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Neutrosophic gsα^*-Open and Closed Maps in Neutrosophic Topological Spaces

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Abstract : The main aim of this paper is to introduce a new concept of $N_{eu}-$ mapping namely $N_{eu}gs\alpha^*-$ open maps and $N_{eu}gs\alpha^*-$ closed maps in $N_{eu}-$ topological spaces . Additionally we relate the properties and characterizations of these mappings with the other mappings in $N_{eu}-$ topological spaces .

Keywords: $N_{eu}gs\alpha^*-$ open set , $N_{eu}gs\alpha^*-$ closed set , $N_{eu}gs\alpha^*-$open map , $N_{eu}gs\alpha^*-$closed map.

1. Introduction

Then the idea of $N_{eu}-$set theory was introduced by F.Smarandache[7] . It includes three components, truth , indeterminancy and false membership function . R.Dhavaseelan and S.Jafari[5] has introduced the concept of $N_{eu}g-$closed sets . A.A.Salama[11] has very first discussed about $N_{eu}-$continuous function and he also discussed about $N_{eu}-$ open and closed mapping . The real life application of $N_{eu}-$topology is applied in Information Systems , Applied Mathematics etc .

In this paper, we introduce some new concepts in $N_{eu}-$topological spaces such as $N_{eu}gs\alpha^*-$closed map and $N_{eu}gs\alpha^*-$open map .

2. Preliminaries

Definition 2.1:[12] Let $\mathbb{P}$ be a non-empty fixed set . A $N_{eu}-$set Ħ on the universe $\mathbb{P}$ is defined as Ħ= $\{\langle \wp,(t_{Ħ}(\wp),i_{Ħ}(\wp),f_{Ħ}(\wp))\rangle : \wp \in \mathbb{P}\}$ where $t_{Ħ}(\wp),i_{Ħ}(\wp),f_{Ħ}(\wp)$ represent the degree of membership $t_{Ħ}(\wp)$, indeterminacy $i_{Ħ}(\wp)$ and non-membership function $f_{Ħ}(\wp)$ respectively for each element $\wp \in \mathbb{P}$ to the set Ħ . Also , $t_{Ħ}$, $i_{Ħ}$, $f_{Ħ}$: $\mathbb{P} \to]^-0,1^+[$ and $^-0 \leq t_{Ħ}(\wp)+i_{Ħ}(\wp)+f_{Ħ}(\wp) \leq 3^+$. Set of all Neutrosophic set over $\mathbb{P}$ is denoted by $N_{eu}(\mathbb{P})$.

Definition 2.2:[12] A neutrosophic topology (N_{eu}T) on a non-empty set $\mathbb{P}$ is a family $\tau_{N_{eu}}$ of $N_{eu}-$sets in $\mathbb{P}$ satisfying the following axioms ,

(i) $0_{N_{eu}}$, $1_{N_{eu}} \in \tau_{N_{eu}}$.

(ii) $ᶏ_1 \cap ᶏ_2 \in \tau_{N_{eu}}$ for any $ᶏ_1$, $ᶏ_2 \in \tau_{N_{eu}}$.

(iii) $\cup ᶏ_i \in \tau_{N_{eu}}$ for every family $\{ ᶏ_i / i \in \Omega\imath \} \subseteq \tau_{N_{eu}}$.

In this case , the ordered pair $(\mathbb{P},\tau_{N_{eu}})$ or simply $\mathbb{P}$ is called a $N_{eu}-$topological space (N_{eu}TS) . The elements of $\tau_{N_{eu}}$ is neutrosophic open set ($N_{eu}-OS$) and $\tau_{N_{eu}}{}^c$ is neutrosophic closed set ($N_{eu}-CS$) .

Definition 2.3:[1] A $N_{eu}-$set ᶏ in a N_{eu}TS $(\mathbb{P},\tau_{N_{eu}})$ is called a neutrosophic generalized semi alpha star closed set ($N_{eu}gs\alpha^*-CS$) if $N_{eu}\alpha-int(N_{eu}\alpha-cl(ᶏ)) \subseteq N_{eu}-int(\mathcal{G})$, whenever $ᶏ \subseteq \mathcal{G}$ and $\mathcal{G}$ is $N_{eu}\alpha^*-OS$.

Definition 2.4:[2] A $N_{eu}TS$ $(\mathbb{P},\tau_{N_{eu}})$ is called a $N_{eu}gs\alpha^* - T_{1/2}$ space if every $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P},\tau_{N_{eu}})$ is a $N_{eu} - CS$ in $(\mathbb{P},\tau_{N_{eu}})$.

Definition 2.5:[5] Let $f_N : (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ be any $N_{eu}-$ function and $\mathbb{A} = \left\{\langle p, \left(t_{\mathbb{A}}(p), i_{\mathbb{A}}(p), f_{\mathbb{A}}(p)\right) \rangle : p \in \mathbb{P}\right\}$ be any $N_{eu}-$ set in $(\mathbb{P},\tau_{N_{eu}})$, then the image of $\mathbb{A}$ under f is denoted by $f_N(\mathbb{A})$, is a $N_{eu}-$ set in $(\mathbb{Q}, \sigma_{N_{eu}})$ and is defined by $f_N(\mathbb{A}) = \left\{\langle q, \left(f\left(t_{\mathbb{A}}(q)\right), f\left(i_{\mathbb{A}}(q)\right), f\left(f_{\mathbb{A}}(q)\right)\right) \rangle : q \in \mathbb{Q}\right\}$,

where $$f_N\left(t_{\mathbb{A}}(q)\right) = \begin{cases} Sup_{p\epsilon f_N^{-1}(q)}\ t_{\mathbb{A}}(p)\ , & if\ f_N^{-1}(q) \neq \emptyset \\ 0 & , \quad otherwise \end{cases}$$

$$f_N\left(i_{\mathbb{A}}(q)\right) = \begin{cases} Sup_{p\epsilon f_N^{-1}(q)}\ i_{\mathbb{A}}(p)\ , & if\ f_N^{-1}(q) \neq \emptyset \\ 0 & , \quad otherwise \end{cases}$$

$$\left(1 - f_N(1 - f_{\mathbb{A}})\right)(q) = \begin{cases} inf_{p\epsilon f_N^{-1}(q)}\ f_{\mathbb{A}}(p)\ , & if\ f_N^{-1}(q) \neq \emptyset \\ 1 & , \quad otherwise \end{cases} .$$

3. Neutrosophic $gs\alpha^*$ – Open and Closed Maps

Definition 3.1: A $N_{eu}-$ function $f_N: (\mathbb{P},\tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ is $N_{eu}gs\alpha^* -$ closed map if the image of every $N_{eu} - CS$ in $(\mathbb{P},\tau_{N_{eu}})$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. (ie) $f_N(\mathbb{A})$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$, for every $N_{eu} - CS$ $\mathbb{A}$ in $(\mathbb{P},\tau_{N_{eu}})$. The complement of $N_{eu}gs\alpha^* -$ closed map is $N_{eu}gs\alpha^* -$ open map .

Theorem 3.2: Every $N_{eu} -$ closed map[10] is $N_{eu}gs\alpha^* -$ closed map , but not conversely.

Example 3.3: Let $\mathbb{P}$ ={p} and $\mathbb{Q}$ ={q} . $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{B}\}$ are N_{eu}TS on $(\mathbb{P},\tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$, $\mathbb{A}$= {⟨p, (0.4,0.6,0.8)⟩} and $\mathbb{B}$= {⟨q, (0.2,0.4,0.6)⟩} . Define a map $f_N: (\mathbb{P},\tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ by $f_N(p) = q$. Let $\mathbb{A}^c$ = {⟨p, (0.8,0.4,0.4)⟩} be a $N_{eu} - CS$ in $(\mathbb{P},\tau_{N_{eu}})$. Then $f_N(\mathbb{A}^c)$ = {⟨q, (0.8 , 0.4 , 0.4)⟩} is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow f_N$ is $N_{eu}gs\alpha^* -$ closed map . But f_N is not $N_{eu} -$ closed map , because $f_N(\mathbb{A}^c)$ is not $N_{eu} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$.

Theorem 3.4: Every $N_{eu}\alpha -$ closed map[10] is $N_{eu}gs\alpha^* -$ closed map , but not conversely.

Example 3.5: Let $\mathbb{P}$ ={p} and $\mathbb{Q}$ ={q} . $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{B}\}$ are N_{eu}TS on $(\mathbb{P},\tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$, $\mathbb{A}$= {⟨p, (0.3,0.8,0.6)⟩} and $\mathbb{B}$= {⟨q, (0.3,0.2,0.8)⟩} . Define a map $f_N: (\mathbb{P},\tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ by $f_N(p) = q$. Let $\mathbb{A}^c$ = {⟨p , (0.6,0.2,0.3)⟩} be a $N_{eu} - CS$ in $(\mathbb{P},\tau_{N_{eu}})$. Then $f_N(\mathbb{A}^c)$ = {⟨ q , (0.6 , 0.2 , 0.3)⟩} is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow f_N$ is $N_{eu}gs\alpha^* -$ closed map . But f_N is not $N_{eu}\alpha -$ closed map , because $f_N(\mathbb{A}^c)$ is not $N_{eu}\alpha - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$.

Theorem 3.6: Every $N_{eu}S -$ closed map[6] is $N_{eu}gs\alpha^* -$ closed map , but not conversely.

Example 3.7: Let $\mathbb{P}$ = {p} and $\mathbb{Q}$ = {q} . $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{B}\}$ are N_{eu}TS on $(\mathbb{P},\tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$, $\mathbb{A}$= {⟨p, (0.2,0.7,0.8)⟩} and $\mathbb{B}$= {⟨q, (0.4,0.3,0.6)⟩} . Define a map $f_N: (\mathbb{P},\tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ by $f_N(p) = q$. Let $\mathbb{A}^c$ = {⟨p , (0.8,0.3,0.2)⟩} be a $N_{eu} - CS$ in $(\mathbb{P},\tau_{N_{eu}})$. Then $f_N(\mathbb{A}^c)$ = {⟨q , (0.8 , 0.3 , 0.2)⟩} is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow f_N$ is $N_{eu}gs\alpha^* -$ closed map . But f_N is not $N_{eu}S -$ closed map , because $f_N(\mathbb{A}^c)$ is not $N_{eu}S - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$.

Theorem 3.8: Every $N_{eu}gs\alpha^* -$ closed map is $N_{eu}\beta -$ closed map[10] , but not conversely.

Example 3.9: Let $\mathbb{P}$ = {p} and $\mathbb{Q}$ = {q} . $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{B}\}$ are N_{eu}TS on $(\mathbb{P},\tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$, $\mathbb{A}$= {⟨p, (0.5,0.6,0.4)⟩} and $\mathbb{B}$= {⟨q, (0.6,0.8,0.4)⟩} . Define a map $f_N: (\mathbb{P},\tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ by $f_N(p) = q$. Let $\mathbb{A}^c$ = {⟨p , (0.4,0.4,0.5)⟩} be a $N_{eu} - CS$ in $(\mathbb{P},\tau_{N_{eu}})$. Then

$f_N(\mathbb{A}^c) = \{\langle q, (0.4,\ 0.4,\ 0.5)\rangle\}$ is $N_{eu}\beta - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow f_N$ is $N_{eu}\beta -$ closed map . But f_N is not $N_{eu}gs\alpha^* -$ closed map , because $f_N(\mathbb{A}^c)$ is not $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$.

Theorem 3.10: Every $N_{eu}gs\alpha^* -$ closed map is $N_{eu}\pi g\beta -$ closed map[9] , but not conversely.

Example 3.11: Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{B}\}$ are N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$, $\mathbb{A} = \{\langle p, (0.2,0.5,0.3)\rangle\}$ and $\mathbb{B} = \{\langle q, (0.4,0.6,0.2)\rangle\}$. Define a map $f_N: (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ by $f_N(p) = q$. Let $\mathbb{A}^c = \{\langle p, (0.3,0.5,0.2)\rangle\}$ be a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Then $f_N(\mathbb{A}^c) = \{\langle q, (0.3, 0.5, 0.2)\rangle\}$ is $N_{eu}\pi g\beta - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow f_N$ is $N_{eu}\pi g\beta -$ closed map . But f_N is not $N_{eu}gs\alpha^* -$ closed map , because $f_N(\mathbb{A}^c)$ is not $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$.

Remark 3.12: The concept of $N_{eu}G^* -$ closed map[4] and $N_{eu}gs\alpha^* -$ closed map are independent .

Example 3.13: Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{B}\}$ are N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$, $\mathbb{A} = \{\langle p, (0.8,0.9,0.7)\rangle\}$ and $\mathbb{B} = \{\langle q, (0.5,0.3,0.8)\rangle\}$. Define a map $f_N: (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ by $f_N(p) = q$. Let $\mathbb{A}^c = \{\langle p, (0.7,0.1,0.8)\rangle\}$ be a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Then $f_N(\mathbb{A}^c) = \{\langle q, (0.7, 0.1, 0.8)\rangle\}$ is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow f_N$ is $N_{eu}gs\alpha^* -$ closed map . But f_N is not $N_{eu}G^* -$ closed map , because $f_N(\mathbb{A}^c)$ is not $N_{eu}G^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$.

Example 3.14: Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{B}\}$ are N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$, $\mathbb{A} = \{\langle p, (0.4,0.3,0.9)\rangle\}$ and $\mathbb{B} = \{\langle q, (0.7,0.4,0.6)\rangle\}$. Define a map $f_N: (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ by $f_N(p) = q$. Let $\mathbb{A}^c = \{\langle p, (0.9,0.7,0.4)\rangle\}$ be a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Then $f_N(\mathbb{A}^c) = \{\langle q, (0.9, 0.7, 0.4)\rangle\}$ is $N_{eu}G^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow f_N$ is $N_{eu}G^* -$closed map . But f_N is not $N_{eu}gs\alpha^* -$ closed map , because $f_N(\mathbb{A}^c)$ is not $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$.

Remark 3.15: The concept of $N_{eu}g -$ closed map[10] and $N_{eu}gs\alpha^* -$ closed map are independent .

Example 3.16: Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{B}\}$ are N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$, $\mathbb{A} = \{\langle p, (0.7,0.7,0.2)\rangle\}$ and $\mathbb{B} = \{\langle q, (0.4,0.3,0.6)\rangle\}$. Define a map $f_N: (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ by $f_N(p) = q$. Let $\mathbb{A}^c = \{\langle p, (0.2,0.3,0.7)\rangle\}$ be a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Then $f_N(\mathbb{A}^c) = \{\langle q, (0.2, 0.3, 0.7)\rangle\}$ is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow f_N$ is $N_{eu}gs\alpha^* -$ closed map . But f_N is not $N_{eu}g -$ closed map , because $f_N(\mathbb{A}^c)$ is not $N_{eu}g - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$.

Example 3.17: Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{B}\}$ are N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$, $\mathbb{A} = \{\langle p, (0.9,0.8,0.8)\rangle\}$ and $\mathbb{B} = \{\langle q, (0.6,0.8,0.4)\rangle\}$. Define a map $f_N: (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ by $f_N(p) = q$. Let $\mathbb{A}^c = \{\langle p, (0.8,0.2,0.9)\rangle\}$ be a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Then $f_N(\mathbb{A}^c) = \{\langle q, (0.8, 0.2, 0.9)\rangle\}$ is $N_{eu}g - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow f_N$ is $N_{eu}g -$closed map . But f_N is not $N_{eu}gs\alpha^* -$closed map , because $f_N(\mathbb{A}^c)$ is not $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$.

Remark 3.18: The concept of $N_{eu}P -$ closed map[10] and $N_{eu}gs\alpha^* -$ closed map are independent .

Example 3.19: Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{B}\}$ are N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$, $\mathbb{A} = \{\langle p, (0.4,0.4,0.5)\rangle\}$ and $\mathbb{B} = \{\langle q, (0.3,0.2,0.8)\rangle\}$. Define a map $f_N: (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ by $f_N(p) = q$. Let $\mathbb{A}^c = \{\langle p, (0.5,0.6,0.4)\rangle\}$ be a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Then $f_N(\mathbb{A}^c) = \{\langle q, (0.5,\ 0.6,\ 0.4)\rangle\}$ is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow f_N$ is $N_{eu}gs\alpha^* -$closed map . But f_N is not $N_{eu}P -$ closed map , because $f_N(\mathbb{A}^c)$ is not $N_{eu}P - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$.

Example 3.20: Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{B}\}$ are N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$, $\mathbb{A} = \{\langle p, (0.8,0.4,0.3)\rangle\}$ and $\mathbb{B} = \{\langle q, (0.7,0.6,0.5)\rangle\}$. Define a map $f_N: (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ by $f_N(p) = q$. Let $\mathbb{A}^c = \{\langle p, (0.3,0.6,0.8)\rangle\}$ be a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Then $f_N(\mathbb{A}^c) = \{\langle q, (0.3,\ 0.6,\ 0.8)\rangle\}$ is $N_{eu}P - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow f_N$ is $N_{eu}P -$closed map. But f_N is not $N_{eu}gs\alpha^* -$closed map , because $f_N(\mathbb{A}^c)$ is not $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$.

Remark 3.21: The concept of $N_{eu}bg -$ closed map[8] and $N_{eu}gs\alpha^* -$ closed map are independent .

Example 3.22: Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, A\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, B\}$ are N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$, $A = \{\langle p, (0.6,0.1,0.7)\rangle\}$ and $B = \{\langle q, (0.5, 0.3,0.8)\rangle\}$. Define a map $f_N: (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ by $f_N(p) = q$. Let $A^c = \{\langle p, (0.7,0.9,0.6)\rangle\}$ be a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Then $f_N(A^c) = \{\langle q, (0.7, 0.9, 0.6)\rangle\}$ is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow f_N$ is $N_{eu}gs\alpha^*$ –closed map . But f_N is not $N_{eu}bg -$ closed map , because $f_N(A^c)$ is not $N_{eu}bg - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$.

Example 3.23: Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, A\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, B\}$ are N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$, $A = \{\langle p, (0.8,0.6,0.6)\rangle\}$ and $B = \{\langle q, (0.7,0.8,0.3)\rangle\}$. Define a map $f_N: (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ by $f_N(p) = q$. Let $A^c = \{\langle p, (0.6,0.4,0.8)\rangle\}$ be a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Then $f_N(A^c) = \{\langle q, (0.6, 0.4, 0.8)\rangle\}$ is $N_{eu}bg - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow f_N$ is $N_{eu}bg -$ closed map . But f_N is not $N_{eu}gs\alpha^*$ –closed map , because $f_N(A^c)$ is not $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$.

Remark 3.24: Let $f: (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ and $g: (\mathbb{Q}, \sigma_{N_{eu}}) \to (\mathbb{R}, \gamma_{N_{eu}})$ be $N_{eu}gs\alpha^* -$ closed map , then $gof : (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{R}, \gamma_{N_{eu}})$ need not be $N_{eu}gs\alpha^* -$ closed map .

Example 3.25: Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, A\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, B\}$ are N_{eu}TS on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$, $A = \{\langle p, (0.6,0.3,0.9)\rangle\}$ and $B = \{\langle q, (0.4,0.5,0.7)\rangle\}$. Define a map $f_N: (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ by $f_N(p) = q$. Let $A^c = \{\langle p, (0.9,0.7,0.6)\rangle\}$ be a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Then $f_N(A^c) = \{\langle q, (0.9, 0.7, 0.6)\rangle\}$ is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow f_N$ is $N_{eu}gs\alpha^* -$ closed map . Let $\mathbb{R} = \{r\}$. Also, $C = \{\langle r, (0.2,0.7,0.8)\rangle\}$ is $N_{eu}(\mathbb{R})$ and $\gamma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, C\}$ is N_{eu}TS on $(\mathbb{R}, \gamma_{N_{eu}})$. Define a map $g_N : (\mathbb{Q}, \sigma_{N_{eu}}) \to (\mathbb{R}, \gamma_{N_{eu}})$ by $g_N(q - 0.2) = r$. Let $B^c = \{\langle q, (0.7,0.5,0.4)\rangle\}$ be a $N_{eu} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $g_N(B^c) = \{\langle r, (0.5, 0.3, 0.2)\rangle\}$ is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{R}, \gamma_{N_{eu}}) \Rightarrow g_N$ is $N_{eu}gs\alpha^* -$ closed map . Define a map $g_N o f_N : (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{R}, \gamma_{N_{eu}})$ by $g_N o f_N(p - 0.2) = r$. But $g_N o f_N$ is not $N_{eu}gs\alpha^* -$ closed map , because $g_N o f_N(A^c)$ is not $N_{eu}gs\alpha^* - CS$ in $(\mathbb{R}, \gamma_{N_{eu}})$.

Theorem 3.26: Let $f_N: (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ and $g_N : (\mathbb{Q}, \sigma_{N_{eu}}) \to (\mathbb{R}, \gamma_{N_{eu}})$ be $N_{eu}gs\alpha^* -$ closed map . Also , $(\mathbb{Q}, \sigma_{N_{eu}})$ is $N_{eu}gs\alpha^* - T_{1/2}$ space , then $g_N o f_N : (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{R}, \gamma_{N_{eu}})$ is $N_{eu}gs\alpha^* -$ closed map .

Proof: Let A be any $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Given f_N is $N_{eu}gs\alpha^* -$ closed map , then $f_N(A)$ is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given $(\mathbb{Q}, \sigma_{N_{eu}})$ is $N_{eu}gs\alpha^* - T_{1/2}$ space , then $f_N(A)$ is $N_{eu} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given g_N is $N_{eu}gs\alpha^* -$ closed map , then $g_N(f_N(A)) = g_N o f_N(A)$ is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{R}, \gamma_{N_{eu}}) \Rightarrow g_N o f_N$ is $N_{eu}gs\alpha^* -$ closed map .

4. Properties of Neutrosophic $gs\alpha^*$ –Open and Closed Maps

Theorem 4.1: Let $f_N: (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ be $N_{eu}\alpha -$ closed map and $g_N : (\mathbb{Q}, \sigma_{N_{eu}}) \to (\mathbb{R}, \gamma_{N_{eu}})$ be $N_{eu}gs\alpha^* -$ closed map . Also , $(\mathbb{Q}, \sigma_{N_{eu}})$ is $N_{eu}gs\alpha^* - T_{1/2}$ space , then $g_N o f_N : (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{R}, \gamma_{N_{eu}})$ is $N_{eu}gs\alpha^* -$ closed map .

Proof: Let A be any $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Given f_N is $N_{eu}\alpha -$ closed map , then $f_N(A)$ is $N_{eu}\alpha - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow f_N(A)$ is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given $(\mathbb{Q}, \sigma_{N_{eu}})$ is $N_{eu}gs\alpha^* - T_{1/2}$ space , then $f_N(A)$ is $N_{eu} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given g_N is $N_{eu}gs\alpha^* -$ closed map , then $g_N(f_N(A)) = g_N o f_N(A)$ is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{R}, \gamma_{N_{eu}}) \Rightarrow g_N o f_N$ is $N_{eu}gs\alpha^* -$ closed map .

Remark 4.1(a): The above theorem is true if we replace $f_N: (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ as $N_{eu}S -$ closed map, $N_{eu}\alpha^* -$ closed map , $N_{eu}R -$ closed map , $N_{eu}S\alpha -$ closed map and $N_{eu}g\alpha -$ closed map .

Theorem 4.2: Let $f_N: (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ be $N_{eu} -$ closed map and $g_N : (\mathbb{Q}, \sigma_{N_{eu}}) \to (\mathbb{R}, \gamma_{N_{eu}})$ be $N_{eu}gs\alpha^* -$ closed map , then $g_N o f_N : (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{R}, \gamma_{N_{eu}})$ is $N_{eu}gs\alpha^* -$ closed map .

Proof: Let $\mathbb{A}$ be any $N_{eu}-CS$ in $(\mathbb{P},\tau_{N_{eu}})$. Given f_N is $N_{eu}-$ closed map, then $f_N(\mathbb{A})$ is $N_{eu}-CS$ in $(\mathbb{Q},\sigma_{N_{eu}})$. Given g_N is $N_{eu}gs\alpha^*-$ closed map, then $g_N(f_N(\mathbb{A}))=g_Nof_N(\mathbb{A})$ is $N_{eu}gs\alpha^*-CS$ in $(\mathbb{R},\gamma_{N_{eu}}) \Rightarrow g_Nof_N$ is $N_{eu}gs\alpha^*-$ closed map.

Theorem 4.3: Let $f_N:(\mathbb{P},\tau_{N_{eu}}) \to (\mathbb{Q},\sigma_{N_{eu}})$ be $N_{eu}\alpha-$ continuous, surjective and $(\mathbb{P},\tau_{N_{eu}})$ be $N_{eu}gs\alpha^*-T_{1/2}$ space. Also, $g_Nof_N:(\mathbb{P},\tau_{N_{eu}}) \to (\mathbb{R},\gamma_{N_{eu}})$ be $N_{eu}gs\alpha^*-$ closed map, then $g_N:(\mathbb{Q},\sigma_{N_{eu}}) \to (\mathbb{R},\gamma_{N_{eu}})$ is $N_{eu}gs\alpha^*-$ closed map.

Proof: Let $\mathbb{A}$ be any $N_{eu}-CS$ in $(\mathbb{Q},\sigma_{N_{eu}})$. Given f_N is $N_{eu}\alpha-$ continuous, then ${f_N}^{-1}(\mathbb{A})$ is $N_{eu}\alpha-CS$ in $(\mathbb{P},\tau_{N_{eu}}) \Rightarrow {f_N}^{-1}(\mathbb{A})$ is $N_{eu}gs\alpha^*-CS$ in $(\mathbb{P},\tau_{N_{eu}})$. Given $(\mathbb{P},\tau_{N_{eu}})$ be $N_{eu}gs\alpha^*-T_{1/2}$ space, then ${f_N}^{-1}(\mathbb{A})$ is $N_{eu}-CS$ in $(\mathbb{P},\tau_{N_{eu}})$. Given $g_Nof_N:(\mathbb{P},\tau_{N_{eu}}) \to (\mathbb{R},\gamma_{N_{eu}})$ is $N_{eu}gs\alpha^*-$ closed map, then $g_Nof_N({f_N}^{-1}(\mathbb{A}))$ is $N_{eu}gs\alpha^*-CS$ in $(\mathbb{R},\gamma_{N_{eu}})$. Given f_N is surjective, then $g_N(\mathbb{A})$ is $N_{eu}gs\alpha^*-CS$ in $(\mathbb{R},\gamma_{N_{eu}}) \Rightarrow g_N$ is $N_{eu}gs\alpha^*-$ closed map.

Remark 4.3(a): The above theorem is true if we replace f_N as $N_{eu}S-$continuous, $N_{eu}\alpha^*-$ continuous, $N_{eu}R-$ continuous, $N_{eu}S\alpha-$ continuous and $N_{eu}g\alpha-$ continuous.

Theorem 4.4: Let $f_N:(\mathbb{P},\tau_{N_{eu}}) \to (\mathbb{Q},\sigma_{N_{eu}})$ be $N_{eu}-$ continuous and surjective. Also, $g_Nof_N:(\mathbb{P},\tau_{N_{eu}}) \to (\mathbb{R},\gamma_{N_{eu}})$ be $N_{eu}gs\alpha^*-$ closed map, then $g_N:(\mathbb{Q},\sigma_{N_{eu}}) \to (\mathbb{R},\gamma_{N_{eu}})$ is $N_{eu}gs\alpha^*-$ closed map.

Proof: Let $\mathbb{A}$ be any $N_{eu}-CS$ in $(\mathbb{Q},\sigma_{N_{eu}})$. Given f_N is $N_{eu}-$ continuous, then ${f_N}^{-1}(\mathbb{A})$ is $N_{eu}-CS$ in $(\mathbb{P},\tau_{N_{eu}})$. Given $g_Nof_N:(\mathbb{P},\tau_{N_{eu}}) \to (\mathbb{R},\gamma_{N_{eu}})$ is $N_{eu}gs\alpha^*-$ closed map, then $g_Nof_N(f^{-1}(\mathbb{A}))$ is $N_{eu}gs\alpha^*-CS$ in $(\mathbb{R},\gamma_{N_{eu}}) \Rightarrow g_N(\mathbb{A})$ is $N_{eu}gs\alpha^*-CS$ in $(\mathbb{R},\gamma_{N_{eu}}) \Rightarrow g_N$ is $N_{eu}gs\alpha^*-$ closed map.

Theorem 4.5: Let $f_N:(\mathbb{P},\tau_{N_{eu}}) \to (\mathbb{Q},\sigma_{N_{eu}})$ and $g_N:(\mathbb{Q},\sigma_{N_{eu}}) \to (\mathbb{R},\gamma_{N_{eu}})$ be two $N_{eu}-$ mappings, such that their composition $g_Nof_N:(\mathbb{P},\tau_{N_{eu}}) \to (\mathbb{R},\gamma_{N_{eu}})$ be $N_{eu}gs\alpha^*-$ closed map. Then the following statements are true.

(1) If g_N is $N_{eu}gs\alpha^*-$ irresolute[2] and injective, then f_N is $N_{eu}gs\alpha^*-$ closed map.

(2) If g_N is strongly $N_{eu}gs\alpha^*-$ continuous[3] and injective, then f_N is $N_{eu}gs\alpha^*-$ closed map.

(3) If g_N is perfectly $N_{eu}gs\alpha^*-$ continuous[3] and injective, then f_N is $N_{eu}gs\alpha^*-$ closed map.

Proof: (1), Let $\mathbb{A}$ be any $N_{eu}-CS$ in $(\mathbb{P},\tau_{N_{eu}})$. Given $g_Nof_N:(\mathbb{P},\tau_{N_{eu}}) \to (\mathbb{R},\gamma_{N_{eu}})$ is $N_{eu}gs\alpha^*-$ closed map, then $g_Nof_N(\mathbb{A})$ is $N_{eu}gs\alpha^*-CS$ in $(\mathbb{R},\gamma_{N_{eu}})$. Given g_N is $N_{eu}gs\alpha^*-$ irresolute, then ${g_N}^{-1}(g_Nof_N(\mathbb{A}))$ is $N_{eu}gs\alpha^*-CS$ in $(\mathbb{Q},\sigma_{N_{eu}})$. Given g_N is injective, then ${g_N}^{-1}(g_Nof_N(\mathbb{A}))=f_N(\mathbb{A})$ is $N_{eu}gs\alpha^*-CS$ in $(\mathbb{Q},\sigma_{N_{eu}}) \Rightarrow f_N$ is $N_{eu}gs\alpha^*-$ closed map.

(2), Let $\mathbb{A}$ be any $N_{eu}-CS$ in $(\mathbb{P},\tau_{N_{eu}})$. Given $g_Nof_N:(\mathbb{P},\tau_{N_{eu}}) \to (\mathbb{R},\gamma_{N_{eu}})$ is $N_{eu}gs\alpha^*-$ closed map, then $g_Nof_N(\mathbb{A})$ is $N_{eu}gs\alpha^*-CS$ in $(\mathbb{R},\gamma_{N_{eu}})$. Given g_N is strongly $N_{eu}gs\alpha^*-$ continuous, then ${g_N}^{-1}(g_Nof_N(\mathbb{A}))$ is $N_{eu}-CS$ in $(\mathbb{Q},\sigma_{N_{eu}})$. Given g is injective, then ${g_N}^{-1}(g_Nof_N(\mathbb{A}))=f_N(\mathbb{A})$ is $N_{eu}-CS$ in $(\mathbb{Q},\sigma_{N_{eu}}) \Rightarrow f_N(\mathbb{A})$ is $N_{eu}gs\alpha^*-CS$ in $(\mathbb{Q},\sigma_{N_{eu}}) \Rightarrow f_N$ is $N_{eu}gs\alpha^*-$ closed map.

(3), Let $\mathbb{A}$ be any $N_{eu}-CS$ in $(\mathbb{P},\tau_{N_{eu}})$. Given $g_Nof_N:(\mathbb{P},\tau_{N_{eu}}) \to (\mathbb{R},\gamma_{N_{eu}})$ is $N_{eu}gs\alpha^*-$ closed map, then $g_Nof_N(\mathbb{A})$ is $N_{eu}gs\alpha^*-CS$ in $(\mathbb{R},\gamma_{N_{eu}})$. Given g_N is perfectly $N_{eu}gs\alpha^*-$ continuous, then ${g_N}^{-1}(g_Nof_N(\mathbb{A}))$ is both $N_{eu}-OS$ and $N_{eu}-CS$ in $(\mathbb{Q},\sigma_{N_{eu}})$ .Given g_N is injective, then ${g_N}^{-1}(g_Nof_N(\mathbb{A}))=f_N(\mathbb{A})$ is both $N_{eu}-OS$ and $N_{eu}-CS$ in $(\mathbb{Q},\sigma_{N_{eu}}) \Rightarrow f_N(\mathbb{A})$ is $N_{eu}gs\alpha^*-CS$ in $(\mathbb{Q},\sigma_{N_{eu}}) \Rightarrow f_N$ is $N_{eu}gs\alpha^*-$ closed map.

Theorem 4.6: Let $f_N: (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ be $N_{eu}gs\alpha^* -$ closed map and $g_N : (\mathbb{Q}, \sigma_{N_{eu}}) \to (\mathbb{R}, \gamma_{N_{eu}})$ be $N_{eu} -$ closed map , then $g_N o f_N : (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{R}, \gamma_{N_{eu}})$ is $N_{eu}gs\alpha^* -$ closed map , if $(\mathbb{Q}, \sigma_{N_{eu}})$ is $N_{eu}gs\alpha^* - T_{1/2}$ space.

Proof: Let $\mathbb{A}$ be any $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Given f_N is $N_{eu}gs\alpha^* -$ closed map , then $f_N(\mathbb{A})$ is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given $(\mathbb{Q}, \sigma_{N_{eu}})$ is $N_{eu}gs\alpha^* - T_{1/2}$ space , then $f_N(\mathbb{A})$ is $N_{eu} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given g_N is $N_{eu} -$ closed map , then $g_N(f_N(\mathbb{A}))$ is $N_{eu} - CS$ in $(\mathbb{R}, \gamma_{N_{eu}}) \Rightarrow g_N o f_N(\mathbb{A})$ is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{R}, \gamma_{N_{eu}}) \Rightarrow g_N o f_N$ is $N_{eu}gs\alpha^* -$ closed map .

Theorem 4.7: Let $f_N: (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ and $g_N : (\mathbb{Q}, \sigma_{N_{eu}}) \to (\mathbb{R}, \gamma_{N_{eu}})$ be two $N_{eu} -$ mappings. Then the following statements are true .

(1) If $g_N o f_N : (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{R}, \gamma_{N_{eu}})$ is $N_{eu}gs\alpha^* -$ open map and f_N is $N_{eu} -$ continuous , then g_N is $N_{eu}gs\alpha^* -$ open map .

(2) If $g_N o f_N : (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{R}, \gamma_{N_{eu}})$ is $N_{eu} -$ closed map and g_N is $N_{eu}gs\alpha^* -$ continuous , then f_N is $N_{eu}gs\alpha^* -$ closed map .

Proof: (1) , Let $\mathbb{A}$ be any $N_{eu} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given f_N is $N_{eu} -$ continuous , then ${f_N}^{-1}(\mathbb{A})$ is $N_{eu} - OS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Given $g_N o f_N$ is $N_{eu}gs\alpha^* -$ open map , then $g_N o f_N\left({f_N}^{-1}(\mathbb{A})\right) = g_N(\mathbb{A})$ is $N_{eu}gs\alpha^* - OS$ in $(\mathbb{R}, \gamma_{N_{eu}}) \Rightarrow g_N$ is $N_{eu}gs\alpha^* -$ open map .

(2) , Let $\mathbb{A}$ be any $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Given $g_N o f_N$ is $N_{eu} -$ closed map , then $g_N o f_N(\mathbb{A})$ is $N_{eu} - CS$ in $(\mathbb{R}, \gamma_{N_{eu}})$. Given g_N is $N_{eu}gs\alpha^*$ –continuous , then ${g_N}^{-1}(g_N o f_N(\mathbb{A})) = f_N(\mathbb{A})$ is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow f_N$ is $N_{eu}gs\alpha^* -$ closed map .

Theorem 4.8: Let $f_N: (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ be a bijective $N_{eu} -$ mapping . Then the following are equivalent.

(1) f_N is $N_{eu}gs\alpha^* -$ open map , (2) f_N is $N_{eu}gs\alpha^* -$ closed map , (3) ${f_N}^{-1}$ is $N_{eu}gs\alpha^*$ –continuous[2] .

Proof: (1) ⇒ (2) , Let $\mathbb{A}$ be any $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow \mathbb{A}^c$ is $N_{eu} - OS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Given f_N is $N_{eu}gs\alpha^* -$ open map , then $f_N(\mathbb{A}^c) = (f_N(\mathbb{A}))^c$ is $N_{eu}gs\alpha^* - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow f_N(\mathbb{A})$ is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow f_N$ is $N_{eu}gs\alpha^* -$ closed map .

(2) ⇒ (3) , Let $\mathbb{A}$ be any $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Given f_N is $N_{eu}gs\alpha^* -$ closed map , then $f_N(\mathbb{A})$ is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given f_N is bijective , then $\left({f_N}^{-1}\right)^{-1}(\mathbb{A}) = f_N(\mathbb{A})$ is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow {f_N}^{-1}$ is $N_{eu}gs\alpha^*$ –continuous .

(3) ⇒ (1) , Let $\mathbb{A}$ be any $N_{eu} - OS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Given ${f_N}^{-1}$ is $N_{eu}gs\alpha^*$ –continuous , then $\left({f_N}^{-1}\right)^{-1}(\mathbb{A}) = f_N(\mathbb{A})$ is $N_{eu}gs\alpha^* - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow f_N$ is $N_{eu}gs\alpha^* -$ open map .

Theorem 4.9: Let $f_N: (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ be $N_{eu}gs\alpha^* -$ closed map and $(\mathbb{Q}, \sigma_{N_{eu}})$ be $N_{eu}gs\alpha^* - T_{1/2}$ space , then f_N is $N_{eu} -$ closed map .

Proof: Let $\mathbb{A}$ be any $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Given f_N is $N_{eu}gs\alpha^* -$ closed map , then $f_N(\mathbb{A})$ is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given $(\mathbb{Q}, \sigma_{N_{eu}})$ is $N_{eu}gs\alpha^* - T_{1/2}$ space , then $f_N(\mathbb{A})$ is $N_{eu} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow f_N$ is $N_{eu} -$ closed map .

Theorem 4.10: Let $f_N: (\mathbb{P}, \tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ be $N_{eu}gs\alpha^* -$ closed map and $(\mathbb{Q}, \sigma_{N_{eu}})$ be $N_{eu}gs\alpha^* - T_{1/2}$ space , then f_N is $N_{eu}\alpha -$ closed map .

Proof: Let $\mathbb{A}$ be any $N_{eu}-CS$ in $(\mathbb{P},\tau_{N_{eu}})$. Given f_N is $N_{eu}gs\alpha^*-$ closed map, then $f_N(\mathbb{A})$ is $N_{eu}gs\alpha^*-CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given $(\mathbb{Q}, \sigma_{N_{eu}})$ is $N_{eu}gs\alpha^*-T_{1/2}$ space, then $f_N(\mathbb{A})$ is $N_{eu}-CS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow f_N(\mathbb{A})$ is $N_{eu}\alpha-CS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow f_N$ is $N_{eu}\alpha-$ closed map.

Remark 4.10(a): The above theorem is true if we replace f as $N_{eu}S-$closed map, $N_{eu}\alpha^*-$ closed map, $N_{eu}R-$ closed map, $N_{eu}S\alpha-$ closed map and $N_{eu}g\alpha-$ closed map.

Theorem 4.11: Let $f_N:(\mathbb{P},\tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ be a bijective $N_{eu}-$mapping and f_N^{-1} is $N_{eu}gs\alpha^*-$ irresolute, then f_N is $N_{eu}gs\alpha^*-$ closed map.

Proof: Let $\mathbb{A}$ be any $N_{eu}-CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow \mathbb{A}$ is $N_{eu}gs\alpha^*-CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Given $f_N{}^{-}{}_1$ is $N_{eu}gs\alpha^*-$ irresolute, then $(f_N{}^{-1})^{-1}(\mathbb{A})$ is $N_{eu}gs\alpha^*-CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given f_N is bijective, then $(f_N{}^{-1})^{-1}(\mathbb{A}) = f_N(\mathbb{A})$ is $N_{eu}gs\alpha^*-CS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow f_N$ is $N_{eu}gs\alpha^*-$closed map.

Theorem 4.12: Let $f_N:(\mathbb{P},\tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ be $N_{eu}gs\alpha^*-$ open map iff $f_N(N_{eu}-int(\mathbb{A})) \subseteq N_{eu}gs\alpha^*-int(f_N(\mathbb{A}))$, for each $N_{eu}-$ set $\mathbb{A}$ in $(\mathbb{P},\tau_{N_{eu}})$.

Proof: Let $\mathbb{A}$ be any $N_{eu}-$ set in $(\mathbb{P},\tau_{N_{eu}}) \Rightarrow N_{eu}-int(\mathbb{A})$ is $N_{eu}-OS$ in $(\mathbb{P},\tau_{N_{eu}})$. Given f_N is $N_{eu}gs\alpha^*-$ open map, then $f_N(N_{eu}-int(\mathbb{A}))$ is $N_{eu}gs\alpha^*-OS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow N_{eu}gs\alpha^*-int\left(f_N(N_{eu}-int(\mathbb{A}))\right) = f_N(N_{eu}-int(\mathbb{A}))$. Given $f_N(N_{eu}-int(\mathbb{A})) \subseteq f_N(\mathbb{A})$, then $f_N(N_{eu}-int(\mathbb{A})) = N_{eu}gs\alpha^*-int\left(f_N(N_{eu}-int(\mathbb{A}))\right) \subseteq N_{eu}gs\alpha^*-int(f_N(\mathbb{A}))$. Conversely, Suppose $\mathbb{A}$ is $N_{eu}-OS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Then by hypothesis, $f_N(\mathbb{A}) = f_N(N_{eu}-int(\mathbb{A})) \subseteq N_{eu}gs\alpha^*-int(f_N(\mathbb{A})) \to ①$. Given $N_{eu}gs\alpha^*-int(f_N(\mathbb{A}))$ is the largest $N_{eu}gs\alpha^*-OS$ which is contained in $f_N(\mathbb{A})$, then $N_{eu}gs\alpha^*-int(f_N(\mathbb{A})) \subseteq f_N(\mathbb{A}) \to ②$. From ① and ②, $f_N(\mathbb{A}) = N_{eu}gs\alpha^*-int(f_N(\mathbb{A})) \Rightarrow f_N(\mathbb{A})$ is $N_{eu}gs\alpha^*-OS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow f_N$ is $N_{eu}gs\alpha^*-$ open map.

Theorem 4.13: Let $f_N:(\mathbb{P},\tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ be $N_{eu}gs\alpha^*-$ closed map iff $f_N(N_{eu}-cl(\mathbb{A})) \supseteq N_{eu}gs\alpha^*-cl(f_N(\mathbb{A}))$, for each $N_{eu}-$ set $\mathbb{A}$ in $(\mathbb{P},\tau_{N_{eu}})$.

Proof: Let $\mathbb{A}$ be any $N_{eu}-$ set in $(\mathbb{P},\tau_{N_{eu}}) \Rightarrow N_{eu}-cl(\mathbb{A})$ is $N_{eu}-CS$ in $(\mathbb{P},\tau_{N_{eu}})$. Given f_N is $N_{eu}gs\alpha^*-$ closed map, then $f_N(N_{eu}-cl(\mathbb{A}))$ is $N_{eu}gs\alpha^*-CS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow N_{eu}gs\alpha^*-cl\left(f_N(N_{eu}-cl(\mathbb{A}))\right) = f_N(N_{eu}-cl(\mathbb{A}))$. Given $f_N(\mathbb{A}) \subseteq f_N(N_{eu}-cl(\mathbb{A}))$, then $f_N(N_{eu}-cl(\mathbb{A})) = N_{eu}gs\alpha^*-cl\left(f_N(N_{eu}-cl(\mathbb{A}))\right) \supseteq N_{eu}gs\alpha^*-cl(f_N(\mathbb{A}))$. Conversely, Suppose $\mathbb{A}$ is $N_{eu}-CS$ in $(\mathbb{P},\tau_{N_{eu}})$. Then by hypothesis, $f_N(\mathbb{A}) = f_N(N_{eu}-cl(\mathbb{A})) \supseteq N_{eu}gs\alpha^*-cl(f_N(\mathbb{A})) \to ①$. Given $N_{eu}gs\alpha^*-cl(f_N(\mathbb{A}))$ is the smallest $N_{eu}gs\alpha^*-CS$ containing $f_N(\mathbb{A})$, then $f_N(\mathbb{A}) \subseteq N_{eu}gs\alpha^*-cl(f_N(\mathbb{A})) \to ②$. From ① and ②, $f_N(\mathbb{A}) = N_{eu}gs\alpha^*-cl(f_N(\mathbb{A})) \Rightarrow f_N(\mathbb{A})$ is $N_{eu}gs\alpha^*-CS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow f_N$ is $N_{eu}gs\alpha^*-$ closed map.

Theorem 4.14: Let $f_N:(\mathbb{P},\tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ be $N_{eu}gs\alpha^*-$ closed map, then $N_{eu}-cl\left(f_N{}^{-1}(\mathbb{A})\right) \supseteq f_N{}^{-1}(N_{eu}gs\alpha^*-cl(\mathbb{A}))$ for every $N_{eu}-$ set $\mathbb{A}$ of $(\mathbb{Q}, \sigma_{N_{eu}})$.

Proof: Let $\mathbb{A}$ be a $N_{eu}-$ set in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow N_{eu}-cl\left(f_N{}^{-1}(\mathbb{A})\right)$ is $N_{eu}-CS$ in $(\mathbb{P},\tau_{N_{eu}})$. Given f_N is $N_{eu}gs\alpha^*-$ closed map, then $f_N\left(N_{eu}-cl\left(f_N{}^{-1}(\mathbb{A})\right)\right)$ is $N_{eu}gs\alpha^*-CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. By theorem 4.13, $f_N\left(N_{eu}-cl\left(f_N{}^{-1}(\mathbb{A})\right)\right) \supseteq N_{eu}gs\alpha^*-cl\left(f_N\left(f_N{}^{-1}(\mathbb{A})\right)\right) \supseteq N_{eu}gs\alpha^*-cl(\mathbb{A}) \quad \Rightarrow N_{eu}-cl\left(f_N{}^{-1}(\mathbb{A})\right) \supseteq f_N{}^{-1}(N_{eu}gs\alpha^*-cl(\mathbb{A}))$.

Theorem 4.15: Let $f_N:(\mathbb{P},\tau_{N_{eu}}) \to (\mathbb{Q}, \sigma_{N_{eu}})$ be $N_{eu}gs\alpha^*-$ open map, then $N_{eu}-int\left(f_N{}^{-1}(\mathbb{A})\right) \subseteq f_N{}^{-1}(N_{eu}gs\alpha^*-int(\mathbb{A}))$ for every $N_{eu}-$ set $\mathbb{A}$ of $(\mathbb{Q}, \sigma_{N_{eu}})$.

Proof: Let $\mathbb{A}$ be a $N_{eu}-$ set in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow N_{eu}-int\left(f_N{}^{-1}(\mathbb{A})\right)$ is $N_{eu}-OS$ in $(\mathbb{P},\tau_{N_{eu}})$. Given f_N is $N_{eu}gs\alpha^*-$ open map, then $f_N\left(N_{eu}-int\left(f_N{}^{-1}(\mathbb{A})\right)\right)$ is $N_{eu}gs\alpha^*-OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Now, $f_N\left(N_{eu}-int\left(f_N{}^{-1}(\mathbb{A})\right)\right) \subseteq N_{eu}gs\alpha^*-int\left(f_N\left(f_N{}^{-1}(\mathbb{A})\right)\right) \subseteq N_{eu}gs\alpha^*-int(\mathbb{A})$ (by theorem 4.12) $\Rightarrow N_{eu}-int\left(f_N{}^{-1}(\mathbb{A})\right) \subseteq f_N{}^{-1}(N_{eu}gs\alpha^*-int(\mathbb{A}))$.

5.Conclusions : In this paper we have discussed about the $N_{eu}gs\alpha^* -$ open and closed map . We had an idea to extend this paper to the next level about $N_{eu}gs\alpha^* -$ homeomorphism and also the application of this paper . In future work , we will discussed and find out the results of this paper application.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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