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A Perspective Note on $\mu_N\ \sigma$ Baire's Space

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Abstract: This paper presents an introduction to many novel types of sets, including μ_N strongly dense sets, μ_N strongly nowhere dense sets, μ_N strongly first category sets, and μ_N strongly nowhere residual sets. The features of these sets are briefly elucidated. In addition, by the use of these techniques, we have successfully obtained the highly Baire space μ_N, and it is imperative to elucidate its inherent features.

Keywords: μ_N Strongly Dense; μ_N Strongly Nowhere Dense; μ_N Strongly First Category Sets.

1. Introduction

The idea of fuzziness introduced by Zadeh has had a significant influence on several disciplines within the realm of mathematics. The concepts introduced by C.L. Chang [1] were subsequently integrated, resulting in the development of fuzzy topological spaces. This fusion of ideas included the principles of fuzziness inside the framework of topological spaces, establishing the foundation for the theory of fuzzy topological spaces. The discovery of intuitionistic fuzzy sets was attributed to K.T. Attanasov [2], who, in collaboration with Stoeva [3], further extended this study by introducing a generalization known as intuitionistic L-fuzzy sets. Smarandache [7] focused his research on the concept of indeterminacy and introduced the concept of neutrosophic sets. Subsequently, the neutrosophic topological spaces were introduced by A.A. Salama and Albowi [13] by the use of neutrosophic sets. The authors [12] developed a novel concept called μ_N TS, which involves the construction of Generalised topological spaces via the use of neutrosophic sets. This approach was inspired by previous studies in the field. The notion of Baire space in μ_NTS was introduced by the authors, and in this study, we further explore the robust properties of μ_N Baire space.

2. Necessities

Definition 2.1 [4-10, 14]: Let X be a non-empty fixed set. A Neutrosophic set [NS for short] A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ which represents the degree of membership function , the degree of indeterminacy and the degree of non-membership function respectively of each element $x \in X$ to the set A.

Remark 2.2.[14] Every intuitionistic fuzzy set A is a non empty set in X is obviously on Neutrosophic sets having the form $A = \{\langle \mu_A(x), 1 - \mu_A(x) + \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$.Since our main purpose is to construct the tools for developing Neutrosophic Set and Neutrosophic topology , we must introduce the neutrosophic sets 0_N and 1_N in X as follows:

0_N may be defined as follows

$(0_1) 0_N = \{\langle x, 0,1,1\rangle : x \epsilon X\}$

1_N may be defined as follows

$(1_1) 1_N = \{\langle x, 1,0,0\rangle : x \epsilon X\}$

Definition 2.3.[14] Let $A = \{\langle \mu_A, \sigma_A, \gamma_A \rangle\}$ be a NS on X, then the complement of the set A [C(A) for short] may be defined

$(C_1)\ C(A) = \{\langle x, \gamma_A(x), 1 - \sigma_A(x), \mu_A(x)\rangle : x \in X\}$

Definition 2.4.[14] Let X be a non-empty set and neutrosophic sets A and B in the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \in X\}$.

$A \subseteq B$ may be defined as :

$(A \subseteq B) \Leftrightarrow \mu_A(x) \le \mu_B(x), \sigma_A(x) \ge \sigma_B(x), \gamma_A(x) \ge \gamma_{\boldsymbol{B}}(x)\ \forall x \in X$

Proposition 2.5. [14] For any neutrosophic set A, the following conditions holds:

$0_N \subseteq$ A,

$A \subseteq 1_N$

Definition 2.6. [14] Let X be a non empty set and $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$

$B = \{\langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \in X\}$ are NSs. Then $A \cap B$ may be defined as :

$(I_1)\ A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle$

$A \cup B$ may be defined as :

$(I_1)\ A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle$

Definition 2.7[12]. A μ_N topology on a non - empty set X is a family of neutrosophic subsets in X $satisfying\ the\ following\ axioms$:

$(\mu_{N_1}) 0_N \in \mu_N$

$(\mu_{N_2}) Union\ of\ any\ number\ of\ \mu_N\ open\ sets\ is\ \mu_N\ open.$

Remark 2.8.[12] The elements of μ_N are μ_N open sets and their complement is called μ_N closed sets.

Definition 2.9.[12]The $\mu_N -$ Closure of A is the intersection of all μ_N closed sets containing A.

Definition 2.10.[12]The $\mu_N -$ Interior of A is the union of all μ_N open sets contained in A.

Definition 2.11.[13]. : A neutrosophic set A in μ_N TS (X, μ_N) is called μ_N dense set if there exists no μ_N closed set B in (X, μ_N) such that $A \subset B \subset 1_N$

Definition 2.12.[13]. The μ_N Topological spaces is said to be μ_N Baire's Space if $\mu_N\ Int(\cup_{i=1}^{\infty} G_i) = 0_N$ where $G_i's$ are μ_N nowhere dense set in (X, μ_N).

Theorem 2.13.[13]: Let (X, μ_N) be a μ_NTS. Then the following are equivalent.

(i) (X, μ_N) is μ_NBaire's Space.

(ii) $\mu_N\ Int(A) = 0_N$, for all μ_N first category set in (X, μ_N).

(iii) $\mu_N\ Cl(A) = 1_N$, μ_N Residual set in (X, μ_N).

3. $\mu_N\ \sigma$ *Nowhere Dense sets*

Definition 3.1: A neutrosophic set A in X is called $\mu_N\ \sigma$ rare set if A is a $\mu_N\ F_\sigma$ set such that $\mu_N Int(A) = 0_N$.

Definition 3.2 :A neutrosophic set A in X is called $\mu_N\ \sigma$ nowhere dense set if A is a $\mu_N\ F_\sigma$ set such that $\mu_N Int(\mu_N\ Cl\ A) = 0_N$.

Remark 3.3 :If A is a $\mu_N\ F_\sigma$ set and μ_N Nowhere dense set in X then A is $\mu_N\ \sigma$ rare set.

Example 3.4: Let $X = \{a\}$ define neutrosophic sets $0_N = \{\langle 0,1,1\rangle\}$, $A = \{\langle 0.1,0.4,0.6\rangle\}, B = \{\langle 0.2,0.3,0.5\rangle\}, C = \{\langle 0.6,0.6,0.1\rangle\}, 1_N = \{\langle 1,0,0\rangle\}$ and we define a μ_N TS $\mu_N = \{0_N, A, C\}$.Here $\overline{A}$ and $\overline{B}$ are $\mu_N\ \sigma$ rare sets.

Theorem 3.5: A neutrosophic set A in X is μ_N σ rare set iff $\overline{A}$ is μ_N dense and μ_N G_δ set.

Proof: Let A be μ_N σ rare set in X.Then A is μ_N F_σ set such that $\mu_N Int(A) = 0_N$ which implies us that $\mu_N Cl(\overline{A}) = 1_N$ and $\overline{A} = \overline{\bigcup_{i=1}^{\infty} A_i} = \bigcap_{i=1}^{\infty} A_i$ where A's are μ_N open sets.Therefore $\overline{A}$ is μ_N dense and μ_N G_δ set in (X, μ_N).Conversely,assume that $\overline{A}$ is μ_N dense and μ_N G_δ set in (X, μ_N). Then $\overline{A} = \bigcap_{i=1}^{\infty} A_i \Rightarrow A = \bigcup_{i=1}^{\infty} A_i$,where A_i's are μ_N closed sets .From this we retrieve that A in (X, μ_N) is μ_N F_σ and also $\mu_N Cl(\overline{A}) = 1_N$ that implies $\mu_N Int(A) = 0_N$.From this we say that A is μ_N σ rare set.

Corollary 3.6: A neutrosophic set A in X is μ_N σ rare set iff μ_N Ext $(\overline{A}) = 0_N$ and $\overline{A}$ is μ_N G_δ set.

Proof: Let A be a μ_N σ rare set in (X, μ_N). Then A is μ_N F_σ set such that μ_N Ext$(A) = 0_N$. Now μ_N Ext $(\overline{A}) = \mu_N$ Ext$(A) = 0_N$ and $\overline{A} = \overline{\bigcup_{i=1}^{\infty} A_i} = \bigcap_{i=1}^{\infty} \overline{A_i}$ where $\overline{A_i} \in \mu_N$ open sets in (X, μ_N). Therefore, μ_N Ext $(\overline{A}) = 0_N$ and $\overline{A}$ is a μ_N G_δ set. Conversely, assume that μ_N Ext $(\overline{A}) = 0_N$ and $\overline{A}$ is a μ_N G_δ set in (X, μ_N). Then $\overline{A} = \bigcap_{i=1}^{\infty} \overline{A_i} \Rightarrow A = \bigcup_{i=1}^{\infty} A_i$ where A_i's is μ_N closed sets in $(X, \mu_N) \Rightarrow$ A in X is μ_N F_σ set. Also μ_N Int $(A) = \mu_N$ Int$(\overline{\overline{A}}) = \mu_N$ Ext $(\overline{A}) = 0_N$. Therefore, A is μ_N σ Rare set.

Theorem 3.7: If a neutrosophic set in X is μ_N σ rare set then μ_N border is a subset of μ_N frontier.

Proof: Suppose A in X is μ_N σ rare set then A is a μ_N F_σ set and μ_N Int $(A) = 0_N$ that implies A $= \bigcup_{i=1}^{\infty} A_i$ where A_i's are μ_N closed sets in (X, μ_N).Now μ_N Br(A)=A $- \mu_N$ Int $(A) = A$ and μ_N Fr(A)= $\mu_N Cl(A) - \mu_N$ Int $(A) = \mu_N$ Cl A. Henceforth μ_N border is a subset of μ_N frontier.

Theorem 3.8: If a neutrosophic set A in X is μ_N σ rare set then A is μ_N strongly first category set.

Proof: Suppose A is μ_N σ rare set then A is a μ_N F_σ set and μ_N Int $(A) = 0_N$ that implies us that A $= \bigcup_{i=1}^{\infty} A_i$ where A_i's are μ_N closed sets in (X, μ_N) and μ_N Int $(A) = 0_N$. We know that $\bigcup_{i=1}^{\infty} \mu_N$ Int $(A) \subseteq \mu_N$ Int$(\bigcup_{i=1}^{\infty} A_i) = \mu_N$ Int $(A) = 0_N \Rightarrow \mu_N$ Int $(A) = 0_N$, where A_i's is μ_N closed sets. We have if A is μ_N closed set with μ_N Int $(A) = 0_N$, then A is a μ_N strongly nowhere dense sets. By using this we get A_i's are μ_N strongly nowhere dense sets and hence A $= \bigcup_{i=1}^{\infty} A_i$ where A_i's are μ_N strongly nowhere dense sets. Therefore then A is a μ_N strongly first category set.

Remark 3.9: The converse of the above theorem not true. Let $X = \{a\}$ define neutrosophic sets $0_N = \{\langle 0,1,1\rangle\}, A = \{\langle 0.1,0.4,0.6\rangle\}, B = \{\langle 0.2,0.3,0.5\rangle\}, C = \{\langle 0.6,0.6,0.1\rangle\}, 1_N = \{\langle 1,0,0\rangle\}$ and we define a μ_N TS $\mu_N = \{0_N, A, C\}$.Here $\overline{A}$ and $\overline{B}$ are μ_N σ rare sets,{A,B,C,D, $0_N, \overline{A}, \overline{B}, \overline{C}$ and $\overline{D}$ are μ_N strongly first category set.We can analyse that $\overline{C}$ and $\overline{D}$ are μ_N strongly first category sets but not μ_N σ rare sets.

Theorem 3.10: Every μ_N σ Nowhere dense set is μ_N σ rare set.

Proof: Let $A \subseteq X$ be a μ_N σ nowhere dense set. Then A is μ_N F_σ set and μ_N nowhere dense set. Using theorem 2.3, A is μ_N F_σ set and μ_N Int $(A) = 0_N$. Hence A is a μ_N σ rare set.

Corollary 3.11: A neutrosophic set A in X is μ_N σ rare set and μ_N closed set then A is μ_N σ nowhere dense set.

Proof: Given that A in X is μ_N σ rare set and μ_N closed set. Then A is μ_N F_σ set with μ_N Int $(A) = 0_N$ and μ_N Cl$(A) = A$, we know Let $A \subseteq X$. If μ_N closed set with μ_N Int $(A) = 0_N$. Then A is μ_N nowhere dense set in μ_NTS.

Remark 3.12: Every μ_N σ nowhere dense set is μ_N nowhere dense set but the reverse is not valid.

Example: Let $X = \{a\}$ define neutrosophic sets $0_N = \{\langle 0,1,1\rangle\}$, $A = \{\langle 0.1,0.4,0.6\rangle\}, B = \{\langle 0.2,0.3,0.5\rangle\}, C = \{\langle 0.6,0.6,0.1\rangle\}$, $D = \{\langle 0.5,0.7,0.2\rangle\} 1_N = \{\langle 1,0,0\rangle\}$ and we define a μ_N TS $\mu_N = \{0_N, A, B\}$.Here the μ_N nowhere dense sets { $C, D, 0_N, \overline{A}, \overline{B}$ } and the μ_N nowhere dense sets {

$\overline{A}, \overline{B}$ }.From this we conclude that Every μ_N nowhere dense set need not be $\mu_N\sigma$ nowhere dense set.

Theorem 3.13: If a neutrosophic set A in (X, μ_N) is $\mu_N\ \sigma$ nowhere dense set then A is μ_N strongly first category set.

Proof: We have "Every $\mu_N\ \sigma$ nowhere dense set is $\mu_N\ \sigma$ rare set." And "If A in (X, μ_N) is $\mu_N\ \sigma$ rare set then A is μ_N strongly first category set." Using these theorem's, we get A is μ_N strongly first category set.

Theorem 3.14: If a neutrosophic set A in X is $\mu_N\ \sigma$ nowhere dense set then $\overline{A}$ is μ_N dense set and $\mu_N G_\delta$ set in (X, μ_N).

Proof: Using Corollary 3.6 and theorem 3.5, μ_N Ext $(\overline{A}) = 0_N$ and $\overline{A}$ is $\mu_N G_\delta$ set.

Theorem 3.15: If a neutrosophic set in (X, μ_N) is $\mu_N\ \sigma$ nowhere dense set then μ_N border is a subset of μ_N Frontier.

Theorem 3.16: (i) Every subset of a $\mu_N\ \sigma$ rare set is $\mu_N\ \sigma$ rare set.

(ii) Every subset of a $\mu_N\ \sigma$ nowhere dense set is $\mu_N\ \sigma$ nowhere dense set.

Definition 3.17: A neutrosophic set A is said to be $\mu_N\ \sigma$- category I set, if $A = \cup_{i=1}^{\infty} A_i$ where A_i's are $\mu_N\ \sigma$ rare set. Remaining sets are called $\mu_N\ \sigma$ category II sets. The complement set of $\mu_N\ \sigma$ first category sets are named as $\mu_N\ \sigma$ complement set.

Theorem 3.18: Every subset of $\mu_N\ \sigma$ category I set is $\mu_N\ \sigma$ category I set.

Theorem 3.19: If A is μ_N dense and $\mu_N G_\delta$ set then $\overline{A}$ is $\mu_N\ \sigma$ category I set.

Theorem 3.20: If A is $\mu_N\ \sigma$ category I set in X then $A \subseteq \eta$ where η is a non-void $\mu_N\ F_\sigma$ set in X.

Theorem 3.21: If A is $\mu_N\ \sigma$ complement set in X then there exists a $\mu_N G_\delta$ set B such that $A \subseteq B$.

Proof: Let A be $\mu_N\ \sigma$ complement set in X. Then $\overline{A}$ is $\mu_N\ \sigma$ first category set by using theorem 3.20, we have there is a non-void $\mu_N\ F_\sigma$ set B in X such that $\overline{A} \subseteq B$. Hence $\overline{A} \subseteq B$ and $\overline{B}$ is a $\mu_N G_\delta$ set. Take $A = \overline{B}$. Therefore we have $A \subseteq B$.

Theorem 3.22:

(i) Every $\mu_N\ \sigma$ category I set is a $\mu_N\ F_\sigma$ set.

(ii) Every $\mu_N\ \sigma$ complement set is a $\mu_N G_\delta$ set.

Definition 3.23: A neutrosophic set A is said to be $\mu_N\ \sigma$-first category in μ_NTS if $A = \cup_{i=1}^{\infty} A_i$ where A_i's are $\mu_N\ \sigma$ nowhere dense sets. Remaining sets are $\mu_N\ \sigma$ second category set. The complement of $\mu_N\ \sigma$ first category set is named as $\mu_N\ \sigma$ residual set.

Theorem 3.24: Every subset of $\mu_N\ \sigma$ first category set is $\mu_N\ \sigma$ first category set.

Theorem 3.25: If A is μ_N dense and $\mu_N G_\delta$ set then $\overline{A}$ is $\mu_N\ \sigma$ first category set.

Theorem 3.26: If A is $\mu_N\ \sigma$ first category set in X then $A \subseteq B$ where A is a non-void $\mu_N\ F_\sigma$ set in X.

Theorem 3.27: If A is $\mu_N\ \sigma$ residual set in X then there exists a $\mu_N G_\delta$ set B such that $A \subseteq B$.

Proof: Let A be a $\mu_N\ \sigma$ residual set in X. Then $\overline{A}$ is $\mu_N\ \sigma$ first category set by using theorem 3.24, we have there is a non-void $\mu_N\ F_\sigma$ set B in X such that $\overline{A} \subseteq B$. Hence $\overline{A} \subseteq B$ and $\overline{B}$ is a $\mu_N G_\delta$ set. Take $A = \overline{B}$. Therefore we have $A \subseteq B$.

Theorem 3.28:

(i) Every $\mu_N\ \sigma$ first category set is a $\mu_N\ F_\sigma$ set.

(ii) Every $\mu_N\ \sigma$ residual set is a $\mu_N G_\delta$ set.

4. $\mu_N\, B_\sigma$ Space and $\mu_N\, \sigma$ Baire Space

Definition 4.1: If $\mu_N\, Int(\bigcup_{i=1}^{\infty} A_i) = 0_N$ where A_i's are $\mu_N\, \sigma$ rare set then X is a $\mu_N\, B_\sigma$ Space.

Definition 4.2: If $\mu_N\, Int(\bigcup_{i=1}^{\infty} A_i) = 0_N$ where A_i's are $\mu_N\, \sigma$ nowhere dense set then X is a $\mu_N\, B_\sigma$ Baire space.

Theorem 4.3: If $\mu_N\, Cl(\bigcap_{i=1}^{\infty} \delta_i) = 1_N$ where δ_i's are μ_N dense set and $\mu_N G_\delta$ set then (X, μ_N) is a $\mu_N\, B_\sigma$ Baire space.

Proof: Given that $\mu_N\, Cl(\bigcap_{i=1}^{\infty} \delta_i) = 1_N$ which gives that $\overline{\mu_N\, Cl(\bigcap_{i=1}^{\infty} \delta_i)} = 0_N \Rightarrow \mu_N\, Int(\bigcup_{i=1}^{\infty} \delta_i) = 0_N$. Take $\mathbf{B}_i = \overline{\delta_i}$. Then $\mu_N\, Int(\bigcup_{i=1}^{\infty} B_i) = \zeta$. Now δ_i's are μ_N dense set and $\mu_N G_\delta$ set in X. Then by theorem 3.8 $\overline{\delta_i}$ is a $\mu_N\, \sigma$ rare set and hence $\mu_N\, Int(\bigcup_{i=1}^{\infty} B_i) = 0_N$ where B_i's are $\mu_N\, \sigma$ rare set. Therefore (X, μ_N) is a $\mu_N\, B_\sigma$ Baire space.

Theorem 4.4: Let (X, μ_N) be μ_NTS. Then the following are equivalent.

(i) (X, μ_N) is a $\mu_N\, B_\sigma$ Baire space.

(ii) $\mu_N\, Int(\delta_i) = 0_N$ for every $\mu_N\, \sigma$ first category set in X.

(iii) $\mu_N\, Cl(\delta_i) = 1_N$ for every $\mu_N\, \sigma$ residual set in X.

Proof: (i)⇒(ii) Let δ_i be μ_N first category set in X. Then $\delta_i = \bigcup_{i=1}^{\infty} \delta_i$ where δ_i's are $\mu_N\, \sigma$ rare set and $\mu_N\, Int(\delta_i) = \mu_N\, Int(\bigcup_{i=1}^{\infty} \delta_i)$ since (X, μ_N) is a $\mu_N\, B_\sigma$ space. $\mu_N\, Int(\delta_i) = 0_N$.

(ii)⇒(iii) Let δ_i be $\mu_N\, \sigma$ complement set in X. Then $\overline{\delta_i}$ is a $\mu_N\, \sigma$ first category set in X. From (ii), $\mu_N\, Int(\delta_i) = 0_N \Rightarrow \overline{\mu_N\, Cl(\delta_i)} = 0_N$. Hence $\mu_N\, Cl(\delta_i) = 1_N$.

(iii)⇒(i) Let δ_i be $\mu_N\, \sigma$ first category set in X. Then $\delta = \bigcup_{i=1}^{\infty} \delta_i$ where δ_i's are $\mu_N\, \sigma$ rare set. We have if δ is $\mu_N\, \sigma$ first category set in X then $\overline{\delta}$ is $\mu_N\, \sigma$ residual set. By (iii) we get $\mu_N\, Cl(\overline{\delta}) = 1_N$ which gives $\overline{\mu_N\, Int(\delta)} = 0_N$. Therefore $\mu_N\, Int(\delta) = 0_N$ and hence $\mu_N\, Int(\bigcup_{i=1}^{\infty} \delta_i) = 0_N$ where δ_i's are $\mu_N\, \sigma$ rare set. Hence (X, μ_N) is a $\mu_N\, B_\sigma$ Baire space.

Theorem 4.5: If $\mu_N\, Int(A) = 0_N$ for each $\mu_N\, F_\sigma$ set A in X then X is a $\mu_N\, B_\sigma$ Baire space.

Proof: Let A be a $\mu_N\, \sigma$ first category set in X. Then $A \subseteq B$ where A is a non-void $\mu_N\, F_\sigma$ set in $X \Rightarrow \mu_N\, Int(A) \subseteq \mu_N\, Int(B) = 0_N$ and hence $\mu_N\, Int(A) = 0_N$ for each μ_N first category set A in X. By theorem 4.4 X is a $\mu_N\, B_\sigma$ Baire space.

Theorem 4.6: If $\mu_N\, Cl(A) = 1_N$ for each $\mu_N G_\delta$ set A in X then X is a $\mu_N\, B_\sigma$ Baire space.

Proof: Let A be a $\mu_N\, \sigma$ first category set in X. Then $A \subseteq B$ where A is a non-empty $\mu_N\, F_\sigma$ set in X. Since B is a $\mu_N\, F_\sigma$ set, $\overline{A}$ is $\mu_N\, G_\delta$ set and then $\mu_N\, Cl(\overline{B}) = 1_N \Rightarrow \mu_N\, Int(A) = 0_N$. Now $A \subseteq B \Rightarrow \mu_N\, Int(A) \subseteq \mu_N\, Int(B) = 0_N$. Hence $\mu_N\, Int(A) = 0_N$. By theorem 4.4, X is a $\mu_N\, B_\sigma$ Baire space.

Theorem 4.7: If $\mu_N\, Int(\bigcup_{i=1}^{\infty} A_i) = 0_N$, where A_i's are μ_N closed set and $\mu_N\, \sigma$ rare set in X, then (X, μ_N) is a $\mu_N\, B_\sigma$ Baire space.

Proof: Given that $\mu_N\, Int(\bigcup_{i=1}^{\infty} A_i) = 0_N$, where A_i's are μ_N closed set and $\mu_N\, \sigma$ rare set. By corollary 3.11, A_i's are $\mu_N\, \sigma$ nowhere dense sets. Therefore $\mu_N\, Int(\bigcup_{i=1}^{\infty} A_i) = 0_N$, where A_i's are $\mu_N\, \sigma$ nowhere dense set and hence (X, μ_N) is a $\mu_N\, B_\sigma$ Baire space.

Remark 4.8: Every $\mu_N\, B_\sigma$ Baire space is a μ_N Baire space if every $\mu_N\, \sigma$ rare set is μ_N closed.

Theorem 4.9: Every $\mu_N\, \sigma$ Baire space isμ_N Baire space.

Theorem 4.10: Let (X, μ_N) be μ_NTS. Then the following are equivalent.

(i) (X, μ_N) is a $\mu_N\, \sigma$ Baire space.

(ii) $\mu_N \, Int(A) = 0_N$ for every $\mu_N \, \sigma$ first category set in X.

(iii) $\mu_N \, Cl(A) = 1_N$ for every $\mu_N \, \sigma$ residual set in X.

Proof: (i) ⇒ (ii) Let A be $\mu_N \, \sigma$ first category set in X. Then $A = \bigcup_{i=1}^{\infty} A_i$ where A_i's are $\mu_N \, \sigma$ nowhere dense sets and $\mu_N \, Int(A) = \mu_N \, Int(\bigcup_{i=1}^{\infty} A_i)$ since (X, μ_N) is a $\mu_N \, \sigma$ Baire space. $\mu_N \, Int(A) = 0_N$.

(ii)⇒(iii) Let A be $\mu_N \, \sigma$ residual set in X. Then $\overline{A}$ is a $\mu_N \, \sigma$ first category set in X. From (ii), $\mu_N \, Int(\overline{A}) = 0_N \Rightarrow \overline{\mu_N \, Cl(A)} = 0_N$. Hence $\mu_N \, Cl(A) = 1_N$.

(iii)⇒(i) Let A be $\mu_N \, \sigma$ first category set in X. Then $\delta A = \bigcup_{i=1}^{\infty} A_i$ where A_i's are $\mu_N \, \sigma$ nowhere dense sets. We have, if A is $\mu_N \, \sigma$ first category set in X then $\overline{A}$ is $\mu_N \, \sigma$ residual set. By (iii) we get $\mu_N \, Cl(\overline{A}) = 1_N$ which gives us that $\overline{\mu_N \, Int(A)} = 1_N$. Therefore $\mu_N \, Int(A) = 0_N$ and hence $\mu_N \, Int(\bigcup_{i=1}^{\infty} A_i) = 0_N$ where A_i's are $\mu_N \, \sigma$ rare set. Hence (X, μ_N) is a $\mu_N \, \sigma$ Baire space.

Theorem 4.11: If $\mu_N \, Int(A) = 0_N$, for each $\mu_N \, F_\sigma$ set A in X, then X is a $\mu_N \, \sigma$ Baire space.

Proof: Let A be $\mu_N \, \sigma$ first category set in X. Then $A \subseteq B$ where B is a non-empty $\mu_N \, F_\sigma$ set in X. $\Rightarrow \mu_N \, Int(A) \subseteq \mu_N \, Int(B) = 0_N$ and so $\mu_N \, Int(A) = 0_N$ for each $\mu_N \, \sigma$ first category set. By theorem 4.10 ⇒ X is a $\mu_N \, \sigma$ Baire space.

Theorem 4.12: If $\mu_N \, Cl(A) = 1_N$, for each $\mu_N G_\delta$ set A in X, then X is a $\mu_N \, \sigma$ Baire space.

Proof: Let A be a $\mu_N \, \sigma$ first category set in X. Then $A \subseteq B$ where B is a non-empty $\mu_N \, F_\sigma$ set in X. Since B is a $\mu_N \, F_\sigma$ set, $\overline{B}$ is $\mu_N \, G_\delta$ set and then $\mu_N \, Cl(\overline{B}) = 1_N \Rightarrow \mu_N \, Int(B) = 0_N$. Now $A \subseteq B \Rightarrow \mu_N \, Int(A) \subseteq \mu_N \, Int(B) = 0_N$. Hence $\mu_N \, Int(A) = 0_N$. By theorem 4.10,X is a $\mu_N \, \sigma$ Baire space.

5. Conclusion

In this paper, we provide many new sorts of sets, including μ_N strongly dense sets, μ_N strongly nowhere dense sets, μ_N strongly first category sets, and μ_N strongly nowhere residual sets, as well as a short explanation of the characteristics that distinguish each of these sets. In addition to this, with their help, we were able to obtain the μ_N powerfully Baire space.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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