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Generalized Double Statistical Convergence Sequences on Ideals in Neutrosophic Normed Spaces

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Abstract: In this present research, having view in the Neutrosophic norm $(\dot{\mu}, \ddot{v}, \dddot{\omega})$, which we presented $\mathcal{J}_2$-lacunary statistical convergence and $\mathcal{J}_2$-lacunary convergence strongly, looked into interactions between them, and made a few findings regarding the respective categories. At least went further to look at how both of such case approaches relate to $\mathcal{J}_2$-statistical convergence within the relevant Neutrosophic normed space.

Keywords: Banach Space; Ideal; Neutrosophic Normed Space; $\mathcal{J}_2$-lacunary Convergence of Statistical; Strongly Convergence.

1. Introduction

Fast invented statistical convergence in a sequence of real number. Research conducted by Das and Ghosal, et al. provide additional information along with applications using ideals. When Zadeh's [18], studied fuzzy set theory began to gain significance to be an academic subject. Atanassov [1] studied intuitionistic fuzzy sets; Atanassov et al. used this idea to analyze administrative decision-making challenges. The concept of an intuitionistic fuzzy metric space was proposed by Park.

Smarandache [15] introduced Neutrosophic Sets (NS) as a development of the IFS. For the situation when the aggregate of the components is 1, in the wake of satisfying the condition by applying the neutrosophic set operators, various results can be acquired by applying the intuitionistic fuzzy operators, whereas the neutrosophic operators are taken into the cognizance of the indeterminacy at a degree akin to truth-membership and falsehood-non membership, the operators disregard the indeterminacy. Jeyaraman et al. [9] developed approximate fixed point theorems for weak contractions on neutrosophic normed spaces in 2022. In the present paper, our aim is to discuss Neutrosophic norm $(\dot{\mu}, \ddot{v}, \dddot{\omega})$, which we presented $\mathcal{J}_2$-lacunary statistical convergence and $\mathcal{J}_2$-lacunary convergence strongly, looked into interactions between them, and made a few findings regarding the respective categories.

2. Preliminaries

The formula $\breve{\delta}(\mathfrak{K}) = \lim_{\hat{n}\to\infty} \frac{1}{\hat{n}} |\{\widetilde{m} \leq \hat{n} : \widetilde{m} \in \mathfrak{K}\}|$, describes the natural density that exists for an integer set $\mathfrak{K}$ that includes positive numbers, whenever $|\widetilde{m} \leq \hat{n} : \widetilde{m} \in \mathfrak{K}|$ represents the maximum value less than or equal to $\widetilde{m}$ with many elements in $\mathfrak{K}$.

Since each value $\breve{\varsigma} > 0$, the numerical sequence $\mathfrak{x} = (\mathfrak{x}_{\widetilde{m}})$ can be considered being statistically convergent in terms of $\mathfrak{L}$.

$$\lim_{\hat{n}\to\infty} \frac{1}{\hat{n}} |\{\widetilde{m} \leq \hat{n} : |\mathfrak{x}_{\widetilde{m}} - \mathfrak{L}| \geq \breve{\varsigma}\}| = 0,$$

i.e., $|\mathfrak{x}_{\widetilde{m}} - \mathfrak{L}| < \breve{\varsigma}$ $(a.a.\ \widetilde{m})$ (1.1)

Here, which we state that $st - lim\, \mathfrak{x}_{\widetilde{m}} = \mathfrak{L}$. As an example, specify $\mathfrak{x}_{\widetilde{m}} = 1$ When $\widetilde{m}$ acts as square, else $\mathfrak{x}_{\widetilde{m}} = 0$. When $|\{\widetilde{m} \leq \hat{n} : \mathfrak{x}_{\widetilde{m}} \neq 0\}| \leq \sqrt{\hat{n}}, thus\ st - lim\, \mathfrak{x}_{\widetilde{m}} = 0$. Remember it $st - lim\, \mathfrak{x}_{\widetilde{m}} = 0$ holds true even if the developers had given $\mathfrak{x}_{\widetilde{m}}$ anything measures upon all if m has become square. Yet $\mathfrak{x}$ never converges nor bounded. This happens obvious which $lim\, \mathfrak{x}_{\widetilde{m}} = \mathfrak{L}$ while inequality of (1.1) exists over every a limited amount of $\widetilde{m}$. Usual convergence naturally generalizes to Statistical Convergence (SC). SC can become thought of just being regular summability convergent in addition to require never remain convergent nor bounded because $lim\, \mathfrak{x}_{\widetilde{m}} = \mathfrak{L}$ yields $st - lim\, \mathfrak{x}_{\widetilde{m}} = \mathfrak{L}$.

A ideal of non-trivial, if $\mathcal{J}$ consists only singletons, and then $\mathcal{J}$ has (i) an admissible ideal over S. The sequence $(\mathfrak{x}_{\widetilde{m}})$ is shown to make themselves ideal convergent towards $\mathfrak{L}$ and every $\breve{\varsigma} > 0$, i.e.,

$$\mathfrak{A}(\breve{\varsigma}) = \{\widetilde{m} \in \mathbb{N} : |\mathfrak{x}_{\widetilde{m}} - \mathfrak{L}| \geq \breve{\varsigma}\} \in \mathcal{J}.$$

Considering $\mathcal{J} = \mathcal{J}_{\breve{\delta}} = \{\mathfrak{A} \subseteq \mathbb{N} : \breve{\delta}(\mathfrak{A}) = 0\}$, while $\breve{\delta}(\mathfrak{A})$ denotes the convergent value for set $\mathfrak{A}$. Convergence of ideal occurs around identical interval as statistical convergence when there is a non-trivial admissible ideal $\mathcal{J}_{\breve{\delta}}$.

Ideal $\mathcal{J}_2$ is a nontrivial on $\mathbb{N} \times \mathbb{N}$ appears to be strongly admissible when $\{i\} \times \mathbb{N}$ and $\mathbb{N} \times \{i\}$ originate from $\mathcal{J}_2$with every case $i \in \mathbb{N}$.

Therefore immediately apparent such strongly admissible ideal remains permissible.

Additionally, the article we take strongly admissible ideal $\mathcal{J}_2$ in $\mathbb{N} \times \mathbb{N}$, and ℓ_∞^2 denotes the space that contains all bounded double sequences.

A sequence of double lacunary represented by a double sequence $\bar{\theta} = \theta_{u\mathcal{s}} = \{(\grave{c}_u, \acute{d}_{\mathcal{s}})\}$ exist two increasing integer sequences $(\grave{c}_u)$ and $(\acute{d}_{\mathcal{s}})$ which means

$$\grave{c}_0 = 0, \hbar_u = \grave{c}_u - \grave{c}_{u-1} \to \infty\ and\ \acute{d}_0 = 0, \bar{\hbar}_{\mathcal{s}} = \acute{d}_{\mathcal{s}} - \acute{d}_{\mathcal{s}-1} \to \infty,\ u, \mathcal{s} \to \infty.$$

We shall utilize the term shown below, $\grave{c}_{u\mathcal{s}} := \grave{c}_u \acute{d}_{\mathcal{s}}, \hbar_{u\mathcal{s}} := \hbar_u \hbar_{\mathcal{s}}\ and\ \theta_{u\mathcal{s}}$ is determined as

$$\mathfrak{J}_{u\mathcal{s}} := \{(\grave{c}, \acute{d}) : \grave{c}_u - 1 < \grave{c} \leq \grave{c}_u \text{ and } \acute{d}_{\mathcal{s}} - 1 < \acute{d} \leq \acute{d}_{\mathcal{s}}\},$$

$$\bar{\bar{q}}_u := \frac{\grave{c}_u}{\grave{c}_u - 1}, \bar{\bar{q}}_{\mathcal{s}} := \frac{\acute{d}_u}{\acute{d}_u - 1}\ \bar{\bar{q}}_{u\mathcal{s}} := \bar{\bar{q}}_u \bar{\bar{q}}_{\mathcal{s}}.$$

All through the research, with $\theta_2 = \theta_{u\mathcal{s}} = \{(\grave{c}_u, \acute{d}_{\mathcal{s}})\}$ that we shall designate a sequence of double lacunary nonnegative real numbers, respectively, wherever a different condition exists.

The double sequence with the integers $\mathfrak{x} = \{\mathfrak{x}_{\widetilde{m}\hat{n}}\}$ can be considered that it is $\mathcal{J}_2$-lacunary statistical convergent as well as $(\mathcal{J}_2)$-convergent towards $\mathfrak{L}$, when for every $\breve{\varsigma} > 0$ and $\breve{\delta} > 0$,

$$\left\{(u, \mathcal{s}) \in \mathbb{N} \times \mathbb{N} : \frac{1}{y_r \overline{y_u}} |\{(\widetilde{m}, \hat{n}) \in \mathfrak{J}_{ru} : |\mathfrak{x}_{\widetilde{m}} - \mathfrak{L}| \geq \breve{\varsigma}\}| \geq \breve{\delta}\right\} \in \mathcal{J}_2.$$

In the above illustration, we have to put

$$\mathfrak{x}_{\widetilde{m}\hat{n}} \to \mathfrak{L}\left(I_{\theta_2}(\mathcal{J}_2)\right)\ or\ I_{\theta_2}(\mathcal{J}_2) - \lim_{\widetilde{m},\hat{n}\to\infty} \mathfrak{x}_{\widetilde{m}\hat{n}} = \mathfrak{L}.$$

Definition 2.1. The 7-tuple $(\Xi, \dot{\mu}, \ddot{v}, \ddot{\omega}, \circledast, \Theta, \Delta)$ is said to be a Neutrosophic Normed Space [*NNS*], if $\circledast$ acts as t-norm which is continuous, Θ and Δ act as t-co norms which are continuous along with Ξ which acts as a real vector space, $\dot{\mu}, \ddot{v}, \ddot{\omega}$ which denote fuzzy sets through $\Xi \times \mathfrak{F}$.

(cn1) $\dot{\mu}\left(\mathfrak{h}, \dot{\varrho}\right) + \ddot{v}\left(\mathfrak{h}, \dot{\varrho}\right) + \ddot{\omega}\left(\mathfrak{h}, \dot{\varrho}\right) \leq 3$,

(cn2) $0 \leq \dot{\mu}\left(\mathfrak{h}, \dot{\varrho}\right) \leq 1; 0 \leq \ddot{v}\left(\mathfrak{h}, \dot{\varrho}\right) \leq 1$ *and* $0 \leq \ddot{\omega}\left(\mathfrak{h}, \dot{\varrho}\right) \leq 1$,

(cn3) $\dot{\mu}\left(\mathfrak{h}, \dot{\varrho}\right) = 0$ for all non-positive real number $\dot{\varrho}$,

(cn4) $\dot{\mu}\left(\mathfrak{h}, \dot{\varrho}\right) = 1$ for all $\dot{\varrho} \in \mathbb{R}^+ \Leftrightarrow \mathfrak{h} = 0$,

(cn5) $\dot{\mu}\left(\gamma\mathfrak{h}, \dot{\varrho}\right) = \dot{\mu}\left(\mathfrak{h}, \frac{\dot{\varrho}}{|\gamma|}\right)$, for all $\gamma \in \mathbb{R}$ and $\gamma \neq 0$,

(cn6) $\dot{\mu}\left(\mathfrak{h} + \mathfrak{z}, \dot{\varrho} + \widetilde{\alpha}\right) \geq min\{\dot{\mu}\left(\mathfrak{h}, \dot{\varrho}\right), \dot{\mu}\left(\mathfrak{z}, \widetilde{\alpha}\right)\}$,

(cn7) $\lim_{\dot{\varrho}\to\infty} \dot{\mu}\left(\mathfrak{h}, \dot{\varrho}\right) = 1$ and $\lim_{\varrho\to\infty} \dot{\mu}\left(\mathfrak{h}, \dot{\varrho}\right) = 0$,

(cn8) $\ddot{v}\left(\mathfrak{h}, \dot{\varrho}\right) = 1$ for all non-positive real number $\dot{\varrho}$,

(cn9) $\ddot{v}\left(\mathfrak{h}, \dot{\varrho}\right) = 0$ for all $\dot{\varrho} \in \mathbb{R}^+ \Leftrightarrow \mathfrak{h} = 0$,

(cn10) $\ddot{v}\left(\gamma\mathfrak{h}, \dot{\varrho}\right) = \ddot{v}\left(\mathfrak{h}, \frac{\dot{\varrho}}{|\gamma|}\right)$, for all $\gamma \in \mathbb{R}$ and $\gamma \neq 0$,

(cn11) $\ddot{v}\left(\mathfrak{h} + \mathfrak{z}, \dot{\varrho} + \widetilde{\alpha}\right) \leq max\{\ddot{v}\left(\mathfrak{h}, \dot{\varrho}\right), \ddot{v}\left(\mathfrak{z}, \widetilde{\alpha}\right)\}$,

(cn12) $\lim_{\dot{\varrho}\to\infty} \ddot{v}\left(\mathfrak{h}, \dot{\varrho}\right) = 0$ and $\lim_{\dot{\varrho}\to\infty} \ddot{v}\left(\mathfrak{h}, \dot{\varrho}\right) = 1$,

(cn13) $\ddot{\omega}\left(\mathfrak{h}, \dot{\varrho}\right) = 1$ for all non-positive real number $\dot{\varrho}$,

(cn14) $\ddot{\omega}\left(\mathfrak{h}, \dot{\varrho}\right) = 0$ for all $\dot{\varrho} \in \mathbb{R}^+ \Leftrightarrow \mathfrak{h} = 0$,

(cn15) $\ddot{\omega}\left(\gamma\mathfrak{h}, \dot{\varrho}\right) = \ddot{\omega}\left(\mathfrak{h}, \frac{\dot{\varrho}}{|\gamma|}\right)$, for all $\gamma \in \mathbb{R}$ and $\gamma \neq 0$,

(cn16) $\ddot{\omega}\left(\mathfrak{h} + \mathfrak{z}, \dot{\varrho} + \widetilde{\alpha}\right) \leq max\{\ddot{\omega}\left(\mathfrak{h}, \dot{\varrho}\right), \ddot{\omega}\left(\mathfrak{z}, \widetilde{\alpha}\right)\}$,

(cn17) $\lim_{\dot{\varrho}\to\infty} \ddot{\omega}\left(\mathfrak{h}, \dot{\varrho}\right) = 0$ and $\lim_{\dot{\varrho}\to\infty} \ddot{\omega}\left(\mathfrak{h}, \dot{\varrho}\right) = 1$.

In the above case, $(\dot{\mu}, \ddot{v}, \ddot{\omega})$ is identified as a *NN* on Ξ. In addition, $(\Xi, \dot{\mu}, \ddot{v}, \ddot{\omega})$ is referred to be a *NNS*.

In *NNS,* we look at generalized sequence a statistical convergence through ideals. In the present article, we pay attention and in addition we have to investigate the interaction among two new ideas, as well as the author's introduction of $\mathcal{J}_2$-Lacunary Statistical Convergence (LSC) and strongly $\mathcal{J}_2$-Lacunary Convergence (LC) in a *NNS*.

3. Main Results

Definition 3.1. Let $(\Xi, \dot{\mu}, \ddot{v}, \ddot{\omega}, \circledast, \Theta, \Delta)$ be a *NNS*, $\mathcal{J}_2 \subseteq 2^{\mathbb{N}\times\mathbb{N}}$ which is a strongly admissible ideal in $\mathbb{N} \times \mathbb{N}$. A sequence $\mathfrak{x} = (\mathfrak{x}_{az})$ is said to be $\mathcal{J}_2$ −Statistically Convergent (SC) to $\grave{\xi} \in \Xi$ relate to the *NN* $(\dot{\mu}, \ddot{v}, \ddot{\omega})$, which is represented by $I(\mathcal{J}_2)^{(\dot{\mu},\ddot{v},\ddot{\omega})} - lim\ \mathfrak{x} = \grave{\xi}$ or $\mathfrak{x}_{a,z} \xrightarrow{(\dot{\mu},\ddot{v},\ddot{\omega})} \grave{\xi}(I(\mathcal{J}_2))$, if for every $\check{\varsigma} > 0$, every $\breve{\delta} > 0$, and $\dot{\varrho} > 0$,

$$\left\{(\widetilde{m},\hat{n}) \in \mathbb{N}\times\mathbb{N} \ : \frac{1}{\widetilde{m}\hat{n}}\left|\left\{\begin{matrix} a \le \widetilde{m}, z \le \hat{n} : \dot{\mu}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) \le 1 - \check{\varsigma} \ or \\ \ddot{\nu}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) \ge \check{\varsigma} \ and \\ \ddot{\omega}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) \ge \check{\varsigma} \end{matrix}\right\}\right| \ge \breve{\delta}\right\} \in \mathcal{I}_2.$$

Definition 3.2. A sequence $\mathfrak{x} = (\mathfrak{x}_{az})$ is said to be $\mathcal{I}_2 -$ LSC to $\grave{\xi} \in \Xi$ relate with the *NN* $(\dot{\mu}, \ddot{\nu}, \ddot{\omega})$ which is denoted by $I_\theta(\mathcal{I}_2)^{(\dot{\mu},\ddot{\nu},\ddot{\omega})} - lim\ \mathfrak{x} = \grave{\xi}$ or $\mathfrak{x}_{a,z} \xrightarrow{(\dot{\mu},\ddot{\nu},\ddot{\omega})} \grave{\xi}\left(I_\theta(\mathcal{I}_2)\right)$, if for every $\check{\varsigma} > 0$, every $\breve{\delta} > 0$, and $\dot{\varrho} > 0$,

$$\left\{(r, u) \in \mathbb{N}\times\mathbb{N} \ : \frac{1}{y_r \overline{y_u}}\left|\left\{\begin{matrix} (a, z) \in J_{ru} : \dot{\mu}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) \le 1 - \check{\varsigma} \ or \\ \ddot{\nu}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) \ge \check{\varsigma} \ and \\ \ddot{\omega}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) \ge \check{\varsigma} \end{matrix}\right\}\right| \ge \breve{\delta}\right\} \in \mathcal{I}_2.$$

Definition 3.3. A sequence $\mathfrak{x} = (\mathfrak{x}_{az})$ is said as a strongly $\mathcal{I}_2 -$ LC to $\grave{\xi}$ or $J_\theta(\mathcal{I}_2)$ −convergent to $\grave{\xi} \in \Xi$ relate to the *NN* $(\dot{\mu}, \ddot{\nu}, \ddot{\omega})$ it can be denoted by $\mathfrak{x}_{az} \xrightarrow{(\dot{\mu},\ddot{\nu},\ddot{\omega})}$ $\grave{\xi}\left(J_\theta(\mathcal{I}_2)\right)$, if for every $\breve{\delta} > 0$ and $\dot{\varrho} > 0$,

$$\left\{\begin{matrix} (r, u) \in \mathbb{N}\times\mathbb{N} \ : \dfrac{1}{y_r \overline{y_u}} \displaystyle\sum_{(a,z) \in \mathfrak{I}_{ru}} \dot{\mu}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) \le 1 - \breve{\delta} \ or \\ \dfrac{1}{y_r \overline{y_u}} \displaystyle\sum_{(a,z) \in \mathfrak{I}_{ru}} \ddot{\nu}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) \ge \breve{\delta} \\ and \dfrac{1}{y_r \overline{y_u}} \displaystyle\sum_{(a,z) \in \mathfrak{I}_{ru}} \ddot{\omega}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) \ge \breve{\delta} \end{matrix}\right\} \in \mathcal{I}_2.$$

Theorem 3.4. *Let* $(\Xi, \dot{\mu}, \ddot{\nu}, \ddot{\omega}, \circledast, \Theta, \Delta)$ *be a NNS, Double Lacunary (DL) sequence* θ, *strongly admissible ideal* $\mathcal{I}_2$ *in* $\mathbb{N}$, *and* $\mathfrak{x} = (\mathfrak{x}_{az}) \in \Xi$, *then*

(i) (a) If $\mathfrak{x}_{az} \xrightarrow{(\dot{\mu},\ddot{\nu},\ddot{\omega})} \grave{\xi}\left(J_\theta(\mathcal{I}_2)\right)$, *then* $\mathfrak{x}_{az} \xrightarrow{(\dot{\mu},\ddot{\nu},\ddot{\omega})} \grave{\xi}\left(I_\theta(\mathcal{I}_2)\right)$.

(b) If $\mathfrak{x} \in \ell_\infty^2(\Xi)$, *be a* Ξ *of all bounded sequence space with* $\mathfrak{x}_{az} \xrightarrow{(\dot{\mu},\ddot{\nu},\ddot{\omega})} \grave{\xi}\left(I_\theta(\mathcal{I}_2)\right)$ *then* $\mathfrak{x}_{az} \xrightarrow{(\dot{\mu},\ddot{\nu},\ddot{\omega})} \grave{\xi}\left(J_\theta(\mathcal{I}_2)\right)$.*n*

(ii) $I_\theta(\mathcal{I}_2)^{(\dot{\mu},\ddot{\nu},\ddot{\omega})} \cap \ell_\infty^2(\Xi) = J_\theta(\mathcal{I}_2)^{(\dot{\mu},\ddot{\nu},\ddot{\omega})} \cap \ell_\infty^2(\Xi)$.

Proof. *(i) – (a).* Given hypothesis, for all $\check{\varsigma} > 0, \breve{\delta} > 0$, and $\dot{\varrho} > 0$, let $\mathfrak{x}_{a,z} \xrightarrow{(\dot{\mu},\ddot{\nu},\ddot{\omega})} \grave{\xi}\left(J_\theta(\mathcal{I}_2)\right)$. Then we can write

$$\sum_{(a,z) \in \mathfrak{I}_{ru}} \left(\dot{\mu}\ \left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\ \right) or\ \ddot{\nu}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\ \right) and\ \ddot{\omega}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\ \right)\right)$$

$$\ge \sum_{\substack{(a,z) \in \mathfrak{I}_{ru} \\ \dot{\mu}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) \le 1 - \check{\varsigma}\ or\ \ddot{\nu}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) \ge \check{\varsigma}\ and \\ \ddot{\omega}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) \ge \check{\varsigma}}} \left(\dot{\mu}\ \left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) or\ \ddot{\nu}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) and\ \ddot{\omega}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right)\right)$$

$$\ge \check{\varsigma}.\left|\left\{(a, z) \in \mathfrak{I}_{ru} : \dot{\mu}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) \le 1 - \check{\varsigma}\ or\ \ddot{\nu}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) \ge \check{\varsigma}\ and\ \ddot{\omega}\ \left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) \ge \check{\varsigma}\ \right\}\right|.$$

Then observe that

$$\frac{1}{y_r \overline{y_u}}\left|\left\{\begin{matrix} (a, z) \in \mathfrak{I}_{ru} : \dot{\mu}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) \le 1 - \check{\varsigma}\ or\ \ddot{\nu}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) \ge \check{\varsigma} \\ and\ \ddot{\omega}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) \ge \check{\varsigma} \end{matrix}\right\}\right| \ge \breve{\delta}$$

and

$$\frac{1}{y_r\overline{y_u}}\sum_{(a,z)\,\in\,\Im_{ru}}\dot{\mu}\,\left(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho}\right)\leq(1-\check{\varsigma}\,)\breve{\delta}\;or\;\frac{1}{y_r\overline{y_u}}\sum_{(a,z)\,\in\,\Im_{ru}}\ddot{v}\,\left(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho}\right)\geq\check{\varsigma}\breve{\delta}$$

$$\text{and }\frac{1}{y_r\overline{y_u}}\sum_{(a,z)\,\in\,\Im_{ru}}\ddot{\omega}\,\left(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho}\right)\geq\check{\varsigma}\breve{\delta},$$

which implies $\left\{(r,u)\in\mathbb{N}\times\mathbb{N}\;:\frac{1}{y_r\overline{y_u}}\left|\left\{\begin{matrix}(a,z)\in\Im_{ru}:\dot{\mu}\left(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho}\right)\leq1-\check{\varsigma}\\ or\;\ddot{v}\left(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho}\right)\geq\check{\varsigma}\\ and\\ \ddot{\omega}\;\left(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho}\right)\geq\check{\varsigma}\end{matrix}\right\}\right|\geq\breve{\delta}\right\}$

$$\subset\left\{(r,u)\in\mathbb{N}\times\mathbb{N}\;:\frac{1}{y_r\overline{y_u}}\geq\check{\varsigma}\breve{\delta}\right\}$$

Since $\mathfrak{x}_{a,z}\xrightarrow{(\dot{\mu},\ddot{v},\ddot{\omega})}\grave{\xi}\big(J_\theta(\mathcal{I}_2)\big)$, we immediately see that $\mathfrak{x}_{a,z}\xrightarrow{(\dot{\mu},\ddot{v},\ddot{\omega})}\grave{\xi}\big(I_\theta(\mathcal{I}_2)\big)$.

$(i)-(b)$. We assume that $\mathfrak{x}_{a,z}\xrightarrow{(\dot{\mu},\ddot{v},\ddot{\omega})}\grave{\xi}\big(I_\theta(\mathcal{I}_2)\big)$ and $\mathfrak{x}\in\ell_\infty^2(\Xi)$. The inequalities $\mathcal{T}$ or hold for all a,z.

Let $\check{\varsigma}>0$ be given. Then we have

$$\frac{1}{y_r\overline{y_u}}\sum_{(a,z)\,\in\,\Im_{ru}}(\dot{\mu}\;\left(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho}\right)or\;\ddot{v}\left(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho}\right)and\;\ddot{\omega}\left(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho}\right))$$

$$=\frac{1}{y_r\overline{y_u}}\sum_{\substack{(a,z)\,\in\,\Im_{ru}\\ \dot{\mu}(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho})\leq1-\check{\varsigma}\;or\;\ddot{v}(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho})\geq\check{\varsigma}\;and\;\ddot{\omega}\;(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho})\geq\check{\varsigma}}}\begin{pmatrix}\dot{\mu}\left(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho}\right)or\;\ddot{v}\left(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho}\right)and\\ \ddot{\omega}\left(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho}\right)\end{pmatrix}$$

$$+\frac{1}{y_r\overline{y_u}}\sum_{\substack{(a,z)\,\in\,\Im_{ru}\\ \dot{\mu}\;(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho})>1-\check{\varsigma}\;or\;\ddot{v}\;(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho})<\check{\varsigma}\;and\;\ddot{\omega}\;(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho})<\check{\varsigma}}}\begin{pmatrix}\dot{\mu}\left(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho}\right)or\;\ddot{v}\left(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho}\right)and\\ \ddot{\omega}\left(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho}\right)\end{pmatrix}$$

$$\leq\frac{\mathfrak{M}}{y_r\overline{y_u}}\left|\left\{\begin{matrix}(a,z)\in\Im_{ru}\;:\dot{\mu}\left(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho}\;\right)\leq1-\check{\varsigma}\;\;or\;\ddot{v}\left(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho}\;\right)\geq\check{\varsigma}\;\;and\\ \ddot{\omega}\left(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho}\right)\geq\check{\varsigma}\end{matrix}\right\}\right|+\check{\varsigma}\,.$$

Note that

$$\mathfrak{A}_{\dot{\mu},\ddot{v},\ddot{\omega}}(\check{\varsigma}\,,\dot{\varrho}\;)=\left\{(r,u)\in\mathbb{N}\times\mathbb{N}\;:\;\frac{1}{y_r\overline{y_u}}\left|\left\{\begin{matrix}(a,z)\in\Im_{ru}\;:\dot{\mu}\left(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho}\right)\leq1-\check{\varsigma}\\ or\;\ddot{v}\left(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho}\;\right)\geq\check{\varsigma}\;\;and\\ \ddot{\omega}\left(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho}\;\right)\geq\check{\varsigma}\end{matrix}\right\}\right|\geq\frac{\check{\varsigma}}{\mathfrak{M}}\right\}$$

belongs to $\mathcal{I}_2$. If $r\in\left(\mathfrak{A}_{\dot{\mu},\ddot{v},\ddot{\omega}}(\check{\varsigma}\,,\dot{\varrho})\right)^c$ then we have

$$\frac{1}{y_r\overline{y_u}}\sum_{(a,z)\,\in\,\Im_{ru}}\dot{\mu}\left(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho}\right)>(1-2\check{\varsigma}\,)$$

$$or\;\frac{1}{y_r\overline{y_u}}\sum_{(a,z)\,\in\,\Im_{ru}}\ddot{v}\left(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho}\right)<2\check{\varsigma}$$

$$and\;\frac{1}{y_r\overline{y_u}}\sum_{(a,z)\,\in\,\Im_{ru}}\ddot{\omega}\left(\mathfrak{x}_{az}-\grave{\xi},\dot{\varrho}\right)<2\check{\varsigma}\,,$$

Now

$$\mathfrak{T}_{\dot{\mu},\ddot{v},\dddot{\omega}}(\breve{\zeta},\dot{\varrho}) = \left\{ \begin{array}{c} (r,u) \in \mathbb{N}\times\mathbb{N} : \frac{1}{y_r\overline{y_u}} \sum_{(a,z)\in\mathfrak{J}_{ru}} \dot{\mu}\left(\mathfrak{x}_{az} - \grave{\xi},\dot{\varrho}\right) \le (1-2\breve{\zeta}) \\ or \frac{1}{y_r\overline{y_u}} \sum_{(a,z)\in\mathfrak{J}_{ru}} \ddot{v}\left(\mathfrak{x}_{az} - \grave{\xi},\dot{\varrho}\right) \ge 2\breve{\zeta} \\ and \frac{1}{y_r\overline{y_u}} \sum_{(a,z)\in\mathfrak{J}_{ru}} \dddot{\omega}\left(\mathfrak{x}_{az} - \grave{\xi},\dot{\varrho}\right) < 2\breve{\zeta}, \end{array} \right\}.$$

Hence $\mathfrak{T}_{\dot{\mu},\ddot{v},\dddot{\omega}}(\breve{\zeta},\dot{\varrho}) \subseteq \mathfrak{A}_{\dot{\mu},\ddot{v},\dddot{\omega}}(\breve{\zeta},\dot{\varrho})$, then along with on an ideal, $\mathfrak{T}_{\dot{\mu},\ddot{v},\dddot{\omega}}(\breve{\zeta},\dot{\varrho}) \in \mathcal{I}_2$.

Therefore, we conclude that $\mathfrak{x}_{a,z} \xrightarrow{(\dot{\mu},\ddot{v},\dddot{\omega})} \grave{\xi}\left(J_\theta(\mathcal{I}_2)\right)$,

(ii) It immediately following (i) – (a) and (i) – (b).

Theorem 3.5. *Let* $(\Xi,\dot{\mu},\ddot{v},\dddot{\omega},\circledast,\Theta,\Delta)$ *be a NNS. When a sequence* θ *of DL with* $\lim\inf_r q_r > 1$, $\lim \inf_u q_u > 1$ *thereafter*

$$\mathfrak{x}_{az} \xrightarrow{(\dot{\mu},\ddot{v},\dddot{\omega})} \grave{\xi}\left(I(\mathcal{I}_2)\right) \Rightarrow \mathfrak{x}_{az} \xrightarrow{(\dot{\mu},\ddot{v},\dddot{\omega})} \grave{\xi}\left(I_\theta(\mathcal{I}_2)\right),$$

Proof. Assume initially that $\liminf_r q_r > 1$, $\lim \inf_u q_u > 1$ then there exists a $\tilde{\varphi},\hat{\psi} > 0$ so that $q_r \ge 1+\alpha, q_u > 1+\beta$ for sufficiently large r,u, which implies that

$$\frac{y_r\overline{y_u}}{z_{ru}} \ge \frac{\tilde{\varphi}\hat{\psi}}{(1+\tilde{\varphi})(1+\hat{\psi})}.$$

If $\mathfrak{x}_{az} \xrightarrow{(\dot{\mu},\ddot{v},\dddot{\omega})} \grave{\xi}\left(I(\mathcal{I}_2)\right)$, then for every $\breve{\zeta} > 0$ and for sufficiently large r,u, we have

$$\frac{1}{z_{ru}}\left|\left\{a \le a_r, z \le z_u : \dot{\mu}\left(\mathfrak{x}_{az} - \grave{\xi},\dot{\varrho}\right) \le 1-\breve{\zeta} \; or \; \ddot{v}\left(\mathfrak{x}_{az} - \grave{\xi},\dot{\varrho}\right) \ge \breve{\zeta} \; and \; \dddot{\omega}\left(\mathfrak{x}_{az} - \grave{\xi},\dot{\varrho}\right) \ge \breve{\zeta}\right\}\right|$$

$$\ge \frac{1}{z_{ru}}\left|\left\{(a,z) \in \mathfrak{J}_{ru} : \dot{\mu}\left(\mathfrak{x}_{az} - \grave{\xi},\dot{\varrho}\right) \le 1-\breve{\zeta} \; or \; \ddot{v}\left(\mathfrak{x}_{az} - \grave{\xi},\dot{\varrho}\right) \ge \breve{\zeta} \; and \; \dddot{\omega}\left(\mathfrak{x}_{az} - \grave{\xi},\dot{\varrho}\right) \ge \breve{\zeta}\right\}\right|$$

$$\ge \frac{\tilde{\varphi}\hat{\psi}}{(1+\tilde{\varphi})(1+\hat{\psi})}\left(\frac{1}{y_r\overline{y_u}}\left|\left\{\begin{array}{c}(a,z) \in \mathfrak{J}_{ru} : \dot{\mu}\left(\mathfrak{x}_{az} - \grave{\xi},\dot{\varrho}\right) \le 1-\breve{\zeta} \; or \\ \ddot{v}\left(\mathfrak{x}_{az} - \grave{\xi},\dot{\varrho}\right) \ge \breve{\zeta} \; and \; \dddot{\omega}\left(\mathfrak{x}_{az} - \grave{\xi},\dot{\varrho}\right) \ge \breve{\zeta}\end{array}\right\}\right|\right)$$

Then for any $\breve{\delta} > 0$, we get

$$\left\{(r,u) \in \mathbb{N}\times\mathbb{N} : \frac{1}{y_r\overline{y_u}}\left|\left\{\begin{array}{c}(a,z) \in \mathfrak{J}_{ru} : \dot{\mu}\left(\mathfrak{x}_{az} - \grave{\xi},\dot{\varrho}\right) \le 1-\breve{\zeta} \; or \; \ddot{v}\left(\mathfrak{x}_{az} - \grave{\xi},\dot{\varrho}\right) \ge \breve{\zeta} \\ and \; \dddot{\omega}\left(\mathfrak{x}_{az} - \grave{\xi},\dot{\varrho}\right) \ge \breve{\zeta}\end{array}\right\}\right| \ge \breve{\delta}\right\}$$

$$\subseteq \left\{(r,u) \in \mathbb{N}\times\mathbb{N} : \frac{1}{z_{ru}}\left|\left\{\begin{array}{c}a \le a_r, z \le z_u : \dot{\mu}\left(\mathfrak{x}_{az} - \grave{\xi},\dot{\varrho}\right) \le 1-\breve{\zeta} \\ or \; \ddot{v}\left(\mathfrak{x}_{az} - \grave{\xi},\dot{\varrho}\right) \ge \breve{\zeta} \\ and \; \dddot{\omega}\left(\mathfrak{x}_{az} - \grave{\xi},\dot{\varrho}\right) \ge \breve{\zeta}\end{array}\right\}\right| \ge \frac{\breve{\delta}\tilde{\varphi}\hat{\psi}}{(1+\tilde{\varphi})(1+\hat{\psi})}\right\}.$$

If $\mathfrak{x}_{az} \xrightarrow{(\dot{\mu},\ddot{\nu},\ddot{\omega})} \dot{\xi}(I(\mathcal{J}_2))$ consequently, the set on the right-hand belongs to $\mathcal{J}_2$ and the set on the left-hand belongs to $\mathcal{J}_2$. It demonstrates that $\mathfrak{x}_{az} \xrightarrow{(\dot{\mu},\ddot{\nu},\ddot{\omega})} \dot{\xi}(I_\theta(\mathcal{J}_2))$.

The following result depends on the hypothesis of the lacunary sequence θ provides satisfaction to the requirement to satisfy each set $\mathcal{C} \in \mathfrak{F}(\mathcal{J}_2)$, $\cup\{\hat{n} : z_{r-1} < \hat{n} \le z_r,\ r \in \mathcal{C}\} \in \mathfrak{F}(\mathcal{J}_2)$.

Theorem 3.6. *Let* $(\Xi, \dot{\mu}, \ddot{\nu}, \ddot{\omega}, \circledast, \Theta, \Delta)$ *be a NNS. When a sequence* θ *of DL with* $\limsup_{r} q_r > \infty$, $\lim\sup_{u} q_u > \infty$ *thereafter*

$$\mathfrak{x}_{az} \xrightarrow{(\dot{\mu},\ddot{\nu},\ddot{\omega})} \xi(I_\theta(\mathcal{J}_2)) \Rightarrow \mathfrak{x}_{az} \xrightarrow{(\dot{\mu},\ddot{\nu},\ddot{\omega})} \xi(I(\mathcal{J}_2)),$$

Proof. If $\lim\sup_{r} q_r > \infty$, $\lim\sup_{u} q_u > \infty$ then without limited uniformity we can assume that there exists a $\mathfrak{M}, \mathfrak{N} > 0$ such that $q_r < \mathfrak{M}$ and $q_u < \mathfrak{N}$ for every r, u. Assume $\mathfrak{x}_{az} \xrightarrow{(\dot{\mu},\ddot{\nu},\ddot{\omega})} \dot{\xi}(I_\theta(\mathcal{J}_2))$and let

$$\mathcal{C}_{ru} := \left|\left\{(a,z) \in \mathfrak{I}_{ru} : \dot{\mu}\left(\mathfrak{x}_{az} - \dot{\xi}, \dot{\varrho}\right) \le 1 - \check{\varsigma} \ or\ \ddot{\nu}\left(\mathfrak{x}_{az} - \dot{\xi}, \dot{\varrho}\right) \ge \check{\varsigma} \ and\ \ddot{\omega}\left(\mathfrak{x}_{az} - \dot{\xi}, \dot{\varrho}\right) \ge \check{\varsigma}\right\}\right|.$$

Since $\mathfrak{x}_{az} \xrightarrow{(\dot{\mu},\ddot{\nu},\ddot{\omega})} \dot{\xi}(I_\theta(\mathcal{J}_2))$, it becomes that for all $\check{\varsigma} > 0$, $\check{\delta} > 0$, and $\dot{\varrho} > 0$,

$$\left\{(r,u) \in \mathbb{N} \times \mathbb{N} : \frac{1}{y_r \overline{y_u}} \left|\left\{\begin{matrix} a, z \in \mathfrak{I}_{ru} : \dot{\mu}\left(\mathfrak{x}_{az} - \dot{\xi}, \dot{\varrho}\right) \le 1 - \check{\varsigma} \ or \\ \ddot{\nu}\left(\mathfrak{x}_{az} - \dot{\xi}, \dot{\varrho}\right) \ge \check{\varsigma} \\ and\ \ddot{\omega}\left(\mathfrak{x}_{az} - \dot{\xi}, \dot{\varrho}\right) \ge \check{\varsigma} \end{matrix}\right\}\right| \ge \check{\delta}\right\} \in \mathcal{J}_2.$$

Hence, we can select a positive integers $r_0, u_0 \in \mathbb{N}$ so that $\frac{\mathcal{C}_{ru}}{y_r \overline{y_u}} < \check{\delta}$, for every $r > r_0, u > u_0$.

Now let $\mathfrak{K} := \max\{\mathcal{C}_{ru} : 1 \le r \le r_0, 1 \le u \le u_0\}$ and let ϱ and v be any integers satisfying $a_{r-1} < \dot{\varrho} \le a_r$ and $z_{u-1} < v \le z_u$. Then, we have

$$\frac{1}{\dot{\varrho} v}\left|\left\{a \le \dot{\varrho}, z \le v : \dot{\mu}\left(\mathfrak{x}_{az} - \dot{\xi}, \dot{\varrho}\right) \le 1 - \check{\varsigma} \ or\ \ddot{\nu}\left(\mathfrak{x}_{az} - \dot{\xi}, \dot{\varrho}\right) \ge \check{\varsigma} \ and\ \ddot{\omega}\left(\mathfrak{x}_{az} - \dot{\xi}, \dot{\varrho}\right) \ge \check{\varsigma}\right\}\right|$$

$$\le \frac{1}{a_{r-1}\, z_{u-1}} \left|\left\{\begin{matrix} a \le a_r, z \le z_u : \dot{\mu}\left(\mathfrak{x}_{az} - \dot{\xi}, \dot{\varrho}\right) \le 1 - \check{\varsigma} \ or\ \ddot{\nu}\left(\mathfrak{x}_{az} - \dot{\xi}, \dot{\varrho}\right) \ge \check{\varsigma} \\ and\ \ddot{\omega}\left(\mathfrak{x}_{az} - \dot{\xi}, \dot{\varrho}\right) \ge \check{\varsigma} \end{matrix}\right\}\right|$$

$$\le \frac{1}{a_{r-1}\, z_{u-1}} \left(\mathcal{C}_{11} + \mathcal{C}_{12} + \mathcal{C}_{21} + \mathcal{C}_{22} + \cdots + \mathcal{C}_{r_0 u_0} + \cdots \mathcal{C}_{ru}\right)$$

$$\le \frac{1}{a_{r-1}\, z_{u-1}} . r_0 u_0 + \frac{1}{a_{r-1}\, z_{u-1}} \left(y_{r_0} \bar{y}_{u_{r_0}+1} \frac{\mathcal{C}_{r_0, u_0+1}}{y_{r_0} \bar{y}_{u_{r_0}+1}} + y_{r_{0+1}} \bar{y}_{u_0} \frac{\mathcal{C}_{r_{0+1}, u_0}}{y_{r_{0+1}} \bar{y}_{u_0}} + \cdots + y_r \bar{y}_u \frac{\mathcal{C}_{ru}}{y_r \bar{y}_u}\right)$$

$$\le \frac{r_0 u_0 . l}{a_{r-1}\, z_{u-1}} + \frac{1}{a_{r-1}\, z_{u-1}} \left(\sup_{r > r_0, u > u_0} \frac{\mathcal{C}_{ru}}{y_r y_u}\right) \left(y_{r_0} \bar{y}_{u_0+1} + y_{r_{0+1}} \bar{y}_{u_0} + \cdots + y_r y_u\right)$$

$$\le \frac{r_0 u_0 . a}{a_{r-1}\, z_{u-1}} + \check{\varsigma} . \frac{(a_r - a_{r_0})(z_u - z_{-1})}{a_{r-1}\, z_{u-1}}$$

$$\leq \frac{r_0 u_0 . z}{a_{r-1}\, z_{u-1}} + \check{\varsigma} . q_r . q_u \leq \frac{r_0 u_0 . l}{a_{r-1}\, z_{u-1}} + \check{\varsigma} . \mathfrak{M}.\mathfrak{N}$$

Since$a_{r-1}\, z_u \to \infty$ as $\dot{\varrho}, v \to \infty$, it follows that

$$\frac{1}{\dot{\varrho} v} \left| \{ a \leq \dot{\varrho}, z \leq v : \dot{\mu}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) \leq 1 - \check{\varsigma} \text{ or } \ddot{v}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) \geq \check{\varsigma} \text{ and } \ddot{\omega}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) \geq \check{\varsigma} \} \right| \to 0$$

and for all $\check{\delta}_1 > 0$, the set

$$\left\{ (\dot{\varrho}, v) \in \mathbb{N} \times \mathbb{N} : \frac{1}{\dot{\varrho} v} \left| \left\{ \begin{array}{c} a \leq \dot{\varrho}, z \leq v : \dot{\mu}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) \leq 1 - \check{\varsigma} \text{ or } \ddot{v}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) \geq \check{\varsigma} \\ \text{and } \ddot{\omega}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) \geq \check{\varsigma} \end{array} \right\} \right| \right\} \in \mathcal{J}_2.$$

This shows that $\mathfrak{x}_{az} \xrightarrow{(\dot{\mu},\ddot{v},\ddot{\omega})} \grave{\xi}\left(I(\mathcal{J}_2)\right)$.

Joining Theorem 3.5 and Theorem 3.6 we get

Theorem 3.7. *Let sequence θ obtain strongly lacunary. NNS. If* $1 < \liminf_r q_r \leq \limsup_r q_r < \infty$, *and* $1 < \lim \inf_u q_u \leq \lim \sup_u q_u < \infty$ *then*

$$\mathfrak{x}_{az} \xrightarrow{(\dot{\mu},\ddot{v},\ddot{\omega})} \grave{\xi}\left(I_\theta(\mathcal{J}_2)\right) \Leftrightarrow \mathfrak{x}_{az} \xrightarrow{(\dot{\mu},\ddot{v},\ddot{\omega})} \grave{\xi}\left(I(\mathcal{J}_2)\right),$$

Proof. *It follows immediately from Theorem 2.2 and Theorem 2.3.*

Theorem 3.8. *Let* $(\Xi, \dot{\mu}, \ddot{v}, \ddot{\omega}, \circledast, \Theta, \Delta)$ *be a NNS so that*

$$\tfrac{1}{4}\check{\varsigma}_{\tilde{m}\hat{n}} \Theta \tfrac{1}{4}\check{\varsigma}_{\tilde{m}\hat{n}} < \tfrac{1}{2}\check{\varsigma}_{\tilde{m}\hat{n}},\ \left(1 - \tfrac{1}{4}\check{\varsigma}_{\tilde{m}\hat{n}}\right) * \left(1 - \tfrac{1}{4}\check{\varsigma}_{\tilde{m}\hat{n}}\right) > 1 - \tfrac{1}{2}\check{\varsigma}_{\tilde{m}\hat{n}} \text{ and } \tfrac{1}{4}\check{\varsigma}_{\tilde{m}\hat{n}} \Delta \tfrac{1}{4}\check{\varsigma}_{\tilde{m}\hat{n}} < \tfrac{1}{2}\check{\varsigma}_{\tilde{m}\hat{n}}$$

If a Banach space Ξ *then closed subset* $I_\theta(\mathcal{J}_2)^{(\dot{\mu},\ddot{v},\ddot{\omega})} \cap \ell_\infty^2(\Xi)$ *of* $\ell_\infty^2(\Xi)$.

Proof. We initially assume that $\left(\mathfrak{x}^{\tilde{m}\hat{n}}\right) = \left(\mathfrak{x}_{az}^{\tilde{m}\hat{n}}\right)$ be a convergent sequence in $I_\theta(\mathcal{J}_2)^{(\dot{\mu},\ddot{v},\ddot{\omega})} \cap \ell_\infty^2(\Xi)$. Suppose $\mathfrak{x}^{(\tilde{m}\hat{n})}$convergent to $\mathfrak{x}$. It is clear $\mathfrak{x} \in \ell_\infty^2(\Xi)$.

We must demonstrate this $\mathfrak{x} \in I_\theta(\mathcal{J}_2)^{(\dot{\mu},\ddot{v},\ddot{\omega})} \cap \ell_\infty^2(\Xi)$. Since $\mathfrak{x}^{\tilde{m}\hat{n}} \in I_\theta(\mathcal{J}_2)^{(\dot{\mu},\ddot{v},\ddot{\omega})} \cap l_\infty^2(\Xi)$ there are some real numbers $\mathfrak{L}_{\tilde{m}\hat{n}}$ in such a way that $\mathfrak{x}_{az}^{\tilde{m}\hat{n}} \xrightarrow{(\dot{\mu},\ddot{v},\ddot{\omega})} \mathfrak{L}_{\tilde{m}\hat{n}}\left(I_\theta(\mathcal{J}_2)\right)$ for $\tilde{m}, \hat{n}$= 1, 2, 3,…

Consider a strictly decreasing positive value double sequence $\{\check{\varsigma}_{\tilde{m}\hat{n}}\}$ converging to zero. For all $\tilde{m}, \hat{n}$ = 1, 2, 3 … that is positive $\mathfrak{N}_{\tilde{m}\hat{n}}$ in such way that if $\tilde{m}, \hat{n} \geq \mathfrak{N}_{\tilde{m}\hat{n}}$ then

$$\sup_{\tilde{m},\hat{n}}, \ddot{v}\left(\mathfrak{x} - \mathfrak{x}^{\tilde{m}\hat{n}}, \dot{\varrho}\right) \leq \frac{\check{\varsigma}_{\tilde{m}\hat{n}}}{\mathbf{4}}.$$

Without limited normality assume that $\mathfrak{N}_{\tilde{m}\hat{n}} = \tilde{m}\hat{n}$ and select a $\check{\delta} > 0$ in such a way that $\check{\delta} < \frac{1}{5}$.

Now set

$$\mathfrak{A}_{\mu,v,\omega}(\check{\varsigma}_{\tilde{m}\hat{n}}, \dot{\varrho}) = \left\{ \begin{array}{c} (r, u) \in \mathbb{N} \times \mathbb{N} \ : \frac{1}{y_r \overline{y_u}} \\ \left| \begin{array}{c} \{(a, z) \in \mathfrak{I}_{ru} : \dot{\mu}\left(\mathfrak{x}_{az}^{\tilde{m}\hat{n}} - \mathfrak{L}_{\tilde{m}\hat{n}}, \dot{\varrho}\right) \leq 1 - \frac{\check{\varsigma}_{\tilde{m}\hat{n}}}{4} or \\ \ddot{v}\left(\mathfrak{x}_{az}^{\tilde{m}\hat{n}} - \mathfrak{L}_{\tilde{m}\hat{n}}, \dot{\varrho}\right) \geq \frac{\check{\varsigma}_{\tilde{m}\hat{n}}}{4} and\ \ddot{\omega}\left(\mathfrak{x}_{az}^{\tilde{m}\hat{n}} - \mathfrak{L}_{\tilde{m}\hat{n}}, \dot{\varrho}\right) \geq \frac{\check{\varsigma}_{\tilde{m}\hat{n}}}{4}\} \end{array} \right| < \check{\delta} \end{array} \right\}$$

belongs to $\mathfrak{F}(\mathcal{J}_2)$ and

$$\mathfrak{B}_{\mu,\nu,\omega}\left(\check{\varsigma}_{\tilde{m}+1,\hat{n}+1}, \dot{\varrho}\right) = \left\{ \begin{array}{c} (r,u) \in \mathbb{N} \times \mathbb{N} \; : \frac{1}{y_r\overline{y_u}} \\ \left| \begin{array}{c} \dot{\mu}\left(\mathfrak{x}_{az}^{\tilde{m}+1,\hat{n}+1} - \mathfrak{L}_{\tilde{m}+1,\hat{n}+1}, \dot{\varrho}\right) \leq 1 - \frac{\check{\varsigma}_{\tilde{m}+1,\hat{n}+1}}{4} \, or \\ \ddot{v}\left(\mathfrak{x}_{az}^{\tilde{m}+1,\hat{n}+1} - \mathfrak{L}_{\tilde{m}+1,\hat{n}+1}, \dot{\varrho}\right) \geq \frac{\check{\varsigma}_{\tilde{m}+1,\hat{n}+1}}{4} \\ and\ \ddot{\omega}\left(\mathfrak{x}_{az}^{\tilde{m}+1,\hat{n}+1} - \mathfrak{L}_{\tilde{m}+1,\hat{n}+1}, \dot{\varrho}\right) \geq \frac{\check{\varsigma}_{\tilde{m}+1,\hat{n}+1}}{4} \end{array} \right| < \check{\delta} \end{array} \right\}$$

belongs to $\mathfrak{F}\,(\mathcal{I}_2)$. Since $\mathfrak{A}_{\dot{\mu},\ddot{v},\ddot{\omega}}(\check{\varsigma}_{\tilde{m}\hat{n}}, \dot{\varrho}) \cap \mathfrak{B}_{\dot{\mu},\ddot{v},\ddot{\omega}}\left(\check{\varsigma}_{\tilde{m}+1,\hat{n}+1}, \dot{\varrho}\right) \in \mathfrak{F}(\mathcal{I}_2)$ and $\emptyset \notin \mathfrak{F}(\mathcal{I}_2)$, we can choose $(r,u) \in \mathfrak{A}_{\dot{\mu},\ddot{v},\ddot{\omega}}(\check{\varsigma}_{\tilde{m}\hat{n}}, \dot{\varrho}) \cap \mathfrak{B}_{\dot{\mu},\ddot{v},\ddot{\omega}}\left(\check{\varsigma}_{\tilde{m}+1,\hat{n}+1}, \dot{\varrho}\right)$. Then

$$\frac{1}{y_r\overline{y_u}} \left| \left\{ \begin{array}{c} (a,z) \in \mathfrak{I}_{ru} : \dot{\mu}\left(\mathfrak{x}_{az}^{\tilde{m}\hat{n}} - \mathfrak{L}_{\tilde{m}\hat{n}}, \dot{\varrho}\right) \leq 1 - \frac{\check{\varsigma}_{\tilde{m}\hat{n}}}{4} \, or \\ \ddot{v}\left(\mathfrak{x}_{az}^{\tilde{m}\hat{n}} - \mathfrak{L}_{\tilde{m}\hat{n}}, \dot{\varrho}\right) \geq \frac{\check{\varsigma}_{\tilde{m}\hat{n}}}{4} \\ and\ \ddot{\omega}\left(\mathfrak{x}_{az}^{\tilde{m}\hat{n}} - \mathfrak{L}_{\tilde{m}\hat{n}}, \dot{\varrho}\right) \geq \frac{\check{\varsigma}_{\tilde{m}\hat{n}}}{4} \\ \vee\, \dot{\mu}\left(\mathfrak{x}_{az}^{\tilde{m}+1,\hat{n}+1} - \mathfrak{L}_{\tilde{m}+1,\hat{n}+1}, \dot{\varrho}\right) \leq 1 - \frac{\check{\varsigma}_{\tilde{m}+1,\hat{n}+1}}{4} \, or \\ \ddot{v}\left(\mathfrak{x}_{az}^{\tilde{m}+1,\hat{n}+1} - \mathfrak{L}_{\tilde{m}+1,\hat{n}+1}, \dot{\varrho}\right) \geq \frac{\check{\varsigma}_{\tilde{m}+1,\hat{n}+1}}{4} \\ and\ \ddot{\omega}\left(\mathfrak{x}_{az}^{\tilde{m}+1,\hat{n}+1} - \mathfrak{L}_{\tilde{m}+1,\hat{n}+1}, \dot{\varrho}\right) \geq \frac{\check{\varsigma}_{\tilde{m}+1,\hat{n}+1}}{4} \end{array} \right\} \right| \leq 2\check{\delta} < 1.$$

Since $y_r\overline{y_u} \to \infty$ and $\mathfrak{A}_{\dot{\mu},\ddot{v},\ddot{\omega}}(\check{\varsigma}_{\tilde{m}\hat{n}}, \dot{\varrho}) \cap \mathfrak{B}_{\dot{\mu},\ddot{v},\ddot{\omega}}\left(\check{\varsigma}_{m+1,n+1}, \dot{\varrho}\right) \in \mathfrak{F}(\mathcal{I}_2)$ is finite, we can select the above r, u so that $y_r\overline{y_u} > 5$.

As a result, there must exist a $(a,z) \in \mathfrak{I}_{ru}$ for whatever we have simultaneously

$\dot{\mu}\left(\mathfrak{x}_{az}^{\tilde{m}\hat{n}} - \mathfrak{L}_{\tilde{m}\hat{n}}, \dot{\varrho}\right) > 1 - \frac{\check{\varsigma}_{\tilde{m}\hat{n}}}{4}$ $or\ \ddot{v}\left(\mathfrak{x}_{az}^{\tilde{m}\hat{n}} - \mathfrak{L}_{\tilde{m}\hat{n}}, \dot{\varrho}\right) < \frac{\check{\varsigma}_{\tilde{m}\hat{n}}}{4}$ and $\ddot{\omega}\left(\mathfrak{x}_{az}^{\tilde{m}\hat{n}} - \mathfrak{L}_{\tilde{m}\hat{n}}, \dot{\varrho}\right) < \frac{\check{\varsigma}_{\tilde{m}\hat{n}}}{4}$,

$\dot{\mu}\left(\mathfrak{x}_{az}^{\tilde{m}+1,\hat{n}+1} - \mathfrak{L}_{\tilde{m}+1,\hat{n}+1}, \dot{\varrho}\right) > 1 - \frac{\check{\varsigma}_{\tilde{m}\hat{n}}}{4}\, or\ \ddot{v}\left(\mathfrak{x}_{az}^{\tilde{m}+1,\hat{n}+1} - \mathfrak{L}_{\tilde{m}+1,\hat{n}+1}, \dot{\varrho}\right) \geq \frac{\check{\varsigma}_{\tilde{m}\hat{n}}}{4}$

$and\ \ddot{\omega}\left(\mathfrak{x}_{az}^{\tilde{m}+1,\hat{n}+1} - \mathfrak{L}_{\tilde{m}+1,\hat{n}+1}, \dot{\varrho}\right) \geq \frac{\check{\varsigma}_{\tilde{m}\hat{n}}}{4}$.

For a given $\check{\varsigma}_{\tilde{m}\hat{n}} > 0$ chose $\frac{\check{\varsigma}_{\tilde{m}\hat{n}}}{2}$ such that

$\left(1 - \frac{1}{2}\check{\varsigma}_{\tilde{m}\hat{n}}\right) * \left(1 - \frac{1}{2}\check{\varsigma}_{\tilde{m}\hat{n}}\right) > 1 - \check{\varsigma}_{\tilde{m}\hat{n}}$, $\frac{1}{2}\check{\varsigma}_{\tilde{m}\hat{n}}\ \Theta\ \frac{1}{2}\check{\varsigma}_{\tilde{m}\hat{n}} < \check{\varsigma}_{\tilde{m}\hat{n}}$ and $\frac{1}{2}\check{\varsigma}_{\tilde{m}\hat{n}}\ \Delta\ \frac{1}{2}\check{\varsigma}_{\tilde{m}\hat{n}} < \check{\varsigma}_{\tilde{m}\hat{n}}$.

Then it follows that

$$\ddot{v}\left(\mathfrak{L}_{\tilde{m}\hat{n}} - \mathfrak{x}_{az}^{\tilde{m}\hat{n}}, \frac{\dot{\varrho}}{2}\right) \Theta\ \ddot{v}\left(\mathfrak{L}_{\tilde{m}+1,\hat{n}+1} - \mathfrak{x}_{az}^{\tilde{m}+1,\hat{n}+1}, \frac{\dot{\varrho}}{2}\right) \leq \frac{\check{\varsigma}_{\tilde{m}\hat{n}}}{4}\ \Theta\ \frac{\check{\varsigma}_{\tilde{m}\hat{n}}}{4} < \frac{\check{\varsigma}_{\tilde{m}\hat{n}}}{2}$$

and

$$\ddot{v}\left(\mathfrak{x}_{az}^{\tilde{m}\hat{n}} - \mathfrak{x}_{az}^{\tilde{m}+1,\hat{n}+1}, \dot{\varrho}\right) \leq \sup_{\tilde{m},\hat{n}} \ddot{v}\left(\mathfrak{x} - \mathfrak{x}^{\tilde{m}\hat{n}}, \frac{\dot{\varrho}}{2}\right)\ \Theta\ \sup_{\tilde{m},\hat{n}} \ddot{v}\left(\mathfrak{x} - \mathfrak{x}^{\tilde{m}+1,\hat{n}+1}, \frac{\dot{\varrho}}{2}\right)$$

$$\leq \frac{\check{\varsigma}_{\tilde{m}\hat{n}}}{4}\ \Theta\ \frac{\check{\varsigma}_{\tilde{m}\hat{n}}}{4} < \frac{\check{\varsigma}_{\tilde{m}\hat{n}}}{2}$$

Hence, we have

$$\ddot{v}\left(\mathfrak{L}_{\widetilde{m}\widehat{n}} - \mathfrak{L}_{\widetilde{m}+1,\widehat{n}+1}, \dot{\varrho}\right) \leq \left[\ddot{v}\left(\mathfrak{L}_{\widetilde{m}\widehat{n}} - \mathfrak{x}_{az}^{\widetilde{m}\widehat{n}}, \frac{\dot{\varrho}}{3}\right) \Theta\, \ddot{v}\left(\mathfrak{x}_{az}^{\widetilde{m}+1,\widehat{n}+1} - \mathfrak{L}_{\widetilde{m}+1,\widehat{n}+1}, \frac{\dot{\varrho}}{3}\right) \Theta\, \ddot{v}\left(\mathfrak{x}_{az}^{\widetilde{m}\widehat{n}} - \mathfrak{x}_{az}^{\widetilde{m}+1,\widehat{n}+1}, \frac{\dot{\varrho}}{3}\right)\right]$$

$$\leq \frac{\check{\varsigma}_{\widetilde{m}\widehat{n}}}{2}\, \Theta\, \frac{\check{\varsigma}_{\widetilde{m}\widehat{n}}}{2} < \check{\varsigma}_{\widetilde{m}\widehat{n}}$$

and similarly $\dot{\mu}\left(\mathfrak{L}_{\widetilde{m}\widehat{n}} - \mathfrak{L}_{\widetilde{m}+1,\widehat{n}+1}, \dot{\varrho}\right) > 1 - \check{\varsigma}_{\widetilde{m}\widehat{n}}$ *and* $\ddot{\omega}\left(\mathfrak{L}_{\widetilde{m}\widehat{n}} - \mathfrak{L}_{\widetilde{m}+1,\widehat{n}+1}, \dot{\varrho}\right) < \check{\varsigma}_{\widetilde{m}\widehat{n}}$.
It implies that $\{\mathfrak{L}_{\widetilde{m}\widehat{n}}\}_{\widetilde{m},\widehat{n} \in \mathbb{N}}$ is a Cauchy sequence in Ξ and let $\mathfrak{L}_{\widetilde{m}\widehat{n}} \to \mathfrak{L} \in \Xi$ as $\widetilde{m}, \widehat{n} \to \infty$.

We will demonstrate this $\mathfrak{x} \xrightarrow{(\dot{\mu},\ddot{v},\ddot{\omega})} \mathfrak{L}_{\widetilde{m}\widehat{n}}\left(I_\theta(\mathcal{J}_2)\right)$.

For any $\check{\varsigma} > 0$ and $\dot{\varrho} > 0$, select $(\widetilde{m}, \widehat{n}) \in \mathbb{N} \times \mathbb{N}$ in such a way that

$$\check{\varsigma}_{\widetilde{m}\widehat{n}} > \frac{1}{4}\check{\varsigma}, \sup_{\widetilde{m},\widehat{n}} \ddot{v}\left(\mathfrak{x} - \mathfrak{x}^{\widetilde{m}\widehat{n}}, \dot{\varrho}\right) < \frac{1}{4}\check{\varsigma}, \qquad \ddot{v}\left(\mathfrak{L}_{\widetilde{m}\widehat{n}} - \mathfrak{L}, \dot{\varrho}\right) > 1 - \frac{1}{4}\check{\varsigma} \; or \; \frac{1}{4}\check{\varsigma}, \ddot{v}\left(\mathfrak{L}_{\widetilde{m}\widehat{n}} - \mathfrak{L}, \dot{\varrho}\right) < 1 - \frac{1}{4}\check{\varsigma}.$$

Now since

$$\frac{1}{y_r\overline{y_u}}\left|\{(a,z) \in \mathfrak{I}_{ru} : \ddot{v}\left(\mathfrak{x}_{az} - \mathfrak{L}, \dot{\varrho}\right) \geq \check{\varsigma}\}\right| \leq \frac{1}{y_r\overline{y_u}}\left|\left\{\begin{matrix} (a,z) \in \mathfrak{I}_{ru} : \ddot{v}\left(\mathfrak{x}_{az} - \mathfrak{x}_{az}^{\widetilde{m}\widehat{n}}, \frac{\dot{\varrho}}{3}\right) \Theta \\ \left[\ddot{v}\left(\mathfrak{x}_{az}^{\widetilde{m}\widehat{n}} - \mathfrak{L}_{\widetilde{m}\widehat{n}}, \frac{\dot{\varrho}}{3}\right) \Theta\, \ddot{v}\left(\mathfrak{L}_{\widetilde{m}\widehat{n}} - \mathfrak{L}, \frac{\dot{\varrho}}{3}\right)\right] \geq \check{\varsigma} \end{matrix}\right\}\right|$$

$$\leq \frac{1}{y_r\overline{y_u}}\left|\left\{(a,z) \in \mathfrak{I}_{ru} : \ddot{v}\left(\mathfrak{x}_{az}^{\widetilde{m}\widehat{n}} - \mathfrak{L}_{\widetilde{m}\widehat{n}}, \frac{\dot{\varrho}}{3}\right) \geq \frac{\check{\varsigma}}{2}\right\}\right|$$

and equivalently

$$\frac{1}{y_r\overline{y_u}}\left|\{(a,z) \in \mathfrak{I}_{ru} : \dot{\mu}\left(\mathfrak{x}_{az} - \mathfrak{L}, \dot{\varrho}\right) \leq 1 - \check{\varsigma}\}\right| > \frac{1}{y_r\overline{y_u}}\left|\left\{(a,z) \in \mathfrak{I}_{ru} : \dot{\mu}\left(\mathfrak{x}_{az}^{\widetilde{m}\widehat{n}} - \mathfrak{L}_{\widetilde{m}\widehat{n}}, \frac{\dot{\varrho}}{3}\right) \leq 1 - \frac{\check{\varsigma}}{2}\right\}\right|$$

and

$$\frac{1}{y_r\overline{y_u}}\left|\{(a,z) \in \mathfrak{I}_{ru} : \ddot{\omega}\left(\mathfrak{x}_{az} - \mathfrak{L}, \dot{\varrho}\right) \geq \check{\varsigma}\}\right| \leq \frac{1}{y_r\overline{y_u}}\left|\left\{(a,z) \in \mathfrak{I}_{ru} : \ddot{\omega}\left(\mathfrak{x}_{az}^{\widetilde{m}\widehat{n}} - \mathfrak{L}_{\widetilde{m}\widehat{n}}, \frac{\dot{\varrho}}{3}\right) \leq \frac{\check{\varsigma}}{2}\right\}\right|.$$

It follows that

$$\left\{(r,u) \in \mathbb{N} \times \mathbb{N} : \frac{1}{y_r\overline{y_u}}\left|\left\{\begin{matrix} (a,z) \in \mathfrak{I}_{ru} : \dot{\mu}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) \leq 1 - \check{\varsigma} \\ or\ \ddot{v}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) \geq \check{\varsigma}\ and\ \ddot{\omega}\left(\mathfrak{x}_{az} - \grave{\xi}, \dot{\varrho}\right) \geq \check{\varsigma} \end{matrix}\right\}\right| \geq \breve{\delta}\right\}$$

$$\subset \left\{(r,u) \in \mathbb{N} \times \mathbb{N} : \frac{1}{y_r\overline{y_u}}\left|\left\{\begin{matrix} (a,z) \in \mathfrak{I}_{ru} : \dot{\mu}\left(\mathfrak{x}_{az}^{\widetilde{m}\widehat{n}} - \mathfrak{L}_{\widetilde{m}\widehat{n}}, \frac{\dot{\varrho}}{3}\right) \leq 1 - \frac{\check{\varsigma}}{2} \\ or\ \ddot{v}\left(\mathfrak{x}_{az}^{\widetilde{m}\widehat{n}} - \mathfrak{L}_{\widetilde{m}\widehat{n}}, \frac{\dot{\varrho}}{3}\right) \geq \frac{\check{\varsigma}}{2}\ and\ \ddot{\omega}\left(\mathfrak{x}_{az}^{\widetilde{m}\widehat{n}} - \mathfrak{L}_{\widetilde{m}\widehat{n}}, \frac{\dot{\varrho}}{3}\right) \geq \frac{\check{\varsigma}}{2} \end{matrix}\right\}\right| \geq \breve{\delta}\right\}$$

for any given $\breve{\delta} > 0$. Hence we have $\mathfrak{x} \xrightarrow{(\dot{\mu},\ddot{v},\ddot{\omega})} \mathfrak{L}_{\widetilde{m}\widehat{n}}\left(I_\theta(\mathcal{J}_2)\right)$..

4. Conclusion

We define the concept of $\mathcal{J}_2$-LSC additionally strong $\mathcal{J}_2$-LC in order to relate towards the *NN*, examine their relationship, and while certain observations regarding these. The research we conducted involving $\mathcal{J}_2$-statistical in addition $\mathcal{J}_2$-LSC about sequences in *NNS* give a technique for approaching convergence problems of fuzzy real number sequences.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

References

1. K. T. Atanassov: Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20 (1986), 87-96.
2. C. Belen and M. Yildirim: On generalized statistical convergence of double sequence via ideals. Ann. Univ. Ferrara, 58(1) (2012), 11-20.
3. C. Cakan, B. Altay and H. Coskun: Double lacunary density and lacunary statistical convergence of double sequences. Studia Sci. Math. Hungar. 47(1) (2010), 35-45.
4. B. Hazarika, A. Alotaibi and S. A. Mohiuddine: Statistical convergence in measure for double sequences of fuzzy-valued functions. Soft Computing, 24 (2020), 6613-6622.
5. M. Jeyaraman: Generalized Hyers-Ulam-Rassias Stability in Neutrosophic Normed Spaces. Octogon Mathematical Magazine 30 (2) (2022), 773 – 792.
6. M. Jeyaraman, AN. Mangayarkkarasi, V. Jeyanthi, R. Pandiselvi: Hyers-Ulam-Rassias Stability for Functional Equation in Neutrosophic Normed Spaces. International Journal of Neutrosophic Science (IJNS) 18 (1) (2022), 127-143.
7. M. Jeyaraman, P. Jenifer: Statistical Δ m-Convergence in Neutrosophic Normed Spaces, Journal of Computational Mathematica 7(1) (2023), 46-60.
8. M. Jeyaraman, P. Jenifer, U. Praveena: New Approach in Logarithmic Summability of Sequences in Neutrosophic Normed Spaces. International Journal of Neutrosophic Science (IJNS) 19 (3) (2022).
9. M. Jeyaraman, A. Ramachandran, VB. Shakila: Approximate fixed point theorems for weak contraction on Neutrosophic normed spaces. Journal of Computational Mathematica 6 (1) (2022), 134-158.
10. O. Kisi: I-lacunary statistical convergence in intuitionistic fuzzy normed spaces: Aku J. Sci. Eng. 20 (2020), 207-212.
11. S. Kumar, V. Kumar and S. S. Bhatia: On ideal version of double sequences, Gen. Math Notes, 17(1) (2013), 32-44.
12. AN. Mangayarkkarasi, M. Jeyaraman, V. Jeyanthi: On Stability of a cubic Functional Equation in Neutrosophic Normed Spaces. Advances and Applications in Mathematical Sciences 21 (4) (2022), 1975 – 1988.
13. S. A. Mohiuddine, B. Hazarika and A. Alotaibi: On statistical convergence of double sequences of fuzzy valued functions. J. Intell. Fuzzy Syst. 32 (2017), 4331-4342.
14. M. Mursaleen and S. A. Mohiuddine: On ideal convergence of double sequences in probabilistic normed spaces. Math. Reports, 12(62)4 (2010), 359-371.
15. F. Smarandache: Neutrosophy. Neutrosophic Probability, Set, and Logic, Pro Quest Information & Learning, Ann Arbor, Michigan, USA (1998).
16. F. Smarandache: Neutrosophic set a generalization of the intuitionistic Fuzzy sets. Inter J Pure Appl Math, 24 (2005), 287297.
17. S. Sowndrarajan, M. Jeyaraman, Florentin Smarandache: Fixed Point theorems in Neutrosophic metric spaces. Neutrosophic Sets a System, 36 (2020), 251-268.
18. L. A. Zadeh: Fuzzy sets. Inform. Control, 8 (1965), 338-353.

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