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Covering Properties via Neutrosophic b-open Sets

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Abstract: The purpose of this article is to study some covering properties in neutrosophic topological spaces using neutrosophic b-open sets. We define neutrosophic b-open cover, neutrosophic b-compactness, neutrosophic countably b-compactness neutrosophic b-Lindelöfness, neutrosophic local b-compactness and study various properties entangled with them. We study some covering properties involving neutrosophic continuous, neutrosophic b-continuous and neutrosophic b^*-continuous functions. Lastly, we define neutrosophic base, neutrosophic subbase, neutrosophic second countability via neutrosophic b-open sets and investigate some properties.

Keywords: Neutrosophic b-open cover; Neutrosophic b-compact space; Neutrosophic countably b-compact space; Neutrosophic local b-compact space; Neutrosophic b-base.

1. Introduction

In 1965, Zadeh [30] introduced the concept of a fuzzy set. K. Atanassov [1], in 1986, extended this notion to intuitionistic fuzzy set. After that, the idea of a neutrosophic set was developed and studied by Florentin Smarandache [20-22]. Later, the theory was studied and taken ahead by many researchers [9,12,26,28]. It had been proved by Smarandache [22] that a neutrosophic set was a generalized form of an intuitionistic fuzzy set. Various applications [4,5,15,29] in different fields were done in a neutrosophic environment.

In the year 1968, C. L. Chang [7] created the notion of a fuzzy topological space and then, in 1997, D. Coker [8] gave the idea of intuitionistic fuzzy topological space. In the year 2012, Salama & Alblowi [23] introduced neutrosophic topological space as a generalization of intuitionistic fuzzy topological space. Afterwards, many studies were done by the researchers [2,3,6,11,16-19,24,25,27] to develop various aspects of neutrosophic topological spaces. The concept of neutrosophic b-open sets was given by Ebenanjar *et al*.[14]. Recently Dey & Ray [10] studied compactness in neutrosophic topological spaces. But compactness via neutrosophic b-open sets has not bee studied so far. In this write-up, we study covering properties using neutrosophic b-open sets.

The article is organized by stating some basic concepts in section 2. In section 3, we define neutrosophic b-open covering, neutrosophic b-compactness, neutrosophic countably b-compactness and neutrosophic b-Lindelöfness and study various properties associated with them. In section 4, we define neutrosophic local b-compactness and try to establish some properties. We define neutrosophic b-base, neutrosophic b-subbase, neutrosophic b-second countability and investigate some covering properties in section 5 and lastly, in section 6, we confer a conclusion.

2. Preliminaries

In this section, we state some basic concepts which will be helpful in the later sections.

2.1. Definition: [20] Let X be the universe of discourse. A neutrosophic set A over X is defined as $A = \{\langle x, \mathrm{T}_A(x), \mathrm{I}_A(x), \mathrm{F}_A(x)\rangle : x \in X\}$, where the functions $\mathrm{T}_A, \mathrm{I}_A, \mathrm{F}_A$ are real standard or non-standard subsets of $]^-0, 1^+[$, i.e., $\mathrm{T}_A: X \to]^-0, 1^+[$, $\mathrm{I}_A: X \to]^-0, 1^+[$, $\mathrm{F}_A: X \to]^-0, 1^+[$ and $^-0 \le \mathrm{T}_A(x) + \mathrm{I}_A(x) + \mathrm{T}_A(x) \le 3^+$.

The neutrosophic set A is characterized by the truth-membership function T_A, indeterminacy-membership function I_A, falsehood-membership function F_A.

2.2. Definition: [28] Let X be the universe of discourse. A single valued neutrosophic set A over X is defined as $A = \{\langle x, \mathrm{T}_A(x), \mathrm{I}_A(x), \mathrm{F}_A(x)\rangle : x \in X\}$, where $\mathrm{T}_A, \mathrm{I}_A, \mathrm{F}_A$ are functions from X to [0,1] and $0 \leq \mathrm{T}_A(x) + \mathrm{I}_A(x) + \mathrm{T}_A(x) \leq 3$.

The set of all single valued neutrosophic sets over X is denoted by $\mathcal{N}(X)$.

Throughout this article, a neutrosophic set (NS, for short) will mean a single-valued neutrosophic set.

2.3. Definition: [16] Let $A, B \in \mathcal{N}(X)$. Then

i) (Inclusion): If $\mathrm{T}_A(x) \leq \mathrm{T}_B(x), \mathrm{I}_A(x) \geq \mathrm{I}_B(x), \mathrm{F}_A(x) \geq \mathrm{F}_B(x)$ for all $x \in X$ then A is said to be a neutrosophic subset of B and which is denoted by $A \subseteq B$.

ii) (Equality): If $A \subseteq B$ and $B \subseteq A$ then $A = B$.

iii) (Intersection): The intersection of A and B, denoted by $A \cap B$, is defined as $A \cap B = \{\langle x, \mathrm{T}_A(x) \wedge \mathrm{T}_B(x), \mathrm{I}_A(x) \vee \mathrm{I}_B(x), \mathrm{F}_A(x) \vee \mathrm{F}_B(x)\rangle : x \in X\}$.

iv) (Union): The union of A and B, denoted by $A \cup B$, is defined as $A \cup B = \{\langle x, \mathrm{T}_A(x) \vee \mathrm{T}_B(x), \mathrm{I}_A(x) \wedge \mathrm{I}_B(x), \mathrm{F}_A(x) \wedge \mathrm{F}_B(x)\rangle : x \in X\}$.

v) (Complement): The complement of the NS A, denoted by A^c, is defined as $A^c = \{\langle x, \mathrm{F}_A(x), 1 - \mathrm{I}_A(x), \mathrm{T}_A(x)\rangle : x \in X\}$

vi) (Universal Set): If $\mathrm{T}_A(x) = 1, \mathrm{I}_A(x) = 0, \mathrm{F}_A(x) = 0$ for all $x \in X$ then A is said to be neutrosophic universal set and which is denoted by $\tilde{X}$.

vii) (Empty Set): If $\mathrm{T}_A(x) = 0, \mathrm{I}_A(x) = 1, \mathrm{F}_A(x) = 1$ for all $x \in X$ then A is said to be neutrosophic empty set and which is denoted by $\tilde{\emptyset}$.

2.4. Definition: [18] Let $\mathcal{N}(X)$ be the set of all neutrosophic sets over X. An NS $P = \{\langle x, \mathrm{T}_A(x), \mathrm{I}_A(x), \mathrm{F}_A(x)\rangle : x \in X\}$ is called a neutrosophic point (NP, for short) iff for any element $y \in X$, $\mathrm{T}_P(y) = \alpha, \mathrm{I}_P(y) = \beta, \mathrm{F}_P(y) = \gamma$ for $y = x$ and $\mathrm{T}_P(y) = 0, \mathrm{I}_P(y) = 1, \mathrm{F}_P(y) = 1$ for $y \neq x$, where $0 < \alpha \leq 1, 0 \leq \beta < 1, 0 \leq \gamma < 1$. A neutrosophic point $P = \{\langle x, \mathrm{T}_A(x), \mathrm{I}_A(x), \mathrm{F}_A(x)\rangle : x \in X\}$ will be denoted by $x_{\alpha,\beta,\gamma}$. For the NP $x_{\alpha,\beta,\gamma}$, x will be called its support. The complement of the NP $x_{\alpha,\beta,\gamma}$ will be denoted by $x^c_{\alpha,\beta,\gamma}$ or $(x_{\alpha,\beta,\gamma})^c$.

2.5. Definition: [16] Let $\tau \subseteq \mathcal{N}(X)$. Then τ is called a neutrosophic topology on X if

i) $\tilde{\emptyset}$ and $\tilde{X}$ belong to τ.

ii) An arbitrary union of neutrosophic sets in τ is in τ.

iii) The intersection of any two neutrosophic sets in τ is in τ.

If τ is a neutrosophic topology on X then the pair (X, τ) is called a neutrosophic topological space (NTS, for short) over X. The members of τ are called neutrosophic open sets in X. If for a neutrosophic set A, $A^c \in \tau$ then A is said to be a neutrosophic closed set in X.

2.6. Definition:[14] Let (X, τ) be an NTS and G be a NS over X. Then G is called a

i) Neutrosophic b-open (NBO, for short) set iff $G \subseteq [int(cl(G))] \cup [cl(int(G))]$.

ii) Neutrosophic b-closed (NBC, for short) set iff $G \supseteq [int(cl(G))] \cup [cl(int(G))]$.

If G is an NBO (resp. NBC) set in (X, τ) then we shall also say that G is a τ-NBO (resp. τ-NBC) set.

2.7. Theorem: [14] Let (X, τ) be an NTS.

i) If $G \in \mathcal{N}(X)$ then G is an NBO set iff G^c is an NBC set.
ii) If $G \in \mathcal{N}(X)$ then G is an NBC set iff G^c is an NBO set.

2.8. Theorem: [13] Let (X, τ) be an NTS and $A \in \mathcal{N}(X)$. Then

i) Every neutrosophic open set in an NTS is an NBO set.
ii) Every neutrosophic closed set in an NTS is an NBC set.

2.9. Definition: [27] Let f be a function from an NTS (X, τ) to the NTS (Y, σ). Then

i) f is called a neutrosophic open function if $f(G) \in \sigma$ for all $G \in \tau$
ii) f is called a neutrosophic continuous function if $f^{-1}(G) \in \tau$ for all $G \in \sigma$.

2.10. Definition: [13] Let f be a function from an NTS (X, τ) to the NTS (Y, σ). Then f is called a neutrosophic

i) b-open function if $f(G)$ is an NBO set in Y for every neutrosophic open set G in X.
ii) b-continuous function if $f^{-1}(G)$ is an NBO set in X for every σ-open NS G in Y.
iii) b^*-continuous function if $f^{-1}(G)$ is an NBO set in X for every NBO set G in Y.

2.11. Proposition: [13] Let $(Y, \tau |_Y)$ be a neutrosophic subspace of the NTS (X, τ). Then

i) $G |_Y$ is a $\tau |_Y$-NBO set in Y for every τ-NBO set G in X.
ii) $G |_Y$ is a $\tau |_Y$-NBC set in Y for every τ-NBC set G in X.

2.12. Definition: [10] Let (X, τ) be an NTS. A collection $\{G_\lambda : \lambda \in \Delta\}$ of neutrosophic sets of X is said to have the finite intersection property (FIP, in short) iff every finite subcollection $\{G_{\lambda_k} : k = 1,2,\cdots,n\}$ of $\{G_\lambda : \lambda \in \Delta\}$ satisfies the condition $\bigcap_{k=1}^{n} G_{\lambda_k} \neq \widetilde{\emptyset}$, where Δ is an index set.

*For neutrosophic function and its properties, please see [25].

3. Neutrosophic b-compactness

3.1. Definition: Let (X, τ) be an NTS and $A \in \mathcal{N}(X)$. A collection $C = \{G_i : i \in \Delta\}$ of NBO sets of X is called a neutrosophic b-open cover (NBOC, in short) of A iff $A \subseteq \cup_{i\in\Delta} G_i$. In particular, C is said to be an NBOC of X iff $\tilde{X} = \cup_{i\in\Delta} G_i$.

Let C be an NBOC of the NS A and $C' \subseteq C$. Then C' is called a neutrosophic b-open subcover (NBOSC, in short) of C if C' is also a NBOC of A.

An NBOC C of an NS A is said to be countable (resp. finite) if C consists of a countable (resp. finite) number of NBO sets.

3.2. Definition: An NS A in an NTS (X, τ) is said to be a neutrosophic b-compact set iff every NBOC of A has a finite NBOSC.

An NS A in an NTS (X, τ) is said to be a neutrosophic b-Lindelöf (resp. neutrosophic countably b-compact) set iff every NBOC (resp. countable NBOC) of A has a countable(resp. finite) NBOSC.

An NTS (X, τ) is said to be a neutrosophic b-compact space iff every NBOC of X has a finite NBOSC.

An NTS (X, τ) is said to be a neutrosophic b-Lindelöf (neutrosophic countably b-compact) space iff every NBOC (countable NBOC) of X has a countable(finite) NBOSC.

3.3. Proposition: Every neutrosophic b-compact space is a neutrosophic countably b-compact space.

Proof: Obvious.

3.4. Proposition: In an NTS, every neutrosophic b-compact set is a neutrosophic compact set.

Proof: Let A be a neutrosophic b-compact set of an NTS (X, τ). Let $C = \{G_i : i \in \Delta\}$ be an NOC of A. Since every neutrosophic open set is an NBO set[by 2.9], so G_i is an NBO set for each $i \in \Delta$. Therefore C is an NBOC of A. Since A is b-compact, so there exists a finite subcollection $\{G_i^1, G_i^2, \ldots, G_i^m\}$, say, of C such that $A \subseteq G_i^1 \cup G_i^2 \cup \ldots \cup G_i^m$. Thus the NOC C of A has a finite NOSC $\{G_i^1, G_i^2, \ldots, G_i^m\}$. Hence A is a neutrosophic compact set.

3.5. Example : Converse of the prop. 3.4 is not true. We establish it by the following example.

Let $X = \{a, b\}$, $B = \{\langle a, 0,1,1\rangle, \langle b, 1,0,0\rangle\}$, $G_n = \{\langle a, 0,1,1\rangle, \langle b, \frac{n}{n+1}, \frac{1}{n}, \frac{1}{n+1}\rangle\}, n \in \mathrm{N} = \{1,2,3, \cdots\}$ and $\tau = \{\tilde{X}, \tilde{\emptyset}, B\}$. Clearly (X, τ) is an NTS and G_n is an NBO set for each $n \in \mathrm{N}$. Obviously B is a neutrosophic compact set. We observe that $\{G_n : n \in \mathrm{N}\}$ is an NBOC of B but it has no NBOSC. Therefore B is not a neutrosophic b-compact set.

3.6. Proposition: Every neutrosophic b-compact space is a neutrosophic compact space.

Proof: Obvious from prop. 3.4.

3.7. Remark: Converse of prop. 3.6 is not true. We establish it by the following example.

Let us consider the NTS (N, τ), where $\tau = \{\tilde{\emptyset}, \tilde{\mathrm{N}}\}, \mathrm{N} = \{1, 2, 3, \cdots\}$. Clearly (N, τ) is a neutrosophic compact space. We show that (N, τ) is not a neutrosophic b-compact space. For $n \in \mathrm{N}$, we define $G_n = \{\langle x, \mathrm{T}_{G_n}(x), \mathrm{I}_{G_n}(x), \mathrm{F}_{G_n}(x) : x \in \mathrm{N}\}$, where $\mathrm{T}_{G_n}(x) = 1, \mathrm{I}_{G_n}(x) = 0, \mathrm{F}_{G_n}(x) = 0$ if $x = n$ and $\mathrm{T}_{G_n}(x) = 0, \mathrm{I}_{G_n}(x) = 1, \mathrm{F}_{G_n}(x) = 1$ if $x \neq n$. Clearly, for each $n \in \mathrm{N}$, G_n is an NBO set in (N, τ). Obviously the collection $C = \{G_n : n \in \mathrm{N}\}$ is an NBOC of N but it has no finite NBOSC. Therefore (N, τ) is not a neutrosophic b-compact space. Thus (N, τ) is a neutrosophic compact space but not a neutrosophic b-compact space.

3.8. Proposition: In an NTS, union of two neutrosophic b-compact sets is neutrosophic b-compact.

Proof: Let A and B be two neutrosophic b-compact sets of an NTS (X, τ). Let $C = \{G_i : i \in \Delta\}$ be an NBOC of $A \cup B$. Then $A \cup B \subseteq \cup_{i \in \Delta} G_i$. Since $A \subseteq A \cup B$, so C is an NBOC of A. Again since A is neutrosophic b-compact, so there exists a finite subcollection $\{G_i^1, G_i^2, \ldots, G_i^m\}$ of C such that $A \subseteq G_i^1 \cup G_i^2 \cup \ldots \cup G_i^m$. Similarly, since B is neutrosophic b-compact, so there exists a finite subcollection $\{H_i^1, H_i^2, \ldots, H_i^n\}$ of C such that $B \subseteq H_i^1 \cup H_i^2 \cup \ldots \cup H_i^n$. Therefore $A \cup B \subseteq G_i^1 \cup G_i^2 \cup \ldots \cup G_i^m \cup H_i^1 \cup H_i^2 \cup \ldots \cup H_i^n$. Thus there exists a finite subcollection $\{G_i^1, G_i^2, \ldots, G_i^m, H_i^1, H_i^2, \ldots, H_i^n\}$ of C such that $\cup B \subseteq G_i^1 \cup G_i^2 \cup \ldots \cup G_i^m \cup H_i^1 \cup H_i^2 \cup \ldots \cup H_i^n$. Therefore $A \cup B$ is neutrosophic b-compact. Hence proved.

3.9. Proposition: In an NTS, finite union of neutrosophic b-compact sets is neutrosophic b-compact.

Proof: Immediate from the prop. 3.8.

3.10. Proposition: In an NTS, union of a neutrosophic b-compact set and a neutrosophic compact set is a neutrosophic compact set.

Proof: Obvious.

3.11. Definition: Let $(Y, \tau |_Y)$ be a neutrosophic subspace of the NTS (X, τ). Then the set of all NBO sets $G |_Y$ in Y for which G is an NBO set in X will be denoted by $NBO(Y)$, i.e., $NBO(Y) = \{G |_Y \subseteq Y : G |_Y$ is an NBO set in Y and $G \subseteq X$ is an NBO set in $X\}$.

3.12. Proposition: Let $(Y, \tau |_Y)$ be a neutrosophic subspace of the NTS (X, τ) and $A \subseteq Y$. Then A is neutrosophic b-compact in X iff every cover of A by the sets in $NBO(Y)$ has a finite subcover.

Proof: Necessary part: Let $C = \{G_i |_Y : i \in \Delta\}$ be a cover of A, where $G_i |_Y \in NBO(Y)$ for each $i \in \Delta$. Then $A \subseteq \cup_{i \in \Delta} G_i |_Y \Rightarrow A \subseteq \cup_{i \in \Delta} G_i$. Clearly G_i is an NBO set in X [by 3.11] for each $G_i |_Y \in C$ and so, $C^* = \{G_i : G_i |_Y \in NBO(Y)\}$ is an NBOC of A in X. Since A is b-compact in X, so there exists a finite subcollection $\{G_{i_k} : k = 1,2,3, \ldots, n\}$ of C^* such that $A \subseteq \cup_{k=1}^{n} G_{i_k} \Rightarrow A \subseteq \left(\cup_{k=1}^{n} G_{i_k}\right) |_Y \Rightarrow A \subseteq \cup_{k=1}^{n} \left(G_{i_k} |_Y\right)$. Thus the cover C of A has a finite subcover $\{G_{i_k} : k = 1,2,3, \ldots, n\}$.

Sufficient part: Let $B = \{G_i: i \in \Delta\}$ be an NBOC of A in X, where G_i is an NBO set in X for each $i \in \Delta$. Then $A \subseteq \cup_{i\in\Delta} G_i \Rightarrow A \subseteq (\cup_{i\in\Delta} G_i) |_Y \Rightarrow A \subseteq \cup_{i\in\Delta} (G_i |_Y)$. Since $G_i |_Y \in NBO(Y)$ for each $G_i \in B$ [by 2.12], so $B^* = \{G_i |_Y: i \in \Delta\}$ is a cover of A by the NBO sets in $NBO(Y)$. Therefore, by hypothesis, there exists a finite subcollection $\{G_{i_k} |_Y: k = 1,2,3,\ldots,n\}$ of B^* such that $A \subseteq \cup_{k=1}^{n} (G_{i_k} |_Y) \Rightarrow A \subseteq (\cup_{k=1}^{n} G_{i_k}) |_Y \Rightarrow A \subseteq \cup_{k=1}^{n} G_{i_k}$. Thus the NBOC B of A has a finite NBOSC $\{G_{i_k}: k = 1,2,3,\ldots,n\}$. Therefore, A is neutrosophic b-compact in X.

3.13. Proposition: Let $(Y, \tau |_Y)$ be a neutrosophic subspace of the NTS (X, τ) and $A \subseteq Y$. Then A is neutrosophic countably b-compact in X iff every countable cover of A by the sets in $NBO(Y)$ has a finite subcover.

Proof: Obvious from the prop. 3.12.

3.14. Proposition: Let $(Y, \tau |_Y)$ be a neutrosophic subspace of the NTS (X, τ) and $A \subseteq Y$. Then A is neutrosophic b-Lindelöf in X iff every cover of A by the sets in $NBO(Y)$ has a countable subcover.

Proof: Obvious from the prop. 3.12.

3.15. Proposition: Let $(Y, \tau |_Y)$ be a neutrosophic subspace of the NTS (X, τ) and $A \subseteq Y$. If A is neutrosophic b-compact in X then A is neutrosophic compact in Y.

Proof: Let $C = \{G_i |_Y: i \in \Delta\}$ be an NOC of A in Y, where $G_i |_Y \in \tau |_Y$ for each $i \in \Delta$. Then $A \subseteq \cup_{i\in\Delta} (G_i |_Y) \Rightarrow A \subseteq \cup_{i\in\Delta} G_i$. Obviously $G_i \in \tau$ and so, G_i is an NBO set in X for each $i \in \Delta$. Therefore, $C^* = \{G_i: G_i |_Y \in C\}$ is an NBOC of A in X. Since A is b-compact in X, so there exists a finite subcollection $\{G_{i_k}: k = 1,2,3,\ldots,n\}$ of C^* such that $A \subseteq \cup_{k=1}^{n} G_{i_k} \Rightarrow A \subseteq (\cup_{k=1}^{n} G_{i_k}) |_Y \Rightarrow A \subseteq \cup_{k=1}^{n} (G_{i_k} |_Y)$. Thus the NOC C of A has a finite NOSC $\{G_{i_k}: k = 1,2,3,\ldots,n\}$. Therefore A is neutrosophic compact in Y.

3.16. Proposition: Let $(Y, \tau |_Y)$ be a neutrosophic subspace of the NTS (X, τ) and $A \subseteq Y$. If A is b-compact in Y then A is b-compact in X.

Proof: Obvious.

3.17. Proposition: If G is an NBC subset of a neutrosophic b-compact space (X, τ) such that $G \cap G^c = \widetilde{\emptyset}$ then G is a neutrosophic b-compact.

Proof: Let $C = \{H_i: i \in \Delta\}$ be an NBOC of G. Then $A \subseteq \cup_{i\in\Delta} H_i$. Since G^c is an NBO set and since $G \cap G^c = \widetilde{\emptyset}$, i.e., $G \cup G^c = \widetilde{X}$, so $D = \{H_i: i \in \Delta\} \cup \{G^c\}$ is an NBOC of X. As X is neutrosophic b-compact, so there exists a finite subcollection $D' = \{H_{i_1}, H_{i_2}, \ldots, H_{i_n}\} \cup \{G^c\}$ of D such that $X \subseteq H_{i_1} \cup H_{i_2} \cup \ldots \cup H_{i_n} \cup G^c$. Therefore $G \subseteq H_{i_1} \cup H_{i_2} \cup \ldots \cup H_{i_n} \cup G^c$. But $G \cap G^c = \widetilde{\emptyset}$, so $G \subseteq H_{i_1} \cup H_{i_2} \cup \ldots \cup H_{i_n}$. Thus the NBOC C of G has a finite NBOSC $\{H_{i_1}, H_{i_2}, \ldots, H_{i_n}\}$. Hence G is a neutrosophic b-compact set.

3.18. Proposition: If G is an NBC subset of a neutrosophic b-compact space (X, τ) such that $G \cap G^c = \widetilde{\emptyset}$ then G is neutrosophic compact.

Proof: Immediate from the prop. 3.17 as b-compactness implies compactness.

3.19. Proposition: If G is a neutrosophic closed subset of a neutrosophic b-compact space (X, τ) such that $G \cap G^c = \widetilde{\emptyset}$ then G is neutrosophic b-compact.

Proof: Immediate from the prop. 3.17 as every neutrosophic closed set is an NBC set.

3.20. Proposition: If G is a neutrosophic closed subset of a neutrosophic b-compact space (X, τ) such that $G \cap G^c = \widetilde{\emptyset}$ then G is neutrosophic compact.

Proof: Immediate from the prop. 3.19.

3.21. Proposition: Let (X, τ) be an NTS. An NS $A = \{\langle x, \mathrm{T}_A(x), \mathrm{I}_A(x), \mathrm{F}_A(x)\rangle: x \in X\}$ over X is neutrosophic b-compact iff for every collection $C = \{G_\lambda: \lambda \in \Delta\}$ of NBO sets of X satisfying $\mathrm{T}_A(x) \leq \vee_{\lambda\in\Delta} \mathrm{T}_{G_\lambda}(x)$, $1 - \mathrm{I}_A(x) \leq \vee_{\lambda\in\Delta} \left(1 - \mathrm{I}_{G_\lambda}(x)\right)$ and $1 - \mathrm{F}_A(x) \leq \vee_{\lambda\in\Delta} \left(1 - \mathrm{F}_{G_\lambda}(x)\right)$, there exists a finite subcollection $\{G_{\lambda_k}: k = 1,2,3,\ldots,n\}$ such that $\mathrm{T}_A(x) \leq \vee_{k=1}^{n} \mathrm{T}_{G_{\lambda_k}}(x)$, $1 - \mathrm{I}_A(x) \leq \vee_{k=1}^{n} \left(1 - \mathrm{I}_{G_{\lambda_k}}(x)\right)$ and $1 - \mathrm{F}_A(x) \leq \vee_{k=1}^{n} \left(1 - \mathrm{F}_{G_{\lambda_k}}(x)\right)$.

Proof: Necessary Part: Let $C = \{G_\lambda: \lambda \in \Delta\}$ be any collection of NBO sets of X satisfying $\mathrm{T}_A(x) \leq \vee_{\lambda\in\Delta} \mathrm{T}_{G_\lambda}(x)$, $1 - \mathrm{I}_A(x) \leq \vee_{\lambda\in\Delta} \left(1 - \mathrm{I}_{G_\lambda}(x)\right)$ and $1 - \mathrm{F}_A(x) \leq \vee_{\lambda\in\Delta} \left(1 - \mathrm{F}_{G_\lambda}(x)\right)$. Now $1 - \mathrm{I}_A(x) \leq$

$\vee_{\lambda\in\Delta}\left(1-\mathrm{I}_{G_\lambda}(x)\right) \Rightarrow 1-\mathrm{I}_A(x)\le 1-\mathrm{I}_{G_\beta}(x)$ for some $\in\Delta \Rightarrow \mathrm{I}_A(x)\ge \mathrm{I}_{G_\beta}(x) \Rightarrow \mathrm{I}_A(x)\ge \wedge_{\lambda\in\Delta}\mathrm{I}_{G_\lambda}(x)$. Similarly $1-\mathrm{F}_A(x)\le \vee_{\lambda\in\Delta}\left(1-\mathrm{F}_{G_\lambda}(x)\right) \Rightarrow \mathrm{F}_A(x)\ge \wedge_{\lambda\in\Delta}\mathrm{F}_{G_\lambda}(x)$. Therefore $A\subseteq\cup_{\lambda\in\Delta} G_\lambda$, i.e., C is an NBOC of A. Since A is neutrosophic b-compact, so C has a finite NBOSC $\{G_{\lambda_k}: k=1,2,3,\cdots,n\}$, say. Therefore $A\subseteq\cup_{k=1}^{n} G_{\lambda_k}$. Then $\mathrm{T}_A(x)\le \vee_{k=1}^{n}\mathrm{T}_{G_{\lambda_k}}(x)$, $\mathrm{I}_A(x)\ge \wedge_{k=1}^{n}\mathrm{I}_{G_{\lambda_k}}(x)$ and $\mathrm{F}_A(x)\ge \wedge_{k=1}^{n}\mathrm{F}_{G_{\lambda_k}}(x)$. Now $\mathrm{I}_A(x)\ge \wedge_{k=1}^{n}\mathrm{I}_{G_{\lambda_k}}(x) \Rightarrow \mathrm{I}_A(x)\ge \mathrm{I}_{G_{\lambda_m}}(x)$ for some $m, 1\le m\le n \Rightarrow 1-\mathrm{I}_A(x)\le 1-\mathrm{I}_{G_{\lambda_m}}(x)$ for some $m, 1\le m\le n \Rightarrow 1-\mathrm{I}_A(x)\le \vee_{k=1}^{n}\left(1-\mathrm{I}_{G_{\lambda_k}}(x)\right)$. Similarly $\mathrm{F}_A(x)\ge \wedge_{k=1}^{n}\mathrm{F}_{G_{\lambda_k}}(x) \Rightarrow 1-\mathrm{F}_A(x)\le \vee_{k=1}^{n}\left(1-\mathrm{F}_{G_{\lambda_k}}(x)\right)$. Thus $\mathrm{T}_A(x)\le \vee_{k=1}^{n}\mathrm{T}_{G_{\lambda_k}}(x)$, $1-\mathrm{I}_A(x)\le \vee_{k=1}^{n}\left(1-\mathrm{I}_{G_{\lambda_k}}(x)\right)$ and $1-\mathrm{F}_A(x)\le \vee_{k=1}^{n}\left(1-\mathrm{F}_{G_{\lambda_k}}(x)\right)$.

Sufficient Part: Let $C=\{G_\lambda:\lambda\in\Delta\}$ be an NBOC of A. Then $A\subseteq\cup_{\lambda\in\Delta} G_\lambda$, i.e., $\mathrm{T}_A(x)\le \vee_{\lambda\in\Delta}\mathrm{T}_{G_\lambda}(x)$, $\mathrm{I}_A(x)\ge \wedge_{\lambda\in\Delta}\mathrm{I}_{G_\lambda}(x)$ and $\mathrm{F}_A(x)\ge \wedge_{\lambda\in\Delta}\mathrm{F}_{G_\lambda}(x)$. Now $\mathrm{I}_A(x)\ge \wedge_{\lambda\in\Delta}\mathrm{I}_{G_\lambda}(x) \Rightarrow \mathrm{I}_A(x)\ge \mathrm{I}_{G_\alpha}(x)$ for some $\in\Delta$ $\Rightarrow 1-\mathrm{I}_A(x)\le 1-\mathrm{I}_{G_\alpha}(x) \Rightarrow 1-\mathrm{I}_A(x)\le \vee_{\lambda\in\Delta}\left(1-\mathrm{I}_{G_\lambda}(x)\right)$. Similarly $\mathrm{F}_A(x)\ge \wedge_{\lambda\in\Delta}\mathrm{F}_{G_\lambda}(x) \Rightarrow 1-\mathrm{F}_A(x)\le \vee_{\lambda\in\Delta}\left(1-\mathrm{F}_{G_\lambda}(x)\right)$. Thus the collection C satisfies the condition $\mathrm{T}_A(x)\le \vee_{\lambda\in\Delta}\mathrm{T}_{G_\lambda}(x)$, $1-\mathrm{I}_A(x)\le \vee_{\lambda\in\Delta}\left(1-\mathrm{I}_{G_\lambda}(x)\right)$ and $1-\mathrm{F}_A(x)\le \vee_{\lambda\in\Delta}\left(1-\mathrm{F}_{G_\lambda}(x)\right)$. By the hypothesis, there exists a finite subcollection $\{G_{\lambda_k}: k=1,2,3,\ldots,n\}$ such that $\mathrm{T}_A(x)\le \vee_{k=1}^{n}\mathrm{T}_{G_{\lambda_k}}(x)$, $1-\mathrm{I}_A(x)\le \vee_{k=1}^{n}\left(1-\mathrm{I}_{G_{\lambda_k}}(x)\right)$ and $1-\mathrm{F}_A(x)\le \vee_{k=1}^{n}\left(1-\mathrm{F}_{G_{\lambda_k}}(x)\right)$. Now $1-\mathrm{I}_A(x)\le \vee_{k=1}^{n}\left(1-\mathrm{I}_{G_{\lambda_k}}(x)\right) \Rightarrow 1-\mathrm{I}_A(x)\le 1-\mathrm{I}_{G_{\lambda_m}}(x)$ for some $, 1\le m\le n \Rightarrow \mathrm{I}_A(x)\ge \mathrm{I}_{G_{\lambda_m}}(x) \Rightarrow \mathrm{I}_A(x)\ge \wedge_{k=1}^{n}\mathrm{I}_{G_{\lambda_k}}(x)$. Similarly, we shall have $\mathrm{F}_A(x)\ge \wedge_{k=1}^{n}\mathrm{F}_{G_{\lambda_k}}(x)$. Therefore $A\subseteq\cup_{k=1}^{n} G_{\lambda_k}$, i.e., the NBOC C of A has a finite NBOSC $\{G_{\lambda_k}: k=1,2,3,\cdots,n\}$. Therefore, A is neutrosophic b-compact set.

Hence proved.

3.22. Proposition: Let (X,τ) be an NTS. Then X is neutrosophic b-compact iff for every collection $C=\{G_\lambda:\lambda\in\Delta\}$ of NBO sets of X satisfying $\vee_{\lambda\in\Delta}\mathrm{T}_{G_\lambda}(x)=1$, $\vee_{\lambda\in\Delta}\left(1-\mathrm{I}_{G_\lambda}(x)\right)=1$ and $\vee_{\lambda\in\Delta}\left(1-\mathrm{F}_{G_\lambda}(x)\right)=1$, there exists a finite subcollection $\{G_{\lambda_k}: k=1,2,3,\ldots,n\}$ such that $\vee_{k=1}^{n}\mathrm{T}_{G_{\lambda_k}}(x)=1$, $\vee_{k=1}^{n}\left(1-\mathrm{I}_{G_{\lambda_k}}(x)\right)=1$ and $\vee_{k=1}^{n}\left(1-\mathrm{F}_{G_{\lambda_k}}(x)\right)=1$.

Proof: Immediate from the prop. 3.21.

3.23. Proposition: An NTS (X,τ) is neutrosophic b-compact iff every collection of NBC sets with FIP has a non-empty intersection.

Proof: Necessary part: Let $A=\{N_i: i\in\Delta\}$ be an arbitrary collection of NBC sets with FIP. We show that $\cap_{i\in\Delta} N_i\ne\widetilde{\emptyset}$. On the contrary, suppose that $\cap_{i\in\Delta} N_i=\widetilde{\emptyset}$. Then $(\cap_{i\in\Delta} N_i)^c=\left(\widetilde{\emptyset}\right)^c \Rightarrow \cup_{i\in\Delta} N_i^c=\tilde{X}$. Therefore $B=\{N_i^c: N_i\in A\}$ is an NBOC of X and so, B has a finite NBOSC $\{N_{i_1}^c, N_{i_2}^c,\ldots,N_{i_k}^c\}$, say. Then $\cup_{j=1}^{k} N_{i_j}^c=\tilde{X} \Rightarrow \cap_{j=1}^{k} N_{i_j}=\widetilde{\emptyset}$, which is a contradiction as A has FIP. Therefore $\cap_{i\in\Delta} N_i\ne\widetilde{\emptyset}$.

Sufficient part: Suppose that X is not neutrosophic b-compact. Then there exists an NBOC $C=\{G_i: i\in\Delta\}$ of X which has no finite NBOSC. Then for every finite subcollection $\{G_{i_1}, G_{i_2},\ldots,G_{i_k}\}$ of C, we have $\cup_{j=1}^{k} G_{i_j}\ne\tilde{X} \Rightarrow \cap_{j=1}^{k} G_{i_j}^c\ne\widetilde{\emptyset}$. Therefore, $\{G_i^c: G_i\in C\}$ is a collection of NBC sets having the FIP. By the assumption, $\cap_{i\in\Delta} G_i^c\ne\widetilde{\emptyset} \Rightarrow \cup_{i\in\Delta} G_i\ne\tilde{X}$. This shows that C is not an NBOC of X, which is a contradiction. Therefore, the NBOC C of X must have a finite NBOSC. Therefore X is neutrosophic b-compact.

Hence proved.

3.24. Proposition: Let f be a neutrosophic b-open function from an NTS (X,τ) to the NTS (Y,σ) and $A\in\mathcal{N}(Y)$. If A is neutrosophic b-compact in Y then $f^{-1}(A)$ is neutrosophic compact in X.

Proof: Let $B=\{G_\lambda:\lambda\in\Delta\}$ be an NOC of $f^{-1}(A)$. Then $f^{-1}(A)\subseteq\cup_{\lambda\in\Delta} G_\lambda \Rightarrow A\subseteq f(\cup_{\lambda\in\Delta} G_\lambda) \Rightarrow A\subseteq \cup_{\lambda\in\Delta} f(G_\lambda)$. Since G_λ is τ-open set, so $f(G_\lambda)$ is σ-NBO set for each $\lambda\in\Delta$ as f is a b-open function. Therefore, $C=\{f(G_\lambda): G_\lambda\in B\}$ is an NBOC of A. Since A is neutrosophic b-compact, so C has a finite NBOSC $\{f(G_{\lambda_1}), f(G_{\lambda_2}), f(G_{\lambda_3}),\ldots,f(G_{\lambda_n})\}$, say. Therefore $A\subseteq\cup_{i=1}^{n} f(G_{\lambda_i}) \Rightarrow A\subseteq$

$f(\cup_{i=1}^{n} G_{\lambda_i}) \Rightarrow f^{-1}(A) \subseteq \cup_{i=1}^{n} G_{\lambda_i}$. Thus the NOC B of $f^{-1}(A)$ has a finite NOSC $\{G_{\lambda_1}, G_{\lambda_2}, G_{\lambda_3}, \ldots, G_{\lambda_n}\}$. Therefore $f^{-1}(A)$ is neutrosophic compact in X. Hence proved.

3.25. Proposition: Let f be a neutrosophic b-open function from an NTS (X, τ) onto the NTS (Y, σ). If (Y, σ) is neutrosophic b-compact (resp. neutrosophic countably b-compact, neutrosophic b-Lindelöf) then (X, τ) is neutrosophic compact (resp. neutrosophic countably compact, neutrosophic Lindelöf).

Proof: Immediate from the prop. 3.24 as f is onto.

3.26. Proposition: Let f be a neutrosophic open function from an NTS (X, τ) onto the NTS (Y, σ). If (Y, σ) is neutrosophic b-compact (resp. neutrosophic countably b-compact, neutrosophic b-Lindelöf) then (X, τ) is neutrosophic compact (resp. neutrosophic countably compact, neutrosophic Lindelöf).

Proof: Obvious as every neutrosophic open set is an NBO set.

3.27. Proposition: Let f be a neutrosophic b-continuous function from an NTS (X, τ) to the NTS (Y, σ). If A is neutrosophic b-compact set in X then $f(A)$ is neutrosophic compact set in Y.

Proof: Let $B = \{G_\lambda : \lambda \in \Delta\}$ be an NOC of $f(A)$. Then $f(A) \subseteq \cup_{\lambda \in \Delta} G_\lambda \Rightarrow f^{-1}(f(A)) \subseteq f^{-1}(\cup_{\lambda \in \Delta} G_\lambda) \Rightarrow f^{-1}(f(A)) \subseteq \cup_{\lambda \in \Delta} f^{-1}(G_\lambda) \Rightarrow A \subseteq \cup_{\lambda \in \Delta} f^{-1}(G_\lambda)$. Since G_λ is σ-open NS in Y, so $f^{-1}(G_\lambda)$ is τ-NBO set in X as f is b-continuous. Therefore $C = \{f^{-1}(G_\lambda) : G_\lambda \in B\}$ is an NBOC of A. Since A is neutrosophic b-compact, so C has a finite NBOSC $\{f^{-1}(G_{\lambda_1}), f^{-1}(G_{\lambda_2}), \ldots, f^{-1}(G_{\lambda_n})\}$, say. Therefore $A \subseteq \cup_{i=1}^{n} f^{-1}(G_{\lambda_i}) \Rightarrow A \subseteq f^{-1}(\cup_{i=1}^{n} G_{\lambda_i}) \Rightarrow f(A) \subseteq \cup_{i=1}^{n} G_{\lambda_i}$. Thus the NOC B of $f(A)$ has a finite NOSC. Therefore $f(A)$ is neutrosophic compact. Hence proved.

3.28. Proposition: Let f be a neutrosophic continuous function from an NTS (X, τ) to the NTS (Y, σ). If f is neutrosophic b-compact in X then $f(A)$ is neutrosophic compact in Y.

Proof: Obvious from the prop. 3.27 as every neutrosophic open set is an NBO set.

3.29. Proposition: Let f be a neutrosophic b-continuous function from an NTS (X, τ) onto the NTS (Y, σ). If (X, τ) is neutrosophic b-compact then (Y, σ) is neutrosophic compact.

Proof: Since f is onto, so $f(\tilde{X}) = \tilde{Y}$. Let $B = \{G_\lambda : \lambda \in \Delta\}$ be an NOC of Y. Then $\cup_{\lambda \in \Delta} G_\lambda = \tilde{Y} \Rightarrow \cup_{\lambda \in \Delta} G_\lambda = f(\tilde{X}) \Rightarrow f^{-1}(\cup_{\lambda \in \Delta} G_\lambda) = \tilde{X} \Rightarrow \cup_{\lambda \in \Delta} f^{-1}(G_\lambda) = \tilde{X}$. Since G_λ is σ-open NS in Y, so $f^{-1}(G_\lambda)$ is τ-NBO set in X as f is b-continuous. Therefore $C = \{f^{-1}(G_\lambda) : G_\lambda \in B\}$ is an NBOC of X. Since X is b-compact, so C has a finite NBOSC $\{f^{-1}(G_{\lambda_1}), f^{-1}(G_{\lambda_2}), \ldots, f^{-1}(G_{\lambda_n})\}$, say. Therefore $\cup_{i=1}^{n} f^{-1}(G_{\lambda_i}) = \tilde{X} \Rightarrow f^{-1}(\cup_{i=1}^{n} G_{\lambda_i}) = \tilde{X} \Rightarrow \cup_{i=1}^{n} G_{\lambda_i} = f(\tilde{X}) \Rightarrow \cup_{i=1}^{n} G_{\lambda_i} = \tilde{Y}$. Thus the NOC B of Y has a finite NOSC. Therefore Y is neutrosophic compact. Hence proved.

3.30. Proposition: Let f be a neutrosophic continuous function from an NTS (X, τ) onto the NTS (Y, σ). If (X, τ) is neutrosophic b-compact then (Y, σ) is neutrosophic compact.

Proof: Obvious from the prop. 3.29 as every neutrosophic open set is an NBO set.

3.31. Proposition: Let f be a neutrosophic b-continuous function from an NTS (X, τ) onto the NTS (Y, σ). If X is neutrosophic countably b-compact then Y is neutrosophic countably compact.

Proof: Since f is onto, so $f(\tilde{X}) = \tilde{Y}$. Let $B = \{G_\lambda : \lambda \in \Delta\}$ be a countable NOC of Y. Then $\cup_{\lambda \in \Delta} G_\lambda = \tilde{Y} \Rightarrow \cup_{\lambda \in \Delta} G_\lambda = f(\tilde{X}) \Rightarrow f^{-1}(\cup_{\lambda \in \Delta} G_\lambda) = \tilde{X} \Rightarrow \cup_{\lambda \in \Delta} f^{-1}(G_\lambda) = \tilde{X}$. Since G_λ is σ-open NS in Y, so $f^{-1}(G_\lambda)$ is τ-NBO set in X as f is b-continuous. Therefore $C = \{f^{-1}(G_\lambda) : G_\lambda \in B\}$ is an NBOC of X. Obviously C is countable as B is countable. Again since X is neutrosophic countably b-compact, so C has a finite NBOSC $\{f^{-1}(G_{\lambda_1}), f^{-1}(G_{\lambda_2}), \ldots, f^{-1}(G_{\lambda_n})\}$, say. Therefore $\cup_{i=1}^{n} f^{-1}(G_{\lambda_i}) = \tilde{X} \Rightarrow f^{-1}(\cup_{i=1}^{n} G_{\lambda_i}) = \tilde{X} \Rightarrow \cup_{i=1}^{n} G_{\lambda_i} = f(\tilde{X}) \Rightarrow \cup_{i=1}^{n} G_{\lambda_i} = \tilde{Y}$. Thus the countable NOC B of Y has a finite NOSC. Hence Y is neutrosophic countably compact.

3.32. Proposition: Let f be a neutrosophic continuous function from an NTS (X, τ) onto the NTS (Y, σ). If X is neutrosophic countably b-compact then Y is neutrosophic countably compact.

Proof: Immediate from the prop. 3.31 as every neutrosophic open set is an NBO set.

3.33. Proposition: Let f be a neutrosophic b-continuous function from an NTS (X, τ) onto the NTS (Y, σ). If X is neutrosophic b-Lindelöf then Y is neutrosophic Lindelöf.

Proof: Since f is onto, so $f(\tilde{X}) = \tilde{Y}$. Let $C = \{G_i : i \in \Delta\}$ be an NOC of Y. Then $\cup_{i \in \Delta} G_i = \tilde{Y} \Rightarrow \cup_{i \in \Delta} G_i = f(\tilde{X}) \Rightarrow f^{-1}(\cup_{i \in \Delta} G_i) = \tilde{X} \Rightarrow \cup_{i \in \Delta} f^{-1}(G_i) = \tilde{X} \Rightarrow \{f^{-1}(G_i) : G_i \in C\}$ is an NBOC of X. Since X

is neutrosophic b- Lindelöf, so $\{f^{-1}(G_i): G_i \in C\}$ has a countable NBOSC $B = \{f^{-1}(G_{i_k}): k = 1,2,3,\dots\}$, say. Therefore, $\tilde{X} = f^{-1}(G_{\lambda_1}) \cup f^{-1}(G_{\lambda_2}) \cup f^{-1}(G_{\lambda_3}) \cup \dots$. This gives $\tilde{X} = f^{-1}(G_{\lambda_1} \cup G_{\lambda_2} \cup G_{\lambda_3} \cup \dots) \Rightarrow f(\tilde{X}) = G_{\lambda_1} \cup G_{\lambda_2} \cup G_{\lambda_3} \cup \dots \Rightarrow \tilde{Y} = G_{\lambda_1} \cup G_{\lambda_2} \cup G_{\lambda_3} \cup \dots \Rightarrow \{G_{i_k}: k = 1,2,3,\dots\}$ is an NOC of Y. Since B is countable so, $\{G_{i_k}: k = 1,2,3,\dots\}$ is also countable. Therefore, the NOC C of Y has a countable NOSC $\{G_{i_k}: k = 1,2,3,\dots\}$ and so, Y is neutrosophic Lindelöf.

3.34. Proposition: Let f be a neutrosophic continuous function from an NTS (X, τ) onto the NTS (Y, σ). If X is neutrosophic b-Lindelöf then Y is neutrosophic Lindelöf.

Proof: Immediate from the prop. 3.33 as every neutrosophic open set is an NBO set.

3.35. Definition: Let f be a neutrosophic function from an NTS (X, τ) to the NTS (Y, σ). Then f is called a neutrosophic b^*-open function if $f(G)$ is an NBO set in Y for every NBO set G in X.

3.36. Proposition: Let f be a neutrosophic b^*-open function from an NTS (X, τ) to the NTS (Y, σ) and $A \in \mathcal{N}(Y)$. If A is neutrosophic b-compact in Y then $f^{-1}(A)$ is neutrosophic b-compact in X.

Proof: Let $B = \{G_i: i \in \Delta\}$ be an NBOC of $f^{-1}(A)$. Then $f^{-1}(A) \subseteq \cup_{i\in\Delta} G_i \Rightarrow A \subseteq f(\cup_{i\in\Delta} G_i) \Rightarrow$ $\mathrm{A} \subseteq \cup_{i\in\Delta} f(G_i)$. Since G_i is a τ-NBO set, so $f(G_i)$ is a σ-NBO set for each $i \in \Delta$ as f is a neutrosophic b^*-open function. Therefore, $C = \{f(G_i): G_i \in B\}$ is an NBOC of A. Since A is neutrosophic b-compact, so C has a finite NBOSC $\{f(G_{\lambda_1}), f(G_{\lambda_2}), f(G_{\lambda_3}), \dots, f(G_{\lambda_n})\}$, say. Therefore, $A \subseteq \cup_{i=1}^{n} f(G_{\lambda_i}) \Rightarrow A \subseteq f(\cup_{i=1}^{n} G_{\lambda_i}) \Rightarrow f^{-1}(A) \subseteq \cup_{i=1}^{n} G_{\lambda_i}$. Thus the NBOC B of $f^{-1}(A)$ has a finite NBOSC $\{G_{\lambda_1}, G_{\lambda_2}, G_{\lambda_3}, \dots, G_{\lambda_n}\}$. Therefore $f^{-1}(A)$ is neutrosophic b-compact in X. Hence proved.

3.37. Proposition: Let f be a neutrosophic b^*-open function from an NTS (X, τ) onto the NTS (Y, σ). If (Y, σ) is neutrosophic b-compact in then (X, τ) is also neutrosophic b-compact.

Proof: Immediate from the prop. 3.36 as f is onto.

3.38. Proposition: Let f be a neutrosophic b^*-open function from an NTS (X, τ) onto the NTS (Y, σ). If (Y, σ) is neutrosophic countably b-compact (neutrosophic b-Lindelöf) then (X, τ) is also neutrosophic countably b-compact (neutrosophic b-Lindelöf).

Proof: Obvious.

3.39. Definition: Let f be a neutrosophic function from an NTS (X, τ) to the NTS (Y, σ). Then f is called a neutrosophic b^*-continuous function if $f^{-1}(G)$ is an NBO set in X for every NBO set G in Y.

3.40. Proposition: Let f be a neutrosophic b^*-continuous function from an NTS (X, τ) to the NTS (Y, σ). If A is neutrosophic b-compact in X then $f(A)$ is also neutrosophic b-compact in Y.

Proof: Let $B = \{G_\lambda: \lambda \in \Delta\}$ be an NBOC of $f(A)$. Then $f(A) \subseteq \cup_{\lambda\in\Delta} G_\lambda \Rightarrow \mathrm{A} \subseteq f^{-1}(\cup_{\lambda\in\Delta} G_\lambda) \Rightarrow \mathrm{A} \subseteq \cup_{\lambda\in\Delta} f^{-1}(G_\lambda)$. Since G_λ is σ-NBO set in Y, so $f^{-1}(G_\lambda)$ is τ-NBO set in X as f is neutrosophic b^*-continuous function. Therefore $C = \{f^{-1}(G_\lambda): G_\lambda \in B\}$ is an NBOC of A. Since A is neutrosophic b-compact in X, so C has a finite NBOSC $\{f^{-1}(G_{\lambda_1}), f^{-1}(G_{\lambda_2}), \dots, f^{-1}(G_{\lambda_n})\}$, say. Therefore $A \subseteq \cup_{i=1}^{n} f^{-1}(G_{\lambda_i}) \Rightarrow A \subseteq f^{-1}(\cup_{i=1}^{n} G_{\lambda_i}) \Rightarrow f(A) \subseteq \cup_{i=1}^{n} G_{\lambda_i}$. Thus the NBOC B of $f(A)$ has a finite NBOSC $\{G_{\lambda_1}, G_{\lambda_2}, G_{\lambda_3}, \dots, G_{\lambda_n}\}$.. Therefore $f(A)$ is neutrosophic b-compact.

3.41. Proposition: Let f be a neutrosophic b^*-continuous function from an NTS (X, τ) onto the NTS (Y, σ). If (X, τ) is neutrosophic b-compact then (Y, σ) is also neutrosophic b-compact.

Proof: Since f is onto, so $f(\tilde{X}) = \tilde{Y}$. Let $B = \{G_\lambda: \lambda \in \Delta\}$ be an NBOC of Y. Then $\cup_{\lambda\in\Delta} G_\lambda = \tilde{Y} \Rightarrow$ $\cup_{\lambda\in\Delta} G_\lambda = f(\tilde{X}) \Rightarrow f^{-1}(\cup_{\lambda\in\Delta} G_\lambda) = \tilde{X} \Rightarrow \cup_{\lambda\in\Delta} f^{-1}(G_\lambda) = \tilde{X}$. Since G_λ is σ-NBO set in Y, so $f^{-1}(G_\lambda)$ is τ-NBO set in X as f is neutrosophic b^*-continuous function. Therefore, $C = \{f^{-1}(G_\lambda): G_\lambda \in B\}$ is an NBOC of X. Since X is neutrosophic b-compact, so C has a finite NBOSC $\{f^{-1}(G_{\lambda_1}), f^{-1}(G_{\lambda_2}), \dots, f^{-1}(G_{\lambda_n})\}$, say. Therefore, $\tilde{X} = \cup_{i=1}^{n} f^{-1}(G_{\lambda_i}) \Rightarrow \tilde{X} = f^{-1}(\cup_{i=1}^{n} G_{\lambda_i}) \Rightarrow f(\tilde{X}) = \cup_{i=1}^{n} G_{\lambda_i} \Rightarrow \tilde{Y} = \cup_{i=1}^{n} G_{\lambda_i}$. Thus the NBOC B of Y has a finite NBOSC $\{G_{\lambda_1}, G_{\lambda_2}, G_{\lambda_3}, \dots, G_{\lambda_n}\}$. Therefore Y is neutrosophic b-compact.

3.42. Proposition: Let f be a neutrosophic b^*-continuous function from an NTS (X,τ) onto the NTS (Y,σ). If (X,τ) is neutrosophic countably b-compact (resp. neutrosophic b-Lindelöf) then (Y,σ) is also neutrosophic countably b-compact (resp. neutrosophic b-Lindelöf).
Proof: Obvious.

4. Neutrosophic local b-compactness
4.1. Definition: An NTS (X,τ) is said to be a neutrosophic locally b-compact space iff for every NP $x_{\alpha,\beta,\gamma}$ in X, there exists an NBO set G in X such that $x_{\alpha,\beta,\gamma} \in G$ and G is neutrosophic b-compact in X.
4.2. Proposition: Every neutrosophic b-compact space is a neutrosophic locally b-compact space.
Proof: Let (X,τ) be a neutrosophic b-compact space and let $x_{\alpha,\beta,\gamma}$ be an NP in X. Since X is neutrosophic b-compact and since $\tilde{X}$ is an NBO set such that $x_{\alpha,\beta,\gamma} \in \tilde{X}$, so, (X,τ) is a neutrosophic locally b-compact space.
4.3. Proposition: Let f be a neutrosophic b^*-open and b^*-continuous function from an NTS space (X,τ) to the NTS (Y,τ). If (Y,τ) neutrosophic locally b-compact then (X,τ) is also a neutrosophic locally b-compact space.
Proof: Let $x_{\alpha,\beta,\gamma}$ be any NP in X. Also let $y_{p,q,r}$ be the NP in Y such that $f(x_{\alpha,\beta,\gamma}) = y_{p,q,r}$. Since $y_{p,q,r} \in Y$ and Y neutrosophic locally b-compact, so there exists a σ-NBO set G such that $y_{p,q,r} \in G$ and G is neutrosophic b-compact in Y. Now $y_{p,q,r} \in G \Rightarrow f(x_{\alpha,\beta,\gamma}) \in G \Rightarrow x_{\alpha,\beta,\gamma} \in f^{-1}(G)$. Since f is neutrosophic b^*-open and G is neutrosophic b-compact in Y, so by the prop. 3.36, $f^{-1}(G)$ is neutrosophic b-compact in X. Again since f is a neutrosophic b^*-continuous function, so $f^{-1}(G)$ is a τ-NBO set. Thus for any any NP $x_{\alpha,\beta,\gamma}$ in X, there exists a τ-NBO set $f^{-1}(G)$ such that $x_{\alpha,\beta,\gamma} \in f^{-1}(G)$ and $f^{-1}(G)$ is neutrosophic b-compact in X. Therefore (X,τ) is neutrosophic locally b-compact space.
4.4. Proposition: Let f be a neutrosophic b^*-open and b^*-continuous function from an NTS space (X,τ) onto the NTS (Y,τ). If (X,τ) neutrosophic locally b-compact then (Y,σ) is also a neutrosophic locally b-compact space.
Proof: Let $y_{p,q,r}$ be any NP in Y. Since f is onto, so there exists an NP $x_{\alpha,\beta,\gamma}$ in X such that $f(x_{\alpha,\beta,\gamma}) = y_{p,q,r}$. Since $x_{\alpha,\beta,\gamma} \in X$ and X neutrosophic locally b-compact, so there exists a τ-NBO set G such that $x_{\alpha,\beta,\gamma} \in G$ and G is neutrosophic b-compact in X. Now $x_{\alpha,\beta,\gamma} \in G \Rightarrow f(x_{\alpha,\beta,\gamma}) \in f(G) \Rightarrow y_{p,q,r} \in f(G)$. Since f is neutrosophic b^*-continuous and G is neutrosophic b-compact in X, so by 3.40, $f(G)$ is neutrosophic b-compact in Y. Again since f is a neutrosophic b^*-open function, so $f(G)$ is a σ-NBO set. Thus for any any NP $y_{p,q,r}$ in Y, there exists a σ-NBO set $f(G)$ such that $y_{p,q,r} \in f(G)$ and $f(G)$ is neutrosophic b-compact in Y. Therefore (Y,σ) is neutrosophic locally b-compact space.

5. Covering properties via neutrosophic b-base
5.1. Definition : Let (X,τ) be an NTS and $NBO(X)$ be the collection of all NBO sets in X. A subcollection B of $NBO(X)$ is called a neutrosophic b-base (Nb-base, for short) for X iff for each $A \in NBO(X)$, there exists a subcollection $\{A_i : i \in \Delta\}$ of B such that $A = \cup \{A_i : i \in \Delta\}$, where Δ is an index set.
A subcollection B_* of $NBO(X)$ is called a neutrosophic b-subbase (Nb-subbase, for short) for X iff the finite intersection of members of B_* forms a neutrosophic b-base for X.
5.2. Definition: An NTS (X,τ) is said to be a neutrosophic b-second countable or neutrosophic $b - C_{II}$ space iff X has a countable neutrosophic b-base.
5.3. Proposition: Let B be an Nb-base for an NTS (X,τ). Then X is neutrosophic b-compact iff every NBOC of X by the members of B has a finite NBOSC.
Proof: Necessary Part: Obvious.
Sufficient Part : Let $B = \{B_\alpha : \alpha \in \Delta\}$ be the Nb-base. Also let $C = \{G_\lambda : \lambda \in \Delta\}$ be an NBOC of X. Then each member G_λ of C is the union of some members of B and the totality of such members of B is

evidently an NBOC of X. By the hypothesis, this collection of members of B has a finite NBOSC $D = \{B_{\alpha_j}: j = 1,2,3,\cdots,n\}$, say. Clearly for each B_{α_j} in D, there is a G_{λ_j} in C such that $B_{\alpha_j} \subseteq G_{\lambda_j}$. Therefore the finite subcollection $\{G_{\lambda_j}: j = 1,2,3,\cdots,n\}$ of C is an NBOC of X, i.e., the NBOC C of X has a finite NBOSC. Therefore X is neutrosophic b-compact.

5.4. Proposition: Let (X,τ) be a neutrosophic countably b-compact space. If X is neutrosophic $b - C_{II}$ then X neutrosophic b-compact.

Proof: Let $D = \{A_i: i \in \Delta\}$ be any NBOC of X. Since X is neutrosophic $b - C_{II}$, so there exists a countable Nb-base $B = \{B_n: n = 1,2,3,\cdots\}$ for X. Then each $A_i \in D$ can be expressed as a countable union of members of B, i.e., for each $A_i \in D$, we have $A_i = \cup_{k=1}^{i_0} B_{n_k}$, where $B_{n_k} \in B$ and i_0 may be infinity. Clearly $B_0 = \{B_{n_k}\}$ is an NBOC of X. Also B_0 is countable as $B_0 \subseteq B$. Therefore, B_0 is a countable NBOC of X. Since X is countably b-compact, so B_0 has a finite NBOSC B', say. Since by construction, each member of B' is contained in one member A_i of D, so these A_i's form a finite NBOC of X. Thus the NBOC D of X has a finite NBOSC. Therefore X is neutrosophic b-compact. Hence Proved.

5.5. Remark: From the propositions 3.3 and 5.4, it is clear that if an NTS (X,τ) is neutrosophic $b - C_{II}$ then neutrosophic b-compactness and neutrosophic countably b-compactness are equivalent.

5.6. Proposition: If an NTS (X,τ) is neutrosophic $b - C_{II}$ then it is neutrosophic b-Lindelöf.

Proof: Let $D = \{A_i: i \in \Delta\}$ be any NBOC of X. Since X is neutrosophic $b - C_{II}$, so there exists a countable Nb-base $B = \{B_n: n = 1,2,3,\cdots\}$ for X. Then each $A_i \in D$ can be expressed as the countable union of members of B, i.e., for each $A_i \in D$, we have $A_i = \cup_{k=1}^{i_0} B_{n_k}$, where $B_{n_k} \in B$ and i_0 may be infinity. Let $B_0 = \{B_{n_k}\}$. Then B_0 is an NBOC of X. Also B_0 is countable as $B_0 \subseteq B$. Therefore, B_0 is a countable NBOC of X. By construction, each member of B_0 is contained in one member A_i of D. So, these A_i's of D form a countable NBOSC of X. Thus the NBOC D of X has a countable NBOSC. Therefore X is neutrosophic b- Lindelöf.

5.7. Proposition: Let β be an Nb-subbase of an NTS (X,τ). Then X is neutrosophic b-compact iff for every collection of NBC sets taken from β^c having the FIP, there is a non-empty intersection.

Proof: Necessary part: Immediate from the prop. 3.23.

Sufficient Part: On the contrary, let us suppose that X is not b-compact. Then by the prop. 3.23, there exists a collection $C = \{G_i: i \in I\}$ of NBC sets of X having FIP such that $\cap_{i\in I} G_i = \widetilde{\emptyset}$. The collection $F = \{C\}$ of all such collections C can be arranged in an order by using the classical inclusion($\subseteq$) and therefore, the collection F will have an upper bound. By Zorn's lemma, there will be a maximal collection of all these collections C. Let $P = \{K_j: j \in J\}$ be the maximal collection. Clearly, this collection P has the following properties:

(i) $\widetilde{\emptyset} \notin P$ (ii) $A \in P, A \subseteq B \Rightarrow B \in P$ (iii) $A, B \in P \Rightarrow A \cap B \in P$ (iv) $\cap (P \cap \beta^c) = \widetilde{\emptyset}$.

Clearly the property (iv) creates a contradiction to the hypothesis. Therefore X is neutrosophic b-compact.

Hence proved.

6. Conclusions

In this article, we have defined neutrosophic b-open cover with the help of neutrosophic b-open sets and then we have defined neutrosophic b-compact, neutrosophic countably b-compact, neutrosophic b-Lindelöf spaces and investigated various covering properties. We have proved that every neutrosophic b-compact space is a neutrosophic compact space but the converse is not true. We have shown that if f is a neutrosophic continuous or a b-continuous function from a neutrosophic b-compact (resp. countably b-compact, b-Lindelöf) space (X,τ) onto a neutrosophic topological space (Y,σ) then (Y,σ) is a neutrosophic compact (resp. countably compact, Lindelöf) space. In 3.41 (resp. 3.42), we have established that neutrosophic b-compactness (resp. countably b-compactness, b-Lindelöfness) is preserved under a neutrosophic b^*-continuous function. We have then defined and studied a few properties of neutrosophic local b-compactness. At last, in section 5, we have defined neutrosophic b-base, b-subbase, neutrosophic b-C_{II} and investigated some properties. We have set up that if a neutrosophic topological space is neutrosophic b-C_{II} then neutrosophic b-compactness and neutrosophic countably b-compactness are equivalent. In 5.7, we have stated and proved "Alexander subbase lemma" in case of a neutrosophic b-compact space. Hope that the findings in this article will assist the research fraternity to move forward for the development of different aspects of neutrosophic topology.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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