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# Similarity Measure of Plithogenic Cubic Vague Sets: Examples and Possibilities

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**Abstract:** The crisp, fuzzy, intuitionistic fuzzy and neutrosophic sets are the extension of the plithogenic set, in which elements are characterized by the number of attributes and each attribute can assume many values. To achieve more accuracy and precise exclusion, a contradiction or dissimilarity degree is specified between each attribute and the values of the dominating attribute. A plithogenic cubic vague set is a combination of a plithogenic cubic set and a vague set. The key tool for resolving problems with pattern recognition and clustering analysis is the similarity measure. In this research, we characterize and investigate the similarities between two Plithogenic Cubic Vague sets (PCVSs) for $(z \equiv F)$, $(z \equiv IF)$ and $(z \equiv N)$. Also, examples are given to examine similarities in the pattern recognition application problems.

**Keywords:** Plithogenic Set, Plithogenic Cubic Vague Set, Pattern Recognition, Similarity Measure.

## 1. Introduction

Zadeh introduced fuzzy set a mathematical theory to deal with uncertainties [1]. It is characterized by the membership value and sometimes it is difficult to assign the value for a fuzzy set. Interval-valued fuzzy set was introduced by Zadeh to overcome this problem. Intuitionistic fuzzy sets (IF) and interval-valued intuitionistic fuzzy sets introduced by Atanassov et al. [2,3] are appropriate to handle this situation. However, it is not enough to handle the unreliable information existing in the belief system. Zulkifli et al. [4] proposed the interval-valued intuitionistic fuzzy vague sets (IVIFVS). Florentin Smarandache [5] introduced a neutrosophic set and provided a mathematical tool to handle difficulties involving inconsistent and indeterminate data. New ideas on neutrosophic sets are introduced by Anitha et al. [6-8]. The interval-valued neutrosophic set was introduced by Jun Ye [9]. Hazwani Hashimcet et al. [10] proposed Interval Neutrosophic Vague Sets. Banik et al. [11,12] studied the MCGDM problem in a pentagonal neutrosophic environment and a novel integrated neutrosophic cosine operator-based linear programming. Haque et al. [13-15] elevated decision-making ideas in interval neutrosophic environment, generalized spherical fuzzy environment, and linguistic generalized spherical fuzzy environment.

The vague set was developed by Gau and Buehrer [16]. The idea of similarity measure of fuzzy sets was introduced by Wang [17] and gave a computational formula. Since then it has attracted many researchers. Fei et al. [18] introduced the similarity between two intuitionistic fuzzy sets. Similarity measures of neutrosophic sets were given by Broumi et al. [19]. Ali et al. [20] introduced neutrosophic cubic set-based decision-making. Shawkat Alkhazaleh [21] studied neutrosophic vague set in 2015. Similarities between vague sets were introduced by Chen S.M [22]. The idea of a cubic set was introduced by Jun [23]. The idea of a cubic vague set was introduced by Khaleed et al. [24] by incorporating a cubic set and a vague set. He also presented a decision-making method based on the similarity measure of cubic vague set.

Smarandache introduced plithogenic set and it may have elements characterized by four or more attributes [25]. A plithogenic multi-criteria decision-making approach to estimate the sustainable supply chain risk management based on order preference and criteria importance through the correlation method is proposed by Abdel and Rehab [26]. Alkhazaleh introduced plithogenic soft set and measured the similarity between two plithogenic soft sets using a set-theoretic approach [27]. Anitha et al. [28] introduced the idea of plithogenic cubic vague set.

In this paper, we introduce the concept of similarity measure between two Plithogenic Cubic Vague sets (PCVSs) $((z \equiv F), (z \equiv IF), (z \equiv N))$. It has the novelty to precisely characterize and model data for real-life occurrences. Since cubic set fails to capture the false membership part to measure the alternative in the decision making method. PCVS has the ability to handle uncertainties and vague information considering the truth and false membership values as the elements are characterized by one or more attributes therefore it is possible to describe the problem. One of the best tool to solve it is similarity measure. Similarity measure of PCVS is a vital concept for measuring entropy in the data. The flow of this paper is as follows. An algorithm to determine the similarities between two PCVS $((z \equiv F), (z \equiv IF), (z \equiv N))$ for a pattern recognition problem is proposed. To illuminate the proposed measure numerical examples are provided.

The organization of the paper is as follows: Section 2 provides some preliminaries for the proposed concept. Section 3 covers the application of the plithogenic cubic vague set and it is divided into three subsections. In 3.1 algorithm and examples of the plithogenic fuzzy cubic vague set, in 3.2 plithogenic intuitionistic fuzzy cubic vague set and in 3.3 plithogenic neutrosophic cubic vague set were presented. In Section 4 discussion is made for the proposed measure. Finally, Section 5 concludes this paper and provides the direction for future studies.

## 2. Preliminaries

**Definition 2.1:** [23] A neutrosophic vague set $A_{NV}$ (NVS in short) on the universe of discourse X written as $A_{NV} = \{\langle x, \hat{T}_{A_{NV}}, \hat{I}_{A_{NV}}, \hat{F}_{A_{NV}} \rangle\, x \in X\}$ whose truth membership, indeterminacy membership and falsity membership functions are defined as $\hat{T}_{A_{NV}}(x) = [T^-, T^+]$, $\hat{I}_{A_{NV}}(x) = [I^-, I^+]$, $\hat{F}_{A_{NV}}(x) = [F^-, F^+]$, where $T^+ = 1 - F^-$, $F^+ = 1 - T^-$ and $0^- \le T^- + I^- + F^- \le 2^+$.

**Definition 2.2:** [16] An interval valued neutrosophic vague set $A_{INV}$ also known as INVS in the universe of discourse E. An IVNVS is characterized by truth membership, indeterminacy membership and falsity membership functions is defined as:

$A_{INV} = \{< e, [\hat{V}_A^L(e), \hat{V}_A^U(e)], [\widehat{W}_A^L(e), \widehat{W}_A^U(e)], [\hat{X}_A^L(e), \hat{X}_A^U(e)] > | e \in E\}$,

$\hat{V}_A^L(e) = [V^{L^-}, V^{L^+}]$ , $\hat{V}_A^U(e) = [V^{U^-}, V^{U^+}]$ , $\widehat{W}_A^L(e) = [W^{L^-}, W^{L^+}]$ , $\widehat{W}_A^L(e) = [W^{U^-}, W^{U^+}]$ , $\hat{X}_A^L(e) = [X^{L^-}, X^{L^+}]$, $\widehat{X}_A^U(e) = [X^{U^-}, X^{U^+}]$ where

$V^{L^+} = 1 - X^{L^-}$, $X^{L^+} = 1 - V^{L^-}$, $V^{U^+} = 1 - X^{U^-}$, $X^{U^+} = 1 - V^{U^-}$ and

$0^- \le V^{L^-} + V^{U^-} + W^{L^-} + W^{U^-} + X^{L^-} + X^{U^-} \le 4^+$, $0^- \le V^{L^+} + V^{U^+} + W^{L^+} + W^{U^+} + X^{L^+} + X^{U^+} \le 4^+$.

**Definition 2.3:** [2] Let U be a universal set. The set $\mathbb{A}_p{}^v = \{\langle x, A_V(x), \lambda_V(x) \rangle : x \in X\}$ is called plithogenic fuzzy cubic vague set in which $A_V$ is an interval valued plithogenic fuzzy vague set in X and $\lambda_V$ is the fuzzy vague set in X.

**Definition 2.4:** [2] Let U be a universal set. The set $\mathbb{A}_p{}^v = \{\langle x, A_V(x), \lambda_V(x) \rangle : x \in X\}$ is called plithogenic intuitionistic fuzzy cubic vague set in which $A_V$ is an interval valued plithogenic intuitionistic fuzzy vague set in X and $\lambda_V$ is the intuitionistic fuzzy vague set in X.

**Definition 2.5:** [2] Let U be a universal set. The set $\mathbb{A}_p{}^v = \{\langle x, A_V(x), \lambda_V(x)\rangle : x \in X\}$ is called plithogenic neutrosophic cubic vague set in which $A_V$ is an interval valued plithogenic neutrosophic vague set in X and $\lambda_V$ is the neutrosophic vague set in X.

## 3. Application of Plithogenic Cubic Vague sets in Pattern Recognition Problem

This section may be divided by subheadings. It should provide a concise and precise description of the experimental results, their interpretation as well as the experimental conclusions that can be drawn.

Here, we introduce the concept of similarity measure between two Plithogenic Cubic Vague sets (PFCVSs) ($z \equiv F$), PIFCVSs ($z \equiv IF$), (PNCVSs ($z \equiv N$) and further results on similarity measure. An example is given to exhibit the effectiveness of the proposed method.

### *3.1 Plithogenic Fuzzy Cubic Vague Set*

**Definition 3.1.1:** Let $\mathbb{A}_{p_1}^v$ and $\mathbb{A}_{p_2}^v$ be any two Plithogenic Fuzzy Cubic Vague sets (PFCVSs). Then,

(1) $0 \leq \left|S\left(\mathbb{A}_{p_1}^v, \mathbb{A}_{p_2}^v\right)\right| \leq 1$,

(2) $S\left(\mathbb{A}_{p_1}^v, \mathbb{A}_{p_2}^v\right) = S\left(\mathbb{A}_{p_2}^v, \mathbb{A}_{p_1}^v\right)$,

(3) $S\left(\mathbb{A}_{p_1}^v, \mathbb{A}_{p_2}^v\right) = 1 \Leftrightarrow \mathbb{A}_{p_1}^v = \mathbb{A}_{p_2}^v$,

(4) $\mathbb{A}_{p_1}^v \subseteq \mathbb{A}_{p_2}^v \subseteq \mathbb{A}_{p_3}^v \Longrightarrow S\left(\mathbb{A}_{p_1}^v, \mathbb{A}_{p_3}^v\right) \leq S\left(\mathbb{A}_{p_2}^v, \mathbb{A}_{p_3}^v\right)$

**Definition 3.1.2:** Let X $= \{x_1, x_2, x_3\}$, $\mathbb{A}_{p_1}^v = \langle A_V^1, \lambda_V^1\rangle$ and $\mathbb{A}_{p_2}^v = \langle A_V^2, \lambda_V^2\rangle$ be two Plithogenic Fuzzy Cubic Vague Sets (PFCVSs) in X. The similarity measure between $\mathbb{A}_{p_1}^v \, and \, \mathbb{A}_{p_2}^v$ is given by $S\left(\mathbb{A}_{p_1}^v, \mathbb{A}_{p_2}^v\right)$, where

$$S\left(\mathbb{A}_{p_1}^v, \mathbb{A}_{p_2}^v\right) = \frac{1}{6n}\sum_{i=1}^{n}\Big(\left|T_{A^1{}_V}^{L^-}(x_i) - T_{A^2{}_V}^{L^-}(x_i)\right| + \left|T_{A^1{}_V}^{U^-}(x_i) - T_{A^2{}_V}^{U^-}(x_i)\right| + \left|T_{A^1{}_V}^{L^+}(x_i) - T_{A^2{}_V}^{L^+}(x_i)\right| + \left|T_{A^1{}_V}^{U^+}(x_i) - T_{A^2{}_V}^{+}(x_i)\right| + \left|T_{\lambda^1{}_V}^{-}(x_i) - T_{\lambda^2{}_V}^{-}(x_i)\right| + \left|T_{\lambda^1{}_V}^{+}(x_i) - T_{\lambda^2{}_V}^{+}(x_i)\right|\Big)$$

**Algorithm**:

***Step 1.*** Construct PFCVS $\mathbb{A}_p{}^v = \langle A_V, \lambda_V\rangle$ as ideal pattern.

***Step 2.*** Then construct PFCVSs $\mathbb{A}_{p_j}^v = \langle A_V^j, \lambda_V^j\rangle$, $j = 1,2 \dots n$ for sample patterns which are to be known.

***Step 3.*** Compute the similarities between ideal pattern $\mathbb{A}_p{}^v = \langle A_V, \lambda_V\rangle$ and the sample pattern $\mathbb{A}_{p_j}^v = \langle A_V^j, \lambda_V^j\rangle$ using definition 3.1.2.

***Step 4.*** The sample pattern $\mathbb{A}_{p_j}^v$ is considered to belong to ideal pattern $\mathbb{A}_p{}^v$ if $S\left(\mathbb{A}_p{}^v, \mathbb{A}_{p_j}^v\right) \leq 0.5$ and sample pattern $\mathbb{A}_{p_j}^v$ is not to be known for an ideal pattern $\mathbb{A}_p{}^v$ if $S\left(\mathbb{A}_p{}^v, \mathbb{A}_{p_j}^v\right) > 0.5$.

**Example 3.1.3:** Consider a simple pattern recognition problem involving three sample patterns and an ideal pattern. Let X $= \{x_1, x_2, x_3\}$. The patterns indicated as pattern 1, pattern 2 and pattern 3 are the selected three sample patterns, whereas pattern 4 is the selected ideal pattern. Also, let $\mathbb{A}_p{}^v$ be PFCVS set of ideal pattern and pattern $\mathbb{A}_{p_j}^v = \langle A_V^j, \lambda_V^j\rangle$ be the PFCVSs of three sample patterns.

***Step 1.*** Create an ideal PFCVS $\mathbb{A}_p{}^v = \langle A_V, \lambda_V\rangle$ on X as,

$$\mathbb{A}_p{}^v = <\{\frac{\langle[0.4,0.6],[0.5,0.5]\rangle}{x_1}, \frac{\langle[0.3,0.8],[0.5,0.6]\rangle}{x_2}, \frac{\langle[0.2,0.6],[0.3,0.6]\rangle}{x_3}\}, \{\frac{[0.2,0.5]}{x_1}, \frac{[0.4,0.7]}{x_2}, \frac{[0.3,0.6]}{x_3}\}>$$

***Step 2.*** Construct PFCVSs $\mathbb{A}_{p_j}^v = \langle A_V^j, \lambda_V^j\rangle$, $j = 1,2,3$ for the sample patterns as;

$$\mathbb{A}_{p_1}^v = <\{\frac{\langle[0.2,0.6],[0.3,0.6]\rangle}{x_1}, \frac{\langle[0.4,0.6],[0.5,0.5]\rangle}{x_2}, \frac{\langle[0.3,0.8],[0.5,0.6]\rangle}{x_3}\}, \{\frac{[0.1,0.7]}{x_1}, \frac{[0.3,0.7]}{x_2}, \frac{[0.1,0.9]}{x_3}\}>$$

$$\mathbb{A}_{p_2}^v = <\{\frac{\langle[0.1,0.7],[0.2,0.4]\rangle}{x_1}, \frac{\langle[0.4,0.6],[0.1,0.2]\rangle}{x_2}, \frac{\langle[0.3,0.8],[0.5,0.6]\rangle}{x_3}\}, \{\frac{[0.5,0.6]}{x_1}, \frac{[0.1,0.7]}{x_2}, \frac{[0.1,0.5]}{x_3}\}>$$

$$\mathbb{A}_{p_3}^v = <\{\frac{\langle[0.3,0.8],[0.5,0.6]\rangle}{x_1}, \frac{\langle[0.4,0.6],[0.1,0.2]\rangle}{x_2}, \frac{\langle[0.4,0.6],[0.5,0.5]\rangle}{x_3}\}, \{\frac{[0.1,0.9]}{x_1}, \frac{[0.1,0.7]}{x_2}, \frac{[0.3,0.7]}{x_3}\}>$$

***Step 3.*** Compute S the degree of similarity between ideal pattern $\mathbb{A}_p{}^v$ and the sample pattern $\mathbb{A}_{p_j}^v$, then the results obtained are:

$$S(\mathbb{A}_p{}^v, \mathbb{A}_{p_1}^v) = 0.13$$
$$S(\mathbb{A}_p{}^v, \mathbb{A}_{p_2}^v) = 0.19$$
$$S(\mathbb{A}_p{}^v, \mathbb{A}_{p_3}^v) = 0.17$$

***Step 4.*** $S(\mathbb{A}_p{}^v, \mathbb{A}_{p_1}^v) \leq 0.5$, $S(\mathbb{A}_p{}^v, \mathbb{A}_{p_2}^v) \leq 0.5$ and $S(\mathbb{A}_p{}^v, \mathbb{A}_{p_3}^v) \leq 0.5$, the sample pattern whose corresponding PFCVS sets are denoted as $\mathbb{A}_{p_1}^v, \mathbb{A}_{p_2}^v$ and $\mathbb{A}_{p_3}^v$ are known as similar patterns of the family of ideal pattern whose PFCVS is represented by $\mathbb{A}_p{}^v$.

### *3.2 Plithogenic Intuitionistic Fuzzy Cubic Vague Set*

**Definition 3.2.1:** Let $\mathbb{A}_{p_1}^v$ and $\mathbb{A}_{p_2}^v$ be two Plithogenic Intuitionistic Fuzzy Cubic Vague Set (PIFCVSs). Then,

(1) $0 \leq |S(\mathbb{A}_{p_1}^v, \mathbb{A}_{p_2}^v)| \leq 1$,

(2) $S(\mathbb{A}_{p_1}^v, \mathbb{A}_{p_2}^v) = S(\mathbb{A}_{p_2}^v, \mathbb{A}_{p_1}^v)$,

(3) $S(\mathbb{A}_{p_1}^v, \mathbb{A}_{p_2}^v) = 1 \Leftrightarrow \mathbb{A}_{p_1}^v = \mathbb{A}_{p_2}^v$,

(4) $\mathbb{A}_{p_1}^v \subseteq \mathbb{A}_{p_2}^v \subseteq \mathbb{A}_{p_3}^v \Longrightarrow S(\mathbb{A}_{p_1}^v, \mathbb{A}_{p_3}^v) \leq S(\mathbb{A}_{p_2}^v, \mathbb{A}_{p_3}^v)$

**Definition 3.2.2:** Let $X = \{x_1, x_2, x_3\}$, $\mathbb{A}_{p_1}^v = \langle A_V^1, \lambda_V^1 \rangle$ and $\mathbb{A}_{p_2}^v = \langle A_V^2, \lambda_V^2 \rangle$ be two Plithogenic Intuitionistic Fuzzy Cubic Vague Sets (PIFCVSs) in X. The similarities between $\mathbb{A}_{p_1}^v$ *and* $\mathbb{A}_{p_2}^v$ is given as $S(\mathbb{A}_{p_1}^v, \mathbb{A}_{p_2}^v)$, where

$$S(\mathbb{A}_{p_1}^v, \mathbb{A}_{p_2}^v) = \frac{1}{12n}\sum_{i=1}^{n}\Big(\left|T_{A^1{}_V}^{L^-}(x_i) - T_{A^2{}_V}^{L^-}(x_i)\right| + \left|T_{A^1{}_V}^{U^-}(x_i) - T_{A^2{}_V}^{U^-}(x_i)\right| + \left|F_{A^1{}_V}^{L^-}(x_i) - F_{A^2{}_V}^{L^-}(x_i)\right| +$$

$$\left|F_{A^1{}_V}^{U^-}(x_i) - F_{A^2{}_V}^{U^-}(x_i)\right| + \left|T_{A^1{}_V}^{L^+}(x_i) - T_{A^2{}_V}^{L^+}(x_i)\right| + \left|T_{A^1{}_V}^{U^+}(x_i) - T_{A^2{}_V}^{+}(x_i)\right| + \left|F_{A^1{}_V}^{L^+}(x_i) - F_{A^2{}_V}^{L^+}(x_i)\right| +$$

$$\left|F_{A^1{}_V}^{U^+}(x_i) - F_{A^2{}_V}^{U^+}(x_i)\right| + \left|T_{\lambda^1{}_V}^{-}(x_i) - T_{\lambda^2{}_V}^{-}(x_i)\right| + \left|F_{\lambda^1{}_V}^{-}(x_i) - F_{\lambda^2{}_V}^{-}(x_i)\right| + \left|T_{\lambda^1{}_V}^{+}(x_i) - T_{\lambda^2{}_V}^{+}(x_i)\right| +$$

$$\left|F_{\lambda^1{}_V}^{+}(x_i) - F_{\lambda^2{}_V}^{+}(x_i)\right|\Big)$$

**Algorithm:**

***Step 1.*** Construct an ideal PIFCVS $\mathbb{A}_p{}^v = \langle A_V, \lambda_V \rangle$.

***Step 2.*** Then construct PIFCVSs $\mathbb{A}_{p_j}^v = \langle A_V^j, \lambda_V^j \rangle$, $j = 1,2 \ldots n$ for sample patterns.

***Step 3.*** Calculate the similarities between ideal pattern $\mathbb{A}_p{}^v = \langle A_V, \lambda_V \rangle$ and sample pattern $\mathbb{A}_{p_j}^v = \langle A_V^j, \lambda_V^j \rangle$ using definition 3.2.2.

***Step 4.*** The sample pattern $\mathbb{A}_{p_j}^v$ is considered to belong to ideal pattern $\mathbb{A}_p{}^v$ if $S\left(\mathbb{A}_p{}^v, \mathbb{A}_{p_j}^v\right) \leq 0.5$ and sample pattern $\mathbb{A}_{p_j}^v$ is not to be known for ideal pattern $\mathbb{A}_p{}^v$ if $S\left(\mathbb{A}_p{}^v, \mathbb{A}_{p_j}^v\right) > 0.5$.

**Example 3.3.3:** Let us consider three simple pattern which are to be known. Let $X = \{x_1, x_2, x_3\}$. Similarly let $\mathbb{A}_p{}^v$ be PIFCVS set of ideal pattern and pattern $\mathbb{A}_{p_j}^v = \langle A_V^j, \lambda_V^j \rangle$ be the PIFCVSs of sample patterns.

***Step 1.*** Create ideal PIFCVS $\mathbb{A}_p{}^v = \langle A_V, \lambda_V \rangle$ on X as,

$$\mathbb{A}_p{}^v = <\{\frac{\langle[0.4,0.6],[0.5,0.5]),([0.4,0.6],[0.5.0.5])\rangle}{x_1}, \frac{\langle[0.3,0.8],[0.5,0.6]),([0.2,0.7],[0.4.0.5])\rangle}{x_2}, \frac{\langle[0.2,0.6],[0.3,0.6]),([0.4,0.8],[0.4.0.7])\rangle}{x_3}\},$$

$$\{\frac{[0.2,0.5],[0.5,0.8]}{x_1}, \frac{[0.4,0.7],[0.3,0.6]}{x_2}, \frac{[0.3,0.6],[0.4,0.7]}{x_3}\} >$$

***Step 2.*** Construct PIFCVSs $\mathbb{A}_{p_j}^{v} = \langle A_V^j, \lambda_V^j \rangle$

$$\mathbb{A}_{p_1}^{v} = <\{\frac{\langle[0.2,0.6],[0.3,0.6]),([0.4,0.8],[0.4.0.7]\rangle}{x_1}, \frac{\langle[0.4,0.6],[0.5,0.5]),([0.4,0.6],[0.5.0.5]\rangle}{x_2}, \frac{\langle[0.3,0.8],[0.5,0.6]),([0.2,0.7],[0.4.0.5]\rangle}{x_3}\},$$

$$\{\frac{[0.1,0.7],[0.2,0.8]}{x_1}, \frac{[0.3,0.7],[0.3,0.6]}{x_2}, \frac{[0.1,0.9],[0.1,0.6]}{x_3}\} >$$

$$\mathbb{A}_{p_2}^{v} = <\{\frac{\langle[0.1,0.7],[0.2,0.4]),([0.3,0.9],[0.6.0.8]\rangle}{x_1}, \frac{\langle[0.4,0.6],[0.1,0.2]),([0.4,0.6],[0.8.0.9]\rangle}{x_2}, \frac{\langle[0.3,0.8],[0.5,0.6]),([0.2,0.7],[0.4.0.5]\rangle}{x_3}\},$$

$$\{\frac{[0.5,0.6],[0.4,0.5]}{x_1}, \frac{[0.1,0.7],[0.2,0.8]}{x_2}, \frac{[0.1,0.5],[0.2,0.5]}{x_3}\} >$$

$$\mathbb{A}_{p_3}^{v} = <\{\frac{\langle[0.3,0.8],[0.5,0.6]),([0.2,0.7],[0.4.0.5]\rangle}{x_1}, \frac{\langle[0.4,0.6],[0.1,0.2]),([0.4,0.6],[0.8.0.9]\rangle}{x_2}, \frac{\langle[0.4,0.6],[0.5,0.5]),([0.4,0.6],[0.5.0.5]\rangle}{x_3}\},$$

$$\{\frac{[0.1,0.9],[0.1,0.6]}{x_1}, \frac{[0.1,0.7],[0.2,0.8]}{x_2}, \frac{[0.3,0.7],[0.3,0.6]}{x_3}\} >$$

***Step 3.*** Calculate the degree of similarity S between the ideal pattern $\mathbb{A}_p{}^{v}$ and the sample pattern $\mathbb{A}_{p_j}^{v}$, then the results obtained are

$$S(\mathbb{A}_p{}^{v}, \mathbb{A}_{p_1}^{v}) = 0.13$$
$$S(\mathbb{A}_p{}^{v}, \mathbb{A}_{p_2}^{v}) = 0.19$$
$$S(\mathbb{A}_p{}^{v}, \mathbb{A}_{p_3}^{v}) = 0.17$$

***Step 4.*** $S(\mathbb{A}_p{}^{v}, \mathbb{A}_{p_1}^{v}) \le 0.5$, $S(\mathbb{A}_p{}^{v}, \mathbb{A}_{p_2}^{v}) \le 0.5$ and $S(\mathbb{A}_p{}^{v}, \mathbb{A}_{p_3}^{v}) \le 0.5$, the sample pattern whose corresponding PIFCVS sets are denoted as $\mathbb{A}_{p_1}^{v}, \mathbb{A}_{p_2}^{v}$ and $\mathbb{A}_{p_3}^{v}$ are known as similar patterns of the family of ideal pattern whose PIFCVS is denoted as $\mathbb{A}_p{}^{v}$.

### *3.3 Plithogenic Neutrosophic Fuzzy Cubic Vague Set*

In this part, we will observe the similarities of two Plithogenic Neutrosophic Cubic Vague Set (PNCVSs) of pattern recognition problem.

**Definition 3.3.1:** Let $\mathbb{A}_{p_1}^{v}$ and $\mathbb{A}_{p_2}^{v}$ be any two Plithogenic Neutrosophic Cubic Vague Set (PNCVSs). Then,

(1) $0 \le |S(\mathbb{A}_{p_1}^{v}, \mathbb{A}_{p_2}^{v})| \le 1$,

(2) $S(\mathbb{A}_{p_1}^{v}, \mathbb{A}_{p_2}^{v}) = S(\mathbb{A}_{p_2}^{v}, \mathbb{A}_{p_1}^{v})$,

(3) $S(\mathbb{A}_{p_1}^{v}, \mathbb{A}_{p_2}^{v}) = 1 \Leftrightarrow \mathbb{A}_{p_1}^{v} = \mathbb{A}_{p_2}^{v}$,

(4) $\mathbb{A}_{p_1}^{v} \subseteq \mathbb{A}_{p_2}^{v} \subseteq \mathbb{A}_{p_3}^{v} \Longrightarrow S(\mathbb{A}_{p_1}^{v}, \mathbb{A}_{p_3}^{v}) \le S(\mathbb{A}_{p_2}^{v}, \mathbb{A}_{p_3}^{v})$

**Definition**: **3.3.2** Let X $= \{x_1, x_2, x_3\}$, $\mathbb{A}_{p_1}^{v} = \langle A_V^1, \lambda_V^1 \rangle$ and $\mathbb{A}_{p_2}^{v} = \langle A_V^2, \lambda_V^2 \rangle$ be two Plithogenic Neutrosophic Cubic Vague Set (PNCVSs). The similarity measure between $\mathbb{A}_{p_1}^{v}$ *and* $\mathbb{A}_{p_2}^{v}$ is given by $S(\mathbb{A}_{p_1}^{v}, \mathbb{A}_{p_2}^{v})$, where

$$S(\mathbb{A}_{p_1}^{v}, \mathbb{A}_{p_2}^{v}) = \frac{1}{18n}\sum_{i=1}^{n}\Big(\Big|T_{A^1{}_V}^{L^-}(x_i) - T_{A^2{}_V}^{L^-}(x_i)\Big| + \Big|T_{A^1{}_V}^{U^-}(x_i) - T_{A^2{}_V}^{U^-}(x_i)\Big| + \Big|I_{A^1{}_V}^{L^-}(x_i) - I_{A^2{}_V}^{L^-}(x_i)\Big| +$$

$$\Big|I_{A^1{}_V}^{U^-}(x_i) - I_{A^2{}_V}^{U^-}(x_i)\Big| + \Big|F_{A^1{}_V}^{L^-}(x_i) - F_{A^2{}_V}^{L^-}(x_i)\Big| + \Big|F_{A^1{}_V}^{U^-}(x_i) - F_{A^2{}_V}^{U^-}(x_i)\Big| + \Big|T_{A^1{}_V}^{L^+}(x_i) - T_{A^2{}_V}^{L^+}(x_i)\Big| +$$

$$\Big|T_{A^1{}_V}^{U^+}(x_i) - T_{A^2{}_V}^{+}(x_i)\Big| + \Big|I_{A^1{}_V}^{L^+}(x_i) - I_{A^2{}_V}^{L^+}(x_i)\Big| + \Big|I_{A^1{}_V}^{U^+}(x_i) - I_{A^2{}_V}^{U^+}(x_i)\Big| + \Big|F_{A^1{}_V}^{L^+}(x_i) - F_{A^2{}_V}^{L^+}(x_i)\Big| +$$

$$\left|F^{U^+}_{A^1{}_V}(x_i) - F^{U^+}_{A^2{}_V}(x_i)\right| + \left|T^-_{\lambda^1{}_V}(x_i) - T^-_{\lambda^2{}_V}(x_i)\right| + \left|I^-_{\lambda^1{}_V}(x_i) - I^-_{\lambda^2{}_V}(x_i)\right| + \left|F^-_{\lambda^1{}_V}(x_i) - F^-_{\lambda^2{}_V}(x_i)\right| +$$

$$\left|T^+_{\lambda^1{}_V}(x_i) - T^+_{\lambda^2{}_V}(x_i)\right| + \left|I^+_{\lambda^1{}_V}(x_i) - I^+_{\lambda^2{}_V}(x_i)\right| + \left|F^+_{\lambda^1{}_V}(x_i) - F^+_{\lambda^2{}_V}(x_i)\right|\Big)$$

**Algorithm:**

***Step 1.*** Construct the ideal PNCVS $\mathbb{A}_p{}^v = \langle A_V, \lambda_V\rangle$.

***Step 2.*** Then construct PNCVSs $\mathbb{A}^v_{p_j} = \langle A^j_V, \lambda^j_V\rangle$

***Step 3.*** Calculate the similarities between ideal pattern $\mathbb{A}_p{}^v = \langle A_V, \lambda_V\rangle$ and the sample pattern $\mathbb{A}^v_{p_j} = \langle A^j_V, \lambda^j_V\rangle$ using definition 3.3.2.

***Step 4.*** The sample pattern $\mathbb{A}^v_{p_j}$ is considered to belong to the ideal pattern $\mathbb{A}_p{}^v$ if $S\left(\mathbb{A}_p{}^v, \mathbb{A}^v_{p_j}\right) \leq 0.5$ and sample pattern $\mathbb{A}^v_{p_j}$ is not to be known for an ideal pattern $\mathbb{A}_p{}^v$ if $S\left(\mathbb{A}_p{}^v, \mathbb{A}^v_{p_j}\right) > 0.5$.

**Example 3.3.3:** Here we consider the example (3.1.3) for PNCVSs.

***Step 1.*** Construct an ideal PNCVS $\mathbb{A}_p{}^v = \langle A_V, \lambda_V\rangle$ on X as,

$$\mathbb{A}_p{}^v =$$

$$<\{\frac{\langle[0.4,0.6],[0.5,0.5]),([0.1,0.8],[0.2,0.6]),([0.4,0.6],[0.5.0.5]\rangle}{x_1}, \frac{\langle[0.3,0.8],[0.5,0.6]),([0.3,0.5],[0.4,0.6]),([0.2,0.7],[0.4.0.5]\rangle}{x_2},$$

$$\frac{\langle[0.2,0.6],[0.3,0.6]),([0.4,0.8],[0.2,0.5]),([0.4,0.8],[0.4.0.7]\rangle}{x_3}\},$$

$$\{\frac{[0.2,0.5],[0.3,0.4],[0.5,0.8]}{x_1}, \frac{[0.4,0.7],[0.2,0.3],[0.3,0.6]}{x_2}, \frac{[0.3,0.6],[0.4,0.5],[0.4,0.7]}{x_3}\}>$$

***Step 2.*** Construct PNCVSs $\mathbb{A}^v_{p_j} = \langle A^j_V, \lambda^j_V\rangle$

$$\mathbb{A}^v_{p_1} =$$

$$<\{\frac{\langle[0.2,0.6],[0.3,0.6]),([0.4,0.8],[0.2,0.5]),([0.4,0.8],[0.4.0.7]\rangle}{x_1}, \frac{\langle[0.4,0.6],[0.5,0.5]),([0.3,0.8],[0.2,0.6]),([0.4,0.6],[0.5.0.5]\rangle}{x_2},$$

$$\frac{\langle[0.3,0.8],[0.5,0.6]),([0.3,0.5],[0.4,0.6]),([0.2,0.7],[0.4.0.5]\rangle}{x_3}\},$$

$$\{\frac{[0.1,0.7],[0.3,0.8],[0.2,0.8]}{x_1}, \frac{[0.3,0.7],[0.2,0.9],[0.3,0.6]}{x_2}, \frac{[0.1,0.9],[0.2,0.7],[0.1,0.6]}{x_3}\}>$$

$$\mathbb{A}^v_{p_2} =$$

$$<\{\frac{\langle[0.1,0.7],[0.2,0.4]),([0.4,0.5],[0.3,0.6]),([0.3,0.9],[0.6.0.8]\rangle}{x_1} \frac{\langle[0.4,0.6],[0.1,0.2]),([0.4,0.5],[0.3,0.6]),([0.4,0.6],[0.8.0.9]\rangle}{x_2},$$

$$\frac{\langle[0.3,0.8],[0.5,0.6]),([0.3,0.5],[0.4,0.6]),([0.2,0.7],[0.4.0.5]\rangle}{x_3}\},$$

$$\{\frac{[0.5,0.6],[0.5,0.6],[0.4,0.5]}{x_1}, \frac{[0.1,0.7],[0.3,0.8],[0.2,0.8]}{x_2}, \frac{[0.1,0.5],[0.2,0.8],[0.2,0.5]}{x_3}\}>$$

$$\mathbb{A}^v_{p_3} =$$

$$<\{\frac{\langle[0.3,0.8],[0.5,0.6]),([0.3,0.5],[0.4,0.6]),([0.2,0.7],[0.4.0.5]\rangle}{x_1}, \frac{\langle[0.4,0.6],[0.1,0.2]),([0.4,0.5],[0.3,0.6]),([0.4,0.6],[0.8.0.9]\rangle}{x_2},$$

$$\frac{\langle[0.4,0.6],[0.5,0.5]),([0.3,0.8],[0.2,0.6]),([0.4,0.6],[0.5.0.5]\rangle}{x_3}\},$$

$$\{\frac{[0.1,0.9],[0.2,0.7],[0.1,0.6]}{x_1}, \frac{[0.1,0.7],[0.3,0.8],[0.2,0.8]}{x_2}, \frac{[0.3,0.7],[0.2,0.9],[0.3,0.6]}{x_3}\}>$$

***Step 3.*** Calculate S, the degree of similarity between the ideal pattern $\mathbb{A}_p{}^v$ and the sample pattern $\mathbb{A}_{p_j}^v$, then the results obtained are

$$S(\mathbb{A}_p{}^v, \mathbb{A}_{p_1}^v) = 0.14$$
$$S(\mathbb{A}_p{}^v, \mathbb{A}_{p_2}^v) = 0.19$$
$$S(\mathbb{A}_p{}^v, \mathbb{A}_{p_3}^v) = 0.16$$

***Step 4.*** $S(\mathbb{A}_p{}^v, \mathbb{A}_{p_1}^v) \leq 0.5$, $S(\mathbb{A}_p{}^v, \mathbb{A}_{p_2}^v) \leq 0.5$ and $S(\mathbb{A}_p{}^v, \mathbb{A}_{p_3}^v) \leq 0.5$, the sample pattern whose corresponding PNCVS sets are denoted by $\mathbb{A}_{p_1}^v, \mathbb{A}_{p_2}^v$ and $\mathbb{A}_{p_3}^v$ are known as the similar patterns of the family of ideal pattern whose PNCVS is denoted by $\mathbb{A}_p{}^v$.

## 4. Discussion

Consider the problem given above to demonstrate the advantage of our projected method of Plithogenic Cubic Vague Set (PCVs) comparing to the set proposed by Jun et al. [17]. Cubic set fails to capture the false membership part to measure the alternative in the decision making method, therefore it is not possible to describe the problem. The elements of PCVS are characterized by one or more attributes and it has the ability to handle uncertainties and vague information considering the truth and false membership values. We have considered a simple pattern recognition problem with three sample patterns and an ideal pattern. We assume that $S\left(\mathbb{A}_p{}^v, \mathbb{A}_{p_j}^v\right) \geq 0.5$ is the ideal pattern and the aim is to find which one among the three sample belongs to the ideal pattern. All three sample pattern of Plithogenic Cubic Vague sets (PFCVSs) $(z \equiv F)$, PIFCVSs $(z \equiv IF)$, (PNCVSs $(z \equiv N)$ is recognized as similar patterns of the family of ideal pattern in the above examples.

## 5. Conclusion

In this paper, we projected a method to measure the similarities between two PCVSs (z≡F, z≡IF, z≡N) and studied some of its properties. Examples are provided to prove the application of similarity measure of Plithogenic Fuzzy Cubic Vague Set (PFCVS), Plithogenic Intuitionistic Fuzzy Cubic Vague Set (PIFCVS) and Plithogenic Neutrosophic Cubic Vague Set (PNCVS) separately in pattern recognition problem. In consecutive research, investigation of AND and OR operations, PCVS to groups, rings and its application in other fields will be carried out.

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**Data availability**
The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

**Conflict of interest**
The authors declare that there is no conflict of interest in the research.

**Ethical approval**
This article does not contain any studies with human participants or animals performed by any of the authors.

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