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Foundation of the SuperHyperSoft Set and the Fuzzy Extension SuperHyperSoft Set: A New Vision

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Abstract: We introduce for the first time the SuperHyperSoft Set and the Fuzzy and Fuzzy Extension SuperHyperSoft Set. Through a theorem we prove that the SuperHyperSoft Set is composed from many HyperSoft Sets.

Keywords: Soft Set, HyperSoft Set, SuperHyperSoft Set, Fuzzy Soft Set, Fuzzy SuperHyperSoft Set, Fuzzy Extension Soft Set, Neutrosophic SuperHyperSoft Set, Fuzzy Extension SuperHyperSoft Set.

1. Definition of Soft Set

Let $\mathcal{U}$ be a universe of discourse, $\mathcal{P}(\mathcal{U})$ the power set of $\mathcal{U}$, and a set of attributes A. Then, the pair (F, $\mathcal{U}$), where $F: A \to \mathcal{P}(\mathcal{U})$ is called a **Soft Set** over $\mathcal{U}$ [1].

2. Definition of HyperSoft Set

Let $\mathcal{U}$ be a universe of discourse, $\mathcal{P}(\mathcal{U})$ the power set of $\mathcal{U}$.

Let a_1, a_2, ..., a_n, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets A_1, A_2, ..., A_n, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \dots, n\}$. Then the pair $(F, A_1 \times A_2 \times A_n)$, where:

$F: A_1 \times A_2 \times \ldots \times A_n \to \mathcal{P}(\mathcal{U})$ is called a **HyperSoft Set** over $\mathcal{U}$ [2].

3. Numerical Example of HyperSoft Set

Let $\mathcal{U} = \{x_1, x_2, x_3, x_4\}$ and a set $\mathcal{M} = \{x_1, x_3\} \subset \mathcal{U}$.

Let the attributes be: a_1 = size, a_2 = color, a_3 = gender, a_4 = nationality, and their attributes' values respectively:

Size = A_1 = {small, medium, tall},

Color = A_2 = {white, yellow, red, black},

Gender = A_3 = {male, female},

Nationality = A_4 = {American, French, Spanish, Italian, Chinese}.

Let the function be:

$F: A_1 \times A_2 \times A_3 \times A_4 \to \mathcal{P}(\mathcal{U})$. This is a HyperSoft Set.

Let's assume:

$F(\{\text{tall}, \text{white}, \text{female}, \text{Italian}\}) = \{x_1, x_3\}$, which means that both x_1 and x_3 are: tall, white, female, and Italian.

4. Definition of SuperHyperSoft Set

The **SuperHyperSoft Set** is an extension of the HyperSoft Set. As for the SuperHyperAlgebra, SuperHyperGraph, SuperHyperTopology and in general for SuperHyperStructure and Neutrosophic SuperHyperStructure (that includes indeterminacy) in any field of knowledge, "Super" stands for working on the powersets (instead of sets) of the attribute value sets.

Let $\mathcal{U}$ be a universe of discourse, $\mathcal{P}(\mathcal{U})$ the powerset of $\mathcal{U}$.
Let $a_1, a_2, \ldots, a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $A_1, A_2, \ldots, A_n$, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, n\}$.

Let $\mathcal{P}(A_1), \mathcal{P}(A_2), \ldots, \mathcal{P}(A_n)$ be the powersets of the sets $A_1, A_2, \ldots, A_n$ respectively. Then the pair $(F, \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n)$, where $\times$ meaning Cartesian product, or: $F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n) \rightarrow \mathcal{P}(\mathcal{U})$ is called a SuperHyperSoft Set.

5. Example of SuperHyperSoft Set

If we define the function: $F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \mathcal{P}(A_3) \times \mathcal{P}(A_4) \rightarrow \mathcal{P}(\mathcal{U})$. we get a *SuperHyperSoft Set*.
Let's assume, from the previous example, that:
$F(\{\text{medium, tall}\}, \{\text{white, red, black}\}, \{\text{female}\}, \{\text{American, Italian}\}) = \{x_1, x_2\}$, which means that: $F(\{\text{medium or tall}\} \text{ and } \{\text{white or red or black}\} \text{ and } \{\text{female}\} \text{ and } \{\text{American or Italian}\}) = \{x_1, x_2\}$.

Therefore, the SuperHyperSoft Set offers a larger variety of selections, so x_1 and x_2 may be: either medium, or tall (but not small), either white, or red, or black (but not yellow), mandatory female (not male), and either American, or Italian (but not French, Spanish, Chinese).

In this example there are: Card{medium, tall} · Card{white, red, black} · Card{female} · Card{American, Italian} = 2·3·1·2 =12 possibilities, where Card{ } means cardinal of the set { }.

This is closer to our everyday life, since for example, when selecting something, we have not been too strict, but accepting some variations (for example: medium or tall, white or red or black, etc.).

6. Fuzzy-Extension-SuperHyperSoft Set

$F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n) \rightarrow \mathcal{P}\left(\mathcal{U}\left(x(d^0)\right)\right)$ where $x(d^0)$ is the fuzzy or any fuzzy-extension degree of appurtenance of the element x to the set $\mathcal{U}$.

Fuzzy-Extensions mean all types of fuzzy sets [3], such as: Fuzzy Set, Intuitionistic Fuzzy Set, Inconsistent Intuitionistic Fuzzy Set (Picture Fuzzy Set, Ternary Fuzzy Set), Pythagorean Fuzzy Set (Atanassov's Intuitionistic Fuzzy Set of second type), Fermatean Fuzzy Set, q-Rung Orthopair Fuzzy Set, Spherical Fuzzy Set, n-HyperSpherical Fuzzy Set, Neutrosophic Set, Spherical Neutrosophic Set, Refined Fuzzy/Intuitionistic Fuzzy/Neutrosophic/other fuzzy extension Sets, Plithogenic Set, etc.

7. Example of Fuzzy Extension SuperHyperSoft Set

In the previous example, taking the degree of a generic element $x(d^0)$ as neutrosophic, one gets the Neutrosophic SuperHyperSoft Set.

Assume, that: $F(\{\text{medium, tall}\}, \{\text{white, red, black}\}, \{\text{female}\}, \{\text{American, Italian}\}) = \{x_1(0.7, 0.4, 0.1), x_2(0.9, 0.2, 0.3)\}$.

Which means that: x_1 with respect to the attribute values $(\{\text{medium or tall}\} \text{ and } \{\text{white or red or black}\} \text{ and } \{\text{female}\}, \text{and } \{\text{American or Italian}\})$ has the degree of appurtenance to the set 0.7, the indeterminate degree of appurtenance 0.4, and the degree of non-appurtenance 0.1.

While x_2 has the degree of appurtenance to the set 0.9, the indeterminate degree of appurtenance 0.2, and the degree of non-appurtenance 0.3.

8. Theorem

The SuperHyperSoft Set is equivalent to a union of the HyperSoft Sets.

Proof

Let's consider the SuperHyperSoft:

$$F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n) \rightarrow \mathcal{P}(\mathcal{U})$$

Assume that the non-empty sets

$B_1 \subseteq A_1, B_2 \subseteq A_2, \ldots, B_n \subseteq A_n$ and $F(B_1, B_2, \ldots, B_n) \in P(U)$

$B_1 = \{b_{11}, b_{12}, \ldots\}, B_2 = \{b_{21}, b_{22}, \ldots\}, \ldots, B_n = \{b_{n1}, b_{n2}, \ldots\}$, therefore

$F(\{b_{11}, b_{12}, \ldots\}, \{b_{21}, b_{22}, \ldots\}, \ldots, \{b_{n1}, b_{n2}, \ldots\})$ can be decomposed in many $F(b_{1k_1}, b_{2k_2}, \ldots, b_{nk_n})$ $\in P(U)$ which are actually HyperSoft Sets.

If we reconsider the previous example, then:

({medium *or* tall} *and* {white *or* red *or* black} *and* {female} *and* {American *or* Italian}) produces *12* possibilities:

1. medium, white, female, American;
2. medium, white, female, Italian;
3. medium, red, female, American;
4. medium, red, female, Italian;
5. medium, black, female, American;
6. medium, black, female, Italian;
7. tall, white, female, American;
8. tall, white, female, Italian;
9. tall, red, female, American;
10. tall, red, female, Italian;
11. tall, black, female, American;
12. tall, black, female, Italian.

Whence F of each of them is equal to $\{x_1, x_2\}$, or:

F(medium, white, female, American) = $\{x_1, x_2\}$

F(medium, white, female, Italian) = $\{x_1, x_2\}$

F(tall, black, female, Italian) = $\{x_1, x_2\}$ and all 12 are HyperSoft Sets.

9. Conclusion

A new type of soft set has been introduced, called SuperHyperSoft Set and an application has been presented. Further work to do is to define the operations (union, intersection, complement) of the SuperHyperSoft Sets.

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Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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