:*** Let $X$ be an ordinary set and $R, P \in SVNR(X)$. If $R, P$ are symmetric then $P\circ R$ may not be symmetric. We shall establish it by a counter example.

Let $X = \{a, b\}$. Also let $R, P \in SVNR(X)$ be given by the Table-19 and Table-20.

Table-19

| $R$ | $a$ | $b$ |
|---|---|---|
| $a$ | (0.6, 0.1, 0.2) | (0.5, 0.6, 0.7) |
| $b$ | (0.5, 0.6, 0.7) | (0.3, 0.2, 0.1) |

Table-20

| $P$ | $a$ | $b$ |
|---|---|---|
| $a$ | (0.5, 0.3, 0.2) | (0.6, 0.4, 0.3) |
| $b$ | (0.6, 0.4, 0.3) | (0.2, 0.5, 0.4) |

Then $\mathcal{T}_{P\circ R}(a,b) = \vee_{y\in X} \frac{\mathcal{T}_R(a,y)+\mathcal{T}_S(y,b)}{2} = \vee \ \{\frac{0.6+0.6}{2}, \frac{0.5+0.2}{2}\} = 0.6$

and $\mathcal{T}_{P\circ R}(b,a) = \vee_{y\in X} \frac{\mathcal{T}_R(b,y)+\mathcal{T}_S(y,a)}{2} = \vee \ \{\frac{0.5+0.5}{2}, \frac{0.3+0.6}{2}\} = 0.5$.

We can see that $\mathcal{T}_{P\circ R}(a,b) = 0.6 \neq 0.5 = \mathcal{T}_{P\circ R}(b,a)$. Therefore $P\circ R$ is not symmetric.

***3.7. Proposition*:** Let $X$ be an ordinary set and $R, S \in SVNR(X\times X)$ are symmetric. Then $S\circ R$ is symmetric iff $S\circ R = R\circ S$.

**Proof:** Since $R$ and $S$ are symmtric, so $R^{-1} = R$ and $S^{-1} = S$ [by 2.10]. First suppose that $S\circ R$ is symmetric. Then $S\circ R = (S\circ R)^{-1} = R^{-1}\circ S^{-1} = R\circ S$. Conversely suppose that $S\circ R = R\circ S$. Then $(S\circ R)^{-1} = R^{-1}\circ S^{-1} = R\circ S$,i.e., $S\circ R$ is symmetric.

***3.8. Proposition:*** Let $X$ be an ordinary set and $R \in SVNR(X \times X)$ be transitive. Then $R \circ R$ is transitive.

**Proof:** Since $R$ is transitive, so $R \circ R \subseteq R$, i.e., $R^2 \subseteq R$. Now

$$\mathcal{T}_{R^2 \circ R^2}(x,z) = \bigvee_{y \in X} \frac{\mathcal{T}_{R^2}(x,y) + \mathcal{T}_{R^2}(y,z)}{2}$$

$$\leq \bigvee_{y \in X} \frac{\mathcal{T}_R(x,y) + \mathcal{T}_R(y,z)}{2}$$

$$= \mathcal{T}_{R \circ R}(x,z)$$
$$= \mathcal{T}_{R^2}(x,z)$$

Similarly we can show that $\mathcal{I}_{R^2 \circ R^2}(x,z) \geq \mathcal{I}_{R^2}(x,z)$ and $\mathcal{F}_{R^2 \circ R^2}(x,z) \geq \mathcal{F}_{R^2}(x,z)$. Therefore $R^2 \circ R^2 \subseteq R^2$. Hence $R^2$, i.e., $R \circ R$ is transitive.

***3.9. Proposition*:** Let $X$ be an ordinary set. If $R \in SVNR(X)$ is transitive $R^{-1}$ is also transitive.

**Proof:** Since $R$ is transitive, so $R \circ R \subseteq R$. Now

$$\mathcal{T}_{R^{-1} \circ R^{-1}}(x,z) = \bigvee_{y \in X} \frac{\mathcal{T}_{R^{-1}}(x,y) + \mathcal{T}_{R^{-1}}(y,z)}{2}$$

$$= \bigvee_{y \in X} \frac{\mathcal{T}_R(y,x) + \mathcal{T}_R(z,y)}{2}$$

$$= \mathcal{T}_{R \circ R}(z,x)$$
$$\leq \mathcal{T}_R(z,x)$$
$$= \mathcal{T}_{R^{-1}}(x,z)$$

Similarly we can show that $\mathcal{I}_{R^{-1} \circ R^{-1}}(x,z) \geq \mathcal{I}_{R^{-1}}(x,z)$ and $\mathcal{F}_{R^{-1} \circ R^{-1}}(x,z) \geq \mathcal{F}_{R^{-1}}(x,z)$. Therefore $R^{-1} \circ R^{-1} \subseteq R^{-1}$ and so, $R^{-1}$ is transitive.

***3.10. Remark:*** Let $X$ be an ordinary set and $R, S \in SVNR(X)$. If $R, S$ are transitive then $R \cup S$ and $R \cap S$ may not be transitive. We shall establish it by a counter example. Let $X = \{a, b\}$. Also let $R, S \in SVNR(X)$ be given by the Table-21 and Table-22.

Table-21

| $R$ | $a$ | $b$ |
|---|---|---|
| $a$ | (0.8, 0.5, 0.4) | (0.6, 0.4, 0.5) |
| $b$ | (0.7, 0.6, 0.2) | (0.7, 0.6, 0.3) |

Table-22

| $S$ | $a$ | $b$ |
|---|---|---|
| $a$ | (0.7, 0.4, 0.2) | (0.4, 0.6, 0.4) |
| $b$ | (0.5, 0.4, 0.3) | (0.5, 0.4, 0.4) |

Clearly $R$ and $S$ are transitive.

Then the relations $R \cup S$ and $R \cap S$ are as given in Table-23 and Table-24.

Table-23

| $R \cup S$ | $a$ | $b$ |
|---|---|---|
| $a$ | (0.8, 0.4, 0.2) | (0.6, 0.4, 0.4) |
| $b$ | (0.7, 0.4, 0.2) | (0.7, 0.4, 0.3) |

Table-24

| $R \cap S$ | $a$ | $b$ |
|---|---|---|
| $a$ | (0.7, 0.5, 0.4) | (0.4, 0.6, 0.5) |
| $b$ | (0.5, 0.6, 0.3) | (0.5, 0.6, 0.4) |

Now,

$$\mathcal{T}_{(R\cup S)\circ(R\cup S)}(a,b) = \vee_{y\in X} \frac{\mathcal{T}_R(a,y)+\mathcal{T}_S(y,b)}{2} = \vee \ \{\frac{0.8+0.6}{2}, \frac{0.6+0.7}{2}\} = 0.7$$

$$\text{and } \mathcal{T}_{(R\cap S)\circ(R\cap S)}(a,b) = \vee_{y\in X} \frac{\mathcal{T}_R(a,y)+\mathcal{T}_S(y,b)}{2} = \vee \ \{\frac{0.5+0.7}{2}, \frac{0.5+0.5}{2}\} = 0.6.$$

We can see that $\mathcal{T}_{(R\cup S)\circ(R\cup S)}(a,b) = 0.7 > 0.6 = \mathcal{T}_{R\cup S}(a,b)$, i.e. $(R\cup S)\circ(R\cup S) \nsubseteq R\cup S$. Therefore $R\cup S$ is not transitive.

We can also see that $\mathcal{T}_{(R\cap S)\circ(R\cap S)}(b,a) = 0.6 > 0.5 = \mathcal{T}_{R\cap S}(b,a)$ , i.e. $(R\cap S)\circ(R\cap S) \nsubseteq R\cap S$ . Therefore $R\cap S$ is not transitive.

## 3. Conclusion

In this article, we have investigated various properties in connection with redefined neutrosophic composite relation. Our investigations into the neutrosophic composite relation provide valuable insights and pave the way for further advancements in the field of neutrosophic algebra. We anticipate that the findings presented in this study will serve as a significant resource for researchers and scholars, enabling them to build upon our work and contribute to the ongoing development and exploration of neutrosophic algebra.

**Data availability**

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

**Conflict of interest**

The authors declare that there is no conflict of interest in the research.

**Ethical approval**

This article does not contain any studies with human participants or animals performed by any of the authors.

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Received: Feb 07, 2023. Accepted: Jun 28, 2023