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A Newfangled Interpretation on Fermatean Neutrosophic Dombi Fuzzy Graphs

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Abstract: Neutrosophic Dombi fuzzy graph is an advancement of the Dombi fuzzy graph and intuitionistic Dombi fuzzy graph. In this paper, we have initiated a new concept of the Fermatean neutrosophic Dombi fuzzy graph. Further, we identified a few products of the direct, cartesian, composition of Fermatean neutrosophic Dombi fuzzy graphs. Also, we examined the related proposition with suitable illustrations with graphs.

Keywords: Neutrosophic Graphs; Neutrosophic Fuzzy Graph; Neutrosophic Fuzzy Edge Graph; Neutrosophic Dombi Fuzzy Graph; Fermatean Neutrosophic Dombi Fuzzy Graph.

1. Introduction

In 1965, Zadeh [17] developed the idea of a fuzzy set and the term degree of membership to address imprecision. Using the degree of non-membership in the fuzzy set idea as an independent variable, Antanassov [5] proposed intuitionistic fuzzy sets (IFS) in 1983. Florentin Samarandache created a neutrosophic set with a degree of indeterminacy in 2005 [15] by utilizing the concept of intuitionistic fuzzy sets. The neutrosophic sets are defined by truth, indeterminacy, and false membership function.

In many fields, including geometry, number theory, topology, optimization, and computer science, graph theory is utilized to solve combinatorial problems. A graph is made up of nodes and arcs. Fuzzy logic is an extension of classical logic, where each and every item has a different grade of membership. The concept of fuzzy graphs was first introduced and explained by Kaufmann in 1975 [10]. IFS relationships and intuitionistic fuzzy graphs were discussed in 2006 by Shannon andAtanassov [14]. Neutrosophic graphs were first developed by Ghoei and Pal[8], and they are used to model a variety of real-world problems.

The Dombi operator with relevant parameter was inaugurated by Dombi [7] in 1982, and the concept of the Dombi fuzzy graph was developed by Ashraf et al. (2018) [4]. The Dombi operator is crucial in simulating and resolving numerous problems encountered in everyday life. In order to take use of this advantage, Mijanur Rahman Seikh and Utpal Mandal (2021) [11] applied Dombi operations to intuitionistic fuzzy graphs and created a multiple attribute group decision-making problem. The neutrosophic Dombi graph was invented and refined by Tejindarsingh lakhwani, Karthick [16].In addition to proposing Pythagorean neutrosophic fuzzy graphs using the Dombi operator, Ajay et al.[1] presented an innovative concept of a Pythagorean neutrosophic fuzzy graph. In this research, we introduced a new emergent notion of the Fermatean neutrosophic Dombi fuzzy graph using the Dombi operator. the primary consideration, In Section 2, we developed and remembered the fundamental concepts and notions used in this part. In Section 3, we described the new concepts of Fermatean neutrosophic Dombi fuzzy graphs and establish their proposition with relatable illustrations and graphs.

2. Preliminaries

2.1. Definition: Let ϑ_D be a non-empty set. A fuzzy set A^{μ} in $\vartheta_D$is distinguished by its membership function $\alpha_{\mu_D}(\vartheta_D)$:→[0,1] and $\mu_D(n)$ is interpreted as the degree of member of elementn^{μ} in a fuzzy set N^{μ} , for each $n^{\mu} \in \vartheta_D$ .It is clear that N^{μ} is determined by the set of tuples of N^{μ} = {(n^{μ},$\alpha_{\mu_D}(\vartheta_D), n^{\mu} \in \vartheta_D$}.

2.2. Definition: An intuitionistic fuzzy set (briefly IFS) N^{μ} is an object of having the form A^{μ}={<n^{μ^*},$\alpha_{\mu_D}(\vartheta_D), \beta_{\mu_D}(\vartheta_D)$> : $n^{\mu} \in R_{N^{\mu}}$} where the functions $\alpha_{\mu_D}(\vartheta_D)$:→[0,1] and $\beta_{\mu_D}(\vartheta_D)$:→[0,1] denote the degree of membership and the degree non-membership of each element $n^{\mu} \in \vartheta_D$to the set G^* respectively, and 0≤ $\alpha_{\mu_D}(\vartheta_D)$+$\beta_{\mu_D}(\vartheta_D) \leq 1$ for each $n^{\mu} \in \vartheta_D$. Denote by IFS($R_{N^{\mu}}$), The set of all intuitionistic fuzzy sets in ϑ_D. An intuitionistic fuzzy set G^* in ϑ_D is simply denoted by N^{μ}= <n^{μ}, $\alpha_{\mu_D}(\vartheta_D), \beta_{\mu_D}(\vartheta_D)$> instead of denoting A^{μ}= {(n^{μ},$\alpha_{\mu_D}(\vartheta_D), \beta_{\mu_D}(\vartheta_D)$) : $n^{\mu} \in \vartheta_D$}.

2.3. Definition Let ϑ_D be a non-empty set. A Neutrosophic set (NS) A^{μ} in $\vartheta_D$is characterized by a truth-membership function α_{μ_D} , an indeterminacy-membership function β_{μ_D} , and a falsity - membership functionγ_{μ_D} is $\alpha_{\mu_D}(\vartheta_D), \beta_{\mu_D}(\vartheta_D), \gamma_{\mu_D}(\vartheta_D)$ are real or non-standard subsets of] $0^-, 1^+$[on ϑ_D.

i.e.)$\alpha_{\mu_D}(\vartheta_D): \vartheta_D \rightarrow \;]0^-, 1^+[$

$\beta_{\mu_D}(\vartheta_D): \vartheta_D \rightarrow \;]0^-, 1^+[$

$\gamma_{\mu_D}(\vartheta_D): \vartheta_D \rightarrow \;]0^-, 1^+[$

2.4. Definition: A fuzzy graph of the graph D^*= (φ_D, ς_D) is a pair of D =(μ_D , ν_D), Where $\mu_D \rightarrow$ [0,1] is a fuzzy set on φ_D and $\nu_D : \varphi_D X \varphi_D \rightarrow$ [0,1] is a fuzzy relation on φ_D such that $\nu_D\,(n,t) \leq$ μ_D (n)∧ μ_D (t),∀(n,t)∈ $\varphi_D X \varphi_D$.

2.5.Definition: A binary function Ŧ :[0,1]X[0,1] → [0,1] is known as triangular norm (t-norm) if for all n,t,s ∈ [0,1], it satisfied the following conditions

(1) (Neutral property or boundary condition) *Ŧ(n,1) = n*

(2) (commutativity) *Ŧ(n,t) = Ŧ(t,n)*

(3) (associativity) *Ŧ(n,(t,s)) = Ŧ((n,t),s)*

(4) (monotonicity) *Ŧ(n,t) ≤Ŧ(s,d) if* $n \leq s$ *and* $t \leq d$

2.6.Definition: A binary function $\mathcal{M}$: [0,1]X[0,1] → [0,1] is known as triangular conorm (t-conorm) if and only if there exists a t-norm *Ŧ* for all *Ŧ(n,t)* ∈ [0,1]X[0,1]

$\mathcal{M}$(n,t)=1-Ŧ(1-n,1-t)

Preferred options for t-norms are:

- The minimum operator $\mathcal{M}$ *(n,t)=MIN(n,t)*
- The product operator $\mathcal{P}$*(n,t)=nt*
- The dombi's t-norm,$\frac{1}{1+([(\frac{1-n}{n})]^{\delta}+[(\frac{1-t}{t})]^{\delta})^{\frac{1}{\delta}}}$, δ >0

Preferred options for related t-conorms are:

- The maximum operator $\mathcal{M}^*$*(n,t)=MAX (n,t)*

- The Probabilistic sum $\mathrm{P}^*(n,t) = n+t-nt$
- The Dombi's t-conorm $\frac{1}{1+([(\frac{1-n}{n})]^{-\delta}+[(\frac{1-t}{t})]^{-\delta})^{-\frac{1}{\delta}}}$, $\delta > 0$

One more pair of $\mathcal{F}$-operator is $\mathcal{F}(n,t)=\frac{nt}{n+t-nt}$

$\mathcal{P}(n,t)=\frac{n+t-2nt}{1-nt}$,which is obtained by substituting $\delta = 1$ in dombi's t-norm and t-conorm. Also

$\mathcal{P}(n,t) \leq \frac{nt}{n+t-nt} \leq \mathcal{M}(n,t)$ and $\mathcal{M}^*(n,t) \leq \frac{n+t-2nt}{1-nt} \leq \mathrm{P}^*(n,t)$.

2.7. Definition: A dombi fuzzy graph with a countable set φ_D as the elementary set is a pair $D=(\mu_D, \nu_D)$,where $\mu_D \to [0,1]$ is a symmetric fuzzy on φ_D such that $\varsigma_D(nt) \leq \frac{\varphi_D(n)\varphi_D(t)}{\varphi_D(n)+\varphi_D(t)-\varphi_D(n)\varphi_D(t)}$, $\forall\, n,t \in \varphi_D$.

2.8. Definition: Let $D^*=(\varphi_D, \varsigma_D)$ be a crisp undirected graph contain no self loop and parallel edges. Let $\mu_D = (\alpha_{\mu_D}, \beta_{\mu_D}, \gamma_{\mu_D})$ such that $\alpha_{\mu_D}: \vartheta_D \to [0,1]$, $\beta_{\mu_D}: \vartheta_D \to [0,1]$, $\gamma_{\mu_D}: \vartheta_D \to [0,1]$ $\alpha_{\nu_D}: \vartheta_D \to [0,1]$, $\beta_{\nu_D}: \vartheta_D \to [0,1]$, $\gamma_{\nu_D}: \vartheta_D \to [0,1]$ .Here $\alpha_{\mu_D}, \alpha_{\nu_D} \to$ The membership function, $\beta_{\mu_D}, \beta_{\nu_D} \to$ The indeterminacy function, $\gamma_{\mu_D}, \gamma_{\nu_D} \to$ The falsity function in the neutrosophic dombi fuzzy graph, $\varsigma_F \subset \vartheta_D X \vartheta_D$ Then the neutrosophic dombi fuzzy graph, $D = (\varphi_D, \mu_D, \nu_D)$

$\alpha_{\varsigma_D}(nt) \leq \frac{\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}{\alpha_{\varphi_D}(n)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}$, $\forall\, nt \in \varsigma_D$

$\beta_{\varsigma_D}(nt) \leq \frac{\beta_{\mu_D}(n)\beta_{\mu_D}(t)}{\beta_{\mu_D}(n)+\beta_{\mu_D}(t)-\beta_{\mu_D}(n)\beta_{\mu_D}(t)}$, $\forall\, nt \in \varsigma_D$

$\gamma_{\varsigma_D}(nt) \leq \frac{\gamma_{\nu_D}(n)+\gamma_{\nu_D}(t)-2\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}{1-\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}$, $\forall\, nt \in \varsigma_D$

3. Fermatean Neutrosophic Dombi Fuzzy Graph

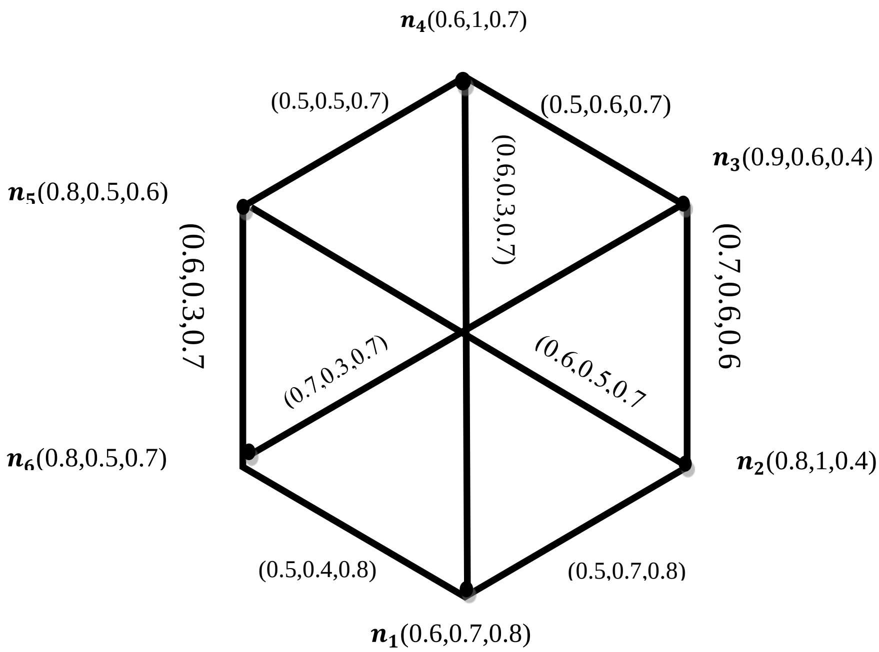
3.1 Definition: A fermatean neutrosophic dombi fuzzy graph is defined [its indicated by *FNDFG*] with a finite elementary set ϑ_D of its order pair $d=(\varphi_D, \varsigma_D)$ where $\varphi_D: \vartheta_D \to [0,1]$.Here we consider, $\mu_D = (\alpha_{\varphi_D}, \beta_{\mu_D}, \gamma_{\nu_D})$ such that $\alpha_{\varphi_D}: \vartheta_D \to [0,1]$, $\beta_{\mu_D}: \vartheta_D \to [0,1]$, $\gamma_{\nu_D}: \vartheta_D \to [0,1]$, $\alpha_{\mu_D}: \vartheta_D \to [0,1]$, $\beta_{\mu_D}: \vartheta_D \to [0,1]$, $\gamma_{\nu_D}: \vartheta_D \to [0,1]$ .Here $\alpha_{\mu_D}, \alpha_{\nu_D} \to$ The membership function, $\beta_{\mu_D}, \beta_{\nu_D} \to$ The indeterminacy function, $\gamma_{\mu_D}, \gamma_{\nu_D} \to$ The falsity function in the Fermatean neutrosophic dombi fuzzy graph, $\varsigma_F \subset \vartheta_D X \vartheta_D$ Then the fermatean neutrosophic dombi fuzzy graph, $d = (\varphi_D, \mu_D, \nu_D)$

$\alpha_{\varsigma_D}(nt) \leq \frac{\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}{\alpha_{\varphi_D}(n)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}$, $\forall\, nt \in \varsigma_D$

$\beta_{\varsigma_D}(nt) \leq \frac{\beta_{\mu_D}(n)\beta_{\mu_D}(t)}{\beta_{\mu_D}(n)+\beta_{\mu_D}(t)-\beta_{\mu_D}(n)\beta_{\mu_D}(t)}$, $\forall\, nt \in \varsigma_D$

$\gamma_{\varsigma_D}(nt) \leq \frac{\gamma_{\nu_D}(n)+\gamma_{\nu_D}(t)-2\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}{1-\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}$, $\forall\, nt \in \varsigma_D$ And $0 \leq \alpha_{\nu_D}{}^3(nt)+\beta_{\nu_D}{}^3(nt)+\gamma_{\nu_D}{}^3(nt) \leq 2$; $0 \leq \alpha_{\nu_D}{}^3(nt)+\gamma_{\nu_D}{}^3(nt) \leq 1$; $0 \leq \beta_{\nu_D}{}^3(nt) \leq 1$.

3.1 Example: Define a graph D=(φ_D, ς_D). Here φ_D ={ $n_1, n_2, n_3, n_4, n_5, n_6$ } and ς_D={$n_1n_2, n_1n_6, n_2n_3, n_2n_5, n_3n_4, n_3n_6, n_4n_5, n_5n_6$}.Let $\emptyset_D$ and δ_D be fermatean neutrosophic dombi fuzzy graph vertex set and fermatean neutrosophic dombi fuzzy edge set specified on φ_D, ς_D respectively.



Fermatean Neutrosophic Dombi Fuzzy graphs

$$\emptyset_D = \begin{bmatrix} \frac{n_1}{0.6}, \frac{n_2}{0.8}, \frac{n_3}{0.9}, \frac{n_4}{0.6}, \frac{n_5}{0.8}, \frac{n_6}{0.8} \\ \frac{n_1}{0.7}, \frac{n_2}{1}, \frac{n_3}{0.6}, \frac{n_4}{1}, \frac{n_5}{0.5}, \frac{n_6}{0.7} \\ \frac{n_1}{0.8}, \frac{n_2}{0.4}, \frac{n_3}{0.4}, \frac{n_4}{0.7}, \frac{n_5}{0.6}, \frac{n_6}{0.7} \end{bmatrix} \quad \varsigma_D = \begin{bmatrix} \frac{n_1n_2}{0.52}, \frac{n_1n_6}{0.52}, \frac{n_2n_3}{0.73}, \frac{n_2n_5}{0.66}, \frac{n_3n_4}{0.56}, \frac{n_3n_6}{0.73}, \frac{n_4n_5}{0.52}, \frac{n_5n_6}{0.66} \\ \frac{n_1n_2}{0.7}, \frac{n_1n_6}{0.41}, \frac{n_2n_3}{0.6}, \frac{n_2n_5}{0.5}, \frac{n_3n_4}{0.6}, \frac{n_3n_6}{0.37}, \frac{n_4n_5}{0.5}, \frac{n_5n_6}{0.33} \\ \frac{n_1n_2}{0.82}, \frac{n_1n_6}{0.86}, \frac{n_2n_3}{0.62}, \frac{n_2n_5}{0.71}, \frac{n_3n_4}{0.75}, \frac{n_3n_6}{0.75}, \frac{n_4n_5}{0.79}, \frac{n_5n_6}{0.79} \end{bmatrix}$$

Definition 3.2: Consider, ζ_D ={nt,$\emptyset_{\zeta_D}(nt)$, $\mu_{\zeta_D}(nt)$,$\nu_{\zeta_D}(nt)$, $nt \in \varsigma_D$} be a fermatean neutrosophic dombi fuzzy edge set in D:

- The order of O_D is established by $\rho(O_D)$ =[$\sum_{n\in\varphi} \emptyset_\varphi$,$\sum_{n\in\varphi} \mu_\varphi$,$\sum_{n\in\varphi} \nu_\varphi$] from illustration 3.1, order of D, $\rho(O_D)$ = [3.4,4.3,2.9]
- The size of O_{s_D} is symbolized by $\rho_S(O_D)$=[$\sum_{n\in\varphi} \emptyset_\varsigma(nt)$, $\sum_{n\in\varphi} \mu_\varsigma(nt)$, $\sum_{n\in\varphi} \nu_\varsigma(nt)$] from illustration 3.1, order of D,$\rho_S(O_D)$ = [5.33,4.71,6.95]
- The degree of vertex n∈ φ_D is denoted by $ɗ_{D_N}$its described by $ɗ_{D_N}$=[$ɗ_{\emptyset_\varphi}(n)$, $ɗ_{\mu_\varphi}(n)$, $ɗ_{\nu_\varphi}(n)$],where

$$ɗ_{\emptyset_\varphi}(n)=\sum_{n,t\neq n\in\varphi} \emptyset_\varsigma(nt) = \sum_{n,t\neq n\in\varphi} \frac{\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}{\alpha_{\varphi_D}(n)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)},$$

$$\sum_{n,t\neq n\in\varphi} \frac{\beta_{\mu_D}(n)\beta_{\mu_D}(t)}{\beta_{\mu_D}(n)+\beta_{\mu_D}(t)-\beta_{\mu_D}(n)\beta_{\mu_D}(t)},$$

$\sum_{n,t\neq n\in\varphi}\frac{\gamma_{\nu_D}(n)+\gamma_{\nu_D}(t)-2\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}{1-\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}$.

From illustration 3.1,$ძ_{D_N}$=$\begin{bmatrix}\frac{n_1}{1.45},\frac{n_2}{1.91},\frac{n_3}{2.023},\frac{n_4}{1.513},\frac{n_5}{1.84},\frac{n_6}{1.91}\\ \frac{n_1}{1.81},\frac{n_2}{1.8},\frac{n_3}{1.575},\frac{n_4}{1.8},\frac{n_5}{1.33},\frac{n_6}{1.16}\\ \frac{n_1}{2.54},\frac{n_2}{2.155},\frac{n_3}{2.125},\frac{n_4}{2.4},\frac{n_5}{2.29},\frac{n_6}{2.4}\end{bmatrix}$

- The Total degree of vertex n $\in\varphi_D$ is specified by T[$ძ_{D_N}$]its explained by T[$ძ_{D_N}$]=[$ძ_{\emptyset_\varphi}(n), ძ_{\mu_\varphi}(n), ძ_{\nu_\varphi}(n)$],where
- $[\text{T}ძ]_{\emptyset_\varphi}(n)$=$\sum_{n,t\neq n\in\varphi}\emptyset_\varsigma(nt)$=$\sum_{n,t\neq n\in\varphi}\frac{\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}{\alpha_{\varphi_D}(n)+\alpha_{\varphi_D}(t)-\alpha_{\varphi_D}(n)\alpha_{\varphi_D}(t)}+\emptyset_{\varphi_D}(n)$,

$\sum_{n,t\neq n\in\varphi}\emptyset_\varsigma(nt)$ =$\sum_{n,t\neq n\in\varphi}\frac{\beta_{\mu_D}(n)\beta_{\mu_D}(t)}{\beta_{\mu_D}(n)+\beta_{\mu_D}(t)-\beta_{\mu_D}(n)\beta_{\mu_D}(t)}$+$\emptyset_{\varphi\mu_D}(n)$,

$\sum_{n,t\neq n\in\varphi}\frac{\gamma_{\nu_D}(n)+\gamma_{\nu_D}(t)-2\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}{1-\gamma_{\nu_D}(n)\gamma_{\nu_D}(t)}$+$\emptyset_{\nu_D}(n)$

from illustration 3.1, $[Tძ]_{D_N}$=$\begin{bmatrix}\frac{n_1}{2.05},\frac{n_2}{2.71},\frac{n_3}{2.723},\frac{n_4}{2.413},\frac{n_5}{2.64},\frac{n_6}{2.8}\\ \frac{n_1}{2.51},\frac{n_2}{2.8},\frac{n_3}{2.375},\frac{n_4}{2.3},\frac{n_5}{1.83},\frac{n_6}{2.99}\\ \frac{n_1}{2.94},\frac{n_2}{2.65},\frac{n_3}{2.725},\frac{n_4}{2.8},\frac{n_5}{2.11},\frac{n_6}{3.1}\end{bmatrix}$

Definition 3.3: Let d = $(\emptyset_D,\mu_{D,}\nu_D)$ and F = $(\emptyset_F,\mu_{F,}\nu_F)$ of the graphs D^*=(φ_D,ς_D) and F^*=(φ_F,ς_F) respectively and its established by the union of two fermatean neutrosophic dombi fuzzy graphare $D\cup$F= $(\emptyset_D\cup\emptyset_F,\mu_D\cup\mu_F,\nu_D\cup\nu_F)$.
$\emptyset_D\cup\emptyset_F$=$[\alpha_{\emptyset_D\cup\emptyset_F},\beta_{\emptyset_D\cup\emptyset_F},\gamma_{\emptyset_D\cup\emptyset_F}]$
$\mu_D\cup\mu_F$=$[\alpha_{\mu_D\cup\mu_F},\beta_{\mu_D\cup\mu_F},\gamma_{\mu_D\cup\mu_F}]$
$\nu_D\cup\nu_F$=$[\alpha_{\nu_D\cup\nu_F},\beta_{\nu_D\cup\nu_F},\gamma_{\nu_D\cup\nu_F}]$ such that

$$[\alpha_{\mu_D\cup\mu_F}](\chi)=\begin{cases}[\alpha_{\mu_D}](\chi),\text{ if }\chi\in\varphi_D-\varphi_F\\ [\alpha_{\mu_F}](\chi),\text{ if }\chi\in\varphi_F-\varphi_D\\ \frac{\alpha_{\mu_D}(\chi)+\alpha_{\mu_D}(\chi)-2\alpha_{\mu_D}(\chi)\alpha_{\mu_D}(\chi)}{1-\alpha_{\mu_D}(\chi)\alpha_{\mu_D}(\chi)},\text{ if }\chi\in\varphi_D\cap\varphi_F\end{cases}$$

$$[\beta_{\mu_D\cup\mu_F}](\chi)=\begin{cases}[\beta_{\mu_D}](\chi),\text{ if }\chi\in\varphi_D-\varphi_F\\ [\beta_{\mu_F}](\chi),\text{ if }\chi\in\varphi_F-\varphi_D\\ \frac{\beta_{\mu_D}(\chi)+\beta_{\mu_D}(\chi)-2\beta_{\mu_D}(\chi)\beta_{\mu_D}(\chi)}{1-\beta_{\mu_D}(\chi)\beta_{\mu_D}(\chi)},\text{ if }\chi\in\varphi_D\cap\varphi_F\end{cases}$$

$$[\gamma_{\mu_D\cup\mu_F}](\chi)=\begin{cases}[\gamma_{\mu_D}](\chi),\text{ if }\chi\in\varphi_D-\varphi_F\\ [\gamma_{\mu_F}](\chi),\text{ if }\chi\in\varphi_F-\varphi_D\\ \frac{\gamma_{\mu_D}(\chi)\gamma_{\mu_D}(\chi)}{\gamma_{\mu_D}(\chi)+\gamma_{\mu_D}(\chi)-\gamma_{\mu_D}(\chi)\gamma_{\mu_D}(\chi)},\text{ if }\chi\in\varphi_D\cap\varphi_F\end{cases}$$

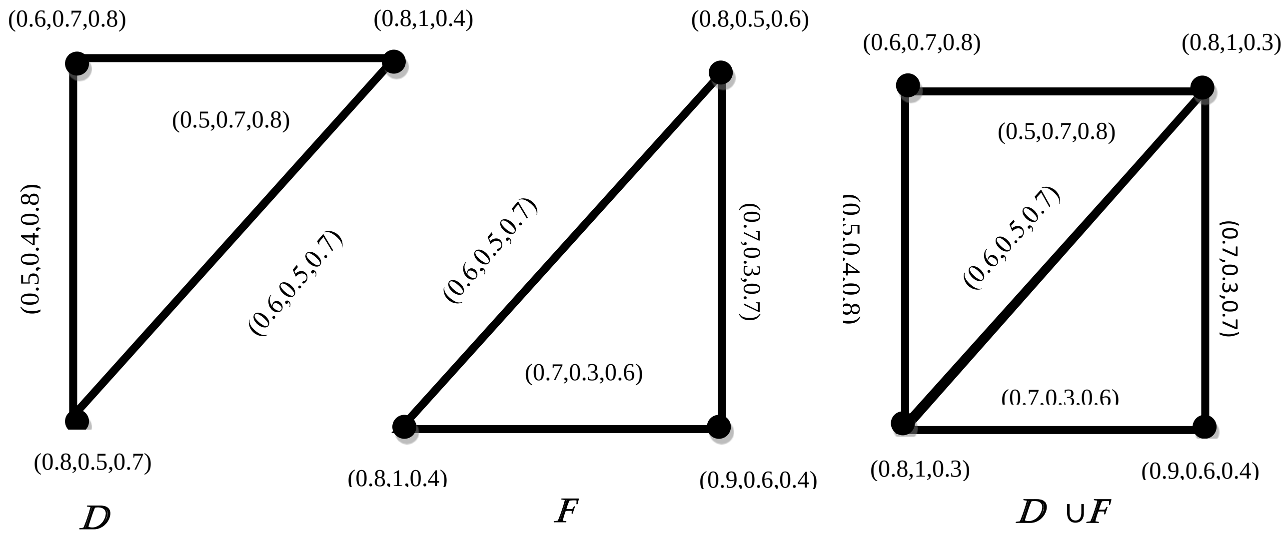
$$[\alpha_{\nu_D\cup\nu_F}](\text{nt})=\begin{cases}[\alpha_{\mu_D}](\text{nt}),\text{ if nt}\in\varsigma_D-\varsigma_F\\ [\alpha_{\mu_F}](\text{nt}),\text{ if nt}\in\varsigma_F-\varsigma_D\\ \frac{\alpha_{\nu_D}(nt)+\alpha_{\nu_D}(nt)-2\alpha_{\nu_D}(nt)\alpha_{\nu_D}(nt)}{1-\alpha_{\nu_D}(nt)\alpha_{\nu_D}(nt)},\text{ if nt}\in\varsigma_D\cap\varsigma_F\end{cases}$$

$$[\beta_{\nu_D \cup \nu_F}](\mathrm{nt}) = \begin{cases} [\beta_{\mu_D}](\mathrm{nt}), \text{if nt} \in \varsigma_D - \varsigma_F \\ [\beta_{\mu_F}](\mathrm{nt}), \text{if nt} \in \varsigma_F - \varsigma_D \\ \frac{\beta_{\nu_D}(nt)+\beta_{\nu_D}(nt)-2\beta_{\nu_D}(nt)\beta_{\nu_D}(nt)}{1-\beta_{\nu_D}(nt)\beta_{\nu_D}(nt)}, \text{if nt} \in \varsigma_D \cap \varsigma_F \end{cases}$$

$$[\gamma_{\nu_D \cup \nu_F}](\mathrm{nt}) = \begin{cases} [\gamma_{\mu_D}](\mathrm{nt}), \text{if nt} \in \varsigma_D - \varsigma_F \\ [\gamma_{\mu_F}](\mathrm{nt}), \text{if nt} \in \varsigma_F - \varsigma_D \\ \frac{\gamma_{\nu_D}(nt)\gamma_{\nu_D}(nt)}{\gamma_{\nu_D}(nt)+\gamma_{\nu_D}(nt)-\gamma_{\nu_D}(nt)\gamma_{\nu_D}(nt)}, \text{if nt} \in \varsigma_D \cap \varsigma_F \end{cases}$$

Where $0 \leq \alpha_{\nu_D}{}^3(\mathrm{nt})+\beta_{\nu_D}{}^3(\mathrm{nt})+\gamma_{\nu_D}{}^3(\mathrm{nt}) \leq 2$

Example 3.3: Let us consider $D = (\emptyset_D, \mu_D, \nu_D)$ and $\boldsymbol{F} = (\emptyset_F, \mu_F, \nu_F)$ $D^* = (\varphi_D, \varsigma_D)$ and $F^* = (\varphi_F, \varsigma_F)$ separately. Here φ_D = { a,b,c}, ς_F ={ab,bc,ca}, φ_F={b,c,d} ,ς_F={bc,cd,da}.Then the union of two neutrosophic dombi fuzzy graphs,$\varphi_D \cup \varphi_F$ is



Definition 3.4: Let $D = (\emptyset_D, \mu_D, \nu_D)$ and $\boldsymbol{F} = (\emptyset_F, \mu_F, \nu_F)$ of the graphs $D^* = (\varphi_D, \varsigma_D)$ and $F^* = (\varphi_F, \varsigma_F)$ respectively and its established by the intersection of two fermatean neutrosophic dombi fuzzy graph $d \cap F$ *is*

$d \cap F = (\emptyset_D \cap \emptyset_F, \mu_D \cap \mu_F, \nu_D \cap \nu_F)$.Here,

$\emptyset_D \cap \emptyset_F = [\alpha_{\emptyset_D \cap \emptyset_F}, \beta_{\emptyset_D \cap \emptyset_F}, \gamma_{\emptyset_D \cap \emptyset_F}]$

$\mu_D \cap \mu_F = [\alpha_{\mu_D \cap \mu_F}, \beta_{\mu_D \cap \mu_F}, \gamma_{\mu_D \cap \mu_F}]$

$\nu_D \cap \nu_F = [\alpha_{\nu_D \cap \nu_F}, \beta_{\nu_D \cap \nu_F}, \gamma_{\nu_D \cap \nu_F}]$ such that

- $[\alpha_{\mu_D \cap \mu_F}](\chi) = \frac{\alpha_{\mu_D}(\chi)+\alpha_{\mu_D}(\chi)-2\alpha_{\mu_D}(\chi)\alpha_{\mu_D}(\chi)}{1-\alpha_{\mu_D}(\chi)\alpha_{\mu_D}(\chi)}$, if $\chi \in \varphi_D \cap \varphi_F$

$$[\beta_{\mu_D \cap \mu_F}](\chi) = \frac{\beta_{\mu_D}(\chi)+\beta_{\mu_D}(\chi)-2\beta_{\mu_D}(\chi)\beta_{\mu_D}(\chi)}{1-\beta_{\mu_D}(\chi)\beta_{\mu_D}(\chi)}, \text{if } \chi \in \varphi_D \cap \varphi_F$$

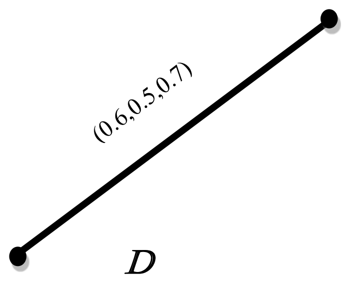
$$[\gamma_{\mu_D \cap \mu_F}](\chi) = \frac{\gamma_{\mu_D}(\chi)\gamma_{\mu_D}(\chi)}{\gamma_{\mu_D}(\chi)+\gamma_{\mu_D}(\chi)-\gamma_{\mu_D}(\chi)\gamma_{\mu_D}(\chi)}, \text{if } \chi \in \varphi_D \cap \varphi_F$$

- $[\alpha_{\nu_D \cap \nu_F}](nt) = \frac{\alpha_{\nu_D}(nt)+\alpha_{\nu_D}(nt)-2\alpha_{\nu_D}(nt)\alpha_{\nu_D}(nt)}{1-\alpha_{\nu_D}(nt)\alpha_{\nu_D}(nt)}$, if nt $\in \varsigma_D \cap \varsigma_F$

  $[\beta_{\nu_D \cap \nu_F}](nt) = \frac{\beta_{\nu_D}(nt)+\beta_{\nu_D}(nt)-2\beta_{\nu_D}(nt)\beta_{\nu_D}(nt)}{1-\beta_{\nu_D}(nt)\beta_{\nu_D}(nt)}$, if nt $\in \varsigma_D \cap \varsigma_F$

  $[\gamma_{\nu_D \cap \nu_F}](nt) = \frac{\gamma_{\nu_D}(nt)\gamma_{\nu_D}(nt)}{\gamma_{\nu_D}(nt)+\gamma_{\nu_D}(nt)-\gamma_{\nu_D}(nt)\gamma_{\nu_D}(nt)}$, if nt $\in \varsigma_D \cap \varsigma_F$

Example 3.4: Let us consider the two neutrosophic dombi fuzzy graphs $D = (\emptyset_D, \mu_{D,} \nu_D)$ and F=$(\emptyset_F, \mu_{F,} \nu_F)$D*= (φ_D, ς_D) and F^*=(φ_F, ς_F) separately. Here φ_D = { a,b,c}, ς_F ={ab,bc,ca}, φ_F={b,c,d} ,ς_F={bc,cd,da}.Then the intersection of $\varphi_D \cap \varphi_F$ is



Definition 3.5:[Direct product] Let $\emptyset_i$ be a fermatean neutrosophic fuzzy subset of φ_D and ς_i be a neutrosophic fuzzy subset $\varsigma_D$of ς_i=1,2,3.The direct product of fermatean neutrosophic dombi fuzzy graph. $d = (\emptyset_D, \mu_{D,} \nu_D)$ and $F = (\emptyset_F, \mu_{F,} \nu_F)$ D*= (φ_D, ς_D) and F^*=(φ_F, ς_F) separately is established by dXF is

$d\,XF = (\emptyset_D X \emptyset_F, \mu_D X \mu_F, \nu_D X \nu_F)$.

(i.e) $\emptyset_D X \emptyset_F$=$[\alpha_{\emptyset_D X \emptyset_F}, \beta_{\emptyset_D X \emptyset_F}, \gamma_{\emptyset_D X \emptyset_F}]$

$\mu_D X \mu_F$=$[\alpha_{\mu_D X \mu_F}, \beta_{\mu_D X \mu_F}, \gamma_{\mu_D X \mu_F}]$

$\nu_D X \nu_F$=$[\alpha_{\nu_D X \nu_F}, \beta_{\nu_D X \nu_F}, \gamma_{\nu_D X \nu_F}]$ such that

$\xi_{D\,XF}$ = $\{(n, n_2), (n, t_2)$: $x \in \varphi_D, n_2 t_2 \in \varsigma_F\} \cup \{(n_1, z), (n_2, z)$: $n_1 n_2 \in \varsigma_D, z \in \varphi_F\}$ such that,

for all $(n_1, n_2) \in \varphi_D$xφ_F ,

- $\alpha_{\emptyset_D X \emptyset_F}(n_1, n_2) = \frac{\alpha_{\emptyset_D}(n_1)\alpha_{\emptyset_F}(n_2)}{\alpha_{\emptyset_D}(n_1)+\alpha_{\emptyset_F}(n_2)-\alpha_{\emptyset_D}(n_1)\alpha_{\emptyset_F}(n_2)}$

  $\beta_{\mu_D X \mu_F}(n_1, n_2) = \frac{\beta_{\mu_D}(n_1)\beta_{\mu_F}(n_2)}{\beta_{\mu_D}(n_1)+\beta_{\mu_F}(n_2)-\beta_{\mu_D}(\chi)\beta_{\mu_F}(n_2)}$

  $\gamma_{\nu_D X \nu_F}(n_1, n_2) = \frac{\gamma_{\nu_D}(n_1)+\gamma_{\nu_F}(n_2)-2\gamma_{\nu_D}(n_1)\gamma_{\nu_F}(n_2)}{\gamma_{\nu_D}(n_1)\gamma_{\nu_F}(n_2)}$

$(\varsigma_D x \varsigma_F)(n_1, n_2)(t_1, t_2) = \frac{\varsigma\alpha_{\emptyset_D}(n_1 t_1)\,\varsigma\alpha_{\emptyset_F}(n_2 t_2)}{\varsigma\alpha_{\emptyset_D}(n_1 t_1)+\varsigma\alpha_{\emptyset_F}(n_2 t_2)-\varsigma\alpha_{\emptyset_D}(n_1 t_1)\,\varsigma\alpha_{\emptyset_F}(n_2 t_2)}$,

$\frac{\varsigma\beta_{\mu_D}(n_1 t_1)\,\varsigma\beta_{\mu_F}(n_2 t_2)}{\varsigma\beta_{\mu_D}(n_1 t_1)+\varsigma\beta_{\mu_F}(n_2 t_2)-\varsigma\beta_{\mu_D}(n_1 t_1)\,\varsigma\beta_{\mu_F}(n_2 t_2)}$,

$\frac{\varsigma\gamma_{\nu_D}(n_1 t_1)\,\varsigma\gamma_{\nu_F}(n_2 t_2)-\varsigma\gamma_{\nu_D}(n_1 t_1)\,\varsigma\gamma_{\nu_F}(n_2 t_2)}{1-\varsigma\gamma_{\nu_D}(n_1 t_1)\,\varsigma\gamma_{\nu_F}(n_2 t_2)}$.

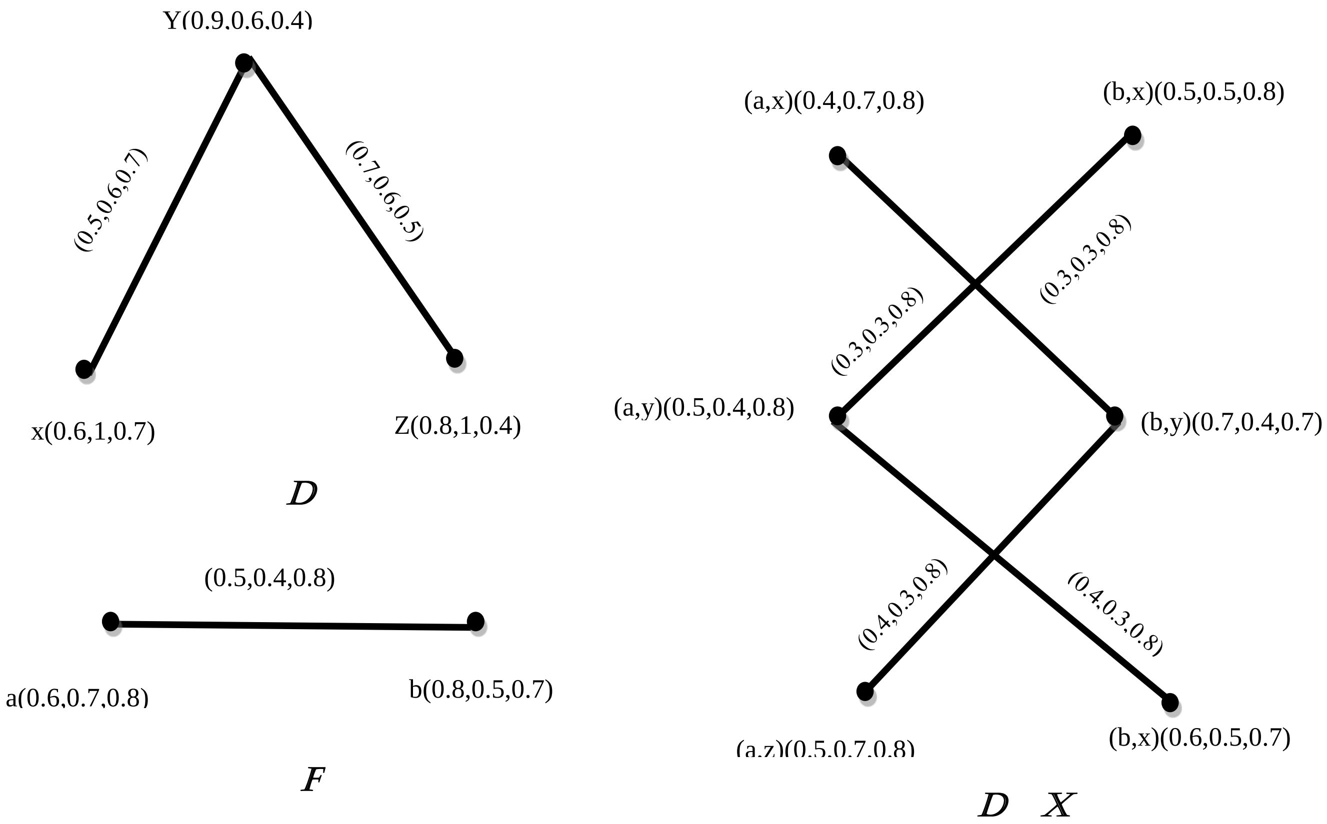
For all $n_1t_1 \in \varsigma_D, n_2t_2 \in \varsigma_F$

Example 3.5: Consider two neutrosophic dombi fuzzy graphs $d = (\emptyset_D, \mu_{D,} \nu_D)$ and $F = (\emptyset_F, \mu_{F,} \nu_F)$.Here φ_D={a,b},φ_F={x,y,z},$\varsigma_D = \{ab\}$, ς_F={xy,yz} where

$\varphi_D = \left\{\frac{a}{(0.6,0.7,0.8)}, \frac{b}{(0.8,0.5,0.7)}\right\}$,$\varphi_F$={$\frac{x}{(0.6,1,0.7)}, \frac{y}{(0.9,0.6,0.4)}, \frac{z}{(0.8,1,0.4)}$},$\varsigma_D = \left\{\frac{ab}{(0.5,0.4,0.8)}\right\}$,

ς_F={$\frac{xy}{(0.5,0.6,0.7)}, \frac{yz}{(0.7,0.6,0.5)}$ }. Then we have

$(\varphi_D x \varphi_F)$(a,b) =(0.5,0.4,0.8), $(\varphi_D x \varphi_F)$(x,y)=(0.5,0.6,0.7) , $(\varphi_D x \varphi_F)$(y,z)=(0.7,0.6,0.5),
$(\varphi_D x \varphi_F)$(a,x)=(0.4,0.7,0.8), $(\varphi_D x \varphi_F)$(a,y)=(0.5,0.4,0.8), $(\varphi_D x \varphi_F)$(a,z)=(0.5,0.7,0.8)
$(\varphi_D x \varphi_F)$(b,x)=(0.5,0.5,0.8), $(\varphi_D x \varphi_F)$(b,y)=(0.7,0.4,0.7), $(\varphi_D x \varphi_F)$(b,z)=(0.6,0.5,0.7)
$(\varsigma_D x \varsigma_F)(a,x)(b,y)$=(0.4,0.3,0.8),$(\varsigma_D x \varsigma_F)(a,y)(b,z) = (0.3,0.3,0.8)$,
$(\varsigma_D x \varsigma_F)(b,x)(a,y)$=(0.4,0.3,0.8), $(\varsigma_D x \varsigma_F)(b,y)(b,z)$=(0.3,0.3,0.8).



Theorem 3.1: The direct product of two fermatean neutrosophic dombi fuzzy graph is a fermatean neutrosophic dombi fuzzy graph.

Proof:

Let d and F be the fermatean neutrosophic dombi fuzzy graphs $d = (\emptyset_D, \mu_{D,} \nu_D)$ and $F = (\emptyset_F, \mu_{F,} \nu_F)$ respectively .

Consider,for all $n_1t_1 \in \varsigma_D, n_2t_2 \in \varsigma_F$ such that

$(\varsigma_D \text{x} \varsigma_F)(n_1,n_2)(t_1,t_2)$

The direct product $d\ XF$= T[$\varsigma_D(n_1t_1)$,$\varsigma_F(n_2t_2)$]

$$\leq \mathrm{T}\begin{bmatrix} \frac{\alpha_{\emptyset_D}(n_1)\alpha_{\emptyset_D}(n_2)}{\alpha_{\emptyset_D}(n_1)+\alpha_{\emptyset_D}(n_2)-\alpha_{\emptyset_D}(n_1)\alpha_{\emptyset_D}(n_2)}, \\ \frac{\beta_{\mu_D}(n_1)\beta_{\mu_D}(n_2)}{\beta_{\mu_D}(n_1)+\beta_{\mu_D}(n_2)-\beta_{\mu_D}(n_2)\beta_{\mu_D}(n_2)}, \\ \frac{\gamma_{\nu_D}(n_1)+\gamma_{\nu_D}(n_2)-2\gamma_{\nu_D}(n_1)\gamma_{\nu_D}(n_2)}{\gamma_{\nu_D}(n_1)\gamma_{\nu_D}(n_2)} \end{bmatrix}$$

$(\varsigma_D \text{x} \varsigma_F)(n_1,n_2)(t_1,t_2)$

$$\leq \begin{cases} \frac{(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_D}(n_1n_2)(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_F}(t_1t_2)}{(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_D}(n_1n_2)+(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_F}(t_1t_2)-(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_D}(n_1n_2)(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_F}(t_1t_2)}, \\ \frac{(\varphi_D \text{x} \varphi_F)\beta_{\mu_D}(n_1n_2)(\varphi_D \text{x} \varphi_F)\beta_{\mu_F}(t_1t_2)}{(\varphi_D \text{x} \varphi_F \beta_{\mu_D}(n_1n_2)+(\varphi_D \text{x} \varphi_F)\beta_{\mu_F}(t_1t_2)-(\varphi_D \text{x} \varphi_F)\beta_{\mu_D}(n_1n_2)(\varphi_D \text{x} \varphi_F)\beta_{\mu_F}(t_1t_2)}, \\ \frac{(\varphi_D \text{x} \varphi_F)\gamma_{\nu_D}(n_1n_2)(+\varphi_D \text{x} \varphi_F)\gamma_{\nu_F}(t_1t_2)-2(\varphi_D \text{x} \varphi_F)\gamma_{\nu_D}(n_1n_2)(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_F}(t_1t_2)}{1-(\varphi_D \text{x} \varphi_F)\gamma_{\nu_D}(n_1n_2)(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_F}(t_1t_2)} \end{cases}$$

Corollary 3.2: The product of two fermatean neutrosophic dombi fuzzy graph is the fermatean neutrosophic dombi fuzzy graph

From the illustration 3.5

$(\varphi_D \text{x} \varphi_F)$(a,x)=(0.4,0.7,0.8), $(\varphi_D \text{x} \varphi_F)$(b,y)=(0.7,0.4,0.7),$(\varsigma_D \text{x} \varsigma_F)(a,y)(b,z) = (0.3,0.3,0.8)$

$(\varsigma_D \text{x} \varsigma_F)(a,y)(b,z) = (0.3,0.3,0.8)$

$$= \begin{cases} \frac{(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_D}(n_1n_2)(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_F}(t_1t_2)}{(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_D}(n_1n_2)+(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_F}(t_1t_2)-(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_D}(n_1n_2)(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_F}(t_1t_2)}, \\ \frac{(\varphi_D \text{x} \varphi_F)\beta_{\mu_D}(n_1n_2)(\varphi_D \text{x} \varphi_F)\beta_{\mu_F}(t_1t_2)}{(\varphi_D \text{x} \varphi_F \beta_{\mu_D}(n_1n_2)+(\varphi_D \text{x} \varphi_F)\beta_{\mu_F}(t_1t_2)-(\varphi_D \text{x} \varphi_F)\beta_{\mu_D}(n_1n_2)(\varphi_D \text{x} \varphi_F)\beta_{\mu_F}(t_1t_2)}, \\ \frac{(\varphi_D \text{x} \varphi_F)\gamma_{\nu_D}(n_1n_2)(+\varphi_D \text{x} \varphi_F)\gamma_{\nu_F}(t_1t_2)-2(\varphi_D \text{x} \varphi_F)\gamma_{\nu_D}(n_1n_2)(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_F}(t_1t_2)}{1-(\varphi_D \text{x} \varphi_F)\gamma_{\nu_D}(n_1n_2)(\varphi_D \text{x} \varphi_F)\alpha_{\emptyset_F}(t_1t_2)} \end{cases}$$

=(0.34,0.34,0.86)

Therefore the product of two fermatean neutrosophic dombi fuzzy graph is fermatean neutrosophic dombi fuzzy graph.

Definition 3.6:[Cartesian product] Let $\emptyset_i$ be a fermatean neutrosophic fuzzy subset of φ_D and ς_i be a neutrosophic fuzzy subset $\varsigma_D$of ς_i=1,2,3.The cartesian product of fermatean neutrosophic dombi fuzzy graph.$\boldsymbol{d}$ = $(\emptyset_D, \mu_{D,}\nu_D)$ and $\boldsymbol{F}$ = $(\emptyset_F, \mu_{F,}\nu_F)$ D*= (φ_D, ς_D) and F^*=(φ_F, ς_F) separately is established by $d \square F$ is defined as

$d \square F$= $(\emptyset_D \square \emptyset_F, \mu_D \square \mu_F, \nu_D \square \nu_F)$.

(i.e)$\emptyset_D \square \emptyset_F$=$[\alpha_{\emptyset_D \square \emptyset_F}, \beta_{\emptyset_D \square \emptyset_F}, \gamma_{\emptyset_D \square \emptyset_F}]$

$\mu_D \square \mu_F$=$[\alpha_{\mu_D \square \mu_F}, \beta_{\mu_D \square \mu_F}, \gamma_{\mu_D \square \mu_F}]$

$\nu_D \square \nu_F$=$[\alpha_{\nu_D \square \nu_F}, \beta_{\nu_D \square \nu_F}, \gamma_{\nu_D \square \nu_F}]$

$\xi_{D\ \square F}$ = $\{(n,n_2),(n,t_2)\text{: } x \in \varphi_D, n_2t_2 \in \varsigma_F\} \cup \{(n_1,z),(n_2,z)\ :\ n_1n_2 \in \varsigma_D, z \in \varphi_F\} \cup$

$\{(n_1,n_2)(t_1,t_2)\text{: } n_1n_2 \in \varsigma_D,, n_2 \neq t_2\}$

For all $(n_1,n_2) \in \varphi_D \text{x} \varphi_F$ such that,

$$i)\ \varphi_D \square \varphi_F(n_1, n_2) = \begin{cases} \frac{\alpha_{\emptyset_D}(n_1)\alpha_{\emptyset_F}(n_2)}{\alpha_{\emptyset_D}(n_1)+\alpha_{\emptyset_F}(n_2)-\alpha_{\emptyset_D}(n_1)\alpha_{\emptyset_F}(n_2)}, \\ \frac{\beta_{\mu_D}(n_1)\beta_{\mu_F}(n_2)}{\beta_{\mu_D}(n_1)+\beta_{\mu_F}(n_2)-\beta_{\mu_D}(n_1)\beta_{\mu_F}(n_2)}, \\ \frac{\gamma_{\nu_D}(n_1)+\gamma_{\nu_F}(n_2)-2\gamma_{\nu_D}(n_1)\gamma_{\nu_F}(n_2)}{\gamma_{\nu_D}(n_1)\gamma_{\nu_F}(n_2)} \end{cases}$$

$$ii)\ (\varsigma_D \square \varsigma_F)\ (\mathrm{n}, n_2)(t, t_2) = \begin{cases} \frac{\varphi_{\alpha_{\emptyset_D}}(n)\,\varsigma_{\alpha_{\emptyset_F}}(n_2t_2)}{\varphi_{\alpha_{\emptyset_D}}(n)+\varsigma_{\alpha_{\emptyset_F}}(n_2t_2)-\varphi_{\alpha_{\emptyset_D}}(n)\,\varsigma_{\alpha_{\emptyset_F}}(n_2t_2)}, \\ \frac{\varphi_{\beta_{\mu_D}}(n)\,\varsigma_{\beta_{\mu_F}}(n_2t_2)}{\varphi_{\beta_{\mu_D}}(n)+\varsigma_{\beta_{\mu_F}}(n_2t_2)-\varphi_{\beta_{\mu_D}}(n)\,\varsigma_{\beta_{\mu_F}}(n_2t_2)}, \\ \frac{\varphi_{\gamma_{\nu_D}}(n)\,\varsigma_{\gamma_{\nu_F}}(n_2t_2)-\varphi_{\gamma_{\nu_D}}(n)\,\varsigma_{\gamma_{\nu_F}}(n_2t_2)}{1-\varphi_{\gamma_{\nu_D}}(n)\,\varsigma_{\gamma_{\nu_F}}(n_2t_2)} \end{cases}$$

For all $n \in \varphi_D$ and $n_2t_2 \in \varsigma_F$

$$(\varsigma_D \square \varsigma_F)\ (n_1, n_2)(t_1, t_2) = \begin{cases} \frac{\varsigma_{\alpha_{\emptyset_D}}(n_1t_1)\,\varsigma_{\alpha_{\emptyset_F}}(n_2t_2)}{\varsigma_{\alpha_{\emptyset_D}}(n_1t_1)+\varsigma_{\alpha_{\emptyset_F}}(n_2t_2)-\varsigma_{\alpha_{\emptyset_D}}(n_1t_1)\,\varsigma_{\alpha_{\emptyset_F}}(n_2t_2)}, \\ \frac{\varsigma_{\beta_{\mu_D}}(n_1t_1)\,\varsigma_{\beta_{\mu_F}}(n_2t_2)}{\varsigma_{\beta_{\mu_D}}(n_1t_1)+\varsigma_{\beta_{\mu_F}}(n_2t_2)-\varsigma_{\beta_{\mu_D}}(n_1t_1)\,\varsigma_{\beta_{\mu_F}}(n_2t_2)}, \\ \frac{\varsigma_{\gamma_{\nu_D}}(n_1t_1)\,\varsigma_{\gamma_{\nu_F}}(n_2t_2)-\varsigma_{\gamma_{\nu_D}}(n_1t_1)\,\varsigma_{\gamma_{\nu_F}}(n_2t_2)}{1-\varsigma_{\gamma_{\nu_D}}(n_1t_1)\,\varsigma_{\gamma_{\nu_F}}(n_2t_2)} \end{cases}$$

For all $n_1t_1 \in \varsigma_D, n_2t_2 \in \varsigma_F$

Example 3.5: Consider two Fermatean neutrosophic dombi fuzzy graphs $d = (\emptyset_D, \mu_{D,}\nu_D)$ and $F = (\emptyset_F, \mu_{F,}\nu_F)$.Here φ_D={x,y,z},φ_F={a,b},ς_D = {xy, yz }, ς_F= $\{ab\}$where

$$\varphi_D = \{\frac{x}{(0.7,0.6,0.5)}, \frac{y}{(0.8,0.7,0.5)}, \frac{z}{(0.9,1,0.6)}\}, \varphi_F = \left\{\frac{a}{(0.7,0.8,0.5)}, \frac{b}{(0.8,0.7,0.6)}\right\},\ \varsigma_F = \left\{\frac{ab}{(0.6,0.5,0.7)}\right\}$$

$\varsigma_D = \left\{\frac{xy}{(0.6,0.5,0.7)}, \frac{yz}{(0.7,0.7,0.7)}\right\}$. Then we have

$\varphi_D \square \varphi_F$(x,y) = (0.6,0.5,0.7), $\varphi_D \square \varphi_F(y,z) = (0.7,0.7,0.7)$, $\varphi_D \square \varphi_F(a,b) = ((0.6,0.5,0.7)$,

$\varphi_D \square \varphi_F(x,a) = (0.5,0.5,0.7)$, $\varphi_D \square \varphi_F(x,b) = (0.6,0.5,0.7)$, $\varphi_D \square \varphi_F(y,a) = (0.6,0.6,0.7)$,

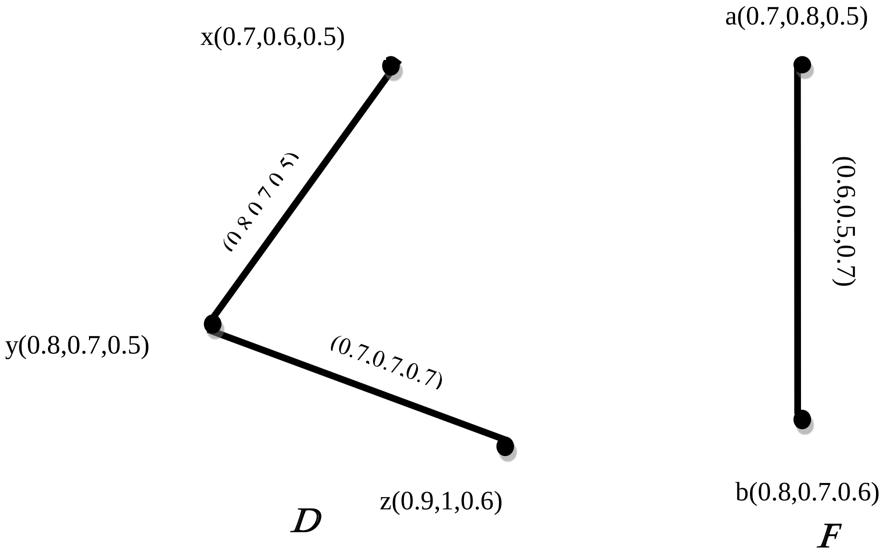
$\varphi_D \square \varphi_F(y,b) = (0.6,0.5,0.7)$, $\varphi_D \square \varphi_F(z,a) = (0.6,0.8,0.7)$, $\varphi_D \square \varphi_F(z,b) = (0.7,0.7,0.8)$,

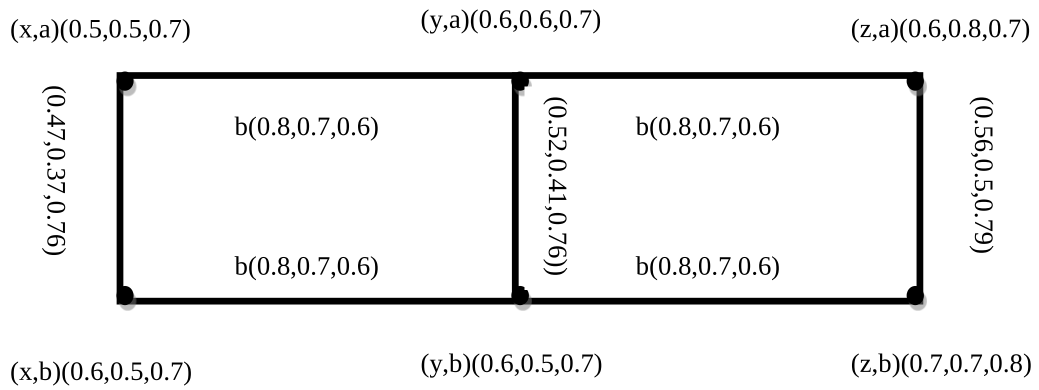
$(\varsigma_D \square \varsigma_F)$(x,a)(x,b) = (0.47,0.37,0.76),$(\varsigma_D \square \varsigma_F)$(y,a)(y,b) = (0.52,0.41,0.76)

$(\varsigma_D \square \varsigma_F)(z,a)(z,b) = (0.56,0.5,0.79)$,$(\varsigma_D \square \varsigma_F)$ (x,a)(y,a)=(0.47,0.44,0.76),

$(\varsigma_D \square \varsigma_F)$(y,a)(z,a)=(0.53,0.6,0.76),$(\varsigma_D \square \varsigma_F)$(x,b)(y,b)=(0.52,0.41,0.79)

$(\varsigma_D \square \varsigma_F)$(y,b)(z,b)=(0.6,0.53,0.79).





D□F

Corollary 3.3: The Cartesian product of two fermatean neutrosophic dombi fuzzy graph is not necessiarily to a fermatean neutrosophic dombi fuzzy graph.

From the graph *d* □ *F*,

$(\varsigma_D \square \varsigma_F)$(y,a)(y,b)=(0.52,0.41,0.76)

$(\varsigma_D \square \varsigma_F)$(y,a)(y,b)

$$==\begin{cases} \frac{(\varphi_D\square\varphi_F)\alpha_{\emptyset_D}(y,a)(\varphi_D\square\varphi_F)\alpha_{\emptyset_F}(y,b)}{(\varphi_D\square\varphi_F)\alpha_{\emptyset_D}(y,a)+(\varphi_D\square\varphi_F)\alpha_{\emptyset_F}(y,b)-(\varphi_D\square\varphi_F)\alpha_{\emptyset_D}(y,a)(\varphi_D\square\varphi_F)\alpha_{\emptyset_F}(y,b)}, \\ \frac{(\varphi_D\square\varphi_F)\beta_{\mu_D}(y,a)(\varphi_D\square\varphi_F)\beta_{\mu_F}(y,b)}{(\varphi_D\square\varphi_F)\beta_{\mu_D}(y,a)+(\varphi_D\square\varphi_F)\beta_{\mu_F}(y,b)-(\varphi_D\square\varphi_F)\beta_{\mu_D}(y,a)(\varphi_D\square\varphi_F)\beta_{\mu_F}(y,b)}, \\ \frac{(\varphi_D\square\varphi_F)\gamma_{\nu_D}(y,a)+(\varphi_D\square\varphi_F)\gamma_{\nu_F}(y,b)-2(\varphi_D\square\varphi_F)\gamma_{\nu_D}(y,a)(\varphi_D\square\varphi_F)\alpha_{\emptyset_F}(y,b)}{1-(\varphi_D\square\varphi_F)\gamma_{\nu_D}(y,a)(\varphi_D\square\varphi_F)\alpha_{\emptyset_F}(y,b)} \end{cases}$$

=(0.42,0.375,0.72)

$(\varsigma_D \square \varsigma_F)$(y,a)(y,b)=(0.52,0.41,0.76)$\nleq$ (0.42,0.375,0.72)

From the illustration, Cartesian of two fermatean neutrosophic dombi fuzzy graph is not neutrosophic dombi fuzzy graph.

Definition 3.7: [Fermatean neutrosophic dombi fuzzy edge graph]

$D^*$is defined the fermatean neutrosophic dombi fuzzy edge graph if a neutrosophic fuzzy fuzzy by a truth-membership function, an indeterminacy-membership function, and a falsity - membership function is attached from (0,1) to each edge of the fermatean neutrosophic dombi fuzzy edge graph $D^*$of a graph d and each vertex φ_D is crisply in d.

Theorem 3.2: The cartesian product of two fermatean neutrosophic dombi fuzzy edge graph is a fermatean neutrosophic dombi fuzzy graph

Proof:

Let d and F be the fermatean neutrosophic dombi fuzzy edge graphs $d = (\emptyset_D, \mu_{D,}\nu_D)$ and $F = (\emptyset_F, \mu_{F,}\nu_F)$ $D^*= (\varphi_D, \varsigma_D)$ and $F^*=(\varphi_F, \varsigma_F)$ separately Then the Cartesian product $d\Box F$

Consider,

i)For all $n \in \varsigma_D, n_2t_2 \in \varsigma_F$ such that

$(\varsigma_D\Box\varsigma_F)(n, n_2)(n, t_2)$= T[$\varphi_D(n)$,$\varsigma_F(n_2t_2)$]=T[1,$\varsigma_F(n_2t_2)$]

$(\varsigma_D\Box\varsigma_F)(n, n_2)(n, t_2)$

$$\leq T\left[\begin{array}{c}\frac{\alpha_{\emptyset_D}\varphi(n)\alpha_{\emptyset_F}\varsigma(n_2t_2)}{\alpha_{\emptyset_D}\varphi(n)+\alpha_{\emptyset_F}\varsigma(n_2t_2)-\alpha_{\emptyset_D}\varphi(n)\alpha_{\emptyset_F}\varsigma(n_2t_2)},\\ \frac{\beta_{\mu_D}\varphi(n)\beta_{\mu_F}\varsigma(n_2t_2)}{\beta_{\mu_D}\varphi(n)+\beta_{\mu_F}\varsigma(n_2t_2)-\beta_{\mu_D}\varphi(n)\beta_{\mu_F}\varsigma(n_2t_2)},\\ \frac{\gamma_{\nu_D}\varphi(n)+\gamma_{\nu_F}\varsigma(n_2t_2)-2\gamma_{\nu_D}\varphi(n)\gamma_{\nu_F}\varsigma(n_2t_2)}{\gamma_{\nu_D}\varphi(n)\gamma_{\nu_F}\varsigma(n_2t_2)}\end{array}\right]$$

$$\varsigma_F(n_2t_2)=\begin{cases}\frac{\alpha_{\emptyset_D}(n_2)\alpha_{\emptyset_F}(t_2)}{\alpha_{\emptyset_D}(n_2)+\alpha_{\emptyset_F}(t_2)-\alpha_{\emptyset_D}(n_2)\alpha_{\emptyset_F}(t_2)},\\ \frac{\beta_{\mu_D}(n_2)\beta_{\mu_F}(t_2)}{\beta_{\mu_D}(n_2)+\beta_{\mu_F}(t_2)-\beta_{\mu_D}(n_2)\beta_{\mu_F}(t_2)},\\ \frac{\gamma_{\nu_D}(n_2)+\gamma_{\nu_F}(t_2)-2\gamma_{\nu_D}(n_2)\gamma_{\nu_F}(t_2)}{\gamma_{\nu_D}(n_2)\gamma_{\nu_F}(t_2)}\end{cases}$$

$$=\begin{cases}\frac{(\varphi_D\Box\varphi_F)\alpha_{\emptyset_D}(n,n_2)(\varphi_D\Box\varphi_F)\alpha_{\emptyset_F}(n,t_2)}{(\varphi_D\Box\varphi_F)\alpha_{\emptyset_D}(n,n_2)+(\varphi_D\Box\varphi_F)\alpha_{\emptyset_F}(n,t_2)-(\varphi_D\Box\varphi_F)\alpha_{\emptyset_D}(n,n_2)(\varphi_D\Box\varphi_F)\alpha_{\emptyset_F}(n,t_2)},\\ \frac{(\varphi_D\Box\varphi_F)\beta_{\mu_D}(n,n_2)(\varphi_D\Box\varphi_F)\beta_{\mu_F}(n,t_2)}{(\varphi_D\Box\varphi_F)\beta_{\mu_D}(n,n_2)+(\varphi_D\Box\varphi_F)\beta_{\mu_F}(n,t_2)-(\varphi_D\Box\varphi_F)\beta_{\mu_D}(n,n_2)(\varphi_D\Box\varphi_F)\beta_{\mu_F}(n,t_2)},\\ \frac{(\varphi_D\Box\varphi_F)\gamma_{\nu_D}(n,n_2)+(\varphi_D\Box\varphi_F)\gamma_{\nu_F}(n,t_2)-2(\varphi_D\Box\varphi_F)\gamma_{\nu_D}(n,n_2)(\varphi_D\Box\varphi_F)\alpha_{\emptyset_F}(n,t_2)}{1-(\varphi_D\Box\varphi_F)\gamma_{\nu_D}(n,n_2)(\varphi_D\Box\varphi_F)\alpha_{\emptyset_F}(n,t_2)}\end{cases}$$

------------------------(1)

For all $n \in \varsigma_D, n_2t_2 \in \varsigma_F$

i) Now consider $n_1t_1 \in \varsigma_D$,z $\in \varphi_F$

$(\varsigma_D\Box\varsigma_F)$ $(n_1, z)(t_1, z)$ = T[$\varsigma_D(n_1t_1)$,φ_F(z)]

$$\varsigma_D(n_1t_1) \leq \begin{cases} \dfrac{\alpha_{\emptyset_D}(n_1)\alpha_{\emptyset_D}(t_1)}{\alpha_{\emptyset_D}(n_1)+\alpha_{\emptyset_D}(t_1)-\alpha_{\emptyset_D}(n_1)\alpha_{\emptyset_D}(t_1)}, \\ \dfrac{\beta_{\mu_D}(n_1)\beta_{\mu_D}(t_1)}{\beta_{\mu_D}(n_1)+\beta_{\mu_D}(t_1)-\beta_{\mu_D}(n_1)\beta_{\mu_D}(t_1)}, \\ \dfrac{\gamma_{\nu_D}(n_1)+\gamma_{\nu_D}(t_1)-2\gamma_{\nu_D}(n_1)\gamma_{\nu_D}(t_1)}{\gamma_{\nu_D}(n_1)\gamma_{\nu_D}(t_1)} \end{cases}$$

$$= \begin{cases} \dfrac{(\varphi_D\Box\varphi_F)\alpha_{\emptyset_D}(n_1,z)(\varphi_D\Box\varphi_F)\alpha_{\emptyset_F}(t_1,z)}{(\varphi_D\Box\varphi_F)\alpha_{\emptyset_D}(n_1,z)+(\varphi_D\Box\varphi_F)\alpha_{\emptyset_F}(t_1,z)-(\varphi_D\Box\varphi_F)\alpha_{\emptyset_D}(n_1,z)(\varphi_D\Box\varphi_F)\alpha_{\emptyset_F}(t_1,z)}, \\ \dfrac{(\varphi_D\Box\varphi_F)\beta_{\mu_D}(n_1,z)(\varphi_D\Box\varphi_F)\beta_{\mu_F}(t_1,z)}{(\varphi_D\Box\varphi_F)\beta_{\mu_D}(n_1,z)+(\varphi_D\Box\varphi_F)\beta_{\mu_F}(t_1,z)-(\varphi_D\Box\varphi_F)\beta_{\mu_D}(n_1,z)(\varphi_D\Box\varphi_F)\beta_{\mu_F}(t_1,z)}, \\ \dfrac{(\varphi_D\Box\varphi_F)\gamma_{\nu_D}(n_1,z)+(\varphi_D\Box\varphi_F)\gamma_{\nu_F}(t_1,z)-2(\varphi_D\Box\varphi_F)\gamma_{\nu_D}(n_1,z)(\varphi_D\Box\varphi_F)\alpha_{\emptyset_F}(t_1,z)}{1-(\varphi_D\Box\varphi_F)\gamma_{\nu_D}(n_1,z)(\varphi_D\Box\varphi_F)\alpha_{\emptyset_F}(t_1,z)} \end{cases}$$

---------------------------(2)

For all $n_1t_1 \in \varsigma_D$ and z $\in \varphi_F$

From (1) & (2)

Every Cartesian product of two fermateanneutrosophicdombi fuzzy edge graph is a fermateanneutrsophicdombi fuzzy edge graph.

Definition 3.8: [Composition or Lexicographic product]: Let $\emptyset_i$ be a fermatean neutrosophic fuzzy subset of φ_D and ς_i be a fermateean neutrosophic fuzzy subset of ς_i=1,2,3.The composition of fermatean neutrosophic dombi fuzzy graphs ***d***= $(\emptyset_D, \mu_{D,}\nu_D)$ and ***F*** = $(\emptyset_F, \mu_{F,}\nu_F)$ D*= (φ_D, ς_D) and F^*=(φ_F, ς_F) separately is established by $d \circ F$and it is defined as

$d \circ F$= $(\emptyset_D \circ \emptyset_F, \mu_D \circ \mu_F, \nu_D \circ \nu_F)$.Here,

$\emptyset_D \circ \emptyset_F$ =$[\alpha_{\emptyset_D \circ \emptyset_F}, \beta_{\emptyset_D \circ \emptyset_F}, \gamma_{\emptyset_D \circ \emptyset_F}]$

$\mu_D \circ \mu_F$=$[\alpha_{\mu_D \circ \mu_F}, \beta_{\mu_D \circ \mu_F}, \gamma_{\mu_D \circ \mu_F}]$

$\nu_D \circ \nu_F$ =$[\alpha_{\nu_D \circ \nu_F}, \beta_{\nu_D \circ \nu_F}, \gamma_{\nu_D \circ \nu_F}]$

$\xi_{\ D \circ F}$ = $\{(n, n_2), (n, t_2)$: $x \in \varphi_D, n_1n_2 \in \varsigma_F\} \cup \{(n_1, z), (n_2, z)\ :\ n_1n_2 \in \varsigma_D, z \in \varphi_F\} \cup$
$\{\ (n_1, n_2)(t_1, t_2)$: $n_1n_2 \in \varsigma_D, , n_2 \neq t_2\}$

$$i)(\varphi_D \circ \varphi_F)(n_1, n_2) = \begin{cases} \dfrac{\alpha_{\emptyset_D}(n_1)\alpha_{\emptyset_F}(n_2)}{\alpha_{\emptyset_D}(n_1)+\alpha_{\emptyset_F}(n_2)-\alpha_{\emptyset_D}(n_1)\alpha_{\emptyset_F}(n_2)}, \\ \dfrac{\beta_{\mu_D}(n_1)\beta_{\mu_F}(n_2)}{\beta_{\mu_D}(n_1)+\beta_{\mu_F}(n_2)-\beta_{\mu_D}(n_1)\beta_{\mu_F}(n_2)} \\ \dfrac{\gamma_{\nu_D}(n_1)+\gamma_{\nu_F}(n_2)-2\gamma_{\nu_D}(n_1)\gamma_{\nu_F}(n_2)}{\gamma_{\nu_D}(n_1)\gamma_{\nu_F}(n_2)} \end{cases}$$

For all $(n_1, n_2) \in \varphi_D \circ \varphi_F$ such that,

$ii)(\varsigma_D \circ \varsigma_F)$ (n, $n_2)(t, t_2)$

$$= \begin{cases} \dfrac{\varphi_{\alpha_{\emptyset_D}}(n)\,\varsigma_{\alpha_{\emptyset_F}}(n_2t_2)}{\varphi_{\alpha_{\emptyset_D}}(n)+\varsigma_{\alpha_{\emptyset_F}}(n_2t_2)-\varphi_{\alpha_{\emptyset_D}}(n)\,\varsigma_{\alpha_{\emptyset_F}}(n_2t_2)}, \\ \dfrac{\varphi_{\beta_{\mu_D}}(n)\,\varsigma_{\beta_{\mu_F}}(n_2t_2)}{\varphi_{\beta_{\mu_D}}(n)+\varsigma_{\beta_{\mu_F}}(n_2t_2)-\varphi_{\beta_{\mu_D}}(n)\,\varsigma_{\beta_{\mu_F}}(n_2t_2)}, \\ \dfrac{\varphi_{\gamma_{\nu_D}}(n)\,\varsigma_{\gamma_{\nu_F}}(n_2t_2)-\varphi_{\gamma_{\nu_D}}(n)\,\varsigma_{\gamma_{\nu_F}}(n_2t_2)}{1-\varphi_{\gamma_{\nu_D}}(n)\,\varsigma_{\gamma_{\nu_F}}(n_2t_2)} \end{cases}$$

For all $n \in \varphi_D$ and $n_2t_2 \in \varsigma_F$

$$iii)(\varsigma_D \circ \varsigma_F)\ (n_1, z)(t_1, z) = \begin{cases} \frac{\alpha_{\emptyset_F}(z)\alpha_{\emptyset_D}(n_1t_1)}{\alpha_{\emptyset_F}(z)+\alpha_{\emptyset_D}(n_1t_1)-\alpha_{\emptyset_F}(z)\alpha_{\emptyset_D}(n_1t_1)}, \\ \frac{\beta_{\mu_F}(z)\beta_{\mu_D}(n_1t_1)}{\beta_{\mu_F}(z)+\beta_{\mu_D}(n_1t_1)-\beta_{\mu_F}(z)\beta_{\mu_D}(n_1t_1)}, \\ \frac{\gamma_{\nu_F}(z)+\gamma_{\nu_D}(n_1t_1)-2\gamma_{\nu_F}(z)\gamma_{\nu_D}(n_1t_1)}{\gamma_{\nu_F}(z)\gamma_{\nu_D}(n_1t_1)} \end{cases}$$

ii) $(\varsigma_D \circ \varsigma_F)\ (n_1, n_2)(t_1, t_2)$

$$= \begin{cases} \frac{\alpha_{\emptyset_F}(n_2)\alpha_{\emptyset_F}(t_2)\varsigma_{\alpha_{\emptyset_D}}(n_1t_1)}{\alpha_{\emptyset_F}(n_2)\alpha_{\emptyset_F}(t_2)+\alpha_{\emptyset_F}(t_2)\varsigma_{\alpha_{\emptyset_D}}(n_1t_1)+\alpha_{\emptyset_F}(n_2)\varsigma_{\alpha_{\emptyset_D}}(n_1t_1)\alpha_{\emptyset_F}(n_2)\alpha_{\emptyset_F}(t_2)\varsigma_{\alpha_{\emptyset_D}}(n_1t_1)}, \\ \frac{\beta_{\mu_F}(n_2)\beta_{\mu_F}(t_2)\varsigma_{\beta_{\mu_D}}(n_1t_1)}{\beta_{\mu_F}(n_2)\beta_{\mu_F}(t_2)+\beta_{\mu_F}(t_2)\varsigma_{\beta_{\mu_D}}(n_1t_1)+\beta_{\mu_F}(n_2)\varsigma_{\beta_{\mu_D}}(n_1t_1)-2\beta_{\mu_F}(n_2)\beta_{\mu_F}(t_2)\varsigma_{\beta_{\mu_D}}(n_1t_1)}, \\ \frac{\gamma_{\nu_F}(n_2)\gamma_{\nu_F}(t_2)+\gamma_{\nu_F}(t_2)\varsigma_{\gamma_{\nu_D}}(n_1t_1)+\gamma_{\nu_F}(n_2)\varsigma_{\gamma_{\nu_D}}(n_1t_1)-2\gamma_{\nu_F}(n_2)\gamma_{\nu_F}(t_2)\varsigma_{\gamma_{\nu_D}}(n_1t_1)}{1-\gamma_{\nu_F}(n_2)\gamma_{\nu_F}(t_2)\varsigma_{\gamma_{\nu_D}}(n_1t_1)} \end{cases}$$

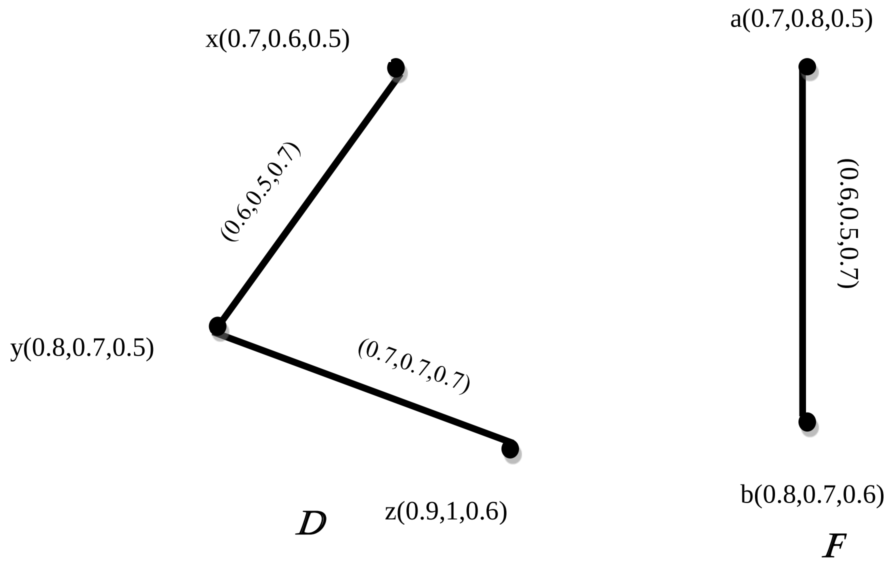
For all $n_1t_1 \in \varsigma_D, n_2 \neq t_2$

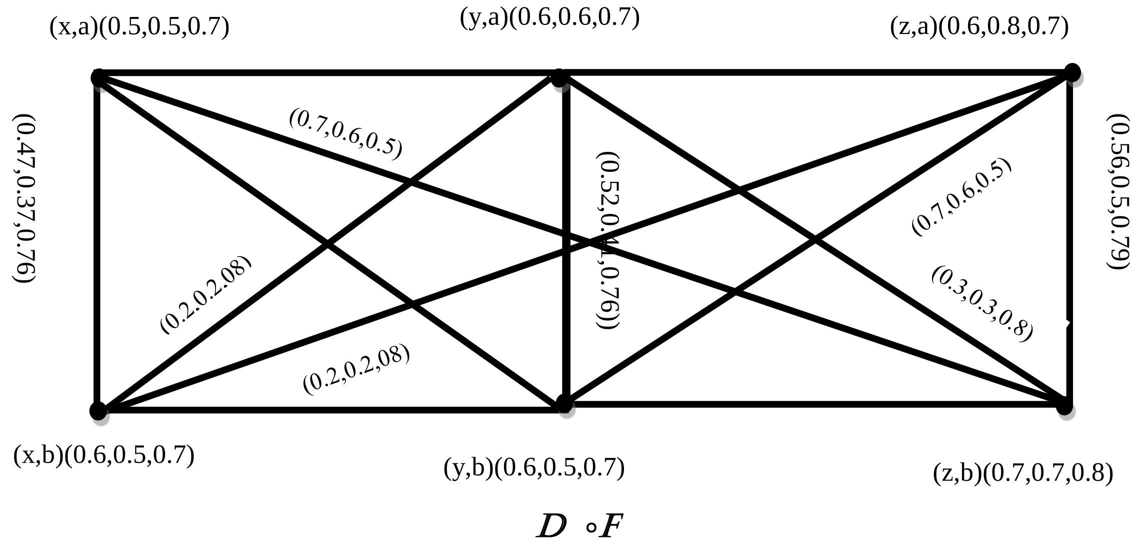
Example 3.6: Consider two Fermatean neutrosophicdombi fuzzy graphs ***d***= $(\emptyset_D, \mu_{D,}\nu_D)$ and ***F***= $(\emptyset_F, \mu_{F,}\nu_F)$.Here φ_D={x,y,z},φ_F={a,b},$\varsigma_D = \{xy, yz\}$, $\varsigma_F = \{ab\}$where

$\varphi_D = \{\frac{x}{(0.7,0.6,0.5)}, \frac{y}{(0.8,0.7,0.5)}, \frac{z}{(0.9,1,0.6)}\}$, $\varphi_F = \left\{\frac{a}{(0.7,0.8,0.5)}, \frac{b}{(0.8,0.7,0.6)}\right\}$, $\varsigma_F = \left\{\frac{ab}{(0.6,0.5,0.7)}\right\}$,

$\varsigma_D = \{\frac{xy}{(0.6,0.5,0.7)}, \frac{yz}{(0.7,0.7,0.7)}\}$, .Then we have

$\varphi_D \circ \varphi_F$(x,y) = (0.6,0.5,0.7), $\varphi_D \circ \varphi_F(y,z) = (0.7,0.7,0.7)$, $\varphi_D \circ \varphi_F(a,b) = (0.6,0.5,0.7)$,
$\varphi_D \circ \varphi_F(x,a) = (0.5,0.5,0.7)$, $\varphi_D \circ \varphi_F(x,b) = (0.6,0.5,0.7)$, $\varphi_D \circ \varphi_F(y,a) = (0.6,0.6,0.7)$,
$\varphi_D \circ \varphi_F(y,b) = (0.6,0.5,0.7)$, $\varphi_D \circ \varphi_F(z,a) = (0.6,0.8,0.7)$, $\varphi_D \circ \varphi_F(z,b) = (0.7,0.7,0.8)$,
$(\varsigma_D \circ \varsigma_F)$(x,a) (x,b) = (0.47,0.37,0.76), $(\varsigma_D \circ \varsigma_F)$(y,a)(y,b) = (0.52,0.41,0.76),
$(\varsigma_D \circ \varsigma_F)(z,a)(z,b) = (0.56,0.5,0.79)$, $(\varsigma_D \circ \varsigma_F)$ (x,a)(y,a) = (0.47,0.44,0.76),
$(\varsigma_D \circ \varsigma_F)$(y,a)(z,a)=(0.53,0.6,0.76), $(\varsigma_D \circ \varsigma_F)$(x,b)(y,b)=(0.52,0.41,0.79),
$(\varsigma_D \circ \varsigma_F)$(y,b)(z,b) = (0.6,0.53,0.79), $(\varsigma_D \circ \varsigma_F)$(x,a)(y,b) = (0.29,0.27,0.82),
$(\varsigma_D \circ \varsigma_F)$(y,a)(z,b)=(0.32,0.32,0.82),$(\varsigma_D \circ \varsigma_F)$(x,b)(z,a)=(0.29,0.29,0.82).





Corollary 3.4: The composition of two fermatean neutrosophic dombi fuzzy graph is not necessarily to a fermatean neutrosophic dombi fuzzy graph. $(\varsigma_D \circ \varsigma_F)$(y,a)(z,a)=(0.53,0.6,0.76)

Edge product of (y,a) and (z,a) is

$$=\left[\frac{0.6X0.6}{0.6+0.6-(0.6X0.6)},\frac{0.6X0.8}{0.6+0.8-(0.6X0.8)},\frac{0.7X0.7}{0.7X0.7-(0.7X0.7)}\right]=(0.42,0.52,0.82)$$

(0.53,0.6,0.76)$\nleq$(0.42,0.52,0.82)

Theorem 3.4: The Composition of two fermatean neutrosophic dombi fuzzy edge graph is a fermatean neutrosophic dombi fuzzy graph.

Proof:

Let d and F be the fermatean neutrosophic dombi fuzzy edge graphs $d = (\emptyset_D, \mu_{D,}\nu_D)$ and $F = (\emptyset_F, \mu_{F,}\nu_F)$ D*= (φ_D, ς_D) and F^*= (φ_F, ς_F) separately Then the Composition product $d \circ F$

Consider, For all $n \in \varsigma_D, n_2 t_2 \in \varsigma_F$ such that

$(\varsigma_D \circ \varsigma_F)(n, n_2)(n, t_2)$= T[$\varphi_D(n)$,$\varsigma_F(n_2 t_2)$]=T[1,$\varsigma_F(n_2 t_2)$]

$(\varsigma_D \circ \varsigma_F)(n, n_2)(n, t_2)$

$$\leq T\left[\begin{matrix} \frac{\alpha_{\emptyset_D}\varphi(n)\alpha_{\emptyset_F}\varsigma(n_2t_2)}{\alpha_{\emptyset_D}\varphi(n)+\alpha_{\emptyset_F}\varsigma(n_2t_2)-\alpha_{\emptyset_D}\varphi(n)\alpha_{\emptyset_F}\varsigma(n_2t_2)}, \\ \frac{\beta_{\mu_D}\varphi(n)\beta_{\mu_F}\varsigma(n_2t_2)}{\beta_{\mu_D}\varphi(n)+\beta_{\mu_F}\varsigma(n_2t_2)-\beta_{\mu_D}\varphi(n)\beta_{\mu_F}\varsigma(n_2t_2)}, \\ \frac{\gamma_{\nu_D}\varphi(n)+\gamma_{\nu_F}\varsigma(n_2t_2)-2\gamma_{\nu_D}\varphi(n)\gamma_{\nu_F}\varsigma(n_2t_2)}{\gamma_{\nu_D}\varphi(n)\gamma_{\nu_F}\varsigma(n_2t_2)} \end{matrix}\right]$$

$$\varsigma_F(n_2t_2)=\begin{cases} \frac{\alpha_{\emptyset_D}(n_2)\alpha_{\emptyset_F}(t_2)}{\alpha_{\emptyset_D}(n_2)+\alpha_{\emptyset_F}(t_2)-\alpha_{\emptyset_D}(n_2)\alpha_{\emptyset_F}(t_2)}, \\ \frac{\beta_{\mu_D}(n_2)\beta_{\mu_F}(t_2)}{\beta_{\mu_D}(n_2)+\beta_{\mu_F}(t_2)-\beta_{\mu_D}(n_2)\beta_{\mu_F}(t_2)}, \\ \frac{\gamma_{\nu_D}(n_2)+\gamma_{\nu_F}(t_2)-2\gamma_{\nu_D}(n_2)\gamma_{\nu_F}(t_2)}{\gamma_{\nu_D}(n_2)\gamma_{\nu_F}(t_2)} \end{cases}$$

$$=\begin{cases} \frac{(\varphi_D\circ\varphi_F)\alpha_{\emptyset_D}(n,n_2)(\varphi_D\circ\varphi_F)\alpha_{\emptyset_F}(n,t_2)}{(\varphi_D\circ\varphi_F)\alpha_{\emptyset_D}(n,n_2)+(\varphi_D\circ\varphi_F)\alpha_{\emptyset_F}(n,t_2)-(\varphi_D\circ\varphi_F)\alpha_{\emptyset_D}(n,n_2)(\varphi_D\circ\varphi_F)\alpha_{\emptyset_F}(n,t_2)}, \\ \frac{(\varphi_D\circ\varphi_F)\beta_{\mu_D}(n,n_2)(\varphi_D\circ\varphi_F)\beta_{\mu_F}(n,t_2)}{(\varphi_D\circ\varphi_F)\beta_{\mu_D}(n,n_2)+(\varphi_D\circ\varphi_F)\beta_{\mu_F}(n,t_2)-(\varphi_D\circ\varphi_F)\beta_{\mu_D}(n,n_2)(\varphi_D\circ\varphi_F)\beta_{\mu_F}(n,t_2)}, \\ \frac{(\varphi_D\circ\varphi_F)\gamma_{\nu_D}(n,n_2)+(\varphi_D\circ\varphi_F)\gamma_{\nu_F}(n,t_2)-2(\varphi_D\circ\varphi_F)\gamma_{\nu_D}(n,n_2)(\varphi_D\circ\varphi_F)\alpha_{\emptyset_F}(n,t_2)}{1-(\varphi_D\circ\varphi_F)\gamma_{\nu_D}(n,n_2)(\varphi_D\circ\varphi_F)\alpha_{\emptyset_F}(n,t_2)} \end{cases}$$

------------------------(3)

For all $n \in \varsigma_D, n_2 t_2 \in \varsigma_F$

i) Now consider $n_1 t_1 \in \varsigma_D$,z $\in \varphi_F$

$(\varsigma_D \circ \varsigma_F)$ $(n_1, z)(t_1, z)$ = T[$\varsigma_D(n_1 t_1)$,φ_F(z)]

$$\varsigma_D(n_1t_1) \leq \begin{cases} \frac{\alpha_{\emptyset_D}(n_1)\alpha_{\emptyset_D}(t_1)}{\alpha_{\emptyset_D}(n_1)+\alpha_{\emptyset_D}(t_1)-\alpha_{\emptyset_D}(n_1)\alpha_{\emptyset_D}(t_1)}, \\ \frac{\beta_{\mu_D}(n_1)\beta_{\mu_D}(t_1)}{\beta_{\mu_D}(n_1)+\beta_{\mu_D}(t_1)-\beta_{\mu_D}(n_1)\beta_{\mu_D}(t_1)}, \\ \frac{\gamma_{\nu_D}(n_1)+\gamma_{\nu_D}(t_1)-2\gamma_{\nu_D}(n_1)\gamma_{\nu_D}(t_1)}{\gamma_{\nu_D}(n_1)\gamma_{\nu_D}(t_1)} \end{cases}$$

$$=\begin{cases} \frac{(\varphi_D\circ\varphi_F)\alpha_{\emptyset_D}(n_1,z)(\varphi_D\circ\varphi_F)\alpha_{\emptyset_F}(t_1,z)}{(\varphi_D\circ\varphi_F)\alpha_{\emptyset_D}(n_1,z)+(\varphi_D\circ\varphi_F)\alpha_{\emptyset_F}(t_1,z)-(\varphi_D\circ\varphi_F)\alpha_{\emptyset_D}(n_1,z)(\varphi_D\circ\varphi_F)\alpha_{\emptyset_F}(t_1,z)}, \\ \frac{(\varphi_D\circ\varphi_F)\beta_{\mu_D}(n_1,z)(\varphi_D\circ\varphi_F)\beta_{\mu_F}(t_1,z)}{(\varphi_D\circ\varphi_F)\beta_{\mu_D}(n_1,z)+(\varphi_D\circ\varphi_F)\beta_{\mu_F}(t_1,z)-(\varphi_D\circ\varphi_F)\beta_{\mu_D}(n_1,z)(\varphi_D\circ\varphi_F)\beta_{\mu_F}(t_1,z)}, \\ \frac{(\varphi_D\circ\varphi_F)\gamma_{\nu_D}(n_1,z)+(\varphi_D\circ\varphi_F)\gamma_{\nu_F}(t_1,z)-2(\varphi_D\circ\varphi_F)\gamma_{\nu_D}(n_1,z)(\varphi_D\circ\varphi_F)\alpha_{\emptyset_F}(t_1,z)}{1-(\varphi_D\circ\varphi_F)\gamma_{\nu_D}(n_1,z)(\varphi_D\circ\varphi_F)\alpha_{\emptyset_F}(t_1,z)} \end{cases}$$

------------(4)

For all $n_1 t_1 \in \varsigma_D$ and z $\in \varphi_F$

Now consider $n_1 t_1 \in \varsigma_D, n_2 = t_2$.

$(\varsigma_D \circ \varsigma_F)$ $[(n_1, n_2)(t_1, t_2)]$ =T[T($\varphi_F(n_2), \varphi_F(t_2)$,$\varsigma_D(n_1 t_1)$)]

=T[(T(1,1),$\varsigma_{\mu_D}(n_1 t_1)$),(T(0,0),$\varsigma_{\nu_F}(n_1 t_1)$)

= T[(T(1,0),T(1,0)),(T($\varsigma_{\mu_D}(n_1 t_1)$,$\varsigma_{\nu_F}(n_1 t_1)$)

$$\leq \begin{cases} \dfrac{(\varphi_D \circ \varphi_F)\alpha_{\emptyset_D}(n_1,n_2)(\varphi_D \circ \varphi_F)\alpha_{\emptyset_F}(t_1,t_2)}{(\varphi_D \circ \varphi_F)\alpha_{\emptyset_D}(n_1,n_2)+(\varphi_D \circ \varphi_F)\alpha_{\emptyset_F}(t_1,t_2)-(\varphi_D \circ \varphi_F)\alpha_{\emptyset_D}(n_1,n_2)(\varphi_D \circ \varphi_F)\alpha_{\emptyset_F}(t_1,t_2)}, \\ \dfrac{(\varphi_D \circ \varphi_F)\beta_{\mu_D}(n_1,n_2)(\varphi_D \circ \varphi_F)\beta_{\mu_F}(t_1,t_2)}{(\varphi_D \circ \varphi_F)\beta_{\mu_D}(n_1,n_2)+(\varphi_D \circ \varphi_F)\beta_{\mu_F}(t_1,t_2)-(\varphi_D \circ \varphi_F)\beta_{\mu_D}(n_1,n_2)(\varphi_D \circ \varphi_F)\beta_{\mu_F}(t_1,t_2)}, \\ \dfrac{(\varphi_D \circ \varphi_F)\gamma_{\nu_D}(n_1,n_2)+(\varphi_D \circ \varphi_F)\gamma_{\nu_F}(t_1,t_2)-2(\varphi_D \circ \varphi_F)\gamma_{\nu_D}(n_1,n_2)(\varphi_D \circ \varphi_F)\alpha_{\emptyset_F}(t_1,t_2)}{1-(\varphi_D \circ \varphi_F)\gamma_{\nu_D}(n_1,n_2)(\varphi_D \circ \varphi_F)\alpha_{\emptyset_F}(t_1,t_2)} \end{cases} \quad \text{--------------(5)}$$

From (3), (4) & (5)

Every Composition of two fermatean neutrosophic dombi fuzzy edge graph is a fermatean neutrsophic dombi fuzzy edge graph.

4. Conclusion

Dombi neutrosophic fuzzy graph is more and more interesting by researches. There are many theoretical and applied results on neutrosophic fuzzy graphs that are built and developed. The concept of neutrosophic graphs can be used in many applications like decision-making problem, transportation, and computer networks. In this paper, we have introduced Fermatean neutrosophic Dombi fuzzy graphs and then we presented and studied some of its properties. Also, we investigated the relationship between the other existing Dombi neutrosophic fuzzy graphs. This shall be extended in the future to investigate the operations of the strong,semi-strong, complement of Fermatean neutrosophic Dombi fuzzy graphs with few real-life applications with relatable illustrations.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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