



Some Types of Neutrosophic Filters in Basic Logic Algebras

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Abstract: The purpose of this article is to study the neutrosophication of prime and Boolean filters in Basic Logic (BL) algebras. We establish the notions of the neutrosophic prime and Boolean filters of BL-algebras with suitable examples and examine a few of their properties. As a result, we can determine the necessary and sufficient conditions and extension properties of both the neutrosophic prime and Boolean filters of BL-algebras. We obtain, C is a neutrosophic prime filter if and only if $T_C(g_1 \rightarrow h_1) = T_C(1)$ or $T_C(h_1 \rightarrow g_1) = T_C(1)$, $I_C(g_1 \rightarrow h_1) = I_C(1)$ or $I_C(h_1 \rightarrow g_1) = I_C(1)$, $F_C(g_1 \rightarrow h_1) = F_C(1)$ or $F_C(h_1 \rightarrow g_1) = F_C(1)$. Also, we prove C_2 is a neutrosophic Boolean filter if $C_1 \subseteq C_2$ and $T_{C_1}(1) = T_{C_2}(1)$, $I_{C_1}(1) = I_{C_2}(1)$, $F_{C_1}(1) = F_{C_2}(1)$, where C_1 is a neutrosophic Boolean filter and C_2 is a neutrosophic filter. In addition, by combining both filters we instigate the concept of the neutrosophic prime Boolean filter of BL-algebras with illustration. In the future, the above study can be extended to soft multiset. Moreover, these filters can be applied to various digital image processing techniques.

Keywords: Basic Logic Algebra; Neutrosophic Filter; Neutrosophic Prime Filter; Neutrosophic Boolean Filter; Neutrosophic Prime Boolean Filter.

1. Introduction

The neutrosophic set was first introduced by Smarandache [1] in 1998, and its central idea is to explain the conception of 'uncertainty' using three mutually independent features. The neutrosophic set is now receiving a lot of attention for its potential to resolve a variety of real-world issues, including uncertainty and indeterminacy. Many novel neutrosophic theories [1, 2] such as the neutrosophic cubic, rough, and soft sets, are also put forth. The algebraic characteristics of the truth-value structure of each many-valued logic serve as a unique identifier [3]. A residuated lattice [4] is a common algebraic construction. The most well-known classes of residuated lattices include Basic Logic (BL), MTL, MV-algebras, and others.

A logical system's structure can be investigated by applying filters with special properties, as is well known. Additionally, there is a significant impact of filter qualities on the algebraic structure properties. The authors [5] introduced the concept of neutrosophic filters in BL-algebras and investigated a few of their associated features in a few instances. Further, the authors [6] discussed many of its properties and extended them to neutrosophic fantastic filters.

By using the prime filters of BL-algebras, Hajek [7] demonstrated the completeness of BL-algebras. In BL-algebras, Turunen [8] proposed the idea of Boolean filters. S. Yahya Mohamed and P.

Umamaheshwari [9] introduced the vague prime and Boolean filter of BL-algebras. However prime and Boolean filters in neutrosophic sets have not been studied so far. This motivated the authors to develop this article. In this article, we explore the ideas of neutrosophic prime and Boolean filters in BL-algebras and a few of their characteristics.

Our major contributions:

- In Section 2, a literature review of a few definitions and concepts regarding the neutrosophic set and filter of BL-algebras is conferred.
- In Section 3, we explore the idea of a neutrosophic prime filter and its features.
- In Section 4, we illustrate the idea of a neutrosophic Boolean and prime Boolean filters with examples.

2. Preliminaries

In this part, a few of the definitions and findings from the literature are referred to evolve the major conclusions.

Definition 2.1: [10, 11] A BL-algebra $(\mathcal{G}, \vee, \wedge, \circ, \rightarrow, 0, 1)$ of type $(2, 2, 2, 2, 0, 0)$ such that the subsequent requirements are persuaded for all $g_1, h_1, i_1 \in \mathcal{G}$,

- (i) $(\mathcal{G}, \vee, \wedge, 0, 1)$ is a bounded lattice,
- (ii) $(\mathcal{G}, \circ, 1)$ is a commutativemonoid,
- (iii) $' \circ '$ and $' \rightarrow '$ form an adjointpair, that is, $i_1 \leq g_1 \rightarrow h_1$ if and only if $g_1 \circ i_1 \leq h_1$
- (iv) $g_1 \wedge h_1 = g_1 \circ (g_1 \rightarrow h_1)$,
- (v) $(g_1 \rightarrow h_1) \vee (h_1 \rightarrow g_1) = 1$.

Proposition 2.2: [8, 10] the succeeding requirements are persuaded in BL-algebra \mathcal{G} for all $g_1, h_1, i_1 \in \mathcal{G}$,

- (i) $h_1 \rightarrow (g_1 \rightarrow i_1) = g_1 \rightarrow (h_1 \rightarrow i_1) = (g_1 \circ h_1) \rightarrow i_1$,
- (ii) $1 \rightarrow g_1 = g_1$,
- (iii) $g_1 \leq h_1$ if and only if $g_1 \rightarrow h_1 = 1$,
- (iv) $g_1 \vee h_1 = ((g_1 \rightarrow h_1) \rightarrow h_1) \wedge ((h_1 \rightarrow g_1) \rightarrow g_1)$,
- (v) $g_1 \leq h_1$ implies $h_1 \rightarrow i_1 \leq g_1 \rightarrow i_1$,
- (vi) $g_1 \leq h_1$ implies $i_1 \rightarrow g_1 \leq i_1 \rightarrow h_1$,
- (vii) $g_1 \rightarrow h_1 \leq (i_1 \rightarrow g_1) \rightarrow (i_1 \rightarrow h_1)$,
- (viii) $g_1 \rightarrow h_1 \leq (h_1 \rightarrow i_1) \rightarrow (g_1 \rightarrow i_1)$,
- (ix) $g_1 \leq (g_1 \rightarrow h_1) \rightarrow h_1$,
- (x) $g_1 \circ (g_1 \rightarrow h_1) = g_1 \wedge h_1$,
- (xi) $g_1 \circ h_1 \leq g_1 \wedge h_1$
- (xii) $g_1 \rightarrow h_1 \leq (g_1 \circ i_1) \rightarrow (h_1 \circ i_1)$,
- (xiii) $g_1 \circ (h_1 \rightarrow i_1) \leq h_1 \rightarrow (g_1 \circ i_1)$,
- (xiv) $(g_1 \rightarrow h_1) \circ (h_1 \rightarrow i_1) \leq g_1 \rightarrow i_1$,
- (xv) $(g_1 \circ g_1^*) = 0$.

Note. In the above sequence, \mathcal{G} is used to intend the BL-algebras, and the operations $' \vee ', ' \wedge ', ' \circ '$ have preference on the way to the operations $' \rightarrow '$.

Note. In a BL-algebra \mathcal{G} , $' * '$ is a complement defined as $g_1^* = g_1 \rightarrow 0$ for all $g_1 \in \mathcal{G}$.

Definition 2.3: [12, 13] A neutrosophic subset C of the universe U is a triple (T_C, I_C, F_C) where $T_C: U \rightarrow [0,1]$, $I_C: U \rightarrow [0,1]$ and $F_C: U \rightarrow [0,1]$ represents truth membership, indeterminacy and false membership functions, respectively where $0 \leq T_C(g_1) + I_C(g_1) + F_C(g_1) \leq 3$, for all $g_1 \in U$.

Definition 2.4: [5] A neutrosophic set C of a BL-algebra \mathcal{G} is called a neutrosophic filter, if it persuades the following:

- (i) $T_C(g_1) \leq T_C(1), I_C(g_1) \geq I_C(1)$ and $F_C(g_1) \geq F_C(1)$,
- (ii) $\min\{T_C(g_1 \rightarrow h_1), T_C(g_1)\} \leq T_C(h_1), \min\{I_C(g_1 \rightarrow h_1), I_C(g_1)\} \geq I_C(h_1)$ and $\min\{F_C(g_1 \rightarrow h_1), F_C(g_1)\} \geq F_C(h_1)$ for all $g_1, h_1 \in \mathcal{G}$.

Proposition 2.5: [5] Let C be a neutrosophic set of BL-algebras \mathcal{G} . C is a neutrosophic filter of \mathcal{G} if and only if

- (i) If $g_1 \leq h_1$ then $T_C(g_1) \leq T_C(h_1), I_C(g_1) \geq I_C(h_1)$ and $F_C(g_1) \geq F_C(h_1)$,
- (ii) $T_C(g_1 \circ h_1) \geq \min\{T_C(g_1), T_C(h_1)\}, I_C(g_1 \circ h_1) \leq \min\{I_C(g_1), I_C(h_1)\}$ and $F_C(g_1 \circ h_1) \leq \min\{F_C(g_1), F_C(h_1)\}$ for all $g_1, h_1 \in \mathcal{G}$.

Proposition 2.6: [5, 6] Let C be a neutrosophic set of BL-algebras \mathcal{G} . Let C be a neutrosophic filter of \mathcal{G} for all $g_1, h_1, i_1 \in \mathcal{G}$ then the following hold.

- (i) $T_C(g_1 \rightarrow h_1) = T_C(1)$, then $T_C(g_1) \leq T_C(h_1), I_C(g_1 \rightarrow h_1) = I_C(1)$, then $I_C(g_1) \geq I_C(h_1), F_C(g_1 \rightarrow h_1) = F_C(1)$, then $F_C(g_1) \geq F_C(h_1)$
- (ii) $T_C(g_1 \wedge h_1) = \min\{T_C(g_1), T_C(h_1)\}, I_C(g_1 \wedge h_1) = \min\{I_C(g_1), I_C(h_1)\}, F_C(g_1 \wedge h_1) = \min\{F_C(g_1), F_C(h_1)\}$
- (iii) $T_C(g_1 \circ h_1) = \min\{T_C(g_1), T_C(h_1)\}, I_C(g_1 \circ h_1) = \min\{I_C(g_1), I_C(h_1)\}, F_C(g_1 \circ h_1) = \min\{F_C(g_1), F_C(h_1)\}$
- (iv) $T_C(0) = \min\{T_C(g_1), T_C(g_1^*)\}, I_C(0) = \min\{I_C(g_1), I_C(g_1^*)\}, F_C(0) = \min\{F_C(g_1), F_C(g_1^*)\}$.

3. Neutrosophic Prime filter

In this segment, we put forward the concept of a neutrosophic prime filter and confer its features with illustrations.

Definition 3.1 Let C be a non-constant neutrosophic filter of a BL-algebra \mathcal{G} . C is called a neutrosophic prime filter, if $T_C(g_1 \vee h_1) \leq \min\{T_C(g_1), T_C(h_1)\},$

$$I_C(g_1 \vee h_1) \geq \min\{I_C(g_1), I_C(h_1)\},$$

$$F_C(g_1 \vee h_1) \geq \min\{F_C(g_1), F_C(h_1)\} \text{ for all } g_1, h_1 \in \mathcal{G}.$$

Example 3.2: Let $C = \{0, g_1, h_1, i_1, j_1, 1\}$. The binary operations are specified by Tables 1 and 2.

Table 1. '°' Operation.

°	0	g_1	h_1	i_1	j_1	1
0	0	0	0	0	0	0
g_1	0	h_1	h_1	j_1	0	g_1
h_1	0	h_1	h_1	0	0	h_1
i_1	0	j_1	0	i_1	j_1	i_1
j_1	0	0	0	j_1	0	j_1
1	0	g_1	h_1	i_1	j_1	1

Table 2. '→' Operation.

→	0	g_1	h_1	i_1	j_1	1
0	1	1	1	1	1	1
g_1	j_1	1	g_1	i_1	i_1	1
h_1	i_1	1	1	i_1	i_1	1
i_1	h_1	g_1	h_1	1	g_1	1
j_1	g_1	1	g_1	1	1	1
1	0	g_1	h_1	i_1	j_1	1

Consider $C = \{(0, [0.6, 0.3, 0.3]), (g_1, [0.5, 0.3, 0.3]), (h_1, [0.5, 0.4, 0.4]), (i_1, [0.5, 0.4, 0.4]), (j_1, [0.5, 0.4, 0.4]), (1, [0.6, 0.3, 0.3])\}$.

It is evident that C assures the Definition 3.1.
Hence, C is a neutrosophic prime filter of \mathcal{G} .

Example 3.3: Let $D = \{0, g_1, h_1, i_1, j_1, 1\}$. The binary operations are specified by Tables 1 and 2.
Consider $D = \{(0, [0.6, 0.3, 0.3]), (g_1, [0.4, 0.3, 0.3]), (h_1, [0.5, 0.4, 0.4]), (i_1, [0.5, 0.4, 0.4]), (j_1, [0.5, 0.4, 0.4]), (1, [0.6, 0.3, 0.3])\}$.

Here, D is not a neutrosophic prime filter of \mathcal{G} .

Since, $T_D(h_1) = 0.5 \not\geq 0.4 = \min\{T_D(g_1), T_D(h_1)\}$.

Proposition 3.4: Let C be a non-constant neutrosophic prime filter of \mathcal{G} if and only if, $(T_C)_{T_C(1)} = \{g_1 / T_C(g_1) \geq T_C(1), g_1 \in \mathcal{G}\}$, $(I_C)_{I_C(1)} = \{g_1 / I_C(g_1) \leq I_C(1), g_1 \in \mathcal{G}\}$, $(F_C)_{F_C(1)} = \{g_1 / F_C(g_1) \leq F_C(1), g_1 \in \mathcal{G}\}$ is a prime filter.

Proof: Let C be a neutrosophic prime filter of \mathcal{G} .

Obviously, $(T_C)_{T_C(1)} = \{g_1 / T_C(g_1) \geq T_C(1), g_1 \in \mathcal{G}\}$.

Since, C is a non-constant neutrosophic filter $T_C(0) \leq T_C(1)$.

That is $0 \notin (T_C)_{T_C(1)}$.

Hence, $(T_C)_{T_C(1)}$ is a prime filter.

Conversely, if $(T_C)_{T_C(1)}$ is a prime filter.

Then, $g_1 \rightarrow h_1 \in (T_C)_{T_C(1)}$ (or) $h_1 \rightarrow g_1 \in (T_C)_{T_C(1)}$ for $g_1, h_1 \in \mathcal{G}$.

This means that, $(g_1 \vee h_1) \rightarrow h_1 = g_1 \rightarrow h_1 \in (T_C)_{T_C(1)}$ (or)

$(g_1 \vee h_1) \rightarrow g_1 = h_1 \rightarrow g_1 \in (T_C)_{T_C(1)}$.

Then, $T_C((g_1 \vee h_1) \rightarrow h_1) = T_C(1)$.

From the Definition 2.4, we have

$$\begin{aligned} T_C(h_1) &\geq T_C((g_1 \vee h_1) \rightarrow h_1) \wedge T_C(g_1 \vee h_1) = T_C(g_1 \vee h_1) \\ T_C(g_1) &\geq T_C((g_1 \vee h_1) \rightarrow g_1) \wedge T_C(g_1 \vee h_1) = T_C(g_1 \vee h_1) \\ \text{Therefore, } T_C(g_1) \wedge T_C(h_1) &\geq T_C(g_1 \vee h_1). \end{aligned}$$

Similarly, we can prove for I_C, F_C .

Hence, C is a neutrosophic prime filter.

Proposition 3.5: Let C be non-constant neutrosophic filter of \mathcal{G} . C is a neutrosophic prime filter if and only if $T_C(g_1 \rightarrow h_1) = T_C(1)$ or $T_C(h_1 \rightarrow g_1) = T_C(1)$, $I_C(g_1 \rightarrow h_1) = I_C(1)$ or $I_C(h_1 \rightarrow g_1) = I_C(1)$, $F_C(g_1 \rightarrow h_1) = F_C(1)$ or $F_C(h_1 \rightarrow g_1) = F_C(1)$.

Proof: Let C be a non-constant neutrosophic filter of \mathcal{G} .

From the Proposition 3.4, C is a neutrosophic prime filter

if and only if $(T_C)_{T_C(1)}$ is a prime filter.

if and only if $g_1 \rightarrow h_1 \in (T_C)_{T_C(1)}$ (or) $h_1 \rightarrow g_1 \in (T_C)_{T_C(1)}$

if and only if $T_C(g_1 \rightarrow h_1) = T_C(1)$ (or) $T_C(h_1 \rightarrow g_1) = T_C(1)$.

Similarly, we can prove for I_C, F_C .

Proposition 3.6: Let C_1 be a non-constant neutrosophic prime filter of \mathcal{G} and C_2 be a non-constant neutrosophic filter of \mathcal{G} . If $C_1 \subseteq C_2$, then $T_{C_1}(1) = T_{C_2}(1)$, $I_{C_1}(1) = I_{C_2}(1)$, $F_{C_1}(1) = F_{C_2}(1)$ then C_2 is also a neutrosophic prime filter.

Proof: Let C_1 be a neutrosophic prime filter of \mathcal{G} .

Then, from the Proposition 3.5, $T_{C_1}(g_1 \rightarrow h_1) = T_{C_1}(1)$ or $T_{C_1}(h_1 \rightarrow g_1) = T_{C_1}(1)$ for all $g_1, h_1 \in \mathcal{G}$.

If $T_{C_1}(g_1 \rightarrow h_1) = T_{C_1}(1)$ by $C_1 \subseteq C_2$ and $T_{C_1}(1) = T_{C_2}(1)$, we have $T_{C_2}(g_1 \rightarrow h_1) = T_{C_2}(1)$. Similarly, if $T_{C_1}(h_1 \rightarrow g_1) = T_{C_1}(1)$, then $T_{C_2}(h_1 \rightarrow g_1) = T_{C_2}(1)$.

Similarly, it can be proved for I_{C_2}, F_{C_2} .

From the Proposition 3.5, we have C_2 is a neutrosophic prime filter.

4. Neutrosophic Boolean and Neutrosophic prime Boolean filters

In this segment, we put forward the notion of neutrosophic Boolean and prime Boolean filters and confer their features with illustrations.

Definition 4.1: Let C be a neutrosophic filter of \mathcal{G} . C is called a neutrosophic Boolean filter if $T_C(g_1 \vee g_1^*) = T_C(1)$, $I_C(g_1 \vee g_1^*) = I_C(1)$, $F_C(g_1 \vee g_1^*) = F_C(1)$ for all $g_1 \in \mathcal{G}$.

Example 4.2: Let $C = \{0, g_1, h_1, i_1, 1\}$. The binary operations are specified by Tables 3 and 4.

Consider $C = \{(0, [0.8,0.2,0.2]), (g_1, [0.8,0.2,0.2]), (h_1, [0.6,0.3,0.3]),$

$(i_1, [0.6,0.3,0.3]), (1, [0.8,0.2,0.2])\}$.

It is evident that C assures the Definition 4.1. Hence, C is a neutrosophic Boolean filter of \mathcal{G} .

Table 3. '°' Operation.

°	0	g_1	h_1	i_1	1
0	0	0	0	0	0
g_1	0	g_1	i_1	i_1	g_1
h_1	0	i_1	h_1	i_1	h_1
i_1	0	i_1	i_1	i_1	i_1
1	0	g_1	h_1	i_1	1

Table 4. '→' Operation.

→	0	g_1	h_1	i_1	1
0	1	1	1	1	1
g_1	0	1	h_1	h_1	1
h_1	0	g_1	1	g_1	1
i_1	0	1	1	1	1
1	0	g_1	h_1	i_1	1

Example 4.3: Let $D = \{0, g_1, h_1, i_1, 1\}$. The binary operations are specified by the Tables 3 and 4.

Consider $D = \{(0, [0.6,0.3,0.3]), (g_1, [0.8,0.2,0.2]), (h_1, [0.6,0.3,0.3]),$

$(i_1, [0.6,0.3,0.3]), (1, [0.8,0.2,0.2])\}$.

Here, D is not a neutrosophic Boolean filter of \mathcal{G} .

Because, $T_D(g_1 \vee g_1^*) = T_D(0) = 0.6 \neq 0.8 = T_D(1)$.

Proposition 4.4 Let C be a neutrosophic Boolean filter of \mathcal{G} if and only if

$$T_C((g_1 \rightarrow g_1^*) \rightarrow g_1^*) = T_C((g_1^* \rightarrow g_1) \rightarrow g_1) = T_C(1),$$

$$I_C((g_1 \rightarrow g_1^*) \rightarrow g_1^*) = I_C((g_1^* \rightarrow g_1) \rightarrow g_1) = I_C(1),$$

$$F_C((g_1 \rightarrow g_1^*) \rightarrow g_1^*) = F_C((g_1^* \rightarrow g_1) \rightarrow g_1) = F_C(1) \text{ for all } g_1 \in \mathcal{G}.$$

Proof: Let C be a neutrosophic Boolean filter of \mathcal{G} .

From the Definition 4.1, we know that $T_C(g_1 \vee g_1^*) = T_C(1)$.

Then, by (iv) of the Proposition 2.2,

$$\text{we have } T_C(g_1 \vee g_1^*) = T_C(((g_1 \rightarrow g_1^*) \rightarrow g_1^*) \wedge ((g_1^* \rightarrow g_1) \rightarrow g_1))$$

$$= T_c((g_1 \rightarrow g_1^*) \rightarrow g_1^*) \wedge T_c((g_1^* \rightarrow g_1) \rightarrow g_1) \text{ [From (ii) of the proposition 2.6]}$$

$$= T_c(1)$$

So, $T_c((g_1 \rightarrow g_1^*) \rightarrow g_1^*) = T_c((g_1^* \rightarrow g_1) \rightarrow g_1) = T_c(1)$ for all $g_1 \in \mathcal{G}$.

Similarly, we can prove for I_c, F_c .

Similarly, the converse part can be proved.

Proposition 4.5: Let $C_1 \subseteq C_2$ and $T_{C_1}(1) = T_{C_2}(1), I_{C_1}(1) = I_{C_2}(1), F_{C_1}(1) = F_{C_2}(1)$, where C_1 is a neutrosophic Boolean filter and C_2 is a neutrosophic filter. Then C_2 is a neutrosophic Boolean filter.

Proof: Let C_1 and C_2 be two neutrosophic filters of \mathcal{G} .

If C_1 is neutrosophic Boolean filter, then $T_{C_1}(g_1 \vee g_1^*) = T_{C_1}(1)$ for all $g_1 \in \mathcal{G}$.

Since, $C_1 \subseteq C_2$ and $T_{C_1}(1) = T_{C_2}(1)$, it follows that $T_{C_2}(g_1 \vee g_1^*) \geq T_{C_2}(1)$.

From (i) of the Definition 2.4, we have $T_{C_2}(g_1 \vee g_1^*) \leq T_{C_2}(1)$.

Hence, $T_{C_2}(g_1 \vee g_1^*) = T_{C_2}(1)$.

Similarly, we can prove for I_{C_2}, F_{C_2} .

Thus, C_2 is a neutrosophic Boolean filter.

Proposition 4.6: Let C be a neutrosophic Boolean filter of \mathcal{G} if it persuades,

$T_c(g_1) = T_c(g_1^* \rightarrow g_1), I_c(g_1) = I_c(g_1^* \rightarrow g_1), F_c(g_1) = F_c(g_1^* \rightarrow g_1)$ for all $g_1 \in \mathcal{G}$.

Proof: Let C be a neutrosophic Boolean filter of \mathcal{G} .

$$\begin{aligned} \text{By the Definition 2.4, } T_c(g_1^* \rightarrow g_1) &\geq \min\{T_c(g_1 \rightarrow (g_1^* \rightarrow g_1)), T_c(g_1)\} \\ &= \min\{T_c(1), T_c(g_1)\} \text{ [Since, } g_1^* = g_1 \rightarrow 0] \\ &\geq T_c(g_1) \text{ and from the Definition 2.4,} \\ T_c(g_1) &\geq \min\{T_c((g_1 \vee g_1^*) \rightarrow g_1), T_c(g_1 \vee g_1^*)\} \\ &= \min\{T_c((g_1 \rightarrow g_1) \wedge (g_1^* \rightarrow g_1)), T_c(1)\} \\ &= \min\{T_c(1 \wedge (g_1^* \rightarrow g_1)), T_c(1)\} \end{aligned}$$

Therefore, $T_c(g_1) \geq T_c(g_1^* \rightarrow g_1)$.

Then, $T_c(g_1) = T_c(g_1^* \rightarrow g_1)$ for all $g_1 \in \mathcal{G}$.

Similarly, $I_c(g_1) = I_c(g_1^* \rightarrow g_1), F_c(g_1) = F_c(g_1^* \rightarrow g_1)$.

Definition 4.7: A neutrosophic filter N is called a neutrosophic prime Boolean filter if it is both a neutrosophic Boolean filter and a neutrosophic prime filter. The set of all neutrosophic prime Boolean filters of \mathcal{G} is denoted by $NPB(\mathcal{G})$.

Example 4.8: Consider the Example 3.2.

Then, from the Example 3.2 C is a neutrosophic prime filter.

Also, by the Definitions 4.1 and 4.7 it is evident that C is a neutrosophic Boolean and prime Boolean filters of \mathcal{G} respectively.

Example 4.9 Let $D = \{0, g_1, h_1, i_1, j_1, 1\}$. The binary operations are specified by the Tables 1 and 2.

Consider $D = \{(0, [0.5, 0.2, 0.2]), (g_1, [0.3, 0.2, 0.2]), (h_1, [0.4, 0.3, 0.3]),$

$(i_1, [0.4, 0.3, 0.3]), (j_1, [0.4, 0.3, 0.3]), (1, [0.5, 0.2, 0.2])\}$.

Here, by the Definition 4.1 D is a neutrosophic Boolean filter of \mathcal{G} .

But D is not a neutrosophic prime filter of \mathcal{G} . Since, $T_D(h_1) = 0.4 \not\leq 0.3 = \min\{T_D(g_1), T_D(h_1)\}$.

Hence, D is not a $NPB(\mathcal{G})$.

Proposition 4.10 Let C and D be two neutrosophic filters of \mathcal{G} . Let $C \subseteq D$ such that $N_C(1) = N_D(1)$. If C is a neutrosophic prime Boolean filter of \mathcal{G} then so is D .

Proof: Let C be a neutrosophic prime Boolean filter of \mathcal{G} .

Since, C is a neutrosophic Boolean filter $N_C(g_1) = N_C(1)$ (or) $N_C(g_1^*) = N_C(1)$ for all $g_1 \in \mathcal{G}$.

By $C \subseteq D$ and $N_C(1) = N_D(1)$, we get $N_D(g_1) = N_D(1)$ (or) $N_D(g_1^*) = N_D(1)$

Hence, D is a neutrosophic Boolean filter.

Since, C is a neutrosophic prime filter $N_C(h_1 \rightarrow g_1) = N_C(1)$ (or) $N_C(g_1 \rightarrow h_1) = N_C(1)$ for all $g_1, h_1 \in \mathcal{G}$.

By $C \subseteq D$ and $N_C(1) = N_D(1)$, we get $N_D(h_1 \rightarrow g_1) = N_D(1)$ (or) $N_D(g_1 \rightarrow h_1) = N_D(1)$

Hence, D is a neutrosophic prime filter. Therefore, D is a neutrosophic prime Boolean filter.

5. Discussion

The key findings from this article are as follows:

- C_2 is a neutrosophic prime filter of \mathcal{G} , if C_1 is a non-constant neutrosophic prime filter of \mathcal{G} where $C_1 \subseteq C_2$, $T_{C_1}(1) = T_{C_2}(1)$, $I_{C_1}(1) = I_{C_2}(1)$, $F_{C_1}(1) = F_{C_2}(1)$. [Extension property]
- C is a neutrosophic Boolean filter of \mathcal{G} if it persuades,
 $T_C(g_1) = T_C(g_1^* \rightarrow g_1)$, $I_C(g_1) = I_C(g_1^* \rightarrow g_1)$, $F_C(g_1) = F_C(g_1^* \rightarrow g_1)$ for all $g_1 \in \mathcal{G}$.
- Suppose C and D are two neutrosophic filters of \mathcal{G} and $C \subseteq D$ such that $N_C = N_D(1)$ where C is a neutrosophic prime Boolean filter of \mathcal{G} then so is D . [Extension property]

6. Conclusions

In the current study, we have put forward the notions of the neutrosophic Boolean and prime filters of a BL-algebra and looked into a few associated features. Additionally, we have inspected a few necessary and adequate criteria for those filters. Also, we have acquired an extension property for both the neutrosophic Boolean and prime filters. Finally, by combining both filters, the notion of a neutrosophic prime Boolean filter is presented with examples. This work stands out in studying the characteristics of prime and Boolean filters in BL-algebras as it mainly concentrates on their neutrosophic nature. In the future, the above study can be extended to deductive, ultra, and transitive filters and used to rectify problems in many other fields. Also, these filters can be applied to various medical diagnoses and image-processing techniques.

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Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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