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3-Dimensional Quartic Bézier Curve Approximation Model by Using Neutrosophic Approach

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Abstract: In a 3-dimensional data collection process, there exists noise data that cannot be included to visualize the process. Therefore, it is difficult to deal with since fuzzy set and intuitionistic fuzzy set theories did not consider the indeterminacy problem. However, using a neutrosophic approach with three memberships: truth, false, and indeterminacy membership function, the error data will be treated as uncertain data by using the indeterminacy degree. Thus, this study will visualize the 3-dimensional quartic Bézier curve model by using neutrosophic set theory. To construct the model, the neutrosophic quartic control point must first be introduced to approximate the neutrosophic quartic Bézier curve. Next, the Bernstein basis function as the methodology of this study will be blended with the neutrosophic fundamental notion. At the end of this paper, a numerical example of the 3-dimensional neutrosophic quartic Bézier curve will be visualized by using the approximation method as the finding of this study. Finally, this study provides significant contributions to making it easier for data collectors to visualize all data, which means that no data will be eliminated since the uncertainty data will also be used.

Keywords: Neutrosophic Set Theory; Bézier Geometric Modelling; Approximation Method; 3-Dimensional Quartic Modelling.

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1. Introduction

Coping with insufficient 3-dimensional data collection is a significant challenge in a variety of areas, including finance, engineering, and research. Statistical inference, Bayesian analysis, fuzzy logic, and neural networks have all been created to deal with uncertainty. These solutions enable the representation of uncertainty in data and can improve the accuracy and dependability of data-driven decisions, then visualize it in 3-dimensional axes. The fuzzy set (FS) model is a theoretical paradigm for dealing with data imprecision and ambiguity. It was created in the 1960s by Lotfi Zadeh to address the inherent ambiguity and vagueness of natural language and human intellect [1]. As a result, FS just analyzes truth and false membership data and ignores the inconsistent data. In 1986, Krassimir Atanassov [2] developed intuitionistic fuzzy set (IFS) theory, which is a generalization of FS that includes true, false, and uncertain information. It is excellent for dealing with ambiguity. Since FS theory only considers whole membership data, the IFS notion is an alternative method for establishing FS when the amount of information recorded is insufficient to classify and process. However, when approaching an advanced problem with intuitive and fuzzy components, it is difficult to deal with, and it is rarely handled in the framework of spline modeling. [3], [4], [5], [6], [7], [8], [9], [10], and [11] contain studies involving fuzzy and intuitionistic fuzzy set theory and spline modeling. As a result, there is a gap in this study since the previous study's limitations include IFS and FS, which cannot deal with the more sophisticated problem. It is also the motivation of this study to use neutrosophic set theory and blend it with the Bernstein basis function to visualize it in 3-dimensional form.

The neutrosophic technique was devised by Florentin Smarandache [12] as a mathematical application of the concept of neutrality that works with uncertain data. The neutrosophic set (NS) idea is defined by membership degrees, non-membership, and indeterminacy. In this sense, an NS refers to the resolution and representation of problems that cover numerous domains. Since true, false, and indeterminate membership degrees are independent in NS theory, an element can have any value at the same time. This enables the modeling of more complex forms of uncertainty and indeterminacy, such as when a statement can be both true and false at the same time. Tas and Topal [13,14] have generated the Bézier curve and surface generally without focusing on the detail of the blending Bernstein function with NS theory, which was published in the top journal namely Neutrosophic Set System (NSS) journal. However, their research does not properly demonstrate the process of blending neutrosophic theory and Bernstein basis function, and the visualization does not clearly show the control points approximate the curve and surface. Meanwhile, Rosli and Zulkifly [15] discuss in detail the application of the B-spline curve interpolation. They also visualize a neutrosophic bicubic B-spline surface interpolation model for uncertainty data [16]. Therefore, this study will visualize the neutrosophic Bézier curve for the quartic version in 3-dimensional by showing the neutrosophic control point approximating the neutrosophic Bézier curve by using a numerical example. This research can contribute to helping data analysts model their data in spline form without wasting any noisy data. For example, in the real case of bathymetry data, there will be uncertainty due to the wave of a lake that the data was collected from by using an echo-sounder on a ship. When the ship's position changes due to a wave, there will be noise data that cannot be determined whether it is the result or not, which means it is true or false, so it will be treated as an indeterminacy degree in this study.

This paper will focus on the 3-dimensional neutrosophic quartic Bézier curve (NQBC) approximation model. The first section of this paper discusses the background of this research and some literature reviews. To visualize the NQBC model, the next section will focus on neutrosophic control point relation (NCPR) that needs to be introduced first by using some properties of the NS such as the fundamental notion NS, neutrosophic relation (NR), neutrosophic point (NP), and neutrosophic point relation (NPR). The third section discusses the blending Bernstein function with NCPR. The fourth section of this paper will visualize the application of the NQBC approximation model by using a numerical example and the algorithm of NQBC. At the end of this paper is the summarization of all sections of this study.

2. Preliminaries

This section discusses the NS, including the core concepts of NS, NR, and NP, and will define the NCP. Smarandache emphasizes that the intuitionistic set in fuzzy systems can handle limited data but not paraconsistent data [12]. "There are three memberships: a truth membership function, *T*, an indeterminacy membership function, *I*, and a falsity membership function, *F*, with the parameter 'indeterminacy' added by the NS specification" [12].

Definition 1:[12]Let Y be the main of conversation, with element in Y denoted as y . The neutrosophic set is an object in the form.

$$\hat{A}=\{\left\langle y:T_{\hat{A}(y)},I_{\hat{A}(y)},F_{\hat{A}(y)}\right\rangle \mid y\in Y\} \tag{1}$$

where the functions $T,I,F:Y\to]^{-}0,1^{+}[$ define, respectively, the degree of truth membership, the degree of indeterminacy, and the degree of false membership of the element $y\in Y$ to the set $\hat{A}$ with the condition;

$$0^{-} \leq T_{\hat{A}}(y) + I_{\hat{A}}(y) + F_{\hat{A}}(y) \leq 3^{+} \tag{2}$$

There is no limit to the amount of $T_{\hat{A}}(y), I_{\hat{A}}(y)$ and $F_{\hat{A}}(y)$

A value is chosen by NS from one of the real standard subsets or one of the non-standard subsets of $]^{-}0,1^{+}[$. The actual value of the interval $[0,1]$, on the other hand, $]^{-}0,1^{+}[$ will be utilized in technical applications since its utilization in real data, such as the resolution of scientific challenges, will be physically impossible. As a direct consequence of this, membership value utilization is increased.

$$\hat{A} = \{\left\langle y : T_{\hat{A}(y)}, I_{\hat{A}(y)}, F_{\hat{A}(y)} \right\rangle \mid y \in Y\} \text{ and } T_{\hat{A}}(y), I_{\hat{A}}(y), F_{\hat{A}}(y) \in [0,1] \tag{3}$$

There is no restriction on the sum of $T_{\hat{A}}(y), I_{\hat{A}}(y), F_{\hat{A}}(y)$ .Therefore,

$$0 \leq T_{\hat{A}}(y) + I_{\hat{A}}(y) + F_{\hat{A}}(y) \leq 3 \tag{4}$$

Definition 2: [13, 14] Let $\hat{B} = \{\left\langle y : T_{\hat{B}(y)}, I_{\hat{B}(y)}, F_{\hat{B}(y)} \right\rangle \mid y \in Y\}$ and $\hat{C} = \{\left\langle z : T_{\hat{C}(z)}, I_{\hat{C}(z)}, F_{\hat{C}(z)} \right\rangle \mid z \in Z\}$ be neutrosophic elements. Thus, $NR = \{\left\langle (y,z) : T_{(y,z)}, I_{(y,z)}, F_{(y,z)} \right\rangle \mid y \in \hat{B}, z \in \hat{C}\}$ is a neutrosophic relation (NR) on $\hat{B}$ and $\hat{C}$.

Definition 3: [13, 14] Neutrosophic set of $\hat{B}$ in space Y is neutrosophic point (NP) and $\hat{B} = \{\hat{B}_i\}$ where $i = 0, \ldots, n$ is a set of NPs where there exists $T_{\hat{B}} : Y \to [0,1]$ as truth membership, $I_{\hat{B}} : Y \to [0,1]$ as indeterminacy membership, and $F_{\hat{B}} : \hat{Y} \to [0,1]$ as false membership with

$$T_{\hat{B}}(\hat{B}) = \begin{cases} 0 & \text{if } \hat{B}_i \notin \hat{B} \\ a \in (0,1) & \text{if } \hat{B}_i \,\hat{\in}\, \hat{B} \\ 1 & \text{if } \hat{B}_i \in \hat{B} \end{cases}$$

$$I_{\hat{B}}(\hat{B}) = \begin{cases} 0 & \text{if } \hat{B}_i \notin \hat{B} \\ b \in (0,1) & \text{if } \hat{B}_i \,\hat{\in}\, \hat{B} \\ 1 & \text{if } \hat{B}_i \in \hat{B} \end{cases} \tag{5}$$

$$F_{\hat{B}}(\hat{B}) = \begin{cases} 0 & \text{if } \hat{B}_i \notin \hat{B} \\ c \in (0,1) & \text{if } \hat{B}_i \,\hat{\in}\, \hat{B} \\ 1 & \text{if } \hat{B}_i \in \hat{B} \end{cases}$$

2.1 Neutrosophic Point Relation

The previous section's discussion of the NS notion, NP, and NR will be used to create the foundation for neutrosophic point relation (NPR). It is a group of Euclid eternal space points. Rosli and Zulkifly [15] describe NPR as follows:

Definition 4. [15] Let N, M be a grouping of elements in global area that are part of a set that is not null and $N, M, O \in \mathbf{R} \times \mathbf{R} \times \mathbf{R}$, then the term "NPR" refers to

$$\hat{R} = \left\{ \begin{array}{l} \left\langle \left(n_i, m_j\right), T_R(n_i, m_j), I_R(n_i, m_j), F_R(n_i, m_j) \right\rangle \\ \mid T_R(n_i, m_j), I_R(n_i, m_j), F_R(n_i, m_j) \in I \end{array} \right\} \tag{6}$$

where $\left(n_i, m_j\right)$ is a set of ordered positions and $\left(n_i, m_j\right) \in N \times M$, while $T_R(n_i, m_j), I_R(n_i, m_j), F_R(n_i, m_j)$ are the truth membership, the indeterminacy membership, and the false membership that follows the condition of the neutrosophic set which is, respectively, $0 \le T_{\hat{N}}(z) + I_{\hat{N}}(z) + F_{\hat{N}}(z) \le 3$.

2.2 *Neutrosophic Control Point Relation*

In computer graphics and mathematical modelling, a control point (CP) is a single point or set of points that impact the shape or behaviour of a curve, surface, or another geometric object. Non-uniform rational B-splines modelling (NURBS), B-splines, and Bézier curves are all examples of techniques that use CPs. The location and properties of the CPs define the geometric object's qualities and deformation. Aside from manipulating data, the form, curvature, and other features of the curve or surface can be changed. The CPs in this work are a group of points used to define the contours of a neutrosophic Bézier curve. It is also critical in geometric modelling for the derivation and fabrication of smooth curves. In this part, the idea of NS and its properties are used to define NCP. The FS idea is utilized to define fuzzy control points based on research in [17, 18]. Therefore, the NCPR for NQBC was introduced based on the idea from Rosli and Zulkifly [15] as follows:

Definition 5: Let $\hat{R}$ be an NPR, then NCPR is viewed as a group of points $n+1$ that denotes a locations and coordinates and is used to describe the curve and is indicated by

$$\begin{aligned} \hat{P}_i^T &= \left\{ \hat{p}_0^T, \hat{p}_1^T, \ldots, \hat{p}_n^T \right\} \\ \hat{P}_i^I &= \left\{ \hat{p}_0^I, \hat{p}_1^I, \ldots, \hat{p}_n^I \right\} \\ \hat{P}_i^F &= \left\{ \hat{p}_0^F, \hat{p}_1^F, \ldots, \hat{p}_n^F \right\} \end{aligned} \tag{7}$$

where $\hat{P}_i^T$, $\hat{P}_i^I$ and $\hat{P}_i^F$ are NCP for truth, false, and indeterminacy membership function and i is one less than n.

Since this study concentrates on **quartic case**, therefore, the $n = 4$ to create the NQBC. Thus, the NCPR is as follows:

$$\begin{aligned} \hat{P}_i^T &= \left\{ \hat{p}_0^T, \hat{p}_1^T, \hat{p}_2^T, \hat{p}_3^T, \hat{p}_4^T \right\} \\ \hat{P}_i^I &= \left\{ \hat{p}_0^I, \hat{p}_1^I, \hat{p}_2^I, \hat{p}_3^I, \hat{p}_4^I \right\} \\ \hat{P}_i^F &= \left\{ \hat{p}_0^F, \hat{p}_1^F, \hat{p}_2^F, \hat{p}_3^F, \hat{p}_4^F \right\} \end{aligned} \tag{8}$$

3. Approximation of Neutrosophic Quartic Bézier curve

Geometric simulation frequently employs Bézier curves, which are parameterized curves guided by a control polygon [19, 20]. The number of data points utilized to construct the curve corresponds to the degree of the polynomial [21]. The following definition illustrates a Bézier curve created by integrating the Bernstein polynomial or basis function using NCPR. NCPR and *Definition 1* are used to build the NQBC, which is then combined in a geometric model with the Bézier blending function. The NQBC model's properties are then discussed. The notion of the Bézier curve for approximation method comes from Piegl and Tiller [22] and is then blended with NCPR as follows:

Definition 3: Let $\hat{P}_i^T = \{\hat{p}_0^T, \hat{p}_1^T, \hat{p}_2^T, \hat{p}_3^T, \hat{p}_4^T\}, \hat{P}_i^I = \{\hat{p}_0^I, \hat{p}_1^I, \hat{p}_2^I, \hat{p}_3^I, \hat{p}_4^I\}$, and $\hat{P}_i^F = \{\hat{p}_0^F, \hat{p}_1^F, \hat{p}_2^F, \hat{p}_3^F, \hat{p}_4^F\}$ where $i = 0,1,2,3,4$ is NCPR. NQBC is defined as $BC(t)$ with the curve position vector depending on the value of the value t, then blending with J_i by Bézier curves and represented as follows:

$$
\begin{aligned}
BC(t)^T &= \sum_{i=0}^{4} \hat{P}_i^T J_{4,i}(t) \\
BC(t)^I &= \sum_{i=0}^{n} \hat{P}_i^I J_{4,i}(t) \\
BC(t)^F &= \sum_{i=0}^{n} \hat{P}_i^F J_{4,i}(t)
\end{aligned}
\tag{9}
$$

where $0 \le t \le 1$ and the blending function is a Bézier or Bernstein basis, J_i:

$$J_{(4,i)}(t) = \binom{4}{i} t^i (1-t)^{4-i} \qquad (0)^0 \equiv 1 \tag{10}$$

with

$$\binom{4}{i} = \frac{4!}{i!(4-1)!} \qquad (0)^0 \equiv 1 \tag{11}$$

Based on the approach by Zaidi and Zulkifly [23], the NQBC equation can also be stated in matrix multiplication. By extending the analytic formulation of the curve into its Bernstein polynomial coefficients and then expressing these coefficients using the polynomial power basis [23], NQBC can be represented as a matrix, as illustrated roughly below:

$$BC(t) = [J][P] \tag{12}$$

where;

$$[J] = [J_{4,0}, J_{4,1}, J_{4,2}, J_{4,3}, J_{4,4}] \tag{13}$$

$$[P]^T = [P_0, P_1, P_2, P_3, P_4] \tag{14}$$

3.1. Properties of Neutrosophic Quartic Bézier curve

A Bézier curve is a specific case that is determined by a control polygon in the context of NURBS curves. Since the Bézier basis is the same as the Bernstein basis, certain properties of Bézier curves are easily recognized. As a result, the NQBC has the following fundamental characteristics from the idea of fundamental Bézier basis features by Piegl and Tiller [22].

- The NQBC's fundamental features are genuine.
- There are less control polygon points than the degree of the polynomial defining the curve segment.
- In most cases, the NQBC will conform to the outline of the control polygon.
- The NQBC's starting and ending positions also happen to be the start and end points of the control polygon.
- The initial and last polygon spans correspond in the direction of the tangent vectors at the ends of the NQBC.

- The NQBC is located inside the largest convex polygon specified by the vertices of the control polygon, also known as the convex hull of the control polygon.
- The NQBC displays the phenomenon of declining variance. This means that the curve does not sway more frequently than the control polygon does around any given straight line.
- Affine transformations have no effect on the NQBC.

4. Visualization of 3-Dimensional Neutrosophic Quartic Bézier curve

In this section, the 3-dimensional NQBC approximation model for truth, false, and indeterminacy will be visualized. **Table 1** shows the NCPR for each membership. All values of NCPR follow the condition of NS, which is $0 \le T_{\hat{B}}(y) + I_{\hat{B}}(y) + F_{\hat{B}}(y) \le 3$.

Table 1. NCPR with its degrees.

NCPR, $\hat{P}_i$	Truth Degree, $\hat{P}_i^T$	False Degree, $\hat{P}_i^F$	Indeterminacy Degree, $\hat{P}_i^I$
$\hat{P}_0 = (3,3)$	0.7	0.4	0.2
$\hat{P}_1 = (8,9)$	0.6	0.4	0.3
$\hat{P}_2 = (12,14)$	0.8	0.3	0.2
$\hat{P}_4 = (18,19)$	0.6	0.2	0.5
$\hat{P}_5 = (26,24)$	0.3	0.4	0.6

Figure 1, Figure 2, and Figure 3 show the 3-dimensional neutrosophic quartic Bézier curves for truth, false, and indeterminacy membership, respectively. The red dot denotes the neutrosophic control point relation, and the yellow dash line denotes the control polygon for NCPR. Figure 4 and Figure 5 visualize the 3-dimensional NQBC for all memberships on one axis and different views of NQBC.

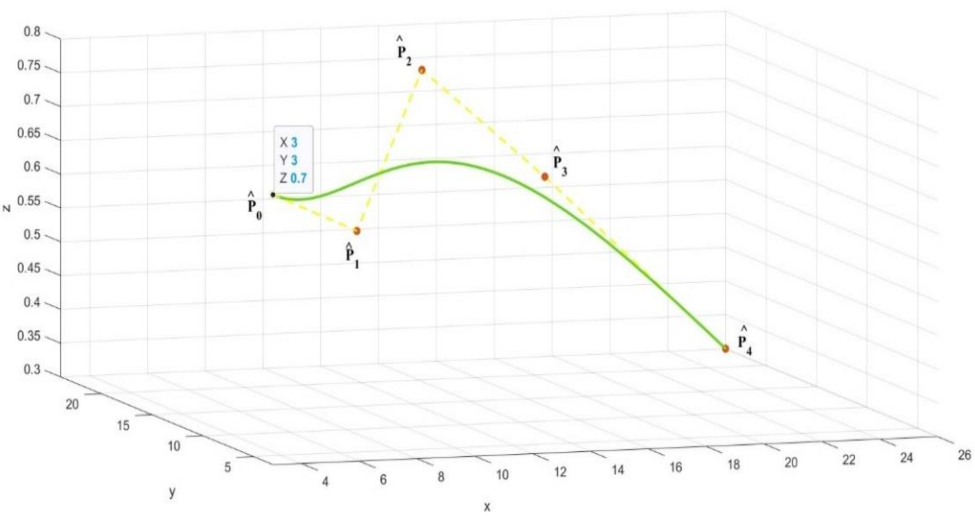

Figure 1. 3-dimensional NQBC for Truth Membership

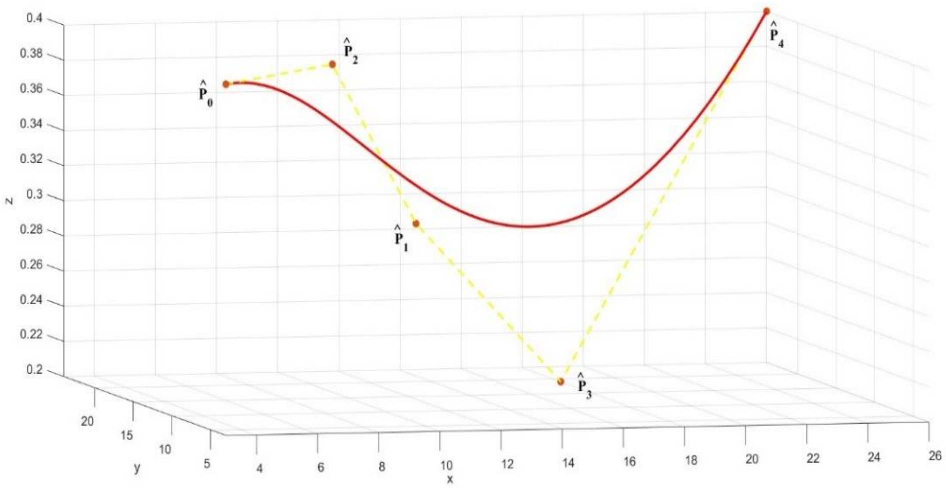

Figure 2. 3-dimensional NQBC for False Membership

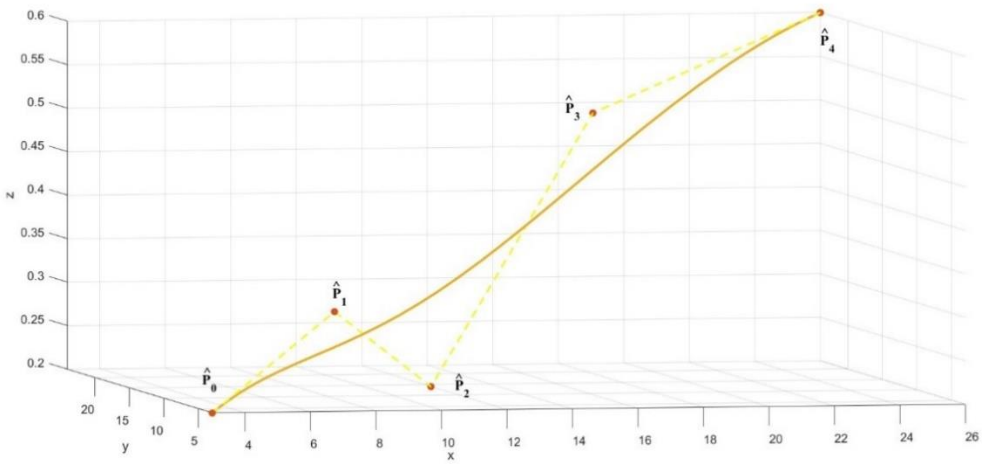

Figure 3. 3-dimensional NQBC for Indeterminacy Membership

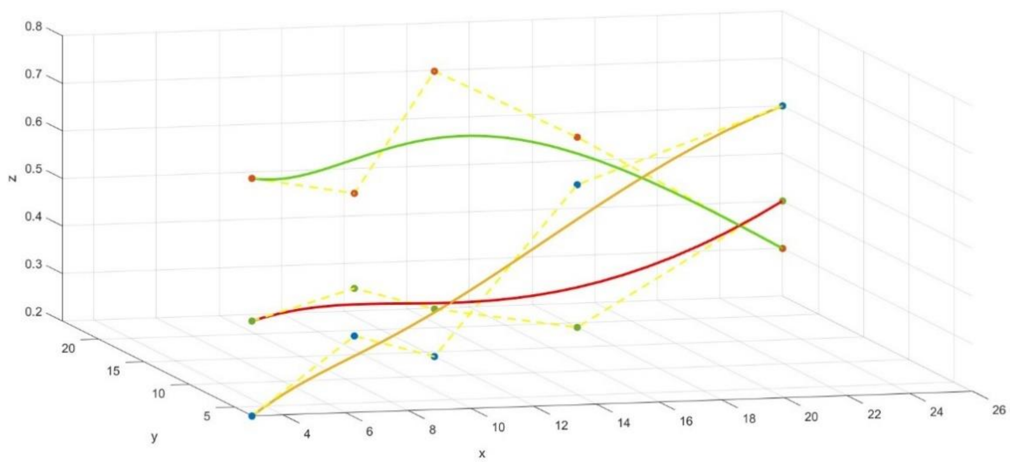

Figure 4. 3-dimensional NQBC for All Membership

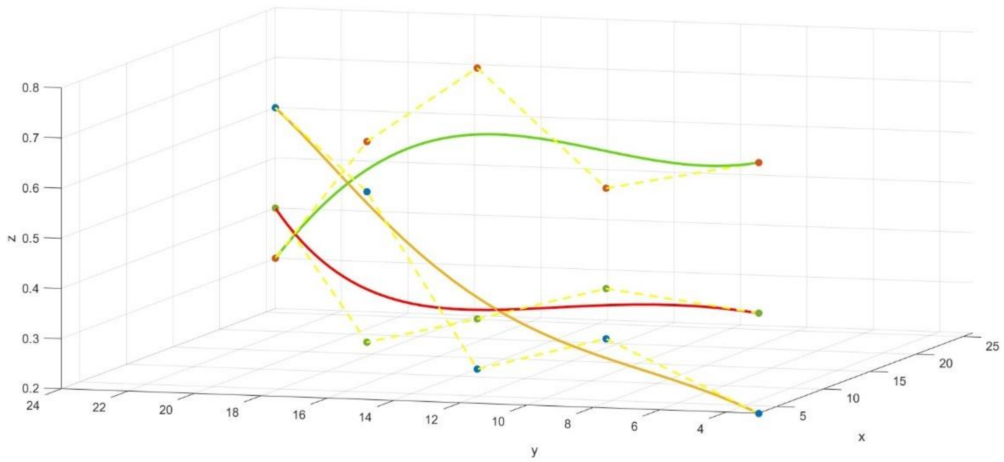

Figure 5. 3-dimensional NQBC in Different View

Based on the results for Figures 1 and 2, which are the truth and falsity membership 3-dimensional NQBC, it can be seen that the falsity membership NQBC is in the opposite direction to the truth membership, while the indeterminacy of NQBC in Figure 4 shows that it does not influence either of them but is clearly in the middle of the truth and falsity of the 3-dimensional NQBCs. This finding also demonstrates the characteristics of NS theory, wherein all memberships are considered to be independent and not influenced by one another. Nevertheless, in order for this study to be sensitive, it is necessary that all degrees comply to the condition of NS, which stands for $0 \le T_{\hat{B}}(y) + I_{\hat{B}}(y) + F_{\hat{B}}(y) \le 3$. According to Table 1, the variables for this study are membership values for truth, falsity, and indeterminacy. Figure 6 shows the algorithm for NQBC construction, the flowchart of this study, and an illustration of the procedure in matrix form:

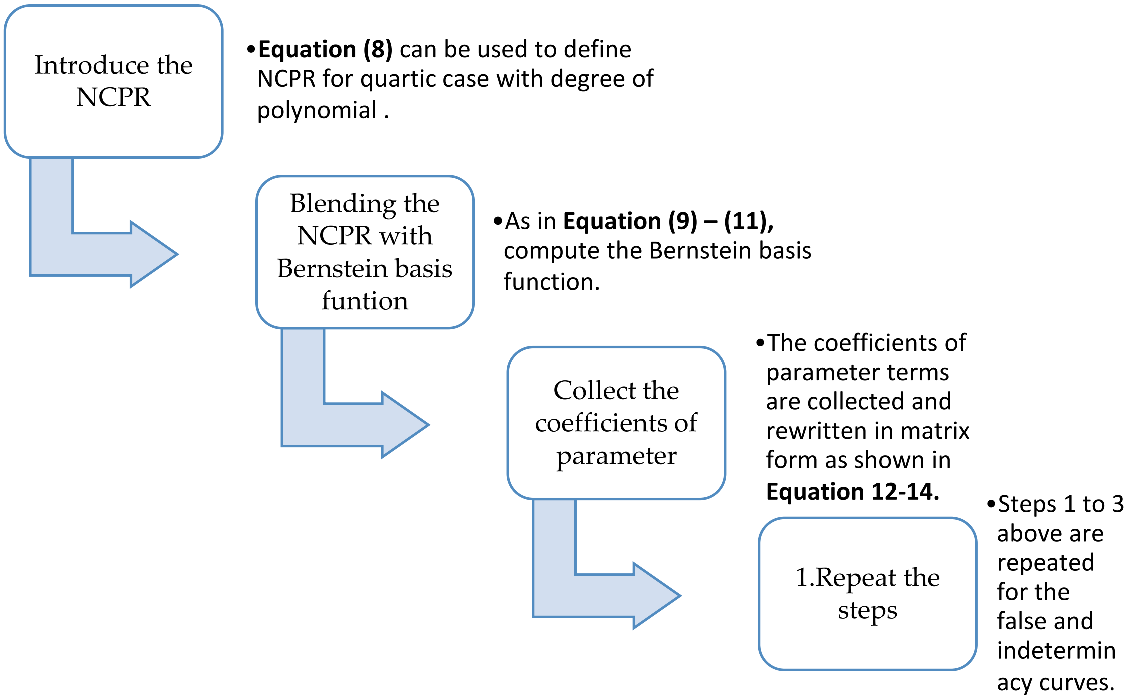

Figure 6. Flowchart of NQBC Construction

The novelties of this study are as follows:

- The NCPR for NQBC was introduced.
- The mathematical representation and properties for NQBC were analyzed and determined.
- The visualization of NQBC and the algorithm for constructing it were presented.

5. Conclusions

In this study, through NCPR, neutrosophic quartic Bézier curves approximation was introduced. Since it has a truth membership function, a false membership function, and an indeterminacy membership function, the NQBC model approximation is an excellent strategy for modeling data with neutrosophic properties. One of the key contributions of this study is that it has demonstrated that all data can be analyzed and processed using these functions. Besides that, the advantage of this model is its capacity to represent 3-dimensional neutrosophic data in the form of a Bézier curve, which is simple for data analysts to interpret and evaluate. Based on Figure 1–5, the neutrosophic data problem can be handled using the NQBC model. However, the limitation of this model is that it uses an approximation method, which simply approximates the curve by using the data, as compared to an interpolation method, which interpolates the curve using the given data. Therefore, this model can also be extended to tackle the neutrosophic data problem by using an interpolation approach on the surface that will be more accurate and precise. Besides that, future studies can also employ the quartic version by using B-spline and NURBS modeling.

Nomenclature:

Abbreviations and the variables

FS – Fuzzy Set

IFS – Intuitionistic Fuzzy Set

NS – Neutrosophic Set, $\hat{A}$

NP – Neutrosophic Point, $\hat{B}_i$

NR – Neutrosophic Relation, NR

NPR – Neutrosophic Point Relation, $\hat{R}$

NCP – Neutrosophic Control Point, $\hat{P}_i$

NCPR – Neutrosophic Control Points Relation, $\left\{\hat{p}_0^{T,I,F}, \hat{p}_1^{T,I,F}, ..., \hat{p}_n^{T,I,F}\right\}$

NQBC – Neutrosophic Quartic Bézier Curve, $BC(t)$

Variables

T - Truth Membership Degree

I - Indeterminacy Membership Degree

F - Falsity Membership Degree

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Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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