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Trigonometric Similarity Measures of Pythagorean Neutrosophic Hypersoft Sets

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Abstract: In our daily life, most problems stem from wrong decisions. Similarity measures (SMs) are very helpful in making good decisions. In this paper, distinct similarities of Pythagorean Neutrosophic Hypersoft Sets (PNHSSs) and its properties are presented. Finally, the proposed SMs applied for converting plastic waste into energy source problems. Comparing various suggested similarities makes decision-making simple, easy and accurate.

Keywords: PNHSS; SM; Tangent Similarity Measure; Cotangent Similarity Measure; Cosine Similarity Measure.

1. Introduction

Recently, many ideas have been introduced to deal with ambiguity and uncertainty (UC). Fuzzy set (FS) theory [1, 2], Intuitionistic fuzzy set (IFS) [3] serve different means when dealing with inconsistent data. However, all of the above theories fail to address the conflicting information that exists in belief systems. In 1998, Smarandache proposed neutrosophic set (NS) [4] theory as a generalization of the theories mentioned above. He considered truth, ambiguity and falsehood separately. Later, Yager [5] was decided to introduce the novel idea of Pythagorean fuzzy sets (PFSs). PFSs have a limitation that their square sum is less than or equal to 1. To overcome unconstrained ambiguity, Molodtsov [6] proposed the concept of soft set (SS) as a new mathematical method. PFSS is derived from the combination of PFS and SS. Smarandache [7] introduced a new technique for dealing with UC. He generalized the SS to the hypersoft set (HSS) by turning the function into a multi-decision function.

In section 2, the basic definitions of Pythagorean Neutrosophic Hypersoft Sets (PNHSSs) are presented. In section 3, six Tangent Similarity Measure (TSM) for PNHSSs are presented. In section 4, given resources were used to determine the accuracy of the similarity measurements.

2. Preliminaries

Definition 2.1: [8] Let $\tilde{\Delta}$ be the universe and $\mathcal{P}(\tilde{\Delta})$ be a power set of $\tilde{\Delta}$. Consider $\dddot{\mathfrak{A}}_1, \dddot{\mathfrak{A}}_2, \ldots, \dddot{\mathfrak{A}}_{\hat{\kappa}}$ for $\hat{\kappa} \geq 1$ be $\hat{\kappa}$ well - defined attributes and attributive values are $\dddot{\mathfrak{G}}_1, \dddot{\mathfrak{G}}_2, \ldots, \dddot{\mathfrak{G}}_{\hat{\kappa}}$ with $\dddot{\mathfrak{G}}_{\hat{\imath}} \cap \dddot{\mathfrak{G}}_{\hat{\jmath}} = \emptyset$, for $\hat{\imath} \neq \hat{\jmath}$, $\hat{\imath}, \hat{\jmath} \,\dot{\in}\, \{1, 2, \ldots, \hat{\kappa}\}$ and their relation $\dddot{\mathfrak{G}}_1 \times \dddot{\mathfrak{G}}_2, \ldots \times \dddot{\mathfrak{G}}_{\hat{\kappa}} = \dddot{\aleph}$, $(\eta, \dddot{\mathfrak{G}}_1 \times \dddot{\mathfrak{G}}_2 \times \ldots \times \dddot{\mathfrak{G}}_{\hat{\kappa}})$ is said to be PNHSS over $\tilde{\Delta}$ where $\eta: \dddot{\mathfrak{G}}_1 \times \dddot{\mathfrak{G}}_2 \times \ldots \times \dddot{\mathfrak{G}}_{\hat{\kappa}} \rightarrow \mathcal{P}(\tilde{\Delta})$ and $\eta(\dddot{\mathfrak{G}}_1 \times \dddot{\mathfrak{G}}_2 \times \ldots \times \dddot{\mathfrak{G}}_{\hat{\kappa}}) = \{(\dddot{\aleph}, < \breve{\varkappa}, \acute{\mathbb{T}}_{\eta(\dddot{\aleph})}(\breve{\varkappa}), \acute{\mathbb{I}}_{\eta(\dddot{\aleph})}(\breve{\varkappa}), \acute{\mathbb{F}}_{\eta(\dddot{\aleph})}(\breve{\varkappa}) >) : \breve{\varkappa} \,\dot{\in}\, \tilde{\Delta}, \dddot{\aleph} \,\dot{\in}\, \dddot{\mathfrak{G}}_1 \times \dddot{\mathfrak{G}}_2 \times \ldots \times \dddot{\mathfrak{G}}_{\hat{\kappa}}\}$ where $\acute{\mathbb{T}}$ is the truthiness, $\acute{\mathbb{I}}$ is the indeterminacy and $\acute{\mathbb{F}}$ is the falseness such that $\acute{\mathbb{T}}_{\eta(\ddot{\aleph})}(\underset{\smile}{\breve{\varkappa}}),\ \acute{\mathbb{I}}_{\eta(\ddot{\aleph})}(\underset{\smile}{\breve{\varkappa}}), \acute{\mathbb{F}}_{\eta(\ddot{\aleph})}(\underset{\smile}{\breve{\varkappa}}) \in [0,1]$ also $0 \leq \left(\acute{\mathbb{T}}_{\eta(\ddot{\aleph})}(\underset{\smile}{\breve{\varkappa}})\right)^2 + \left(\acute{\mathbb{I}}_{\eta(\ddot{\aleph})}(\underset{\smile}{\breve{\varkappa}})\right)^2 + \left(\acute{\mathbb{F}}_{\eta(\ddot{\aleph})}(\underset{\smile}{\breve{\varkappa}})\right)^2 \leq 2$.

3. Trigonometric Similarity Measures for Pythagorean Neutrosophic Hypersoft Sets

Definition 3.1: Let $\underset{\smile}{\ddot{\mathcal{W}}} = \left(\ddot{\aleph}, < \underset{\smile}{\breve{\varkappa}}, \acute{\mathcal{T}}^{\underset{\smile}{\ddot{\mathcal{W}}}}_{\eta(\ddot{\aleph})}(\underset{\smile}{\breve{\varkappa}}), \acute{\mathcal{I}}^{\underset{\smile}{\ddot{\mathcal{W}}}}_{\eta(\ddot{\aleph})}(\underset{\smile}{\breve{\varkappa}}), \acute{\mathcal{F}}^{\underset{\smile}{\ddot{\mathcal{W}}}}_{\eta(\ddot{\aleph})}(\underset{\smile}{\breve{\varkappa}}) >: \underset{\smile}{\breve{\varkappa}} \in \tilde{\Delta}\right)$;
$\underset{\smile}{\ddot{\mathcal{V}}} = \left(\ddot{\aleph}, < \underset{\smile}{\breve{\varkappa}}, \acute{\mathcal{T}}^{\underset{\smile}{\ddot{\mathcal{V}}}}_{\eta(\ddot{\aleph})}(\underset{\smile}{\breve{\varkappa}}), \acute{\mathcal{I}}^{\underset{\smile}{\ddot{\mathcal{V}}}}_{\eta(\ddot{\aleph})}(\underset{\smile}{\breve{\varkappa}}), \acute{\mathcal{F}}^{\underset{\smile}{\ddot{\mathcal{V}}}}_{\eta(\ddot{\aleph})}(\underset{\smile}{\breve{\varkappa}}) >: \underset{\smile}{\breve{\varkappa}} \in \tilde{\Delta}\right)$ be PNHSSs. The TSMs between $\underset{\smile}{\ddot{\mathcal{W}}}$, $\underset{\smile}{\ddot{\mathcal{V}}}$ is,

$\acute{\mathbb{T}}_{PNHSS}(\underset{\smile}{\ddot{\mathcal{W}}}, \underset{\smile}{\ddot{\mathcal{V}}}) =$

$$\left(\ddot{\aleph}, < \underset{\smile}{\breve{\varkappa}},\ \frac{1}{\acute{n}}\sum_{i=1}^{n}\left[1 - tan\left(\frac{\pi\left(\left|\acute{\mathcal{T}}^{\underset{\smile}{\ddot{\mathcal{W}}}^2}_{(\eta(\ddot{\aleph}))_i}(\underset{\smile}{\breve{\varkappa}}) - \acute{\mathcal{T}}^{\underset{\smile}{\ddot{\mathcal{V}}}^2}_{(\eta(\ddot{\aleph}))_i}(\underset{\smile}{\breve{\varkappa}})\right| + \left|\acute{\mathcal{I}}^{\underset{\smile}{\ddot{\mathcal{W}}}^2}_{(\eta(\ddot{\aleph}))_i}(\underset{\smile}{\breve{\varkappa}}) - \acute{\mathcal{I}}^{\underset{\smile}{\ddot{\mathcal{V}}}^2}_{(\eta(\ddot{\aleph}))_i}(\underset{\smile}{\breve{\varkappa}})\right| + \left|\acute{\mathcal{F}}^{\underset{\smile}{\ddot{\mathcal{W}}}^2}_{(\eta(\ddot{\aleph}))_i}(\underset{\smile}{\breve{\varkappa}}) - \acute{\mathcal{F}}^{\underset{\smile}{\ddot{\mathcal{V}}}^2}_{(\eta(\ddot{\aleph}))_i}(\underset{\smile}{\breve{\varkappa}})\right|\right)}{12}\right)\right] >\right) \quad (1)$$

Proposition 1:
The TSM $\acute{\mathbb{T}}_{PNHSS}(\underset{\smile}{\ddot{\mathcal{W}}}, \underset{\smile}{\ddot{\mathcal{V}}})$ satisfies the following properties:

(1) $0 \leq \acute{\mathbb{T}}_{PNHSS}(\underset{\smile}{\ddot{\mathcal{W}}}, \underset{\smile}{\ddot{\mathcal{V}}}) \leq 1$

(2) $\acute{\mathbb{T}}_{PNHSS}(\underset{\smile}{\ddot{\mathcal{W}}}, \underset{\smile}{\ddot{\mathcal{V}}}) = 1$ iff $\underset{\smile}{\ddot{\mathcal{W}}} = \underset{\smile}{\ddot{\mathcal{V}}}$

(3) $\acute{\mathbb{T}}_{PNHSS}(\underset{\smile}{\ddot{\mathcal{W}}}, \underset{\smile}{\ddot{\mathcal{V}}}) = \acute{\mathbb{T}}_{PNHSS}(\underset{\smile}{\ddot{\mathcal{V}}}, \underset{\smile}{\ddot{\mathcal{W}}})$

(4) If $\underset{\smile}{\ddot{\mathfrak{Q}}}$ is a PNHSS set and $\underset{\smile}{\ddot{\mathcal{W}}} \subset \underset{\smile}{\ddot{\mathcal{V}}} \subset \underset{\smile}{\ddot{\mathfrak{Q}}}$ then $\acute{\mathbb{T}}_{PNHSS}(\underset{\smile}{\ddot{\mathcal{W}}}, \underset{\smile}{\ddot{\mathfrak{Q}}}) \leq \acute{\mathbb{T}}_{PNHSS}(\underset{\smile}{\ddot{\mathcal{W}}}, \underset{\smile}{\ddot{\mathcal{V}}})$ and $\acute{\mathbb{T}}_{PNHSS}(\underset{\smile}{\ddot{\mathcal{W}}}, \underset{\smile}{\ddot{\mathfrak{Q}}}) \leq \acute{\mathbb{T}}_{PNHSS}(\underset{\smile}{\ddot{\mathcal{V}}}, \underset{\smile}{\ddot{\mathfrak{Q}}})$.

Proof:

(1) Since tangent values of PNHSSs are in the interval [0, 1]. Hence $0 \leq \acute{\mathbb{T}}_{PNHSS}(\underset{\smile}{\ddot{\mathcal{W}}}, \underset{\smile}{\ddot{\mathcal{V}}}) \leq 1$.

(2) If $\underset{\smile}{\ddot{\mathcal{W}}} = \underset{\smile}{\ddot{\mathcal{V}}}$, then $\acute{\mathcal{T}}^{\underset{\smile}{\ddot{\mathcal{W}}}}_{\eta(\ddot{\aleph})}(\underset{\smile}{\breve{\varkappa}}) = \acute{\mathcal{T}}^{\underset{\smile}{\ddot{\mathcal{V}}}}_{\eta(\ddot{\aleph})}(\underset{\smile}{\breve{\varkappa}})$, $\acute{\mathcal{I}}^{\underset{\smile}{\ddot{\mathcal{W}}}}_{\eta(\ddot{\aleph})}(\underset{\smile}{\breve{\varkappa}}) = \acute{\mathcal{I}}^{\underset{\smile}{\ddot{\mathcal{V}}}}_{\eta(\ddot{\aleph})}(\underset{\smile}{\breve{\varkappa}})$, $\acute{\mathcal{F}}^{\underset{\smile}{\ddot{\mathcal{W}}}}_{\eta(\ddot{\aleph})}(\underset{\smile}{\breve{\varkappa}}) = \acute{\mathcal{F}}^{\underset{\smile}{\ddot{\mathcal{V}}}}_{\eta(\ddot{\aleph})}(\underset{\smile}{\breve{\varkappa}})$. Hence $\left|\acute{\mathcal{T}}^{\underset{\smile}{\ddot{\mathcal{W}}}^2}_{(\eta(\ddot{\aleph}))_i}(\underset{\smile}{\breve{\varkappa}}) - \acute{\mathcal{T}}^{\underset{\smile}{\ddot{\mathcal{V}}}^2}_{(\eta(\ddot{\aleph}))_i}(\underset{\smile}{\breve{\varkappa}})\right| = 0$, $\left|\acute{\mathcal{I}}^{\underset{\smile}{\ddot{\mathcal{W}}}^2}_{(\eta(\ddot{\aleph}))_i}(\underset{\smile}{\breve{\varkappa}}) - \acute{\mathcal{I}}^{\underset{\smile}{\ddot{\mathcal{V}}}^2}_{(\eta(\ddot{\aleph}))_i}(\underset{\smile}{\breve{\varkappa}})\right| = 0$, $\left|\acute{\mathcal{F}}^{\underset{\smile}{\ddot{\mathcal{W}}}^2}_{(\eta(\ddot{\aleph}))_i}(\underset{\smile}{\breve{\varkappa}}) - \acute{\mathcal{F}}^{\underset{\smile}{\ddot{\mathcal{V}}}^2}_{(\eta(\ddot{\aleph}))_i}(\underset{\smile}{\breve{\varkappa}})\right| = 0$. Thus $\acute{\mathbb{T}}_{PNHSS}(\underset{\smile}{\ddot{\mathcal{W}}}, \underset{\smile}{\ddot{\mathcal{V}}}) = 1$. Conversely, if $\acute{\mathbb{T}}_{PNHSS}(\underset{\smile}{\ddot{\mathcal{W}}}, \underset{\smile}{\ddot{\mathcal{V}}}) = 1$ then $\left|\acute{\mathcal{T}}^{\underset{\smile}{\ddot{\mathcal{W}}}^2}_{(\eta(\ddot{\aleph}))_i}(\underset{\smile}{\breve{\varkappa}}) - \acute{\mathcal{T}}^{\underset{\smile}{\ddot{\mathcal{V}}}^2}_{(\eta(\ddot{\aleph}))_i}(\underset{\smile}{\breve{\varkappa}})\right| = 0$, $\left|\acute{\mathcal{I}}^{\underset{\smile}{\ddot{\mathcal{W}}}^2}_{(\eta(\ddot{\aleph}))_i}(\underset{\smile}{\breve{\varkappa}}) - \acute{\mathcal{I}}^{\underset{\smile}{\ddot{\mathcal{V}}}^2}_{(\eta(\ddot{\aleph}))_i}(\underset{\smile}{\breve{\varkappa}})\right| = 0$, $\left|\acute{\mathcal{F}}^{\underset{\smile}{\ddot{\mathcal{W}}}^2}_{(\eta(\ddot{\aleph}))_i}(\underset{\smile}{\breve{\varkappa}}) - \acute{\mathcal{F}}^{\underset{\smile}{\ddot{\mathcal{V}}}^2}_{(\eta(\ddot{\aleph}))_i}(\underset{\smile}{\breve{\varkappa}})\right| = 0$. Since, tan (0) = 0. $\acute{\mathcal{T}}^{\underset{\smile}{\ddot{\mathcal{W}}}}_{\eta(\ddot{\aleph})}(\underset{\smile}{\breve{\varkappa}}) = \acute{\mathcal{T}}^{\underset{\smile}{\ddot{\mathcal{V}}}}_{\eta(\ddot{\aleph})}(\underset{\smile}{\breve{\varkappa}})$, $\acute{\mathcal{I}}^{\underset{\smile}{\ddot{\mathcal{W}}}}_{\eta(\ddot{\aleph})}(\underset{\smile}{\breve{\varkappa}}) = \acute{\mathcal{I}}^{\underset{\smile}{\ddot{\mathcal{V}}}}_{\eta(\ddot{\aleph})}(\underset{\smile}{\breve{\varkappa}})$, $\acute{\mathcal{F}}^{\underset{\smile}{\ddot{\mathcal{W}}}}_{\eta(\ddot{\aleph})}(\underset{\smile}{\breve{\varkappa}}) = \acute{\mathcal{F}}^{\underset{\smile}{\ddot{\mathcal{V}}}}_{\eta(\ddot{\aleph})}(\underset{\smile}{\breve{\varkappa}})$. Hence $\underset{\smile}{\ddot{\mathcal{W}}} = \underset{\smile}{\ddot{\mathcal{V}}}$.

(3) Proof is Straightforward.

(4) If $\underbrace{\ddot{\mathcal{W}}} \subset \underbrace{\ddot{\mathcal{V}}} \subset \underbrace{\ddot{\mathfrak{Q}}}$, $\acute{\mathcal{T}}^{\ddot{\mathcal{W}}}_{\eta(\ddot{\aleph})}(\breve{\varkappa}) \leq \acute{\mathcal{T}}^{\ddot{\mathcal{V}}}_{\eta(\ddot{\aleph})}(\breve{\varkappa}) \leq \acute{\mathcal{T}}^{\ddot{\mathfrak{Q}}}_{\eta(\ddot{\aleph})}(\breve{\varkappa})$, $\acute{\mathcal{J}}^{\ddot{\mathcal{W}}}_{\eta(\ddot{\aleph})}(\breve{\varkappa}) \geq \acute{\mathcal{J}}^{\ddot{\mathcal{V}}}_{\eta(\ddot{\aleph})}(\breve{\varkappa}) \geq \acute{\mathcal{J}}^{\ddot{\mathfrak{Q}}}_{\eta(\ddot{\aleph})}(\breve{\varkappa})$, $\acute{\mathcal{F}}^{\ddot{\mathcal{W}}}_{\eta(\ddot{\aleph})}(\breve{\varkappa}) \geq \acute{\mathcal{F}}^{\ddot{\mathcal{V}}}_{\eta(\ddot{\aleph})}(\breve{\varkappa}) \geq \acute{\mathcal{F}}^{\ddot{\mathfrak{Q}}}_{\eta(\ddot{\aleph})}(\breve{\varkappa})$.

$$\left|\acute{\mathcal{T}}^{\ddot{\mathcal{W}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa}) - \acute{\mathcal{T}}^{\ddot{\mathcal{V}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa})\right| \leq \left|\acute{\mathcal{T}}^{\ddot{\mathcal{W}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa}) - \acute{\mathcal{T}}^{\ddot{\mathfrak{Q}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa})\right|,$$
$$\left|\acute{\mathcal{T}}^{\ddot{\mathcal{V}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa}) - \acute{\mathcal{T}}^{\ddot{\mathfrak{Q}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa})\right| \leq \left|\acute{\mathcal{T}}^{\ddot{\mathcal{W}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa}) - \acute{\mathcal{T}}^{\ddot{\mathfrak{Q}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa})\right|,$$
$$\left|\acute{\mathcal{J}}^{\ddot{\mathcal{W}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa}) - \acute{\mathcal{J}}^{\ddot{\mathcal{V}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa})\right| \geq \left|\acute{\mathcal{J}}^{\ddot{\mathcal{W}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa}) - \acute{\mathcal{J}}^{\ddot{\mathfrak{Q}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa})\right|,$$
$$\left|\acute{\mathcal{J}}^{\ddot{\mathcal{V}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa}) - \acute{\mathcal{J}}^{\ddot{\mathfrak{Q}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa})\right| \geq \left|\acute{\mathcal{J}}^{\ddot{\mathcal{W}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa}) - \acute{\mathcal{J}}^{\ddot{\mathfrak{Q}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa})\right|,$$
$$\left|\acute{\mathcal{F}}^{\ddot{\mathcal{W}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa}) - \acute{\mathcal{F}}^{\ddot{\mathcal{V}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa})\right| \geq \left|\acute{\mathcal{F}}^{\ddot{\mathcal{W}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa}) - \acute{\mathcal{F}}^{\ddot{\mathfrak{Q}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa})\right|,$$
$$\left|\acute{\mathcal{F}}^{\ddot{\mathcal{V}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa}) - \acute{\mathcal{F}}^{\ddot{\mathfrak{Q}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa})\right| \geq \left|\acute{\mathcal{F}}^{\ddot{\mathcal{W}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa}) - \acute{\mathcal{F}}^{\ddot{\mathfrak{Q}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa})\right|.$$

Thus, $\grave{\mathbb{T}}_{PNHSS}(\ddot{\mathcal{W}}, \ddot{\mathfrak{Q}}) \leq \grave{\mathbb{T}}_{PNHSS}(\ddot{\mathcal{W}}, \ddot{\mathcal{V}})$; $\grave{\mathbb{T}}_{PNHSS}(\ddot{\mathcal{W}}, \ddot{\mathfrak{Q}}) \leq \grave{\mathbb{T}}_{PNHSS}(\ddot{\mathcal{V}}\ \ddot{\mathfrak{Q}})$.

Definition 3.2:

Let $\ddot{\mathcal{W}} = \left(\ddot{\ddot{\aleph}}, < \breve{\varkappa}, \acute{\mathcal{T}}^{\ddot{\mathcal{W}}}_{\eta(\ddot{\aleph})}(\breve{\varkappa}), \acute{\mathcal{J}}^{\ddot{\mathcal{W}}}_{\eta(\ddot{\aleph})}(\breve{\varkappa}), \acute{\mathcal{F}}^{\ddot{\mathcal{W}}}_{\eta(\ddot{\aleph})}(\breve{\varkappa}) >: \breve{\varkappa} \dot{\in} \tilde{\tilde{\Delta}}\right)$;

$\ddot{\mathcal{V}} = \left(\ddot{\ddot{\aleph}}, < \breve{\varkappa}, \acute{\mathcal{T}}^{\ddot{\mathcal{V}}}_{\eta(\ddot{\aleph})}(\breve{\varkappa}), \acute{\mathcal{J}}^{\ddot{\mathcal{V}}}_{\eta(\ddot{\aleph})}(\breve{\varkappa}), \acute{\mathcal{F}}^{\ddot{\mathcal{V}}}_{\eta(\ddot{\aleph})}(\breve{\varkappa}) >: \breve{\varkappa} \dot{\in} \tilde{\tilde{\Delta}}\right)$ be PNHSSs. The Cotangent Similarity Measure (CTSM) based on the co-tangent function between $\ddot{\mathcal{W}}$, $\ddot{\mathcal{V}}$ is, $CT^1{}_{PNHSS}(\ddot{\mathcal{W}}, \ddot{\mathcal{V}}) =$

$$\left(\ddot{\ddot{\aleph}}, < \breve{\varkappa}, \ \frac{1}{\acute{n}} \sum_{i=1}^{n} cot\left[\frac{\pi}{4} + \frac{\pi}{4}\left(\left|\acute{\mathcal{T}}^{\ddot{\mathcal{W}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa}) - \acute{\mathcal{T}}^{\ddot{\mathcal{V}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa})\right| \vee \left|\acute{\mathcal{J}}^{\ddot{\mathcal{W}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa}) - \acute{\mathcal{J}}^{\ddot{\mathcal{V}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa})\right| \vee \left|\acute{\mathcal{F}}^{\ddot{\mathcal{W}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa}) - \acute{\mathcal{F}}^{\ddot{\mathcal{V}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa})\right|\right)\right] \quad (2)$$

$$CT^2{}_{PNHSS}(\ddot{\mathcal{W}}, \ddot{\mathcal{V}}) = \left(\ddot{\ddot{\aleph}}, < \breve{\varkappa}, \ \frac{1}{\acute{n}} \sum_{i=1}^{n} cot\left[\frac{\pi}{4} + \frac{\pi}{12}\left(\left|\acute{\mathcal{T}}^{\ddot{\mathcal{W}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa}) - \acute{\mathcal{T}}^{\ddot{\mathcal{V}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa})\right| \vee \left|\acute{\mathcal{J}}^{\ddot{\mathcal{W}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa}) - \acute{\mathcal{J}}^{\ddot{\mathcal{V}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa})\right| \vee \left|\acute{\mathcal{F}}^{\ddot{\mathcal{W}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa}) - \acute{\mathcal{F}}^{\ddot{\mathcal{V}}^2}_{(\eta(\ddot{\aleph}))_i}(\breve{\varkappa})\right|\right)\right] \quad (3)$$

Here $\vee$ denotes Max Operator.

Proposition 2:

The CTSMs $CT^{1,2}_{PNHSS}(\ddot{\mathcal{W}}, \ddot{\mathcal{V}})$ satisfies,

(1) $0 \leq CT^{1,2}_{PNHSS}(\ddot{\mathcal{W}}, \ddot{\mathcal{V}}) \leq 1$

(2) $CT^{1,2}_{PNHSS}(\ddot{\mathcal{W}}, \ddot{\mathcal{V}}) = 1$ iff $\ddot{\mathcal{W}} = \ddot{\mathcal{V}}$

(3) $CT^{1,2}_{PNHSS}\left(\ddot{\mathcal{W}},\ddot{\mathcal{V}}\right) = CT^{1,2}_{PNHSS}\left(\ddot{\mathcal{V}},\ddot{\mathcal{W}}\right)$

(4) If $\ddot{\mathfrak{Q}}$ is a PNHSS set and $\ddot{\mathcal{W}} \subset \ddot{\mathcal{V}} \subset \ddot{\mathfrak{Q}}$ then $CT^{1,2}_{PNHSS}\left(\ddot{\mathcal{W}},\ddot{\mathfrak{Q}}\right) \leq CT^{1,2}_{PNHSS}\left(\ddot{\mathcal{W}},\ddot{\mathcal{V}}\right)$;

$CT^{1,2}_{PNHSS}\left(\ddot{\mathcal{W}},\ddot{\mathfrak{Q}}\right) \leq CT^{1,2}_{PNHSS}\left(\ddot{\mathcal{V}},\ddot{\mathfrak{Q}}\right)$.

Proof: Proof is similar to Prop 1.

Definition 3.3:

Let $\ddot{\mathcal{W}} = \left(\ddot{\aleph}, < \breve{\varkappa}, \mathcal{T}^{\ddot{\mathcal{W}}}_{\eta(\ddot{\aleph})}\left(\breve{\varkappa}\right), \mathcal{I}^{\ddot{\mathcal{W}}}_{\eta(\ddot{\aleph})}\left(\breve{\varkappa}\right), \mathcal{F}^{\ddot{\mathcal{W}}}_{\eta(\ddot{\aleph})}\left(\breve{\varkappa}\right) >: \breve{\varkappa} \dot{\in} \tilde{\Delta}\right)$;

$\ddot{\mathcal{V}} = \left(\ddot{\aleph}, < \breve{\varkappa}, \mathcal{T}^{\ddot{\mathcal{V}}}_{\eta(\ddot{\aleph})}\left(\breve{\varkappa}\right), \mathcal{I}^{\ddot{\mathcal{V}}}_{\eta(\ddot{\aleph})}\left(\breve{\varkappa}\right), \mathcal{F}^{\ddot{\mathcal{V}}}_{\eta(\ddot{\aleph})}\left(\breve{\varkappa}\right) >: \breve{\varkappa} \dot{\in} \tilde{\Delta}\right)$ be PNHSSs. The Cosine Similarity Measures (CSMs) between $\ddot{\mathcal{W}}$, $\ddot{\mathcal{V}}$ by using A.M is given by, $\grave{C}^1{}_{PNHSS}\left(\ddot{\mathcal{W}},\ddot{\mathcal{V}}\right) =$

$$\left(\ddot{\aleph}, < \breve{\varkappa}, \frac{1}{\acute{n}}, \sum_{i=1}^{n} \frac{\left(\mathcal{T}^{\ddot{\mathcal{W}}^2}_{(\eta(\ddot{\aleph}))_i}\left(\breve{\varkappa}\right)\right)\left(\mathcal{T}^{\ddot{\mathcal{V}}^2}_{(\eta(\ddot{\aleph}))_i}\left(\breve{\varkappa}\right)\right) + \left(\mathcal{I}^{\ddot{\mathcal{W}}^2}_{(\eta(\ddot{\aleph}))_i}\left(\breve{\varkappa}\right)\right)\left(\mathcal{I}^{\ddot{\mathcal{V}}^2}_{(\eta(\ddot{\aleph}))_i}\left(\breve{\varkappa}\right)\right) + \left(\mathcal{F}^{\ddot{\mathcal{W}}^2}_{(\eta(\ddot{\aleph}))_i}\left(\breve{\varkappa}\right)\right)\left(\mathcal{F}^{\ddot{\mathcal{V}}^2}_{(\eta(\ddot{\aleph}))_i}\left(\breve{\varkappa}\right)\right)}{\sqrt{\mathcal{T}^{\ddot{\mathcal{W}}^4}_{(\eta(\ddot{\aleph}))_i}\left(\breve{\varkappa}\right) + \mathcal{I}^{\ddot{\mathcal{W}}^4}_{(\eta(\ddot{\aleph}))_i}\left(\breve{\varkappa}\right) + \mathcal{F}^{\ddot{\mathcal{W}}^4}_{(\eta(\ddot{\aleph}))_i}\left(\breve{\varkappa}\right)}\sqrt{\mathcal{T}^{\ddot{\mathcal{V}}^4}_{(\eta(\ddot{\aleph}))_i}\left(\breve{\varkappa}\right) + \mathcal{I}^{\ddot{\mathcal{V}}^4}_{(\eta(\ddot{\aleph}))_i}\left(\breve{\varkappa}\right) + \mathcal{F}^{\ddot{\mathcal{V}}^4}_{(\eta(\ddot{\aleph}))_i}\left(\breve{\varkappa}\right)}} >\right) \text{------} \quad (4)$$

Proposition 3:

The CSMs $\grave{C}^1{}_{PNHSS}\left(\ddot{\mathcal{W}},\ddot{\mathcal{V}}\right)$ satisfies,

(1) $0 \leq \grave{C}^1{}_{PNHSS}\left(\ddot{\mathcal{W}},\ddot{\mathcal{V}}\right) \leq 1$

(2) $\grave{C}^1{}_{PNHSS}\left(\ddot{\mathcal{W}},\ddot{\mathcal{V}}\right) = 1$ iff $\ddot{\mathcal{W}} = \ddot{\mathcal{V}}$

(3) $\grave{C}^1{}_{PNHSS}\left(\ddot{\mathcal{W}},\ddot{\mathcal{V}}\right) = \grave{C}^1{}_{PNHSS}\left(\ddot{\mathcal{V}},\ddot{\mathcal{W}}\right)$.

Proof:

(1) Value of the Cosine function lies between [0, 1]. Hence $0 \leq \grave{C}^1{}_{PNHSS}\left(\ddot{\mathcal{W}},\ddot{\mathcal{V}}\right) \leq 1$.

(2) If $\ddot{\mathcal{W}} = \ddot{\mathcal{V}}$, then $\mathcal{T}^{\ddot{\mathcal{W}}}_{(\eta(\ddot{\aleph}))_i}\left(\breve{\varkappa}\right) = \mathcal{T}^{\ddot{\mathcal{V}}}_{(\eta(\ddot{\aleph}))_i}\left(\breve{\varkappa}\right)$, $\mathcal{I}^{\ddot{\mathcal{W}}}_{(\eta(\ddot{\aleph}))_i}\left(\breve{\varkappa}\right) = \mathcal{I}^{\ddot{\mathcal{V}}}_{(\eta(\ddot{\aleph}))_i}\left(\breve{\varkappa}\right)$, $\mathcal{F}^{\ddot{\mathcal{W}}}_{(\eta(\ddot{\aleph}))_i}\left(\breve{\varkappa}\right) = \mathcal{F}^{\ddot{\mathcal{V}}}_{(\eta(\ddot{\aleph}))_i}\left(\breve{\varkappa}\right)$

for i = 1,2,…n. Hence, $\grave{C}^1{}_{PNHSS}\left(\ddot{\mathcal{W}},\ddot{\mathcal{V}}\right) = 1$.

(3) Proof is Straightforward.

Definition 3.4:

Let $\ddot{\mathcal{W}} = \left(\ddot{\aleph}, < \breve{\varkappa}, \mathcal{T}^{\ddot{\mathcal{W}}}_{\eta(\ddot{\aleph})}\left(\breve{\varkappa}\right), \mathcal{I}^{\ddot{\mathcal{W}}}_{\eta(\ddot{\aleph})}\left(\breve{\varkappa}\right), \mathcal{F}^{\ddot{\mathcal{W}}}_{\eta(\ddot{\aleph})}\left(\breve{\varkappa}\right) >: \breve{\varkappa} \dot{\in} \tilde{\Delta}\right)$;

$\ddot{\mathcal{V}} = \left(\ddot{\aleph}, < \breve{\varkappa}, \mathcal{T}^{\ddot{\mathcal{V}}}_{\eta(\ddot{\aleph})}\left(\breve{\varkappa}\right), \mathcal{I}^{\ddot{\mathcal{V}}}_{\eta(\ddot{\aleph})}\left(\breve{\varkappa}\right), \mathcal{F}^{\ddot{\mathcal{V}}}_{\eta(\ddot{\aleph})}\left(\breve{\varkappa}\right) >: \breve{\varkappa} \dot{\in} \tilde{\Delta}\right)$ be PNHSSs.

The CSMs between $\ddot{\underset{\smile}{\mathcal{W}}}$, $\ddot{\underset{\smile}{\mathcal{V}}}$ based on the cosine function is

$$\grave{C}^2{}_{PNHSS}\left(\ddot{\underset{\smile}{\mathcal{W}}},\ddot{\underset{\smile}{\mathcal{V}}}\right) = \left(\ddot{\aleph}, < \underset{\smile}{\varkappa}, \frac{1}{\acute{n}} \sum_{i=1}^{n} cos\left[\frac{\pi}{2}\left(\left|\acute{\mathcal{T}}^{\ddot{\underset{\smile}{\mathcal{W}}}^2}_{\left(\eta\left(\ddot{\aleph}\right)\right)_i}\left(\underset{\smile}{\varkappa}\right) - \acute{\mathcal{T}}^{\ddot{\underset{\smile}{\mathcal{V}}}^2}_{\left(\eta\left(\ddot{\aleph}\right)\right)_i}\left(\underset{\smile}{\varkappa}\right)\right| \vee \left|\acute{\jmath}^{\ddot{\underset{\smile}{\mathcal{W}}}^2}_{\left(\eta\left(\ddot{\aleph}\right)\right)_i}\left(\underset{\smile}{\varkappa}\right) - \acute{\jmath}^{\ddot{\underset{\smile}{\mathcal{V}}}^2}_{\left(\eta\left(\ddot{\aleph}\right)\right)_i}\left(\underset{\smile}{\varkappa}\right)\right| \vee \left|\acute{\mathcal{F}}^{\ddot{\underset{\smile}{\mathcal{W}}}^2}_{\left(\eta\left(\ddot{\aleph}\right)\right)_i}\left(\underset{\smile}{\varkappa}\right) - \acute{\mathcal{F}}^{\ddot{\underset{\smile}{\mathcal{V}}}^2}_{\left(\eta\left(\ddot{\aleph}\right)\right)_i}\left(\underset{\smile}{\varkappa}\right)\right|\right)\right] \quad (5)$$

$$\grave{C}^3{}_{PNHSS}\left(\ddot{\underset{\smile}{\mathcal{W}}},\ddot{\underset{\smile}{\mathcal{V}}}\right) = \left(\ddot{\aleph}, < \underset{\smile}{\varkappa}, \frac{1}{\acute{n}} \sum_{i=1}^{n} cos\left[\frac{\pi}{6}\left(\left|\acute{\mathcal{T}}^{\ddot{\underset{\smile}{\mathcal{W}}}^2}_{\left(\eta\left(\ddot{\aleph}\right)\right)_i}\left(\underset{\smile}{\varkappa}\right) - \acute{\mathcal{T}}^{\ddot{\underset{\smile}{\mathcal{V}}}^2}_{\left(\eta\left(\ddot{\aleph}\right)\right)_i}\left(\underset{\smile}{\varkappa}\right)\right| \vee \left|\acute{\jmath}^{\ddot{\underset{\smile}{\mathcal{W}}}^2}_{\left(\eta\left(\ddot{\aleph}\right)\right)_i}\left(\underset{\smile}{\varkappa}\right) - \acute{\jmath}^{\ddot{\underset{\smile}{\mathcal{V}}}^2}_{\left(\eta\left(\ddot{\aleph}\right)\right)_i}\left(\underset{\smile}{\varkappa}\right)\right| \vee \left|\acute{\mathcal{F}}^{\ddot{\underset{\smile}{\mathcal{W}}}^2}_{\left(\eta\left(\ddot{\aleph}\right)\right)_i}\left(\underset{\smile}{\varkappa}\right) - \acute{\mathcal{F}}^{\ddot{\underset{\smile}{\mathcal{V}}}^2}_{\left(\eta\left(\ddot{\aleph}\right)\right)_i}\left(\underset{\smile}{\varkappa}\right)\right|\right)\right] \quad (6)$$

Proposition 4:
The CSMs $\grave{C}^{2,3}_{PNHSS}\left(\ddot{\underset{\smile}{\mathcal{W}}},\ddot{\underset{\smile}{\mathcal{V}}}\right)$ satisfies the following properties:

(1) $0 \leq \grave{C}^{2,3}_{PNHSS}\left(\ddot{\underset{\smile}{\mathcal{W}}},\ddot{\underset{\smile}{\mathcal{V}}}\right) \leq 1$

(2) $\grave{C}^{2,3}_{PNHSS}\left(\ddot{\underset{\smile}{\mathcal{W}}},\ddot{\underset{\smile}{\mathcal{V}}}\right) = \grave{C}^{2,3}_{PNHSS}\left(\ddot{\underset{\smile}{\mathcal{V}}},\ddot{\underset{\smile}{\mathcal{W}}}\right)$.

(3) $\grave{C}^{2,3}_{PNHSS}\left(\ddot{\underset{\smile}{\mathcal{W}}},\ddot{\underset{\smile}{\mathcal{V}}}\right) = 1$ iff $\ddot{\underset{\smile}{\mathcal{W}}} = \ddot{\underset{\smile}{\mathcal{V}}}$

(4) If $\ddot{\underset{\smile}{\mathfrak{Q}}}$ is a PNHSS set and $\ddot{\underset{\smile}{\mathcal{W}}} \subset \ddot{\underset{\smile}{\mathcal{V}}} \subset \ddot{\underset{\smile}{\mathfrak{Q}}}$ then $\grave{C}^{2,3}_{PNHSS}\left(\ddot{\underset{\smile}{\mathcal{W}}},\ddot{\underset{\smile}{\mathfrak{Q}}}\right) \leq \grave{C}^{2,3}_{PNHSS}\left(\ddot{\underset{\smile}{\mathcal{W}}},\ddot{\underset{\smile}{\mathcal{V}}}\right)$ and

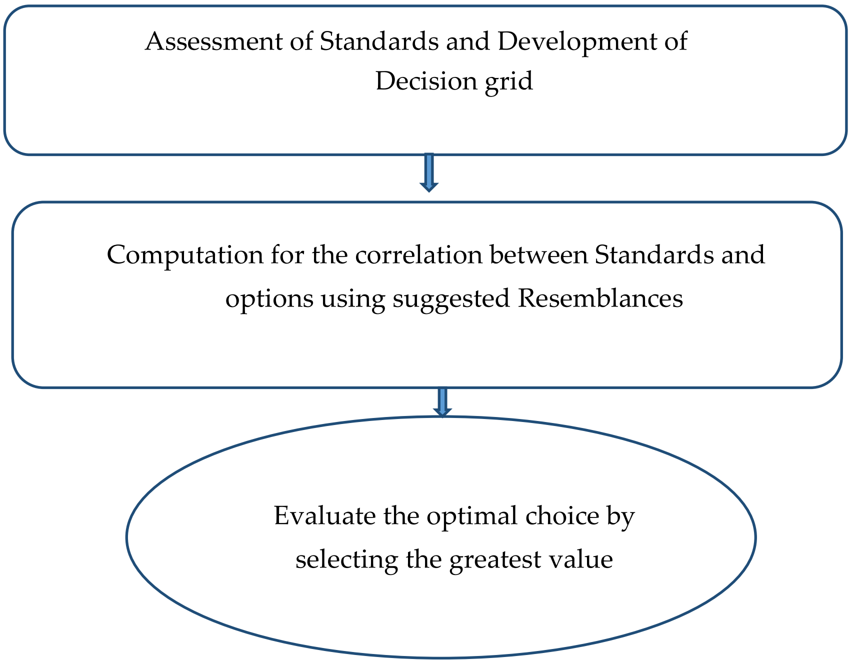
$\grave{C}^{2,3}_{PNHSS}\left(\ddot{\underset{\smile}{\mathcal{W}}},\ddot{\underset{\smile}{\mathfrak{Q}}}\right) \leq \grave{C}^{2,3}_{PNHSS}\left(\ddot{\underset{\smile}{\mathcal{V}}},\ddot{\underset{\smile}{\mathfrak{Q}}}\right)$.

Proof: The proof is similar to Prop.1.

4. Application of TSMs for PNHSS

All countries have been using different types of plastic like PETE, HDPE, PVC, LDPE, PP, PS and Mix plastic. Few countries are converting plastic waste into energy in the form of solid, liquid and gaseous fuels. Also, it's possible to convert waste plastics into Hydrogen, Methane and Ethylene. Both Hydrogen and Methane can be used for clean fuels. Few states are currently sending their collected plastic waste to cement plants for Co-Processing. The world is affected a lot due to usage of plastic. Plastic things are prohibited by many countries. But still we could not minimize as expected. Several techniques are used for converting plastic waste into energy. Pyrolysis is a common technique used to convert plastic waste into energy. We try to develop a mathematical model to overcome this world problem.

4.1 Methodology



Let $\underbrace{\acute{\mathcal{C}}}$ = $\left\{\underbrace{\acute{\mathcal{C}}^1}, \underbrace{\acute{\mathcal{C}}^2}, \underbrace{\acute{\mathcal{C}}^3}, \underbrace{\acute{\mathcal{C}}^4}, \underbrace{\acute{\mathcal{C}}^5}, \underbrace{\acute{\mathcal{C}}^6}, \underbrace{\acute{\mathcal{C}}^7}, \underbrace{\acute{\mathcal{C}}^8}, \underbrace{\acute{\mathcal{C}}^9}, \underbrace{\acute{\mathcal{C}}^{10}}, \underbrace{\acute{\mathcal{C}}^{11}}, \underbrace{\acute{\mathcal{C}}^{12}}\right\}$ be a set of Countries and $\underbrace{\acute{\mathrm{P}}}$ = $\{\mathit{Slow}\ \text{Pyrolysis}(SP), \mathit{Intermediate}\ \text{Pyrolysis}(IP), \mathit{Ultra\ Fast}\ \text{Pyrolysis(UFP)}\}$ be a types of M Pyrolysis process.
The collection of attributes to $\underbrace{\acute{\mathcal{C}}}$ & $\underbrace{\acute{\mathrm{P}}}$ be,

$$\underbrace{\ddot{\mathfrak{C}}} = \left\{\begin{array}{c} \underbrace{\ddot{\mathfrak{C}}}^1 \ (\text{Mean Amount of plastic usage[PU] (MT/Day)}) \\ \underbrace{\ddot{\mathfrak{C}}}^2 \ (\text{Typical Amount of PU (MT/ Year)}) \\ \underbrace{\ddot{\mathfrak{C}}}^3 \ (\text{Average Amount of PU (Grams/ Week)}) \\ \underbrace{\ddot{\mathfrak{C}}}^4 \ (\text{Typical Amount of PU}(kg/person)) \\ \underbrace{\ddot{\mathfrak{C}}}^5 \ (\text{Mean Amount of PU } (g/person)) \end{array}\right\}$$

Sub-Attributes are $\underbrace{\ddot{\mathfrak{C}}}^1 = \{< 1.5\ \text{M.T}, 1.5 - 2.5\ \text{M.T}, 2.5 - 3.5\ \text{M.T}, > 3.5\ \text{M.T}\}$; $\underbrace{\ddot{\mathfrak{C}}}^2 = \{< 10\ \text{M.T}, 10 - 15\ \text{M.T}, 15 - 16.5\ \text{M.T}\}$; $\underbrace{\ddot{\mathfrak{C}}}^3 = \{0.1 - 2\ \text{G}, 2 - 5\ \text{G}, > 5\ \text{G}\}$;$\underbrace{\ddot{\mathfrak{C}}}^4 = \{< 5\ \text{kgs}, 5 - 10\ \text{kgs}, 10 - 15\ \text{kgs}\}$;$\underbrace{\ddot{\mathfrak{C}}}^5 = \{0.1 - 2\ \text{g}, 2 - 5\ \text{g}, > 5\ \text{g}\}$. The PNHSS be $\eta: \left(\underbrace{\ddot{\mathfrak{C}}}^1 \times \underbrace{\ddot{\mathfrak{C}}}^2 \times \underbrace{\ddot{\mathfrak{C}}}^3 \times \underbrace{\ddot{\mathfrak{C}}}^4 \times \underbrace{\ddot{\mathfrak{C}}}^5\right) \rightarrow \mathcal{P}\left(\underbrace{\acute{\mathcal{C}}}\right)$ and $\ddot{\gamma}: \left(\underbrace{\ddot{\mathfrak{C}}}^1 \times \underbrace{\ddot{\mathfrak{C}}}^2 \times \underbrace{\ddot{\mathfrak{C}}}^3 \times \underbrace{\ddot{\mathfrak{C}}}^4 \times \underbrace{\ddot{\mathfrak{C}}}^5\right) \rightarrow \mathcal{P}\left(\underbrace{\acute{\mathrm{P}}}\right)$.
Let $\left(\eta, \acute{\zeta}\right) = \{2.5\ \text{M} - 3.5\ \text{M}, 10 - 15\ \text{M.T}, 2 - 5\ \text{G}, 10 - 15\ \text{kgs}, > 5\ \text{g}\}$.

Now using the proposed several SMs for PNHSSs, we will decide which country is widely using mentioned energy techniques.
For this purpose, we should first provide the relationship between $\left\{\underbrace{\acute{\mathcal{C}}^2}, \underbrace{\acute{\mathcal{C}}^3}, \underbrace{\acute{\mathcal{C}}^5}, \underbrace{\acute{\mathcal{C}}^{11}}\right\}$ and $\{2.5\ \text{M} - 3.5\ \text{M}, 10 - 15\ \text{M.T}, 2 - 5\ \text{G}, 10 - 15\ \text{kgs}, > 5\ \text{g}\}$ in terms of PNHSSs.
In the 2nd step, we should provide the relationship between $\{2.5\ \text{M} - 3.5\ \text{M}, 10 - 15\ \text{M.T}, 2 - 5\ \text{G}, 10 - 15\ \text{kgs}, > 5\ \text{g}\}$ and $\{(SP), (IP), (\text{UFP})\}$.
In Step 3, we should find the correlation between $\left\{\underbrace{\acute{\mathcal{C}}^2}, \underbrace{\acute{\mathcal{C}}^3}, \underbrace{\acute{\mathcal{C}}^5}, \underbrace{\acute{\mathcal{C}}^{11}}\right\}$ and $\{(SP), (IP), (\text{FP}), (\text{UFP})\}$.
In step 4, The association is determined with the proposed TSMs for PNHSS by Equations (1-6).

In step 5, Finding the best selection.

Table 1. Relation between Regions and criteria

Regions	$\mathbf{2.5\,M - 3.5\,M}$	$\mathbf{10 - 15\,M.T}$	$\mathbf{2 - 5\,G}$	$\mathbf{10 - 15\,kgs}$	$\mathbf{> 5\,g}$
$\underbrace{\acute{C}^{2}}$	(.5, .3, .4)	(.5, .4, .6)	(.9, .4, .3)	(.7, .3, .4)	(.6, .3, .7)
$\underbrace{\acute{C}^{3}}$	(.6, .4, .5)	(.7, .4, .5)	(.8, .4, .1)	(.8, .3, .5)	(.8, .2, .6)
$\underbrace{\acute{C}^{5}}$	(.8, .3, .2)	(.9, .3, .1)	(.6, .7, .8)	(.7, .5, .6)	(.5, .4, .6)
$\underbrace{\acute{C}^{11}}$	(.7, .6, .1)	(.5, .2, .6)	(.4, .6, .7)	(.4, .5, .7)	(.7, .3, .6)

Table 2. Relation between sources and criteria.

Sources	$\mathbf{2.5\,M - 3.5\,M}$	$\mathbf{10 - 15\,M.T}$	$\mathbf{2 - 5\,G}$	$\mathbf{10 - 15\,kgs}$	$\mathbf{> 5\,g}$
SP	(.6, .3, .5)	(.8, .6, .4)	(.7, .2, .3)	(.8, .6, .5)	(.7, .6, .2)
IP	(.7, .5, .3)	(.7, .6, .4)	(.8, .6, .1)	(.6, .3, .7)	(.9, .7, .2)
UFP	(.7, .2, .6)	(.5, .6, .7)	(.9, .1, .2)	(.8, .7, .5)	(.4, .2, .9)

Table 3. SMs using $\breve{\mathbb{T}}_{PNHSS}\left(\underbrace{\ddot{\mathcal{W}}},\underbrace{\ddot{\mathcal{V}}}\right)$.

SMs	**Regions**	**SP**	**IP**	**UFP**
$\breve{\mathbb{T}}_{PNHSS}\left(\underbrace{\ddot{\mathcal{W}}},\underbrace{\ddot{\mathcal{V}}}\right)$	$\underbrace{\acute{C}^{2}}$	.85229	.82273	**.88249**
	$\underbrace{\acute{C}^{3}}$	**.88170**	.87670	.70570
	$\underbrace{\acute{C}^{5}}$	**.82234**	.79836	.75526
	$\underbrace{\acute{C}^{11}}$	.78589	**.82361**	.76558

Table 4. SMs using $CT^1{}_{PNHSS}\left(\underbrace{\ddot{\mathcal{W}}},\underbrace{\ddot{\mathcal{V}}}\right)$.

SMs	Regions	SP	IP	UFP
$\boldsymbol{CT^1{}_{PNHSS}\left(\underbrace{\ddot{\mathcal{W}}},\underbrace{\ddot{\mathcal{V}}}\right)}$	$\underbrace{\acute{\mathcal{C}}^2}$	.61163	.62552	**.71484**
	$\underbrace{\acute{\mathcal{C}}^3}$	**.72953**	.66488	.63983
	$\underbrace{\acute{\mathcal{C}}^5}$	**.60579**	.56037	.50495
	$\underbrace{\acute{\mathcal{C}}^{11}}$	.53976	**.62016**	.51110

Table 5: SM using $CT^2{}_{PNHSS}\left(\underbrace{\ddot{\mathcal{W}}},\underbrace{\ddot{\mathcal{V}}}\right)$.

SMs	Regions	SP	IP	UFP
$\boldsymbol{CT^2{}_{PNHSS}\left(\underbrace{\ddot{\mathcal{W}}},\underbrace{\ddot{\mathcal{V}}}\right)}$	$\underbrace{\acute{\mathcal{C}}^2}$	.85186	.85851	**.87225**
	$\underbrace{\acute{\mathcal{C}}^3}$	**.90033**	.87437	.86319
	$\underbrace{\acute{\mathcal{C}}^5}$	**.84881**	.82829	.80667
	$\underbrace{\acute{\mathcal{C}}^{11}}$	.82248	**.85499**	.78950

Table 6. SM using $\grave{C}^1{}_{PNHSS}\left(\underbrace{\ddot{\mathcal{W}}},\underbrace{\ddot{\mathcal{V}}}\right)$.

SMs	Regions	SP	IP	UFP
$\boldsymbol{\grave{C}^1{}_{PNHSS}\left(\underbrace{\ddot{\mathcal{W}}},\underbrace{\ddot{\mathcal{V}}}\right)}$	$\underbrace{\acute{\mathcal{C}}^2}$	.86637	.82112	**.94452**
	$\underbrace{\acute{\mathcal{C}}^3}$	**.92401**	.89219	.85379
	$\underbrace{\acute{\mathcal{C}}^5}$	**.79727**	.80323	.71249
	$\underbrace{\acute{\mathcal{C}}^{11}}$	.67766	**.77675**	.66510

Table 7: SM using $\grave{C}^2{}_{PNHSS}\left(\underbrace{\ddot{\mathcal{W}}},\underbrace{\ddot{\mathcal{V}}}\right)$

SMs	Regions	SP	IP	UFP
$\grave{C}^2{}_{PNHSS}\left(\underbrace{\ddot{\mathcal{W}}},\underbrace{\ddot{\mathcal{V}}}\right)$	$\underbrace{\acute{C}^2}$	.87026	.88792	**.94036**
	$\underbrace{\acute{C}^3}$	**.94101**	.90718	.88230
	$\underbrace{\acute{C}^5}$	**.86286**	.79998	.82065
	$\underbrace{\acute{C}^{11}}$	.82876	**.87008**	.74815

Table 8: SM using $\grave{C}^3{}_{PNHSS}\left(\underbrace{\ddot{\mathcal{W}}},\underbrace{\ddot{\mathcal{V}}}\right)$.

SMs	Regions	SP	IP	UFP
$\grave{C}^3{}_{PNHSS}\left(\underbrace{\ddot{\mathcal{W}}},\underbrace{\ddot{\mathcal{V}}}\right)$	$\underbrace{\acute{C}^2}$	.98518	.98724	**.99328**
	$\underbrace{\acute{C}^3}$	**.99335**	.98943	.98651
	$\underbrace{\acute{C}^5}$	**.9842**	.97647	.97527
	$\underbrace{\acute{C}^{11}}$	.98039	**.98511**	.97056

The Highest Measure (Table values 3,4,5,6,7,8) reflects Region $\underbrace{\acute{C}^2}$ should be selected for UFP, Region $\underbrace{\acute{C}^3}$ should be selected for SP, Region $\underbrace{\acute{C}^5}$ should be selected for SP, Region $\underbrace{\acute{C}^{11}}$ should be selected for IP.

5. Conclusions

The aim of this paper is to establish Tangent, Cotangent and Cosine SMs of PNHSSs. The extension is very applicable to decision-making problems. We introduced six TSMs for PNHSSs with properties. Also, applied them to Energy source selection problem.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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