



An Efficient Optimal Solution Method for Neutrosophic Transport Models: Analysis, Improvements, and Examples

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Abstract: Transport issues aim to determine the number of units that will be transferred from the production centers to consumption areas so that the cost of transportation is as low as possible, taking into account the conditions of supply and demand. Due to the great importance of these issues and to obtain more accurate results that take into account all circumstances, we conducted two research studies. In the first research, we presented a formulation of neutrosophic transport issues, and in the second research, we presented some ways to find a preliminary solution to these issues, but we do not know whether the preliminary solution is optimal or not, so we will present in this research a study whose purpose is to shed light on some important methods used to improve the optimal solution to transportation issues and then reformulating them using the concepts of neutrosophic science, a science that leaves nothing to chance or circumstances but rather provides solutions with neutrosophic values. Unspecified values take into account the best and worst conditions.

Keywords: Neutrosophic Science; Optimal Solution; Transportation; Stepping-Stone Method; Modified Distribution Method.

1. Introduction

The linear programming method is one of the most important operations research methods that companies and institutions have benefited from in their workflows by building linear models for which science has provided ways to find the optimal solution. Given this importance, we have reformulated these models in two previous studies and found one of the most important ways to solve them using the concepts of neutrosophic science [1, 2]. The issue of transportation is one of the most important issues that have been dealt with using the linear programming method because transportation problems appear frequently in practical life. We need to transfer materials from production centers to consumption centers to secure the regions; we need the lowest possible cost. To solve these recurring and daily problems, we use operations research methods, specifically the linear programming method, where we transform the issue data into a classical linear mathematical model when the data are classical and neutrosophic model when the data are neutrosophic. And in the research [3], there is a full explanation of the neutrosophic transport issues. Since these models are linear neutrosophic, we can get an optimal solution by using the direct simplex neutrosophic method described in the research [1].

But we know that these models have special characteristics, in terms of restrictions and objective function, which enabled scientists and researchers to find special methods, the methods in which we get preliminary solutions, and we explained how to obtain a preliminary solution for the neutrosophic transport in the research [4], and we recall that the neutrosophic transport issues are

transport issues in which the required quantities and the available quantities are neutrosophic values of the form $Na_i \in a_i + \varepsilon_i$, where ε_i the indeterminacy of the produced quantities can take the forms $\varepsilon_i \in [\lambda_{i1}, \lambda_{i2}]$. In addition, the required quantities of neutrosophic values of the form $Nb_j \in b_j + \delta_j$, where δ_j is the indeterminacy of the quantities produced we take it as one of the forms $\delta_j \in [\mu_{j1}, \mu_{j2}]$. and the cost of transportation is neutrosophic values form $Nc_{ij} \in c_{ij} + \alpha_{ij}$, where α_{ij} is the indeterminacy of the cost of transportation, we take it as one of the form $c_{2j} \in \{\alpha_{1,2j}, \alpha_{2,2j}\}$. In order to review the basic concepts of neutrosophic science and its stages of development and the most important topics of operations research that have been reformulated using the concepts of this science, the following research can be found [5-16].

2. Methods

The purpose of this research, as we mentioned in the abstract, is to highlight some of the methods used according to classical logic to improve the preliminary solution to transportation issues, where we will present:

- i. The Stepping-Stone Method
- ii. Modified Distribution Method

As stated in some references [17-20], with a focus on the scientific basis on which these methods were based, and then we will reformulate them using the concepts of neutrosophic science and use them to improve the solution of neutrosophic models after finding a preliminary solution for them using one of the methods mentioned in the reference [4].

2.1 The Stepping-Stone Method

To reach the optimal solution in this way, we follow the following steps:

- i. We find the preliminary solution by one of the three aforementioned methods, then we calculate the total cost according to the preliminary solution.
- ii. We identify the basic variables from the non-basic variables from the preliminary solution table.
- iii. We determine the indirect cost by finding closed paths, as each closed path has its beginning and end as a non-basic variable and consists of horizontal and vertical lines whose pillars are basic variables, as it happens that there are two basic variables in the way of the path, so we deviate from the basic non-basic variable and in general, the closed path represents in the following Figure 1:

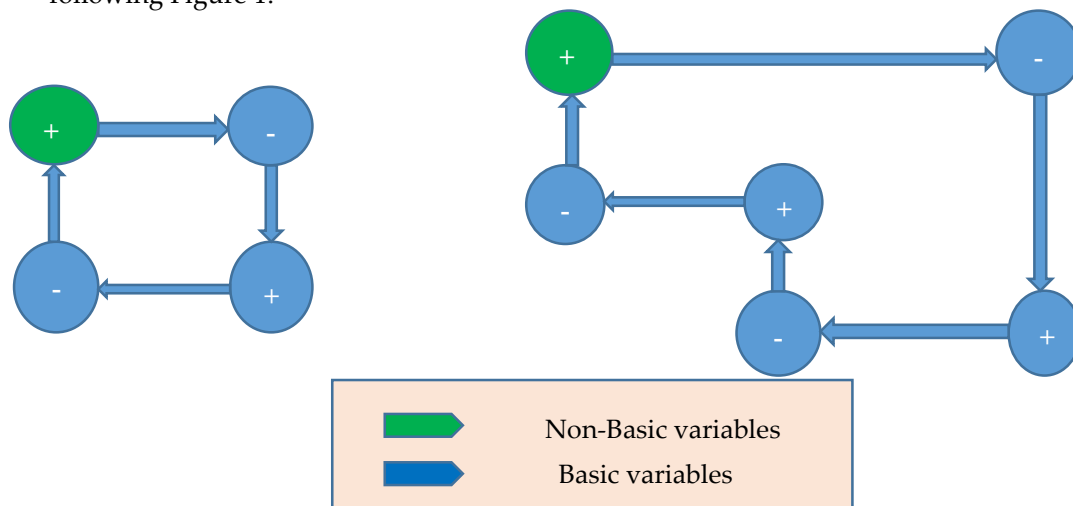


Figure 1. A representation of closed pathways.

We calculate the indirect cost of each non-basic variable by giving the cost of the non-basic variable a positive sign, and the cost of the basic variables we give it alternating negative and then positive signs, and so on. If the basic variables are positive or zero, this means that the solution that

we got is optimal and we stop. But if at least one of the indirect costs is negative, then we must develop the solution by choosing one of the non-basic variables to become basic and the exit of one of the basic variables.

Note:

To determine the basic internal variable, we take the non-basic variable that achieved the most negative in the indirect cost, and to make the solution the best possible, we try to pass in it the largest possible amount, we explain the above through the following example 1:

Example 1:

The following Table 1 represents the cost of transporting goods from sources $A_i; i = 1, 2, 3, 4$ to distribution centers $B_j; j = 1, 2, 3, 4$ it is required to use the mobile quarantine method to improve the solution and obtain the ideal solution:

Table 1. Issue data.

Consumption center Production centers	B_1	B_2	B_3	Available Quantities
A_1	2	4	0	150
A_2	{3, 4.5}	{1, 2}	{5, 8}	200
A_3	6	2	4	325
A_4	1	7	9	25
Required quantities	180	320	200	700

In this example, the cost of transportation of the product at the production center A_2 is neutrosophic values we take in forms $c_{2j} \in \{\alpha_{1,2j}, \alpha_{2,2j}\}$.

The solution:

We find the initial solution using the least cost method; we get the following preliminary solution Table 2.

Table 2. Preliminary solution.

Consumption center Production centers	B_1	B_2	B_3	Available Quantities
A_1	2	4	0 150	150
A_2	{3, 4.5}	{1, 2} 200	{5, 8}	200
A_3	6 155	2 120	4 50	325 170 50
A_4	1 25	7	9	25
Required quantities	180 155	320 120	200 50	700

We note that the number of occupied squares is equal to $m + n - 1 = 6$

The total transportation cost according to the preliminary solution is:

$$Z_1 = 0 \times 150 + \{1,2\} \times 200 + 6 \times 155 + 2 \times 120 + 4 \times 50 + 1 \times 25$$

For $c_{22} = 1 \Rightarrow Z_1 = 1595$

For $c_{22} = 2 \Rightarrow Z_1 = 1795$

That is, against this preliminary solution, we have a neutrosophic value for the total transportation cost:

$$Z_1 \in \{1595, 1795\}$$

Now we see whether this solution is an optimal solution or not?

For this we define basic variables and non-basic variables, it is clear that

The basic variables are:

$$x_{13}, x_{22}, x_{31}, x_{32}, x_{33}, x_{41}$$

The non-basic variables are:

$$x_{11}, x_{12}, x_{21}, x_{23}, x_{42}, x_{43}$$

We have six basic variables and six non-basic variables, so we get six closed paths are formed it is in Figure 2:

Note: The non-basic variables are green.

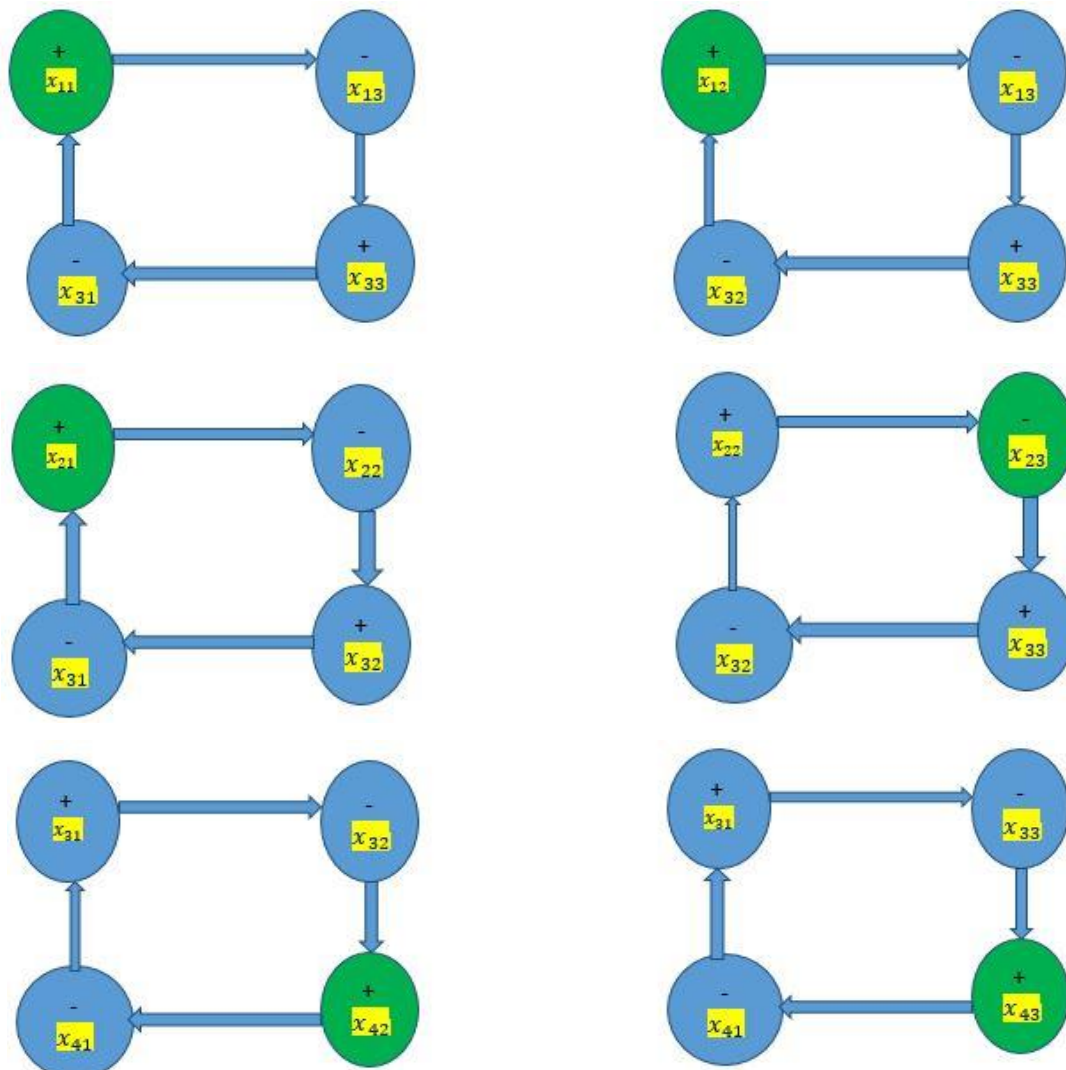


Figure 2. Possible closed paths after finding the initial solution.

The following Table 3, shows how the path is formed:

Table 3. Closed path identification.

Consumption center Production centers	B_1	B_2	B_3	Available Quantities
A_1	2	4	0	150
A_2	{3,4,5}	{1,2}	{5,8}	200
A_3	6	2	4	325
A_4	1	7	9	25
Required quantities	180	320	200	700

We calculate the indirect cost, we find:

From the previous table and according to the drawn path, we explain how to calculate the indirect cost:

We start from the room (A_1B_1) we have the cost $c_{11} = 2$, we take it with a positive sign because it is the room of the non-basic variable, then to the room (A_1B_3) we have the cost $c_{13} = 0$ We take here the minus sign and the variable in this room is a basic variable then to the room (A_3B_3) we have the cost $c_{33} = 4$, we take here the sign is positive, and the variable in this room is a basic variable then to the room (A_3B_1) we have the cost $c_{31} = 6$ We take here the minus sign, and the variable in this room is a basic variable, So the indirect cost of the non-essential variable is x_{11} is :

$$x_{11} : 2 - 0 + 4 - 6 = 0$$

In the same way, we calculate the cost for all non-essential variables we find:

$$x_{12} : 4 - 0 + 4 - 2 = 6$$

$$x_{21} : \{3,4,5\} - \{1,2\} + 2 - 6 = \{-2, -3, -0.5, -1,5\}$$

$$x_{23} : \{5,8\} - 4 + 2 - \{1,2\} = \{2,1,5,4\}$$

$$x_{42} : 7 - 1 + 6 - 2 = 10$$

$$x_{43} : 9 - 1 + 6 - 4 = 10$$

We note that the indirect cost corresponding to the basic variable x_{21} is a negative amount and it is the only one, so we enter this variable and it becomes one of the basic variables and we exit instead of x_{31}

We notice that we can pass the quantity $x_{21} = 155$, then it becomes:

$$x_{31} = 0, \quad x_{32} = 275, \quad x_{22} = 45$$

We get the following Table 4:

Table 4. The first improvement.

Consumption center Production centers	B_1	B_2	B_3	Available Quantities
A_1	2	4	0	150
A_2	{3, 4.5}	{1, 2}	{5, 8}	200
A_3	6	2	4	325
A_4	1	7	9	25
Required quantities	180	320	200	700

We note that the transportation cost is according to the previous solution:

$$Z_2 \in (0 \times 150 + \{3,4.5\} \times 155 + \{1,2\} \times 45 + 2 \times 275 + 4 \times 50 + 1 \times 25)$$

For $c_{21} = 3$ and $c_{22} = 1 \Rightarrow Z_2 = 1285$

For $c_{21} = 3$ and $c_{22} = 2 \Rightarrow Z_2 = 1330$

For $c_{21} = 4.5$ and $c_{22} = 1 \Rightarrow Z_2 = 1517.5$

For $c_{21} = 4.5$ and $c_{22} = 2 \Rightarrow Z_2 = 1562.5$

Therefore:

$$Z_2 \in \{1285, 1330, 1517.5, 1562.5\}$$

$$\forall Z_2 \in \{1285, 1330, 1517.5, 1562.5\} \Rightarrow Z_2 < Z_1 \in \{1595, 1795\}$$

That is, this solution is better than the previous one, the question now is whether this solution is the optimal solution, for this we define the basic variables and the non-basic variables we find:

The basic variables are:

$$x_{41}, x_{33}, x_{32}, x_{22}, x_{21}, x_{13}$$

The non-basic variables are:

$$x_{43}, x_{42}, x_{31}, x_{23}, x_{12}, x_{11}$$

We have six basic variables and six non-basic variables, so we get six closed paths; we form the closed paths for the six non-basic variables in Figure 3:

Note: The non-basic variables are green.

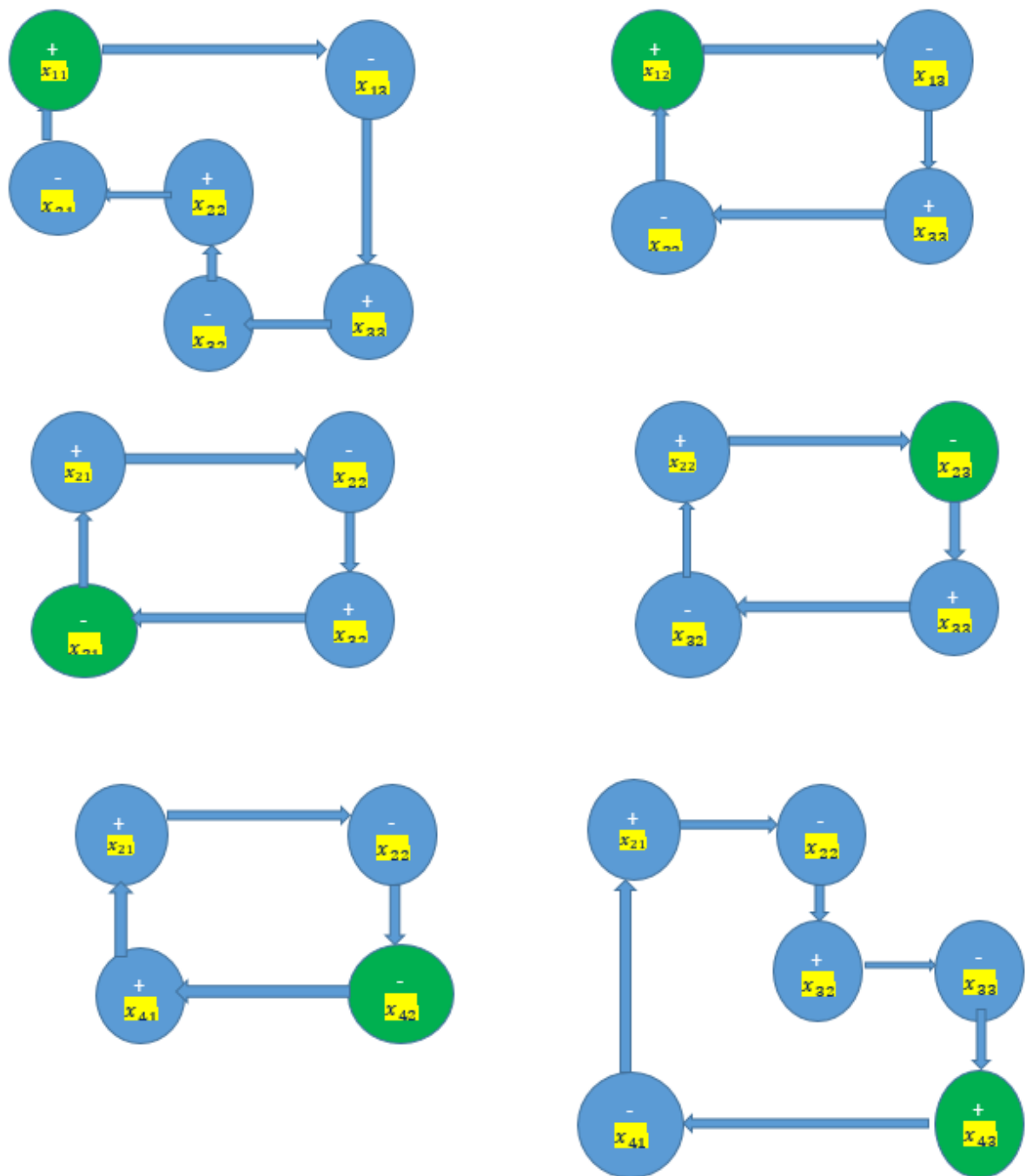


Figure 3. Possible closed paths after the first optimization.

We calculate the indirect cost:

$$x_{11} : 2 - 0 + 4 - \{1,2\} + 1 - 3 = \{3,2\}$$

$$x_{12} : 4 - 0 + 4 - 2 = 8$$

$$x_{32} : 6 - 2 + \{1,2\} - \{3,4,5\} = \{2,0.5,3,1.5\}$$

$$x_{42} : 7 - \{1,2\} + \{3,4,5\} - 1 = \{8,9.5,7,8.5\}$$

$$x_{43} : 9 - 4 + 2 - \{1,2\} + \{3,4,5\} - 1 = \{8,9.5,7,8.5\}$$

We note that the indirect cost for each non-basic variable is positive, and therefore we cannot introduce any non-basic variable to the basic rule. Therefore, the solution that we obtained in the first

improvement is an optimal vinegar, and the minimum transportation cost is the one that we obtained previously:

Therefore the optimal solution is:

$$x_{13} = 150, x_{21} = 155, x_{22} = 45, x_{32} = 275, x_{33} = 50, x_{41} = 25$$

The minimum cost of transportation is:

$$Z_2 \in \{1285, 1330, 1517.5, 1562.5\}$$

2.2 Modified Distribution Method

This method is another method of finding the optimal solution for transportation issues, and it is also similar to the previous method (the mobile stone method) conjunction.

To find the optimal solution to the transportation issue according to this method, we follow the following steps:

- i. We find the initial solution in one of the previously mentioned ways
- ii. We define the essential variables and non-basic variables for the solution
- iii. We associate with each line i multiplied by u_i , and with each column j multiplied, we call it v_j , so it is:

For each basic variable x_{ij} we have:

$$u_i + v_j = c_{ij} \quad (*)$$

Where c_{ij} is the cost from A_i to B_j :

Since the number of basic variables is $m + n - 1$, we get the $m + n - 1$ equation from the previous Figure (*) and by solving these equations we must find the values of u_i, v_j which have $m + n$ so we must give one of these multipliers an optional value, then we solve these equations according to this value.

After we found the values u_i, v_j , for each non-basic variable x_{ij} we calculate the quantities $c_q = c_{ij} - u_i - v_j$

In a similar way to the moving stone method, but if one of these quantities is negative, then we must introduce a non-basic variable to the set of basic variables and output instead of a basic variable, and the primary variable entered is chosen in the same previous way:

Example 2:

Let's take the previous example, where we found the preliminary solution according to the cost method:

Table 5. The preliminary solution.

Consumption center Production centers		v_1	v_2	v_3	Available Quantities
		B_1	B_2	B_3	
u_1	A_1	2	4	0	150
				150	
u_2	A_2	{3,4,5}	{1,2}	{5,8}	200
			200		
u_3	A_3	6	2	4	325
		155	120	50	
u_4	A_4	1	7	9	25
		25			
Required quantities		180	320	200	700 / 700

Transportation cost is: $Z_1 = 1595$

Basic variables:

$$x_{13}, x_{22}, x_{31}, x_{32}, x_{33}, x_{41}$$

Non- basic variables:

$$x_{11}, x_{12}, x_{21}, x_{23}, x_{42}, x_{43}$$

Multipliers is: $u_i ; i = 1,2,3,4$ and $v_j ; j = 1,2,3$.

For basic variables, we have:

For x_{13} , we have $u_1 + v_3 = 0$

For x_{22} , we have $u_2 + v_2 = \{1,2\}$

For x_{31} , we have $u_3 + v_1 = 6$

For x_{32} , we have $u_3 + v_2 = 2$

For x_{33} , we have $u_3 + v_3 = 4$

For x_{41} , we have $u_4 + v_1 = 1$

It is six equations with seven unknowns. To solve them, we impose $u_1 = 0$, so we find the rest of the variables:

$$u_1 = 0, u_2 = 3, u_3 = 4, \quad u_4 = -1$$

$$v_1 = 2, v_2 = -2, v_3 = 0$$

For non-basic variables, we have:

$$x_{11}, x_{12}, x_{21}, x_{23}, x_{42}, x_{43}$$

For x_{11} it is: $\bar{c}_{11} = c_{11} - u_1 - v_1 = 2 - 0 - 2 = 0$

For x_{12} it is: $\bar{c}_{12} = c_{12} - u_1 - v_2 = 4 - 0 + 2 = 6$

For x_{21} it is: $\bar{c}_{21} = c_{21} - u_2 - v_1 = \{3,4,5\} - 3 - 2 = \{-2, -3,5\}$

For x_{23} it is: $\bar{c}_{23} = c_{23} - u_2 - v_3 = \{5,8\} - 3 - 0 = \{2,5\}$

For x_{42} it is: $\bar{c}_{42} = c_{42} - u_4 - v_2 = 7 + 1 + 2 = 10$

For x_{43} it is: $\bar{c}_{43} = c_{43} - u_4 - v_3 = 9 + 1 - 0 = 10$

We note that the quantity $\bar{c}_{21} = -2$ is a negative value, and therefore the initial solution that we got is not optimal, we must develop this solution, and for that, we form the closed path for the non-basic variable x_{21} , so we find it from the Figure 4:

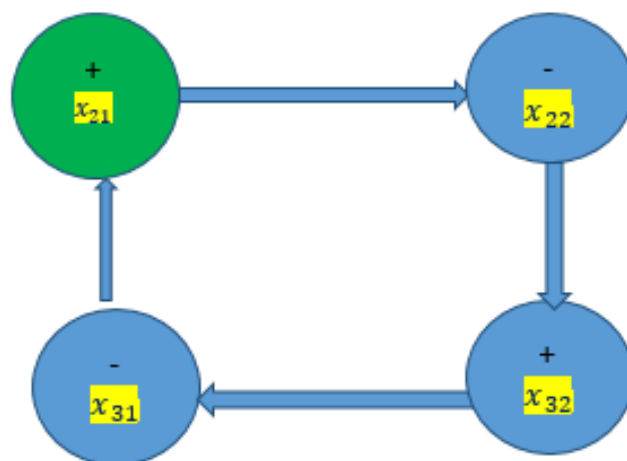


Figure 4. Possible closed pathways for the non-basic variable x_{21} .

We enter x_{21} into the set of basic variables, by giving it the value $x_{21} = 155$, and we remove the variable x_{31} so it becomes a non-basic variable, and then it becomes $x_{22} = 45$ and $x_{32} = 275$, so we get the following Table 6:

Table 6. The first improvement.

Consumption center Production centers		v_1	v_2	v_3	Available quantities
		B_1	B_2	B_3	
u_1	A_1	2	4	0	150
				150	
u_2	A_2	{3,4,5}	{1,2}	{5,8}	200
		155	45		
u_3	A_3	6	2	4	325
			275	50	
u_4	A_4	1	7	9	25
		25			
Required quantities		180	320	200	700 / 700

We enter x_{21} into the set of basic variables, by giving it the value $x_{21} = 155$, and we remove the variable x_{31} so it becomes a non-basic variable, and then it becomes $x_{22} = 45$ and $x_{32} = 275$, so we get the following Table 6:

The new transfer cost is:

$$Z_2 \in \{1285, 1330, 1517.5, 1562.5\}$$

$$\forall Z_2 \in \{1285, 1330, 1517.5, 1562.5\} \Rightarrow Z_2 < Z_1 \in \{1595, 1795\}$$

This solution is better than the previous solution, but is it the optimal solution?

For basic variables, we have:

For x_{13} we have $u_1 + v_3 = 0$

For x_{21} we have $u_2 + v_1 = 3$

For x_{22} we have $u_2 + v_2 = 1$

For x_{32} we have $u_3 + v_2 = 2$

For x_{33} we have $u_3 + v_3 = 4$

For x_{41} we have $u_4 + v_1 = 1$

We assume $u_1 = 0$ and solve the system of equations we find:

$$u_1 = 0, u_2 = 3, u_3 = 4, u_4 = 1$$

$$v_3 = 0, v_2 = -2, v_1 = 0,$$

For non-basic variables:

For x_{11} it is: $\bar{c}_{11} = c_{11} - u_1 - v_1 = 2 - 0 - 0 = 2$

For x_{12} it is: $\bar{c}_{12} = c_{12} - u_1 - v_2 = 4 - 0 + 2 = 6$

For x_{21} it is: $\bar{c}_{21} = c_{21} - u_2 - v_1 = \{3,4,5\} - 3 + 0 = \{0,1,5\}$

For x_{23} it is: $\bar{c}_{23} = c_{23} - u_2 - v_3 = \{5,8\} - 3 + 0 = \{2,5\}$

For x_{42} then: $\bar{c}_{42} = c_{42} - u_4 - v_2 = 7 - 1 + 2 = 8$

For x_{43} it is: $\bar{c}_{43} = c_{43} - u_4 - v_3 = 9 - 1 - 0 = 8$

We note that all quantities \bar{c}_{ij} are positive quantities, so the solution that we got is optimal, and the minimum cost of transportation is:

$$Z_2 \in \{1285, 1330, 1517.5, 1562.5\}$$

$$\forall Z_2 \in \{1285, 1330, 1517.5, 1562.5\} \Rightarrow Z_2 < Z_1 \in \{1595, 1795\}$$

Therefore the optimal solution is:

$$x_{13} = 150, x_{21} = 155, x_{22} = 45, x_{32} = 275, x_{33} = 50, x_{41} = 25$$

The minimum cost of transportation is:

$$Z_2 \in \{1285, 1330, 1517.5, 1562.5\}$$

3. Conclusion

In many practical issues, we encounter cases in which we are unable to provide confirmed data on the reality of the state of the system under study; the data is affected by circumstances surrounding the working environment, and this matter affects the future and may cause large losses. In the example that was presented in this research, we noticed the study gave us a neutrosophic transport cost suitable for all conditions because it was obtained through neutrosophic data.

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Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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