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On b-anti-Open Sets: A Formal Definition, Proofs, and Examples

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Abstract: The concepts of open sets, closed sets, the interior of a set, and the exterior of a set are the most basic concepts in the study of topological spaces in any setting. When we turn our attention to the concept of anti-topological spaces, we encounter analogous fundamental concepts, such as the definition of anti-open sets, anti-closed sets, anti-interior, anti-exterior, etc. These concepts have already been introduced and studied by mathematicians worldwide. In this article, we introduce and study the concepts of b-anti-open set, b-anti-closed set, anti-b-interior, and anti-b-closure in the context of anti-topological spaces and investigate some of their basic properties.

Keywords: b-anti-Open Set; b-anti-Closed Set; b-anti-Interior; b-anti-Closure.

1. Introduction

In the age of artificial intelligence (AI), decision-making assumes a pivotal role within this technological landscape. AI technologies like cognitive computing and machine learning have the capacity to enhance the decision-making process by scrutinizing extensive data sets, identifying patterns, and suggesting the most advantageous solutions. These capabilities prove invaluable for decision-makers grappling with intricate situations, be it in the realm of medical diagnosis or strategic planning.

Many mathematicians from around the world are actively engaged in the development of decision-making theories utilizing the concept of neutrosophic logic. Haque et al. [13, 16] have adeptly employed neutrosophic logic in the formulation of decision-making theories. Furthermore, recent research by Banik et al. [14, 15, 17] has leveraged both fuzzy logic and neutrosophic logic in various modeling applications within the field of agriculture science. Neutrosophic logic also proves valuable in medical science, as exemplified by its application in [17] and several other studies.

Since the introduction of neutrosophic logic in 1995 by Florentin Smarandache [18], along with the subsequent development of neutrosophic topological spaces, various applications of neutrosophic theories have emerged in the literature. Similarly, with the theoretical advancement of anti-topological spaces and anti-algebra, we anticipate similar applications in the near future. Thus, we are also motivated to delve into the study of anti-topological spaces and their associated concepts with the aim of yielding future benefits.

In the year 2021, Şahin et al. [11] introduced the notion of anti-topological spaces. Subsequently, Witczak [12] conducted a comprehensive study on anti-topological spaces, providing valuable insights into the emerging field. In that work, the author introduced the concepts of anti-interior and anti-closure of a set, accompanied by a thorough examination of various properties associated with these notions. Furthermore, the author defined anti-dense sets and anti-nowhere-dense sets, shedding light on their essential properties. Additionally, the concept of anti-continuity was explored within this framework.

Over the years, researchers have introduced and investigated a multitude of open and closed sets [1, 2, 3, 4, 5, 7, 8, 9, 10] within various settings. Witczak [12] extended this line of research by introducing anti-semi-open sets, pseudo-anti-open sets, and anti-genuine sets. More recently, Khaklary and Ray [6] introduced and studied a diverse range of open sets, including anti-pre-open sets, anti-pre-closed sets, regular open sets, regular closed sets, α-open sets, α-closed sets, and more, in the context of anti-topological spaces.

In this article, we further advance the field by introducing the novel concepts of b-anti-open sets and b-anti-closed sets within the realm of anti-topological spaces. We delve into a comprehensive study of their properties, offering fresh insights into this intriguing domain. Figure 1 presents the flowchart of the proposed work.

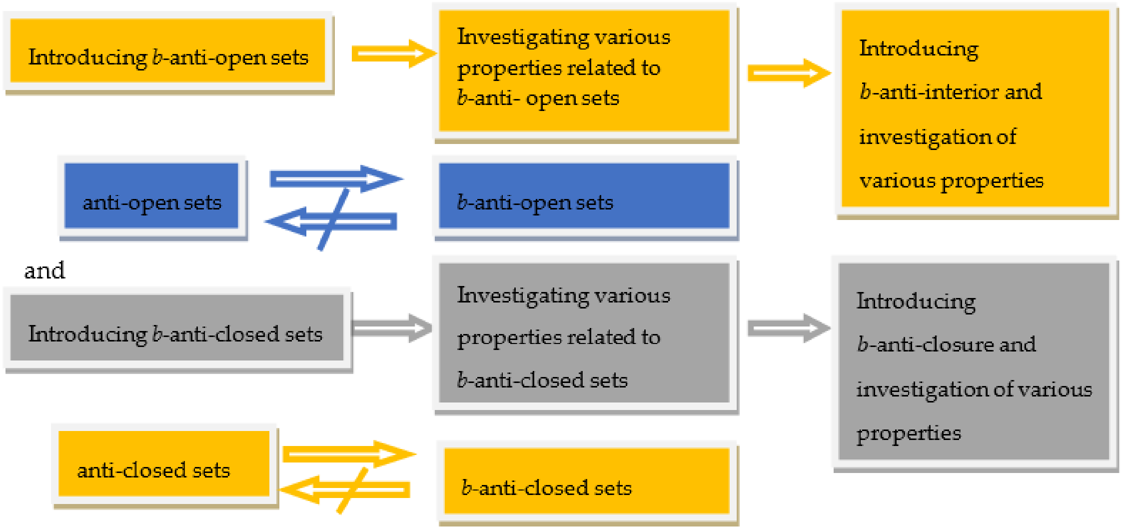

Figure 1. Flowchart of the proposed work.

2. Preliminaries

Definition 2.1: [11] Let X be a non-empty universe and τ be a collection of subsets of X. Then τ is called an anti-topology on X and (X, τ) is called an anti-topological space if the following three conditions are satisfied.

(i) $\varphi, X \notin \tau$
(ii) For all $q_1, q_2, \ldots q_n \in \tau$, then $\bigcap_{i=1}^{n} q_i \notin \tau$ when any n is finite.
(iii) For all $q_1, q_2, \ldots q_n \in \tau$, $\bigcup_{i \in I} q_i \notin \tau$.

Definition 2.2: [12] Let X be a non-empty universe and τ be a collection of subsets of X. We say (X, τ) is an anti-topological space if the following conditions are satisfied.

(i) $\varphi, X \notin \tau$
(ii) For any $n \in \mathbb{N}$, if $A_1, A_2, \ldots A_n \in \tau$, then $\bigcap_{i=1}^{n} A_i \notin \tau$ (with the assumption that the sets in question are not all identical, i.e. the intersection is non-trivial).
(iii) For any collection $\{A_i\}_{i \in J \neq \varphi}$ such that $A_i \in \tau$ for each $i \in J$, $\bigcup_{i \in J} A_i \notin \tau$(with the assumption that the sets in question are not all identical, i.e. the union is non-trivial).

The elements of τ are called anti-open sets, while their complements are anti-closed sets. The set of all anti-closed sets will be denoted by τ_{Cl}. We say that every anti-topology is anti-closed under finite intersections and arbitrary unions (this refers respectively to condition (ii) and condition (iii) above). It is assumed that the property of being anti-closed refers only to non-trivial unions or intersections. The notion of non-trivial family is used to speak about those families of sets which contain at least two (different) sets.

Definition 2.3: [12] Let (X, τ) be an anti-topological space and $A \subseteq X$. Then anti-interior of A, denoted by $aInt(A)$, is defined as $aInt(A) = \bigcup\{U: U \subseteq A \text{ and } U \in \tau\}$.

Definition 2.4: [12] Let (X, τ) be an anti-topological space and $A \subseteq X$. Then anti-closure of A, denoted by $aCl(A)$, is defined as $aCl(A) = \bigcap\{F: A \subseteq F \text{ and } A \in \tau_{Cl}\}$.

Theorem 2.1: Let (X, τ) be an anti-topological space and $A, B \subseteq X$. Then the following hold:

(i) $aInt(A) \subseteq A$
(ii) If $A \in \tau$ then $aInt(A) = A$
(iii) $A \subseteq B$ then $aInt(A) \subseteq aInt(B)$
(iv) $aInt(aInt(A)) = aInt(A)$
(v) $A \subseteq aCl(A)$
(vi) If A is an anti-closed set then $aCl(A) = A$
(vii) $A \subseteq B$ then $aCl(A) \subseteq aCl(B)$
(viii) $aCl(aCl(A)) = aCl(A)$

Definition 2.5: [6] Let (X, τ) be an anti-topological space and $A \subseteq X$. Then A will be called an anti-pre-open set if $A \subseteq aInt(aCl(A))$.

Definition 2.6: [12] Let (X, τ) be an anti-topological space and $A \subseteq X$. Then A will be called an anti-semi-open set if $A \subseteq aCl(aInt(A))$.

3. b-anti-open sets

Definition 3.1: Let (X, τ) be an anti-topological space. A subset A of X will be called a b-anti-open set iff $A \subseteq aInt(aCl(A)) \cup aCl(aInt(A))$.

Example 3.2:

(i) Let $X = \{1,3,5,7,9\}$, $\tau = \{\{3\}, \{1,5,7\}, \{7,9\}\}$. Clearly (X, τ) is an anti-topological space and $\tau_{Cl} = \{\{1,5,7,9\}, \{3,9\}, \{1,3,5\}\}$. Let us take $A = \{1,5,7\} \subseteq X$. Now, $aInt(aCl(A)) \cup aCl(aInt(A)) = aInt(\{1,5,7,9\}) \cup aCl(\{1,5,7\}) = \{1,5,7,9\} \cup \{1,5,7,9\} = \{1,5,7,9\}$. Therefore, $A \subseteq aInt(aCl(A)) \cup aCl(aInt(A))$, i.e., A is a b-anti-open set.

(ii) Let $X = \{1,3,5,7,9\}$, $\tau = \{\{3\}, \{1,5,7\}, \{7,9\}\}$. Clearly, (X, τ) is an anti-topological space and $\tau_{Cl} = \{\{1,5,7,9\}, \{3,9\}, \{1,3,5\}\}$.Let $A = \{1,3,5\} \subseteq X$. Then $aInt(aCl(A)) \cup aCl(aInt(A)) = aInt(\{1,3,5\}) \cup aCl(\{3\}) = \{3\} \cup \{3\} = \{3\}$.Clearly, $A \nsubseteq aInt(aCl(A)) \cup aCl(aInt(A))$.So,$A$ is not a b-anti-open set.

Proposition 3.1: In an anti-topological space, every anti-open set is a b-anti-open set.

Proof: Let (X, τ) be an anti-topological space and $A \subseteq X$ such that A is anti-open. Since A is anti-open, so we have, $A \in \tau \Rightarrow aInt(A) = A$. Now, we have $A \subseteq aCl(A) \Rightarrow aInt(A) \subseteq aInt(aCl(A)) \Rightarrow A \subseteq aInt(aCl(A)) \Rightarrow A \subseteq aInt(aCl(A)) \cup aCl(aInt(A)) \Rightarrow A$ is a b-anti-open set. Thus, every anti-open set is a b-anti-open set.

Remark 3.1: Converse of the prop. 3.1 is not true. We establish it by the following counterexample. Let $X = \{1,2,3,4,5\}$, $\tau = \{\{1\}, \{4\}, \{2,3\}, \{3,5\}\}$. Clearly (X, τ) is an anti-topological space and the anti-closed sets of X are $\{2,3,4,5\}, \{1,2,3,5\}, \{1,4,5\}, \{1,2,4\}$. Let us take $A = \{2,3,4\} \subseteq X$. Clearly, A is not an anti-open set. Now $aInt(aCl(A)) \cup aCl(aInt(A)) = aInt(\{2,3,4,5\}) \cup aCl(\{2,3,4\}) = \{2,3,4,5\} \cup \{2,3,4,5\} = \{2,3,4,5\}$. Clearly, $A \subseteq aInt(aCl(A)) \cup aCl(aInt(A))$ and so, A is a b-anti-open set. Thus A is not a b-anti-open set but not an anti-open set.

Proposition 3.2: In an anti-topological space, union of an arbitrary number of b-anti-open sets is a b-anti-open set.

Proof: Let (X,τ) be an anti-topological space and $\{A_i: i \in \Delta\}$ be an arbitrary collection of b-anti-open sets in X where Δ is an index set. Let $x \in \bigcup_{i\in\Delta} A_i \Rightarrow x \in A_k$, for some $k \in \Delta$. Since A_k is a b-anti-open set, so $A_k \subseteq aInt(aCl(A_k)) \cup aCl(aInt(A_k))$ and so, $x \in aInt(aCl(A_k)) \cup aCl(aInt(A_k))$. Now $A_k \subseteq \bigcup_{i\in\Delta} A_i \Rightarrow aCl(A_k) \subseteq aCl(\bigcup_{i\in\Delta} A_i) \Rightarrow aInt(aCl(A_k)) \subseteq aInt(aCl(\bigcup_{i\in\Delta} A_i))$. Similarly, $aCl(aInt(A_k)) \subseteq aCl(aInt(\bigcup_{i\in\Delta} A_i))$. Therefore, $A_k \subseteq aInt(aCl(A_k)) \cup aCl(aInt(A_k)) \subseteq aInt(aCl(\bigcup_{i\in\Delta} A_i)) \cup aCl(aInt(\bigcup_{i\in\Delta} A_i)) \Rightarrow x \in aInt(aCl(\bigcup_{i\in\Delta} A_i)) \cup aCl(aInt(\bigcup_{i\in\Delta} A_i))$. This gives $\bigcup_{i\in\Delta} A_i \subseteq aInt(aCl(\bigcup_{i\in\Delta} A_i)) \cup aCl(aInt(\bigcup_{i\in\Delta} A_i))$, i.e., $\bigcup_{i\in\Delta} A_i$ is a b-anti-open set. Hence proved.

Remark 3.2: In an anti-topological space, intersection of two b-anti-open sets may not be a b-anti-open set.
Let $X = \{1,2,3,4,5\}, \tau = \{\{1\},\{4\},\{2,3\},\{3,5\}\}$. Clearly (X, τ) is an anti-topological space and the anti-closed sets of X are $\{2,3,4,5\},\{1,2,3,5\},\{1,4,5\},\{1,2,4\}$. Let us consider the subsets $A = \{2,3,4\}$ and $B = \{2,4,5\}$ of X. Obviously A and B are b-anti-open sets. Now $A \cap B = \{2,4\}$ and $aInt(aCl(A \cap B)) \cup aCl(aInt(A \cap B)) = aInt(\{2,4\}) \cup aCl(\{4\}) = \{4\} \cup \{4\} = \{4\}$. Therefore, $A \cap B \nsubseteq aInt(aCl(A \cap B)) \cup aCl(aInt(A \cap B))$, i.e., $A \cap B$ is not a b-anti-open set.

Proposition 3.3: In an anti-topological space,

(i) Every anti-pre-open set is a b-anti-open set.
(ii) Every anti-semi-open set is a b-anti-open set.

Proof:

(i) Let (X,τ) be an anti-topological space and let A be an anti-pre-open subset of X. Then $A \subseteq aInt(aCl(A)) \Rightarrow A \subseteq aInt(aCl(A)) \cup aCl(aInt(A)) \Rightarrow A$ is a b-anti-openset.
(ii) Let (X,τ) be an anti-topological space and let A be an anti-semi-open subset of X. Then $A \subseteq aCl(aInt(A)) \Rightarrow A \subseteq aInt(aCl(A)) \cup aCl(aInt(A)) \Rightarrow A$ is a b-anti-open set.

Definition3.2: Let (X,τ) be an anti-topological space. A subset A of X will be called a b-anti-closed set if $aInt(aCl(A)) \cap aCl(aInt(A)) \subseteq A$.

Example 3.1:

(i) Let $X = \{a,b,c,d,e\}$. Clearly, $\tau = \{\{a\},\{b,c\},\{c,d,e\}\}$ is an anti-topology for X and $\tau_{Cl} = \{\{b,c,d,e\},\{a,d,e\},\{a,b\}\}$. Let us take $A = \{a,b\} \subseteq X$. Then $aInt(aCl(A)) \cap aCl(aInt(A)) = \{a\} \subseteq A$. Therefore, A is a b-anti-closed set.
(ii) Let $X = \{1,2,3,4,5\}$. Clearly, $\tau = \{\{1\},\{4\},\{2,3\},\{3,5\}\}$ is an anti-topology for X and $\tau_{Cl} = \{\{1,2,3,5\},\{1,4,5\},\{1,2,4\}\}$. Let us take $B = \{1,3,5\} \subseteq X$. Then $aInt(aCl(B)) \cap aCl(aInt(B)) = \{1,2,3,5\} \nsubseteq B$. Therefore, B is not a b-anti-closed set.

Proposition 3.4: Let (X,τ) be an anti-topological space and $A \subseteq X$. Then A is a b-anti-open set iff A^c is a b-anti-closed set.

Proof: A is a b-anti-open set

$$\Leftrightarrow A \subseteq aInt(aCl(A)) \cup aCl(aInt(A))$$
$$\Leftrightarrow A^c \supseteq [aInt(aCl(A)) \cup aCl(aInt(A))]^c$$
$$\Leftrightarrow A^c \supseteq [aInt(aCl(A))]^c \cap [aCl(aInt(A))]^c$$
$$\Leftrightarrow A^c \supseteq [aCl(aCl(A))^c] \cap [aInt(aInt(A))^c]$$
$$\Leftrightarrow A^c \supseteq aCl(aInt(A^c)) \cap aInt(aCl(A^c))$$
$$\Leftrightarrow A^c \text{ is a } b\text{-anti-closed set. Hence proved.}$$

Proposition 3.5: In an anti-topological space, every anti-closed set is a b-anti-closed set.

Proof: Let (X,τ) be an anti-topological space and $A \subseteq X$ such that A is anti-closed. Then A^c is anti-open set and from the proposition 3.1, it follows that A^c is a b-anti-open set. Therefore, by the proposition 3.4, A is a b-anti-closed set. Hence proved.

Remark 3.3: Converse of the prop. 3.5 is not true. We establish it by the following counterexample. Let $X=\{1,2,3,4,5\}$. Clearly, $\tau=\{\{1\},\{4\},\{2,3\},\{3,5\}\}$ is an anti-topology for X and $\tau_{Cl}=\{\{1,2,3,5\},\{1,4,5\},\{1,2,4\}\}$. Let us take $A=\{1,5\} \subseteq X$. Obviously A is not an anti-closed set. Now $aInt(aCl(A)) \cap aCl(aInt(A))=\{1\} \subseteq A$. Therefore, A is a b-anti-closed set. Thus A is a b-anti-closed set but not an anti-closed set.

Proposition 3.6: In an anti-topological space,

(i) Every anti-pre-closed set is a b-anti-closed set.

(ii) Every anti-semi-closed set is b-anti-closed set.

Proof:

(i) Let A be an anti-pre-closed subset of X. Then $aCl(aInt(A)) \subseteq A \Rightarrow aCl(aInt(A)) \cap aInt(aCl(A)) \subseteq A \Rightarrow A$ is a b-anti-closed set.

(ii) Let A be an anti-semi-closed subset of X. Then $aInt(aCl(A)) \subseteq A \Rightarrow aInt(aCl(A)) \cap aInt(aCl(A)) \subseteq A \Rightarrow A$ is a b-anti-closed set.

Proposition 3.7: In an anti-topological space, intersection of arbitrary number of b-anti-closed sets is b-anti-closed.

Proof: Let (X,τ) be an anti-topological space and $\{A_i : i \in \Delta\}$ be an arbitrary collection of b-anti-closed sets in X where Δ is an index set. Then ${A_i}^c$ is a b-anti-open set for each $i \in \Delta \Rightarrow \bigcup_{i\in\Delta} {A_i}^c$ is a b-anti-open set [by prop.3.2] $\Rightarrow (\bigcap_{i\in\Delta} A_i)^c$ is a b-anti-open set $\Rightarrow \bigcap_{i\in\Delta} A_i$ is an$b$-anti-closed set. Hence proved.

Remark 3.4: In an anti-topological space, union of two b-anti-closed sets may not be a b-anti-closed set. We establish it by the following counterexample:
Let $X=\{1,2,3,4,5\}$. Clearly, $\tau=\{\{1\},\{4\},\{2,3\},\{3,5\}\}$ is an anti-topology for X and $\tau_{Cl}=\{\{1,2,3,5\},\{1,4,5\},\{1,2,4\}\}$. Let us take $A=\{1,5\}$and $B=\{1,3\} \subseteq X$. Clearly, A and B are two b-anti-closed sets in X. Now $A \cup B=\{1,3,5\}$ and $aInt(aCl(A\cup B)) \cap aCl(aInt(A\cup B))=\{1,2,3,5\} \nsubseteq A \cup B$. Therefore, $A \cup B$ is not a b-anti-closed set. Thus, the union of two b-anti-closed sets may not be a b-anti-closed set.

Definition 3.3: Let (X,τ) be an anti-topological space and $A \subseteq X$. Then the b-anti-interior of A, denoted by $b-aInt(A)$, is defined as $b-aInt(A)=\bigcup\{G: G \text{ is } b-\text{anti}-\text{open set in } X \text{ and } G \subseteq A\}$.

Proposition 3.8: Let (X,τ) be an anti-topological space and $A \subseteq X$. Then the following hold:

(i) $b-aInt(A)$ is a b-anti-open set.

(ii) $b-aInt(A) \subseteq A$.

(iii) A is b-anti-open set iff $b-aInt(A)=A$.

(iv) $b-aInt(b-aInt(A))= b-aInt(A)$.

Proof:

(i) Since $b-aInt(A)=\bigcup\{G: G \text{ is } b-\text{anti}-\text{open set in } X \text{ and } G \subseteq A\}$ and union of arbitrary number of b-anti-open sets is a b-open set, so $b-aInt(A)$ is a b-anti-open set.

(ii) Since $b-aInt(A)$ is the union of all b-anti-open sets contained in A, so $b-aInt(A) \subseteq A$.

(iii) Let A be a b-anti-open set. Since $b-aInt(A)$ is the union of all b-anti-open sets which are contained in A and since, A is a b-anti-open set contained in A, so $A \subseteq b-aInt(A)$. Also from (ii), $b-aInt(A) \subseteq A$. Therefore, $b-aInt(A)=A$. Conversely let $b-aInt(A)=A$. Since $b-aInt(A)$ is a b-anti-open set, so A is also a b-anti-open set.

(iv) Since $b - aInt(A)$ is a b-anti-open set, so by (iii), $b - aInt(b - aInt(A)) = b - aInt(A)$.

Proposition 3.9: Let (X, τ) be an anti-topological space and A, B be subsets of X. Then the following hold:

(i) $A \subseteq B \Rightarrow b - aInt(A) \subseteq b - aInt(B)$.

(ii) $b - aInt(A \cup B) \supseteq b - aInt(A) \cup b - aInt(B)$.

(iii) $b - aInt(A \cap B) \subseteq b - aInt(A) \cap b - aInt(B)$.

Proof:

(i) We have $x \in b - aInt(A) \Rightarrow x \in \bigcup\{G: G \text{ is } b - \text{anti} - \text{open set in } X \text{ and } G \subseteq A\} \Rightarrow x \in \bigcup\{G: G \text{ is } b - \text{anti} - \text{open set in } X \text{ and } G \subseteq B\}$ ($\because A \subseteq B$) $\Rightarrow x \in b - aInt(B)$.

(ii) $A \subseteq A \cup B \Rightarrow b - aInt(A) \subseteq b - aInt(A \cup B)$. Similarly, $b - aInt(B) \subseteq b - aInt(A \cup B)$. Therefore, $b - aInt(A \cup B) \supseteq b - aInt(A) \cup b - aInt(B)$.

(iii) $A \cap B \subseteq A \Rightarrow b - aInt(A \cap B) \subseteq b - aInt(A)$. Similarly, $b - aInt(A \cap B) \subseteq b - aInt(B)$. Therefore, $b - aInt(A \cap B) \subseteq b - aInt(A) \cap b - aInt(B)$.

Definition 3.4: Let (X, τ) be an anti-topological space and $A \subseteq X$. Then the b-anti-closure of A, denoted by $b - aCl(A)$, is defined as $b - aCl(A) = \bigcap\{G: G \text{ isa } b - \text{anti} - \text{closed set in } X \text{ and } A \subseteq G\}$.

Proposition 3.10: Let (X, τ) be an anti-topological space and $A \subseteq X$. Then the following hold:

(i) $b - aCl(A)$is a $b - \text{anti} - \text{closed}$ set.

(ii) $A \subseteq b - aCl(A)$.

(iii) A is b-anti-closed set iff $b - aCl(A) = A$.

(iv) $b - aCl(b - aCl(A)) = b - aCl(A)$.

Proof:

(i) Since $b - aCl(A) = \bigcap\{G: G \text{ is a } b - \text{anti} - \text{closed set in } X \text{ and } A \subseteq G\}$ and intersection of arbitrary number of b-anti-closed sets is a b-anti-closed, so $b - aCl(A)$ is a b-anti-closed set.

(ii) Since $b - aCl(A)$ is the intersection of all b-anti-closed sets containing$A$, so $A \subseteq b - aCl(A)$.

(iii) Let A be a b -anti-closed set. Since $b - aCl(A) = \bigcap\{G: G \text{ is a } b - \text{anti} - \text{closed set in } X \text{ and } A \subseteq G\}$ and since A is also a b-anti-closed set so, $A \in \{G: G \text{ is a } b - \text{anti} - \text{closed set in } X \text{ and } A \subseteq G\}$ and therefore, $b - aCl(A) \subseteq A$. Also from (ii), $A \subseteq b - aCl(A)$. Hence, $b - aCl(A) = A$. Conversely, suppose that $b - aCl(A) = A$. Since $b - aCl(A)$ is a b-anti-closed set, so A is also a b-anti-closed set.

(iv) Since $b - aCl(A)$ is a b-anti-closed, so by (iii), $b - aCl(b - aCl(A)) = b - aCl(A)$.

Proposition 3.11: Let (X, τ) be an anti-topological space and A, B be subsets of X. Then the following hold.

(i) $A \subseteq B \Rightarrow b - aCl(B) \subseteq b - aCl(A)$.

(ii) $b - aCl(A \cup B) \subseteq b - aCl(A) \cup b - aCl(B)$.

(iii) $b - aCl(A \cap B) \supseteq b - aCl(A) \cap b - aCl(B)$

Proof:

(i) We have $x \in b - aCl(B) \Rightarrow x \in \bigcap\{G: G \text{ is } b - \text{anti} - \text{closed set in } X \text{ and } B \subseteq G\} \Rightarrow x \in \bigcap\{G: G \text{ is } b - \text{anti} - \text{closed set in } X \text{ and } A \subseteq G\}$ ($\because A \subseteq B$) $\Rightarrow x \in b - aCl(A)$. Hence $b - aCl(B) \subseteq b - aCl(A)$.

(ii) $A \subseteq A \cup B \Rightarrow b - aCl(A \cup B) \subseteq b - aCl(A)$. Similarly,$b - aCl(A \cup B) \subseteq b - aCl(B)$. Therefore, $b - aCl(A \cup B) \subseteq b - aCl(A) \cup b - aCl(B)$.

(iii) $A \cap B \subseteq A \Rightarrow b - aCl(A) \subseteq b - aCl(A \cap B)$. Similarly, $b - aCl(B) \subseteq b - aCl(A \cap B)$. Therefore, $b - aCl(A \cap B) \supseteq b - aCl(A) \cap b - aCl(B)$.

Proposition 3.12: Let (X,τ) be an anti-topological space and A,B be two subsets of X. Then the following hold:

(i) $\left(b-aCl(A)\right)^{c}=b-aInt(A^{c})$.

(ii) $b-aCl(A^{c})=\left(b-aInt(A)\right)^{c}$

Proof:

(i) We have $x\in\left(b-aCl(A)\right)^{c}$

$\Rightarrow x\in(\bigcap\{G: G \text{ is a } b-\text{anti}-\text{closed set in } X \text{ and } A\subseteq G\})^{c}$

$\Rightarrow x\in\bigcup\{G^{c} : G^{c} \text{ is a } b-\text{anti}-\text{open set in } X \text{ and } G^{c}\subseteq A^{c}\}$

$\Rightarrow x\in b-aInt(A^{c})$.

Hence, $\left(b-aCl(A)\right)^{c}\subseteq b-aInt(A^{c})$.

Again, let $x\in b-aInt(A^{c})$

$\Rightarrow x\in\bigcup\{G: G \text{ is } b-\text{anti}-\text{open set in } X \text{ and } G\subseteq A^{c}\}$

$\Rightarrow x\in(\bigcap\{G^{c} : G^{c} \text{ is } b-\text{anti}-\text{closed set in } X \text{ and } A\subseteq G^{c}\})^{c}$

$\Rightarrow x\in\left(b-aCl(A)\right)^{c}$.

Therefore, $b-aInt(A^{c})\subseteq\left(b-aCl(A)\right)^{c}$.

and so, $b-aInt(A^{c})=\left(b-aCl(A)\right)^{c}$.

(ii) Replacing A by A^{c} in (i), we get $\left(b-aCl(A^{c})\right)^{c}=b-aInt((A^{c})^{c})$

$\Rightarrow\left(b-aCl(A^{c})\right)^{c}=b-aInt(A)\Rightarrow b-aCl(A^{c})=\left(b-aInt(A)\right)^{c}$.

4. Discussion

In this study, we explored the intricate relationships within anti-topological spaces, shedding light on the properties of *b*-anti-open sets and *b*-anti-closed sets. Notably, the observation that every anti-open (resp. anti-closed) set is a *b*-anti-open (resp. *b*-anti-closed) set suggests a broader characterization of *b*-anti-open sets (*b*-anti-closed sets). The exploration also uncovered nuanced aspects, such as the non-preservation of the *b*-anti-open property under the intersection of two *b*-anti-open sets, challenging conventional notions. Counterexamples, particularly the non-closure of unions of *b*-anti-closed sets, highlighted the counterintuitive nature of these spaces, prompting careful consideration in their analysis. The study further revealed intriguing properties regarding closure and interior operations. The observed reversal of conventional inclusions in the closure operation introduces a noteworthy departure from typical topological expectations. Unlike the standard relationship where the closure of a subset is contained within the closure of its superset, our findings reveal a reversal: if *A* is a subset of *B* then the *b*-anti-closure of *B* is a subset of the *b*-anti-closure of *A*. This counterintuitive result challenges the conventional understanding of closure operations and prompts a reevaluation of the underlying principles governing these relationships. Similarly, the outcomes concerning the closure of unions and intersections add another layer of complexity. These findings deepen our understanding of anti-topological spaces, revealing their complexities and inviting further exploration into their properties and applications.

5. Conclusion

In this article, we have introduced the concepts of b-anti-open sets and b-anti-closed sets in connection with anti-topological spaces and then explored their fundamental properties. Furthermore, we have defined the b-anti-interior and b-anti-closure of a set, delving into an in-depth analysis of their associated properties. From the above discussion, we have found that classes of b-anti-open sets and b-anti-closed sets in anti-topological spaces are finer than classes of anti-open sets and anti-closed sets, respectively. Also, the deviations from standard topological expectations signify the unique characteristics of the anti-topological space under consideration.

As we move forward, our future research endeavors will aim to investigate novel concepts and ideas related to anti-topological spaces. We anticipate that the insights presented in this article will contribute to the advancement of various facets within the field of anti-topological spaces, aiding researchers in their exploration and development of this intriguing domain.

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Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

References

1. Azad, K. K. (1981). On fuzzy semicontinuity, fuzzy almost continuity and fuzzy weakly continuity. Journal of mathematical analysis and applications, 82(1), 14-32. https://doi.org/10.1016/0022-247X(81)90222-5.
2. Alabdulsada, L. M. H. (2019). On B-Open Sets. European Journal of Pure and Applied Mathematics, 12(2), 358–369. https://doi.org/10.29020/nybg.ejpam.v12i2.3401.
3. Benchalli, S. S., & Karnel, J. J. (2010). On fuzzy b-open sets in fuzzy topological spaces. Journal of Computer and Mathematical Sciences, 1(2), 103-273.
4. Ebenanjar, P. E., Immaculate, H. J., & Wilfred, C. B. (2018, December). On neutrosophic b-open sets in neutrosophic topological space. In Journal of Physics: Conference Series (Vol. 1139, No. 1, p. 012062). IOP Publishing. https://doi.org/10.1088/1742-6596/1139/1/012062.
5. Iswarya, P., & Bageerathi, K. (2016). On neutrosophic semi-open sets in neutrosophic topological spaces. International Journal of Math. Trends and Tech., 37(3), 214-223. https://doi.org/10.14445/22315373/IJMTT-V37P529.
6. Khaklary, J. K., & Ray, G. C. (2023). A new class of NeutroOpen, NeutroClosed, AntiOpen and AntiClosed sets in NeutroTopological and AntiTopological spaces. International Journal of Neutrosophic Science (IJNS), 20(2). https://doi.org/10.54216/IJNS.200206.
7. Levine, N. (1963). Semi-open sets and semi-continuity in topological spaces. The American mathematical monthly, 70(1), 36-41. https://doi.org/10.1080/00029890.1963.11990039.
8. Mashhour, A. S. (1982). On precontinuous and weak precontinuous mappings. In Proc. Math. Phys. Soc. Egypt. (Vol. 53, pp. 47-53). https://doi.org/10.1080/00029890.1963.11990039.
9. Rao, V. V., & Rao, Y. S. (2017). Neutrosophic Pre-open sets and Pre-closed sets in Neutrosophic Topology. International Journal of Chem Tech Research, 10(10), 449-458.
10. Shahna, A. S. B. (1991). On fuzzy strong semicontinuity and fuzzy precontinuity. Fuzzy sets and systems, 44(2), 303-308. https://doi.org/10.1016/0165-0114(91)90013-G.
11. Şahin, M., Kargın, A., & Yücel, M. (2021). Neutro-topological space and anti-topological space. NeutroAlgebra Theory, 1, 16.
12. Witczak, T. (2022). Some remarks on anti-topological spaces. arXiv preprint arXiv:2202.10013, Asia Mathematika, 6(1), 23-34. https://doi.org/10.48550/arXiv.2202.10013.

13. Haque, T. S., Chakraborty, A., Alrabaiah, H., & Alam, S. (2022). Multiattribute decision-making by logarithmic operational laws in interval neutrosophic environments. Granular Computing, 7(4), 837-860. https://doi.org/10.1007/s41066-021-00299-7.
14. Banik, B., Alam, S., & Chakraborty, A. (2023). Comparative study between GRA and MEREC technique on an agricultural-based MCGDM problem in pentagonal neutrosophic environment. International Journal of Environmental Science and Technology, 1-16. https://doi.org/10.1007/s13762-023-04768-1.
15. Banik, B., Alam, S., & Chakraborty, A. (2023). A novel integrated neutrosophic cosine operator based linear programming ANP-EDAS MCGDM strategy to select anti-pegasus software. International Journal of Information Technology & Decision Making, 1-37. https://doi.org/10.1142/S0219622023500529.
16. Haque, T. S., Chakraborty, A., Mondal, S. P., & Alam, S. (2020). Approach to solve multi-criteria group decision-making problems by exponential operational law in generalised spherical fuzzy environment. CAAI Transactions on Intelligence Technology, 5(2), 106-114. https://doi.org/10.1049/trit.2019.0078.
17. Haque, T. S., Alam, S., & Chakraborty, A. (2022). Selection of most effective COVID-19 virus protector using a novel MCGDM technique under linguistic generalised spherical fuzzy environment. Computational and Applied Mathematics, 41(2), 84. https://doi.org/10.1007/s40314-022-01776-8.
18. Smarandache, F. (2010). Neutrosophic logic-a generalization of the intuitionistic fuzzy logic. Multispace & multistructure. Neutrosophic transdisciplinarity (100 collected papers of science), 4, 396.

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