






Interval-Valued Fermatean Neutrosophic Shortest Path Problem via Score Function

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Abstract: Contemporary mathematical techniques have been crafted to address the uncertainty of numerous real-world settings, including Fermatean neutrosophic fuzzy set theory. Fermatean neutrosophic fuzzy set is an extension of combining Fermatean and neutrosophic sets. Fermatean neutrosophic set was developed to enable the analytical management of ambiguous data from relatively typical real-world decision-making scenarios. Decision-makers find it challenging to determine the degree of membership (MG) and non-membership (NG) with sharp values due to the insufficient data provided. Intervals MG and NG are suitable options in these circumstances. In this article, the shortest route issue is formulated using an interval set of values in a Fermatean neutrosophic setting. A de-neutrosophication technique utilizing a scoring function is then suggested. A mathematical version is also included to show the framework's usefulness and viability in more detail.

Keywords: Fermatean Neutrosophic Shortest Path Problem; Fermatean Neutrosophic Fuzzy Set; Shortest Path Problem; Network.

1. Introduction

For processing conceptual data, graph models are frequently used in a variety of domains, including operations science, networks, data analysis, pattern discovery, the field of finance, and visual design. In 1965, Zadeh [1] presented the Fuzzy Set (FS) as a magic solution to uncertainty and ambiguity. The FS theory is demonstrated in a variety of real-world problems in numerous practical applications. Atanassov [2] first presented the Intuitionistic Fuzzy Set (IFS) model in 1986. In IFS, membership and non-membership are used to characterize every item (totals are always capped at 1). Yager has introduced the Pythagorean fuzzy set (PFS) notion as a generalization of the intuitionistic fuzzy set (IFS) [3] to manage the complex imprecision and uncertainty in real-world decision-making difficulties.

By relaxing the requirement that the square root of the sum of the membership degree and non-membership degree must be greater than one, the Pythagorean fuzzy model varies from other fuzzy models. Neutrosophic sets, a concept first put forth by Smarandache [4] in 1995, can be used to overcome issues including insufficient, ambiguous, and inaccurate information. Senapati and Yager [5] introduce the idea of Fermatean fuzzy sets (FFS) with the restriction that the sum of the cubes expressing membership and non-membership degrees cannot be greater than one. The FFS is a useful method to accept ambiguity and vagueness since it increases the relative volume of membership and non-membership in fuzzy and PFSs. The Fermatean fuzzy TOPSIS approach with Dombi aggregation

operators was presented by Aydemer et al. in 2020 [6]. Barraza et al. [7] in 2020 provide an application of Fermatean fuzzy matrices in the co-design of urban projects. Broumi et al. [8] proposed the concept of a complex Fermatean Neutrosophic graph and its use in decision-making in 2023.

The theory and applications of Fermatean Neutrosophic Graphs were provided in 2022 by Broumi et al. [9]. In 2022, Ganie [10] presented a method for multi-criteria decision-making based on the distance and knowledge measures of FFSs. Broumi also looks into bipolar single-valued neutrosophic graphs in [11], as well as the associated properties. In [12, 13], Sundareswaran et al. described and looked into the neutrosophic environment's susceptible features. A correlation metric for Pythagorean Neutrosophic Sets was presented in 2019 by Jansi et al. [14]. A New Decision-Making Approach Based on FFSs and WASPAS for Green Construction Supplier Evaluation was given by Keshavarz-Ghorabae et al. [15] in 2020. The Fermatean fuzzy WASPAS method-based multi-criteria healthcare waste disposal location selection was proposed by Mishra. On interval neutrosophic sets, Broumi et al. [16] studied some operations on interval-valued Fermatean neutrosophic sets and their use in multicriteria decision-making. Broumi et al. [17] proposed a new intelligent algorithm for trapezoidal interval-valued neutrosophic network analysis. In 2016, Dey et al. [18] researched the fuzzy version of the shortest path problem (SPP). Using interval-valued triangular fuzzy arc weight, Ebrahimnejad et al. [19] suggested an optimization method for unraveling SPPs in 2020. In 2020, Singh [20] presented a fuzzy SPP from the perspective of a startup founder. SPP was first proposed by Jan A et al. [21] in 2022 using Pythagorean fuzzy components with interval values.

A new emergent concept of Fermatean neutrosophic was introduced by Antony and Jansi [22] in 2021 by fusing the concepts of Neutrosophic sets and FFSs. To determine the shortest path, the Fermatean neutrosophic graphs are examined in this work. Asim Bash et al. [23] provide a solution for neutrosophic Pythagorean fuzzy shortest path in a network. In 2023, Sasikala [24] presented her interpretation of Fermatean Neutrosophic Dombi Fuzzy Graphs. Mary et al. [25] provide a solution approach to the minimum spanning tree problem under the Fermatean fuzzy environment. Fermatean fuzzy hypergraph and some of its characteristics were proposed by Thamizhendhi [26] in 2021. By Vidhya [27] in 2022, an enhanced A search algorithm for the shortest path in a Pythagorean fuzzy environment with interval values. Broumi et al. [28] studied the concept of interval-valued Fermatean Neutrosophic graphs. Raut et al. [29] studied the problem of the shortest path on Fermatean Neutrosophic Networks. To the best of our knowledge, there is no study on interval Fermatean Neutrosophic Networks.

The organization of this paper is as follows: Section 1 covers the context and significant applications that provided inspiration for the proposed study. Section 2 provides a list of some fundamental definitions. A framework for the interval-valued Fermatean neutrosophic SPP is provided in Section 3, a quantified example is given in Section 4, and the study is summarized, possible future directions are discussed, and the benefits of the suggested work are emphasized in Section 5.

2. Preliminaries

In the following, some basic concepts and definitions of PFS, FFS, interval Fermatean neutrosophic sets, and interval valued Fermatean neutrosophic graph are reviewed from the literature.

Definition 2.1 [3] A PFS A on a universe of discourse X , is a structure having the form as

$$A_{PFS} = \{(x, T_A(x), F_A(x)) \mid x \in X\}$$

where $T_A(x): X \rightarrow [0,1]$ indicates the degree of membership and $F_A(x): X \rightarrow [0,1]$ indicates the degree of non-membership of every element $x \in X$ to the set A , respectively, with the constraints: $0 \leq (T_A(x))^2 + (F_A(x))^2 \leq 1$.

Senapati et al. [5] suggested the idea of FFS considering more possible types of uncertainty. These are defined below,

Definition 2.2: [5] A FFS A on a universe of discourse X is a structure defined as,

$$A_{FFS} = \{ \langle x, T_A(x), F_A(x) \rangle \mid x \in X \}$$

where $T_A(x): X \rightarrow [0,1]$ indicates the degree of membership, and $F_A(x): X \rightarrow [0,1]$ indicates the degree of non-membership of the element $x \in X$ to the set A, respectively, with the constraints:

$$0 \leq (T_A(x))^3 + (F_A(x))^3 \leq 1.$$

Definition 2.3: [16] An interval valued Fermatean neutrosophic number $A = \langle [T_A^L, T_A^U], [I_A^L, I_A^U], [F_A^L, F_A^U] \rangle$ is supposedly. Zero interval valued Fermatean neutrosophic number if and only if

$$T_A^L = 0, T_A^U = 0, I_A^L = 1, I_A^U = 1, F_A^L = 1 \text{ and } F_A^U = 1.$$

Definition 2.4: [16] An interval-valued Fermatean neutrosophic set (IVFNS) A on the universe of discourse X is of the structure: $A_{IVFNS} = \{ \langle x, \tilde{T}_A(x), \tilde{I}_A(x), \tilde{F}_A(x) \rangle \mid x \in X \}$, where, $\tilde{T}_A(x) = [T_A^L(x), T_A^U(x)]$, $\tilde{I}_A(x) = [I_A^L(x), I_A^U(x)]$ and $\tilde{F}_A(x) = [F_A^L(x), F_A^U(x)]$ represents the truth-membership degree, indeterminacy-membership degree and falsity-membership degree, respectively. Consider the mapping $\tilde{T}_A(x): X \rightarrow D[0,1]$, $\tilde{I}_A(x): X \rightarrow D[0,1]$, $\tilde{F}_A(x): X \rightarrow D[0,1]$ and $0 \leq (T_A^U(x))^3 + (F_A^U(x))^3 \leq 1$ and $0 \leq (I_A^U(x))^3 \leq 1$,

$$0 \leq (T_A^U(x))^3 + (I_A^U(x))^3 + (F_A^U(x))^3 \leq 2, \forall x \in X.$$

Definition 2.5: [28] An Interval-Valued Fermatean Neutrosophic Graph (IVFNG) of a graph $G = (V, E)$ we mean a pair $G = (A, B)$, where $A = \langle [T_A^L, T_A^U], [I_A^L, I_A^U], [F_A^L, F_A^U] \rangle$ is an interval-valued Fermatean neutrosophic set on V; and $B = \langle [T_B^L, T_B^U], [I_B^L, I_B^U], [F_B^L, F_B^U] \rangle$ is an interval valued Fermatean neutrosophic relation on E satisfying the following condition:

- $V = \{ v_1, v_2, \dots, v_n \}$, such that $T_A^L: V \rightarrow [0, 1]$, $T_A^U: V \rightarrow [0, 1]$, $I_A^L: V \rightarrow [0, 1]$, $I_A^U: V \rightarrow [0, 1]$ and $F_A^L: V \rightarrow [0, 1]$, $F_A^U: V \rightarrow [0, 1]$ denote the degree of truth-membership, the degree of indeterminacy-membership and falsity-membership of the element $y \in V$, respectively, and $0 \leq (T_A(v_i))^3 + (I_A(v_i))^3 + (F_A(v_i))^3 \leq 2$, for all $v_i \in V$, $(i = 1, 2, \dots, n)$ means $0 \leq (T_A^U(v_i))^3 + (I_A^U(v_i))^3 + (F_A^U(v_i))^3 \leq 2, \forall x \in X$.
- The functions $T_B^L: V \times V \rightarrow [0,1]$, $T_B^U: V \times V \rightarrow [0,1]$, $I_B^L: V \times V \rightarrow [0,1]$, $I_B^U: V \times V \rightarrow [0,1]$ and $F_B^L: V \times V \rightarrow [0,1]$, $F_B^U: V \times V \rightarrow [0,1]$ are such that $T_B^L(\{v_i, v_j\}) \leq \min[T_A^L(v_i), T_A^L(v_j)]$, $T_B^U(\{v_i, v_j\}) \leq \min[T_A^U(v_i), T_A^U(v_j)]$, $I_B^L(\{v_i, v_j\}) \geq \max[I_A^L(v_i), I_A^L(v_j)]$, $I_B^U(\{v_i, v_j\}) \geq \max[I_A^U(v_i), I_A^U(v_j)]$, $F_B^L(\{v_i, v_j\}) \geq \max[F_A^L(v_i), F_A^L(v_j)]$, $F_B^U(\{v_i, v_j\}) \geq \max[F_A^U(v_i), F_A^U(v_j)]$ denoting the degree of truth-membership, indeterminacy-membership and falsity membership of the edge $(v_i, v_j) \in E$ respectively, where $0 \leq (T_B(v_i, v_j))^3 + (I_B(v_i, v_j))^3 + (F_B(v_i, v_j))^3 \leq 2$ for all $\{v_i, v_j\} \in E$ $(i, j = 1, 2, \dots, n)$ means $0 \leq (T_B^U(v_i, v_j))^3 + (I_B^U(v_i, v_j))^3 + (F_B^U(v_i, v_j))^3 \leq 2, \forall x \in X$.

Definition 2.6: [16] Broumi et al. [29] defined the average possible membership degree of element x to interval valued Fermatean neutrosophic set $A = \langle [T_A^L(x), T_A^U(x)], [I_A^L(x), I_A^U(x)], [F_A^L(x), F_A^U(x)] \rangle$ as follows:

$$S_{Broumi}(x) = \frac{(T_A^L(x))^3 + (T_A^U(x))^3 + (I_A^L(x))^3 + (I_A^U(x))^3 + (F_A^L(x))^3 + (F_A^U(x))^3}{2}$$

Definition 2.7: [16] Let $A = \langle [T^L, T^U], [I^L, I^U], [F^L, F^U] \rangle$, $A_1 = [T_1^L, T_1^U], [I_1^L, I_1^U], [F_1^L, F_1^U]$ and $A_2 = \langle [T_2^L, T_2^U], [I_2^L, I_2^U], [F_2^L, F_2^U] \rangle$ be three interval valued fermatean neutrosophic numbers and $\lambda > 0$. Then, the operations rules are described as follows;

- $A_1 \oplus A_2 = \langle \left[\sqrt[3]{T_1^L{}^3 + T_2^L{}^3 - T_1^L{}^3 T_2^L{}^3}, \sqrt[3]{T_1^U{}^3 + T_2^U{}^3 - T_1^U{}^3 T_2^U{}^3} \right], [I_1^L I_2^L, I_1^U I_2^U], [F_1^L F_2^L, F_1^U F_2^U] \rangle$
- $A_1 \otimes A_2 = \langle [T_1^L T_2^L, T_1^U T_2^U], \left[\sqrt[3]{I_1^L{}^3 + I_2^L{}^3 - I_1^L{}^3 I_2^L{}^3}, \sqrt[3]{I_1^U{}^3 + I_2^U{}^3 - I_1^U{}^3 I_2^U{}^3} \right], \left[\sqrt[3]{F_1^L{}^3 + F_2^L{}^3 - F_1^L{}^3 F_2^L{}^3}, \sqrt[3]{F_1^U{}^3 + F_2^U{}^3 - F_1^U{}^3 F_2^U{}^3} \right] \rangle$
- $\lambda A = \langle \left[\sqrt[3]{1 - (1 - T^L{}^\lambda)}, \sqrt[3]{1 - (1 - T^U{}^\lambda)} \right], [I^{L\lambda}, I^{U\lambda}], [F^{L\lambda}, F^{U\lambda}] \rangle$
- $A^\lambda = \langle [T^{L\lambda}, T^{U\lambda}], \left[\sqrt[3]{1 - (1 - I^L{}^\lambda)}, \sqrt[3]{1 - (1 - I^U{}^\lambda)} \right], \left[\sqrt[3]{1 - (1 - F^L{}^\lambda)}, \sqrt[3]{1 - (1 - F^U{}^\lambda)} \right] \rangle$

3. Fermatean Neutrosophic Shortest Path Algorithm

One of the prominent graph theory puzzles is the shortest path problem. The shortest path problem has been extensively examined with respect to almost every fuzzy structure in fuzzy graph theory. The novelty of the suggested method is in its capacity to deal with problems arising in interval-valued Fermatean neutrosophic numbers. The algorithm we employed is relatively simple to use and yields results much faster than other methodologies. This strategy can be applied to any type of neutrosophic structure. Whether in the context of machine learning, shipping, computerized systems, labs or manufacturing facilities, etc., this algorithmic rule can be used to meet the demand for shortest path explanations.

A technique for figuring out the shortest path between each node and its predecessor is suggested in this portion of the article. In practical applications, this approach can be employed to determine the shortest path in a network.

Step 1: Prioritize v_1 and v_n as the destination's first and last nodes, respectively.

Step 2: Considering that node 1 is not isolated from itself by any distance, let $d_1 = \langle [0,0], [1,1], [1,1] \rangle$ Additionally, add the label $(\langle [0,0], [1,1], [1,1] \rangle, -)$ to the first node.

Step 3: Find $d_j = \min\{d_i \oplus d_{ij}\}$. For $j = 2, 3 \dots n$. use the Score function for de-neutrosophication of IVFNS.

$$S_{Broumi}(x) = \frac{(T_A^L(x))^3 + (T_A^U(x))^3 + (I_A^L(x))^3 + (I_A^U(x))^3 + (F_A^L(x))^3 + (F_A^U(x))^3}{2}, \text{ where } score(A) \in [0,1].$$

Step 4: If a unique distance value is encountered at $i = r$. hence j is thus designated as $[d_j, r]$.

If there is no unique match between the distance measurements.

It indicates that there are several IVFNS pathways leading from a node.

Use the score feature of IVFNS to find the shortest path out of multiple options.

Step 5: Let the destination node be labeled as $[d_n, k]$. where d_n is the shortest displacement between initial and final node.

Step 6: Therefore, we check the label of node k to get the IVFN shortest path from the first to the last node. Let it be. Next, we evaluate node l 's label of node l , and so forth.

To obtain the initial node, repeat the steps above.

Step 7: Consequently, step 6 can be used to determine the IVFN shortest path.

4. Numerical Example

Presume a network of IVFNG shown in Figure 1. The shortest path is computed using the proposed technique in the approach shown below.

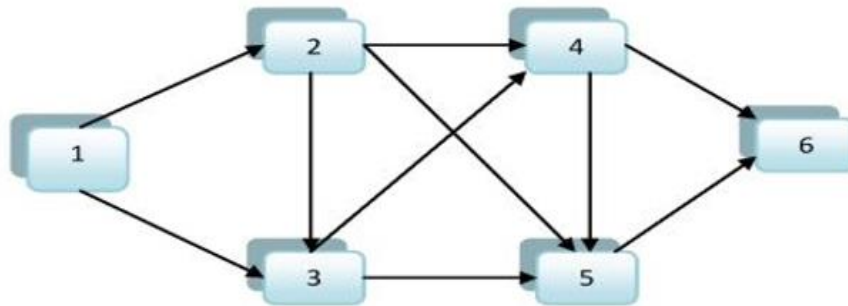


Figure 1. IVFN network.

In Table 1, IVFNG s is utilized to illustrate the path between each pair of nodes.

Table 1. Distance between the nodes in IVFN Network Edges.

Edges	Distance
(1, 2)	$\langle [0.4, 0.6], [0.1, 0.3] [0.2, 0.3] \rangle$
(1, 3)	$\langle [0.2, 0.7], [0.1, 0.5], [0.1, 0.3] \rangle$
(2, 3)	$\langle [0.1, 0.7], [0.2, 0.4], [0.3, 0.5] \rangle$
(2, 4)	$\langle [0.4, 0.5], [0.7, 0.8], [0.1, 0.2] \rangle$
(2, 5)	$\langle [0.5, 0.6], [0.5, 0.7], [0.3, 0.4] \rangle$
(3, 4)	$\langle [0.6, 0.7], [0.4, 0.6], [0.3, 0.5] \rangle$
(3, 5)	$\langle [0.6, 0.7], [0.3, 0.6], [0.2, 0.5] \rangle$
(4, 5)	$\langle [0.4, 0.7], [0.5, 0.8], [0.1, 0.6] \rangle$
(4, 6)	$\langle [0.3, 0.5], [0.3, 0.8], [0.1, 0.2] \rangle$
(5, 6)	$\langle [0.5, 0.8], [0.5, 0.6], [0.2, 0.4] \rangle$

In Table 1, IVFNG s is utilized to illustrate the path between each pair of nodes.

Now, utilizing the methodology described, we determine the shortest path as specified:

The destination node being 6, $n = 6$.

If you mark the source node as $(\langle [0,0], [1,1], [1,1] \rangle, -)$ (let's say node 1) and set $d_1 = \langle [0,0], [1,1], [1,1] \rangle$ to those coordinates, you can find d_j as follows.

Iteration 1: Since node 2 has only one predecessor, we set $i = 1$ and $j = 2$, which results in d_2 as

$$\begin{aligned}
 d_2 &= \min \{d_1 \oplus d_{12}\} \\
 &= \min (\langle [0,0], [1,1], [1,1] \rangle \oplus \langle [0.4,0.6], [0.1,0.3] [0.2,0.3] \rangle) \\
 &= \langle [0.4,0.6], [0.1,0.3], [0.2,0.3] \rangle
 \end{aligned}$$

When $i = 1$, the minimum value is attained. Thus, vertex 2 is labeled as

$$\langle [0.4,0.6], [0.1,0.3], [0.2,0.3] \rangle, -1$$

Iteration 2: Set $i = 1, 2$ and $j = 3$, since node 3's predecessors are 1 and 2.

$$\begin{aligned} d_3 &= \min \{d_1 \oplus d_{13}, d_2 \oplus d_{23}\} \\ &= \min \left\{ \begin{aligned} &\langle [0,0], [1,1], [1,1] \rangle \oplus \langle [0.2,0.7], [0.1,0.5], [0.1,0.3] \rangle, \\ &\langle [0.4,0.6], [0.1,0.3], [0.2,0.3] \rangle \oplus \langle [0.1,0.7], [0.2,0.4], [0.3,0.5] \rangle \end{aligned} \right\} \\ &= \min \left\{ \begin{aligned} &\langle [0.2,0.7], [0.1,0.5], [0.1,0.3] \rangle, \\ &\langle [0.0216, 0.1616], [0.02, 0.12], [0.06, 0.15] \rangle \end{aligned} \right\} \end{aligned}$$

Score function enables us to identify the absolute minimum:

$$\begin{aligned} S_{Broumi}(x) &= \frac{(T_A^L(x))^3 + (T_A^U(x))^3 + (I_A^L(x))^3 + (I_A^U(x))^3 + (F_A^L(x))^3 + (F_A^U(x))^3}{2} \\ S(\langle [0.2,0.7], [0.1,0.5], [0.1,0.3] \rangle) &= 0.2525, \text{ and} \\ S(\langle [0.0216, 0.1616], [0.02, 0.12], [0.06, 0.15] \rangle) &= 0.048 \\ \text{So, the } d_3 &= \langle [0.1,0.7], [0.2,0.4], [0.3,0.5] \rangle \end{aligned}$$

When $i = 2$, the minimum value is attained. Thus, vertex 3 is labeled as $\langle [0.1,0.7], [0.2,0.4], [0.3,0.5] \rangle, 2$.

Iteration 3: Set $i = 2, 3$, and $j = 4$, since node 4's predecessors are 2 and 3.

$$\begin{aligned} d_4 &= \min \{d_2 \oplus d_{24}, d_3 \oplus d_{34}\} \\ &= \min \left\{ \begin{aligned} &\langle [0.4,0.6], [0.1,0.3], [0.2,0.3] \rangle \oplus \langle [0.4,0.5], [0.7,0.8], [0.1,0.2] \rangle, \\ &\langle [0.1,0.7], [0.2,0.4], [0.3,0.5] \rangle \oplus \langle [0.6,0.7], [0.4,0.6], [0.3,0.5] \rangle \end{aligned} \right\} \\ &= \min \left\{ \begin{aligned} &\langle [0.0413, 0.1047], [0.1146, 0.1751], [0.003, 0.0116] \rangle, \\ &\langle [0.0723, 0.1895], [0.0238, 0.887], [0.018, 0.0781] \rangle \end{aligned} \right\} \end{aligned}$$

Score function enables us to identify the absolute minimum:

$$\begin{aligned} S(\langle [0.0413, 0.1047], [0.1146, 0.1751], [0.003, 0.0116] \rangle) &= 0.004, \text{ and} \\ S(\langle [0.0723, 0.1895], [0.0238, 0.887], [0.018, 0.0781] \rangle) &= 0.0042 \\ \text{Hence } d_4 &= \langle [0.0413, 0.1047], [0.1146, 0.1751], [0.003, 0.0116] \rangle \end{aligned}$$

When $i = 2$, the minimum value is attained. Thus, vertex 4 is labeled as $\langle [0.0413, 0.1047], [0.1146, 0.1751], [0.003, 0.0116] \rangle, 2$.

Iteration 4: Set $i = 2, 3, 4$ and $j = 5$, since node 5's predecessors are 2, 3 and 4.

$$\begin{aligned} d_5 &= \min \{d_2 \oplus d_{25}, d_3 \oplus d_{35}, d_4 \oplus d_{45}\} \\ &= \min \left\{ \begin{aligned} &\langle [0.4,0.6], [0.1,0.3], [0.2,0.3] \rangle \oplus \langle [0.5,0.6], [0.5,0.7], [0.3,0.4] \rangle, \\ &\langle [0.1,0.7], [0.2,0.4], [0.3,0.5] \rangle \oplus \langle [0.6,0.7], [0.3,0.6], [0.2,0.5] \rangle, \\ &\langle [0.0413, 0.1047], [0.1146, 0.1751], [0.003, 0.0116] \rangle \oplus \langle [0.4,0.7], [0.5,0.8], [0.1,0.6] \rangle \end{aligned} \right\} \\ &= \min \left\{ \begin{aligned} &\langle [0.0603, 0.1284], [0.042, 0.1202], [0.012, 0.0298] \rangle, \\ &\langle [0.0723, 0.1895], [0.0116, 0.0887], [0.012, 0.0781] \rangle, \\ &\langle [0.0214, 0.1146], [0.0421, 0.1715], [0.0003, 0.072] \rangle \end{aligned} \right\} \end{aligned}$$

Score function enables us to identify the absolute minimum:

$$\begin{aligned} S(\langle [0.0603, 0.1284], [0.042, 0.1202], [0.012, 0.0298] \rangle) &= 0.0021 \\ S(\langle [0.0723, 0.1895], [0.0116, 0.0887], [0.012, 0.0781] \rangle) &= 0.0042, \text{ and} \\ S(\langle [0.0214, 0.1146], [0.0421, 0.1715], [0.0003, 0.072] \rangle) &= 0.0035 \end{aligned}$$

So, the $d_5 \langle [0.0603, 0.1284], [0.042, 0.1202], [0.012, 0.0298] \rangle$

When $i = 2$, the minimum value is attained. Thus, vertex 5 is labeled as

$$\langle [0.0603, 0.1284], [0.042, 0.1202], [0.012, 0.0298] \rangle, 2].$$

Iteration 5: Set $i = 4, 5$ and $j = 6$, since node 6's predecessors are 4 and 5.

$$d_6 = \min \{ d_4 \oplus d_{46}, d_5 \oplus d_{56} \}$$

$$= \min \left\{ \langle [0.0413, 0.1047], [0.1146, 0.1751], [0.003, 0.0116] \rangle \oplus \langle [0.3, 0.5], [0.3, 0.8], [0.1, 0.2] \rangle, \right. \\ \left. \langle [0.0603, 0.1284], [0.042, 0.1202], [0.012, 0.0298] \rangle \oplus \langle [0.5, 0.8], [0.5, 0.6], [0.2, 0.4] \rangle \right\}$$

$$= \min \left\{ \langle [0.009, 0.042], [0.0095, 0.1715], [0.0003, 0.0027] \rangle, \right. \\ \left. \langle [0.0417, 0.171], [0.0417, 0.0725], [0.0027, 0.0213] \rangle \right\}$$

Score function enables us to identify the absolute minimum:

$$S(\langle [0.009, 0.042], [0.0095, 0.1715], [0.0003, 0.0027] \rangle) = 0.0026, \text{ and}$$

$$S(\langle [0.0417, 0.171], [0.0417, 0.0725], [0.0027, 0.0213] \rangle) = 0.0028$$

So, the $d_6 = \langle [0.009, 0.042], [0.0095, 0.1715], [0.0003, 0.0027] \rangle$

When $i = 4$, the minimum value is attained.

Thus, vertex 6 is labeled as $\langle [0.009, 0.042], [0.0095, 0.1715], [0.0003, 0.0027] \rangle, 4]$.

Since d_6 is the final destination. So, the shortest displacement is specified as proceeding from vertex one to six.

$$\langle [0.009, 0.042], [0.0095, 0.1715], [0.0003, 0.0027] \rangle$$

The shortest way can be determined as follows:

Node 6 is labeled $\langle [0.009, 0.042], [0.0095, 0.1715], [0.0003, 0.0027] \rangle, 4]$.

Node 5 is labeled as $\langle [0.0603, 0.1284], [0.042, 0.1202], [0.012, 0.0298] \rangle, 2]$.

Node 4 is labeled as $\langle [0.0413, 0.1047], [0.1146, 0.1751], [0.003, 0.0116] \rangle, 2]$.

Node 3 is labeled as $\langle [0.1, 0.7], [0.2, 0.4], [0.3, 0.5] \rangle, 2]$.

Consequently, the shortest route is $1 \rightarrow 2 \rightarrow 4 \rightarrow 6$ with the IVFN value of distance being $\langle [0.009, 0.042], [0.0095, 0.1715], [0.0003, 0.0027] \rangle$

The shortest path is depicted in Figure 2 by the dotted line, and the paths of various nodes are shown in Table 2.

Table 2. Shortest Path of the above network.

Nodes No.(j)	d_i	Shortest path from 1st node to j^{th} node
2	$\langle [0.4, 0.6], [0.1, 0.3], [0.2, 0.3] \rangle$	$1 \rightarrow 2$
3	$\langle [0.1, 0.7], [0.2, 0.4], [0.3, 0.5] \rangle$	$1 \rightarrow 3$
4	$\langle [0.0413, 0.1047], [0.1146, 0.1751], [0.003, 0.0116] \rangle$	$1 \rightarrow 2 \rightarrow 4$
5	$\langle [0.0603, 0.1284], [0.042, 0.1202], [0.012, 0.0298] \rangle$	$1 \rightarrow 2 \rightarrow 5$
6	$\langle [0.009, 0.042], [0.0095, 0.1715], [0.0003, 0.0027] \rangle, 4]$	$1 \rightarrow 2 \rightarrow 4 \rightarrow 6$

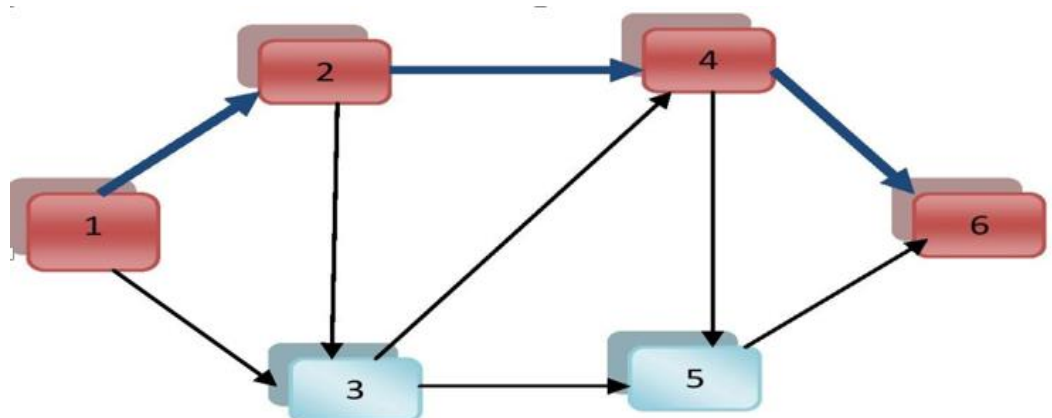


Figure 2. Shortest path IVFN network

5. Conclusions

The paper explores the idea of an Interval-Valued Fermatean Neutrosophic graph. The shortest path of an IVFNG has been determined via an algorithm. The suggested approach is employed to identify the network's shortest path across all possible paths in a numerical example. This research will be highly helpful to researchers who want to provide fresh approaches to the shortest path problem. New frameworks and algorithms will be created in the future to determine the best path for a specific network in various fixed contexts under various neutrosophic environments utilizing the findings of the present study.

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Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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