Neutrosophic Systems with Applications, Vol. 14, 2024
https://doi.org/10.61356/j.nswa.2024.104

Fuzzy Inference Full Implication Method Based on Single Valued Neutrosophic t-representable t-norm: Purposes, Strategies, and a Proof-of-Principle Study

Minxia Luo [1,*], Ziyang Sun [1], Donghui Xu [1], and Lixian Wu [1]

[1] Department of Information and Computing Science, China Jiliang University, Hangzhou, 310018, People's Republic of China.

Emails: mxluo@cjlu.edu.cn; s21080701014@cjlu.edu.cn; s1908070110@cjlu.edu.cn; s1608070106@cjlu.edu.cn.

* Correspondence: mxluo@cjlu.edu.cn.

Abstract: As a generalization of intuitionistic fuzzy sets, single-valued neutrosophic sets have certain advantages in solving indeterminate and inconsistent information. In this paper, we study the fuzzy inference full implication method based on single-valued neutrosophic t-representable t-norm. Firstly, single-valued neutrosophic fuzzy inference triple I principles for fuzzy modus ponens and fuzzy modus tollens are given. Then, single-valued neutrosophic R-type triple I solutions for FMP and FMT are given. Finally, the robustness of the full implication triple I method based on the left-continuous single-valued neutrosophic t-representable t-norm is investigated. As a special case of the main results, the sensitivity of full implication triple I solutions based on three special single-valued neutrosophic t-representable t-norms are given.

Keywords: Single Valued Neutrosophic Set; Single Valued Neutrosophic; t-representable t-norm; Full Implication Triple I Method.

1. Introduction

Fuzzy sets have been applied to deal with uncertain, vague, inaccurate information in the real world. However, it is widely known that fuzzy reasoning plays an important role in fuzzy set theory. Especially, the most basic forms of fuzzy reasoning are Fuzzy Modus Ponens (FMP for short) and Fuzzy Modus Tollens (FMT for short), which can be shown as follows [1, 2]:

FMP (A, B, A^*): given the fuzzy rule and premise A^*, attempt to reason a suitable fuzzy consequent B^*.

FMT (A, B, B^*): given the fuzzy rule and premise B^*, attempt to reason a suitable fuzzy consequent A^*.

In the above models, and $B, B^* \in F(Y)$, where and denote fuzzy subsets of the universes and respectively.

The most famous method to solve the above models is the Compositional Rule of Inference (CRI for short), which is presented by Zadeh [2, 3]. However, the CRI method lacks clear logic semantics and reductivity. To overcome this shortcoming, Wang [1] proposed the fuzzy reasoning full implication triple I method, which can bring fuzzy reasoning into the framework of logical semantic [4]. In recent years, many scholars have studied the fuzzy reasoning full implication method. Wang et al. [5] gave a unified form for fuzzy reasoning full implication method based on normal implication and regular implication. Pei [6] gave a unified form fuzzy reasoning full implication method based on residual implication induced by left continuous t-norms. Moreover, Pei [7] established the solid logical foundation for the fuzzy reasoning full implication method based on left continuous t-norms.

Liu et al. [8] gave the unified form of the solutions for fuzzy reasoning full implication method. Luo and Yao [9] studied the fuzzy reasoning triple I method based on Schweizer-Sklar operators.

Although fuzzy set theory has been successfully applied in many fields, there are some defects in dealing with fuzzy and incomplete information. Atanassov [10] introduced intuitionistic fuzzy sets (IFSs), which are represented by a membership and a non-membership function. Intuitionistic fuzzy sets can represent not only the positive and negative aspects of the given information but also the hesitant information. Meanwhile, Gorzalczany [11] and Turksen [12] proposed interval-valued fuzzy sets, which represent a subinterval in the membership function. Intuitionistic fuzzy sets and interval-valued fuzzy sets are equivalent [13]. In recent years, some research results on intuitionistic fuzzy reasoning and interval-valued fuzzy reasoning have been achieved. Zheng et al. [14] extended the triple I method on intuitionistic fuzzy sets. Li et al. [15] extended the CRI method on interval-valued fuzzy sets. Luo et al. [16-19] studied interval-value fuzzy reasoning full implication triple I method and reverse triple I method based on the interval-valued associated t-norm. Moreover, Luo et al.[20] studied fuzzy reasoning triple I method based on the interval-value t-representable t-norm.

Although an intuitionistic fuzzy set has some advantages in dealing with fuzzy and incomplete information, it has defects in dealing with fuzzy, incomplete, and inconsistent information. To deal with this case, Smarandache [21] proposed a neutrosophic set, which is represented by a truth-membership function, an indeterminacy-membership function, and a falsity-membership function. The neutrosophic set represents uncertain, incomplete, and inconsistent information in the real world. However, truth-membership, indeterminacy-membership, and falsity-membership functions are nonstandard fuzzy subsets, which are difficult to apply in practice. Smarandache [22] and Wang et al [23] proposed a single-valued neutrosophic set, the truth-membership, indeterminacy-membership, and falsity-membership degrees are a real number in the unit interval [0,1]. The single-valued neutrosophic set can be considered as a generalization intuitionistic fuzzy set. In recent years, Scholars have paid attention to the study of single-valued neutrosophic sets. Smarandache [21] studied a unifying field in logic. Smarandache [24] proposed n-norm and n-conorm in neutrosophic logic. Rivieccio [25] investigated neutrosophic logic. Alkhazaleh [26] gives some norms and conforms based on the neutrosophic set. Zhang et al. [27] gave a new inclusion relation for neutrosophic sets. Hu and Zhang [28] constructed the residuated lattices based on the neutrosophic t-norms and neutrosophic residual implications. So far, there is little research on fuzzy reasoning methods based on single-valued neutrosophic sets. In [29], Ghorai et al. studied the operations of the Cartesian product, composition, and union of two image fuzzy digraphs. In [30], Ghorai et al. proposed a bipolar fuzzy incidence graph and analyzed the properties of a bipolar fuzzy incidence graph. In [31], Ghorai et al. analyzed the properties of the complexity function and its importance in the network field and applied the complexity function to identify the period of COVID-19. Zhao et al. [32] study reverse triple I algorithms based on single-valued neutrosophic fuzzy inference.

Therefore, we consider researching the fuzzy reasoning triple I method based on a class single valued neutrosophic triangular norm. An important criterion for judging an algorithm is whether the algorithm has a logical basis. Therefore, this paper proposes a logic-based fuzzy reasoning algorithm based on a class single valued neutrosophic triangular norm. The algorithm proposed in this paper is a new neutrosophic set fuzzy inference algorithm with a logical basis.

1.1 The organization of the work

The organization of this paper is as follows: some basic concepts for single-valued neutrosophic sets are reviewed in section 2. In section 3, we give fuzzy inference triple I principles based on left-continuous single-valued neutrosophic t-representable t-norms for fuzzy modus ponens and fuzzy modus tollens, and the corresponding solutions of single-valued neutrosophic triple I methods. In section 4, the robustness of the triple I method based on left-continuous single-valued neutrosophic t-representable t-norm is investigated. Finally, the conclusions are given in Section 5.

2. Preliminaries

In this section, we review some basic concepts for triangular norm, triangular conorm, and single-valued neutrosophic set, which will be used in this article.

Definition 2.1. [33] A mapping T: $[0,1]\times[0,1]\to[0,1]$ is called a triangular norm (t-norm), if it satisfies associativity, commutativity, monotonicity, and boundary condition $T(x,1)$=x for any $x\in[0,1]$. A mapping S: is called a triangular conorm (t-conorm), if it satisfies associativity, commutativity, monotonicity, and boundary condition $S(x,0)$=x for any $x\in[0,1]$. A t-norm is called the dual t-norm of the t-conorm if $T(x,y)=1-S(1-x,1-y)$. Similarly, a t-conorm is called the dual t-conorm of the t-norm, if $S(x,y)=1-T(1-x,1-y)$.

Definition 2.2. [33] A t-norm T is called left-continuous (resp., right-continuous), if for any $(x_0,y_0)\in[0,1]^2$, and for each $\varepsilon>0$ there is a $\delta>0$ such that $T(x,y)>T(x_0,y_0)-\varepsilon$, whenever $(x,y)\in(x_0-\delta,x_0]\times(y_0-\delta,y_0]$ (resp., $T(x,y)<T(x_0,y_0)+\varepsilon$, whenever $(x,y)\in[x_0,x_0+\delta]\times[y_0,y_0+\delta]$).

Proposition 2.1. [33] A t-norm T is a left-continuous t-norm if and only if there exists a binary operation R_T such that (T,R_T) satisfies the residual principle, i.e., $T(x,z)\le y$ iff $z\le R_T(x,y)$ for all $x,y,z\in[0,1]$, where $R_T(x,y)=\sup\{z|T(x,z)\le y\}$ is called a residual implication induced by t-norm T.

Proposition 2.2. [33] A t-conorm S is a right-continuous t-conorm if and only if there exists a binary operation R_S on L such that (S,R_S) forms a co-adjoint pair, i.e., $x\le S(y,z)$ iff $R_S(x,y)\le z$ for all $x,y,z\in[0,1]$, where $R_S(x,y)=\inf\{z|x\le S(y,z)\}$ is called a coresidual implication induced by t-conorm S.

Example 1. Three important t-norms and their residual implication, t-conorms, and their coresidual implication [32, 33] are in Table 1.

Table 1. t-norms and their residual implications, t-conorms and their coresidual implications.

Name	t-norms	Residual Implications	t-conorms	Coresidual Implications
Łukasiewicz	$T_L(x,y)$ =$0\vee(x+y-1)$	$R_{T_L}(x,y)=1\wedge(1-x+y)$	$S_L(x,y)=(x+y)\wedge 1$	$R_{S_L}(x,y)=(x-y)\vee 0$
Gougen	$T_{Go}(x,b)\ =\ xy$	$R_{T_{Go}}(x,y)=1\wedge\dfrac{y}{x}$	$S_{Go}(x,y)$= $x+y-xy$	$R_{S_{Go}}(x,y)=\dfrac{x-y}{1-y}\vee 0$
Gödel	$T_G(x,y)\ =\ x\wedge y$	$R_{T_G}(x,y)=\begin{cases}1, & if\quad x\le y,\\ y, & if\quad x>y.\end{cases}$	$S_G(x,y)=x\vee y$	$R_{S_{Go}}(x,y)=\begin{cases}0, & if\quad x\le y,\\ x, & if\quad x>y.\end{cases}$

Definition 2.3. [22] Let X be a universal set. A neutrosophic set A on X is characterized by three functions, i.e., a truth-membership function $t_A(x)$, an indeterminacy-membership function $i_A(x)$ and a falsity-membership function $f_A(x)$. Then, a neutrosophic set A can be defined as follows:

$$A=\{\langle x,t_A(x),i_A(x),f_A(x)\rangle\mid x\in X\},$$

where $t_A(x):X\to]^-0,1^+[$, $i_A(x):X\to]^-0,1^+[$, $f_A(x):X\to]^-0,1^+[$, such that $0^-\le t_A(x)+i_A(x)+f_A(x)\le 3^+$, $t_A(x),i_A(x),f_A(x)\in[0,1]$ and satisfy the condition $0\le t_A(x)+i_A(x)+f_A(x)\le 3$ for each x in X.

The family of all single valued neutrosophic sets is denoted by $SVNS(X)$.

Definition 2.4. [22] Let A, B be two single valued neutrosophic sets on universal X, the following relations are defined as follows:

(i). $A \subseteq B$ if and only $t_A(x) \leq t_B(x)$, $i_A(x) \geq i_B(x)$ and $f_A(x) \geq f_B(x)$ for all $x \in X$;

(ii). $A = B$ if and only $A \subseteq B$ and $B \subseteq A$;

(iii). $A \cap B = \langle \min(t_A(x), t_B(x)), \max(i_A(x), i_B(x)), \max(f_A(x), f_B(x)) \rangle$ for all $x \in X$ for all $x \in X$;

(iv). $A \cup B = \langle \max(t_A(x), t_B(x)), \min(i_A(x), i_B(x)), \min(f_A(x), f_B(x)) \rangle$ for all $x \in X$;

(v). $A^c = \{\langle f_A(x), 1 - i_A(x), t_A(x) \rangle | x \in X\}$.

Remark 2.1. For arbitrary single valued neutrosophic set $A \in SVNS(X)$, we can obtain:

(i). If $t_A(x) + i_A(x) + f_A(x) = 1$, then a single-valued neutrosophic set A reduces to an intuitionistic fuzzy set.

(ii). If $t_A(x) + i_A(x) + f_A(x) = 1$ and $i_A(x) = 0$, then a single-valued neutrosophic set A reduces to a fuzzy set.

The set of all single valued neutrosophic numbers denoted by $SVNN$, i.e. $SVNN = \{$Žt$, i, f$ž $\mid t, i, f \in [0,1]\}$. Let $\alpha = \langle t_\alpha, i_\alpha, f_\alpha \rangle$, $\beta = \langle t_\beta, i_\beta, f_\beta \rangle \in SVNN$, an ordering on $SVNN$ as $\alpha \leq \beta$ if and only if $t_\alpha \leq t_\beta, i_\alpha \geq i_\beta, f_\alpha \geq f_\beta$, $\alpha = \beta$ iff $\alpha \leq \beta$ and $\beta \leq \alpha$ .Obviously, $\alpha \wedge \beta = \langle t_\alpha \wedge t_\beta, i_\alpha \vee i_\beta, f_\alpha \vee f_\beta \rangle$, $\alpha \vee \beta = \langle t_\alpha \vee t_\beta, i_\alpha \wedge i_\beta, f_\alpha \wedge f_\beta \rangle$, $\wedge_{i \in I} \alpha_i = \langle \wedge_{i \in I} t_{\alpha_i}, \vee_{i \in I} i_{\alpha_i}, \vee_{i \in I} f_{\alpha_i} \rangle$, $\vee_{i \in I} \alpha_i = \langle \vee_{i \in I} t_{\alpha_i}, \wedge_{i \in I} i_{\alpha_i}, \wedge_{i \in I} f_{\alpha_i} \rangle 0^* = \langle 0,1,1 \rangle$ and $1^* = \langle 1,0,0 \rangle$ are the smallest element and the greatest element in $SVNN$, respectively. It is easy to verify that $(SVNN, \leq)$ is a complete lattice [29].

After introducing single-valued neutrosophic numbers, we will then introduce the properties of single-valued neutrosophic t-norm.

Definition 2.5. [28] A function $\mathcal{T}$: $SVNN \times SVNN \rightarrow SVNN$ is called a single-valued neutrosophic t-norm if the following four axioms are satisfied, for all $\alpha, \beta, \gamma \in SVNN$,

(i). $\mathcal{T}(\alpha, \beta) = \mathcal{T}(\beta, \alpha)$, (commutativity)

(ii). $\mathcal{T}((\alpha, \beta), \gamma) = \mathcal{T}(\alpha, (\beta, \gamma))$, (associativity)

(iii). $\mathcal{T}(\alpha, \gamma) \leq \mathcal{T}(\beta, \gamma)$ if $\alpha \leq \beta$, (monotonicity)

(iv). $\mathcal{T}(\alpha, 1^*) = \alpha$. (boundary condition)

Example 2. [32] The function $\mathcal{T}$: $SVNN \times SVNN \rightarrow SVNN$ defined by $\mathcal{T}(\alpha, \beta) = \langle T(t_\alpha, t_\beta), S(i_\alpha, i_\beta), S(f_\alpha, f_\beta) \rangle$ is a single-valued neutrosophic t-norm, which is called a single-valued neutrosophic t-representable t-norm, where T is a t-norm and S is its dual t-conorm on [0, 1]. $\mathcal{T}$ is called a left-continuous single valued neutrosophic t-representable t-norm if T is left-continuous and S is right-continuous.

Definition 2.6. [32] A single valued neutrosophic residual implication is defined by $\mathcal{R}_\mathcal{T}(\alpha, \beta) = \sup\{\gamma \in SVNN \mid \mathcal{T}(\gamma, \alpha) \leq \beta\}$, $\forall \alpha, \beta \in SVNN$, where $\mathcal{T}$ is a left-continuous single valued neutrosophic t-representable t-norm.

Proposition 2.3. [32] Let $\mathcal{T}$ be a single-valued neutrosophic t-representable t-norm, the following statements are equivalent:

(i). $\mathcal{T}$ is left-continuous;

(ii). $\mathcal{T}$ and $\mathcal{R}_\mathcal{T}$ form an adjoint pair, i.e., they satisfy the following residual principle

$$\mathcal{T}(\gamma, \alpha) \leq \beta \Leftrightarrow \gamma \leq \mathcal{R}_\mathcal{T}(\alpha, \beta), \alpha, \beta, \gamma \in SVNN.$$

Proposition 2.4. [32] Let $\alpha = \langle t_\alpha, i_\alpha, f_\alpha \rangle$, $\beta = \langle t_\beta, i_\beta, f_\beta \rangle \in SVNN$, then $\mathcal{R}_{\mathcal{T}}(\alpha,\beta) = \langle R_T(t_\alpha, t_\beta), R_S(i_\beta, i_\alpha), R_S(f_\beta, f_\alpha)\rangle$, which is the single-valued neutrosophic residual implication induced by left-continuous single-valued neutrosophic t-representable t-norm, where R_T is residual implication induced by left-continuous t-norm T, R_S is coresidual implication induced by right-continuous t-conorm S.

Proposition 2.5. Let $\mathcal{R}_{\mathcal{T}}$ be single valued neutrosophic residual implication induced by left-continuous single valued neutrosophic t-representable t-norm $\mathcal{T}$, then

(i). $\mathcal{R}_{\mathcal{T}}(\alpha,\beta) = 1^*$ **iff** $\alpha \leq \beta$;

(ii). $\gamma \leq \mathcal{R}_{\mathcal{T}}(\alpha,\beta)$ iff $\alpha \leq \mathcal{R}_{\mathcal{T}}(\gamma,\beta)$;

(iii). $\mathcal{R}_{\mathcal{T}}(1^*,\alpha) = \alpha$;

(iv). $\mathcal{R}_{\mathcal{T}}(\alpha, \mathcal{R}_{\mathcal{T}}(\mathcal{R}_{\mathcal{T}}(\alpha,\beta),\beta) = 1^*$;

(v). $\mathcal{R}_{\mathcal{T}}(\bigvee_{i\in I}\beta_i, \alpha) = \bigwedge_{i\in I}\mathcal{R}_{\mathcal{T}}(\beta_i,\alpha)$;

(vi). $\mathcal{R}_{\mathcal{T}}(\beta, \bigwedge_{i\in I}\alpha) = \bigwedge_{i\in I}\mathcal{R}_{\mathcal{T}}(\beta,\alpha_i)$;

(vii). $\mathcal{R}_{\mathcal{T}}$ is antitone in the first variable and isotone in the second variable.

After introducing the properties of single-valued neutrosophic t-representable t-norm, to better understand its usage, we will use the following examples to introduce three important single-valued neutrosophic t-representable t-norms and their residual implications.

Example 3. [32] The following are three important single-valued neutrosophic t-representable t-norms and their residual implications.

(i). The single valued neutrosophic Łukasiewicz t-norm and its residual implication:

$$\mathcal{T}_L(\alpha,\beta) = \langle (t_\alpha + t_\beta - 1) \vee 0, (i_\alpha + i_\beta) \wedge 1, (f_\alpha + f_\beta) \wedge 1\rangle$$
$$\mathcal{R}_{\mathcal{T}_L}(\alpha,\beta) = \langle 1 \wedge (1 - t_\alpha + t_\beta), (i_\beta - i_\alpha) \vee 0, (f_\beta - f_\alpha) \vee 0\rangle.$$

(ii). The single valued neutrosophic Gougen t-norm and its residual implication:

$$\mathcal{T}_{Go}(\alpha,\beta) = \langle t_\alpha t_\beta, i_\alpha + i_\beta - i_\alpha i_\beta, f_\alpha + f_\beta - f_\alpha f_\beta\rangle.$$

$$\mathcal{R}_{\mathcal{T}_{Go}}(\alpha,\beta) = \begin{cases} \langle 1,0,0\rangle, & if \quad t_\alpha \leq t_\beta, i_\beta \leq i_\alpha, f_\beta \leq f_\alpha, \\ \langle 1,0,\dfrac{f_\beta - f_\alpha}{1-f_\alpha}\rangle, & if \quad t_\alpha \leq t_\beta, i_\beta \leq i_\alpha, f_\alpha < f_\beta, \\ \langle 1,\dfrac{i_\beta - i_\alpha}{1-i_\alpha},0\rangle, & if \quad t_\alpha \leq t_\beta, i_\alpha < i_\beta, f_\beta \leq f_\alpha, \\ \langle 1,\dfrac{i_\beta - i_\alpha}{1-i_\alpha},\dfrac{f_\beta - f_\alpha}{1-f_\alpha}\rangle, & if \quad t_\alpha \leq t_\beta, i_\alpha < i_\beta, f_\alpha < f_\beta, \\ \langle \dfrac{t_\beta}{t_\alpha},0,0\rangle, & if \quad t_\beta < t_\alpha, i_\beta \leq i_\alpha, f_\beta \leq f_\alpha, \\ \langle \dfrac{t_\beta}{t_\alpha},0,\dfrac{f_\beta - f_\alpha}{1-f_\alpha}\rangle, & if \quad t_\beta < t_\alpha, i_\beta \leq i_\alpha, f_\alpha < f_\beta, \\ \langle \dfrac{t_\beta}{t_\alpha},\dfrac{i_\beta - i_\alpha}{1-i_\alpha},0\rangle, & if \quad t_\beta < t_\alpha, i_\alpha < i_\beta, f_\beta \leq f_\alpha, \\ \langle \dfrac{t_\beta}{t_\alpha},\dfrac{i_\beta - i_\alpha}{1-i_\alpha},\dfrac{f_\beta - f_\alpha}{1-f_\alpha}\rangle, & if \quad t_\beta < t_\alpha, i_\alpha < i_\beta, f_\alpha < f_\beta. \end{cases}$$

(iii). The single valued neutrosophic t-norm and its residual implication:

$$\mathcal{T}_G(\alpha,\beta) = \langle t_\alpha \wedge t_\beta, i_\alpha \vee i_\beta, f_\alpha \vee f_\beta\rangle.$$

$$\mathcal{R}_{\mathcal{T}_G}(\alpha,\beta)=\begin{cases}\langle 1,0,0\rangle, & if \quad t_\alpha\le t_\beta, i_\beta\le i_\alpha, f_\beta\le f_\alpha,\\ \langle 1,0,f_\beta\rangle, & if \quad t_\alpha\le t_\beta, i_\beta\le i_\alpha, f_\alpha< f_\beta,\\ \langle 1,i_\beta,0\rangle, & if \quad t_\alpha\le t_\beta, i_\alpha< i_\beta, f_\beta\le f_\alpha,\\ \langle 1,i_\beta,f_\beta\rangle, & if \quad t_\alpha\le t_\beta, i_\alpha< i_\beta, f_\alpha< f_\beta,\\ \langle t_\beta,0,0\rangle, & if \quad t_\beta< t_\alpha, i_\beta\le i_\alpha, f_\beta\le f_\alpha,\\ \langle t_\beta,0,f_\beta\rangle, & if \quad t_\beta< t_\alpha, i_\beta\le i_\alpha, f_\alpha< f_\beta,\\ \langle t_\beta,i_\beta,0\rangle, & if \quad t_\beta< t_\alpha, i_\alpha< i_\beta, f_\beta\le f_\alpha,\\ \langle t_\beta,i_\beta,f_\beta\rangle, & if \quad t_\beta< t_\alpha, i_\alpha< i_\beta, f_\alpha< f_\beta.\end{cases}$$

To further demonstrate the robustness of single-valued neutrosophic t-norm, we will now introduce a distance metric d.

Definition 2.7. [34] A metric space is an ordered pair (X,d), where X is a set and d is a metric on X, i.e., a function $d\colon X\times X\to[0,+\infty)$ such that for any $x,y,z\in X$, the following holds:
$(D1)$ $d(x,y)\ge$ **0;**
$(D2)$ $d(x,y)$ **= 0 if and only if** $x=y$**;**
$(D3)$ $d(x,y)\le d(x,z)+d(y,z)$**.**
The function d is called a distance.

3. Single-Valued Neutrosophic Fuzzy Inference Triple I Method

In this section, we will study the single-valued neutrosophic fuzzy inference triple I method based on left-continuous single-valued neutrosophic t-representable t-norm $\mathcal{T}$. Suppose $\mathcal{R}$ is a single-valued neutrosophic residuated implication induced by left-continuous single-valued neutrosophic t-representable t-norm $\mathcal{T}$. A single valued neutrosophic set A on universe X is called normal if there exists $x_0\in X$ such that $A(x_0)=1^*$. A single valued neutrosophic set A on universe X is called co-normal if there exists $x_0\in X$ such that $A(x_0)=0^*$.

Definition 3.1. (Single valued neutrosophic fuzzy inference triple I principle for FMP) Suppose that $\mathcal{R}$ is a single-valued neutrosophic residual implication induced by a left-continuous single-valued neutrosophic t-representable t-norm $\mathcal{T}$, $A,A^*\in SVNS(X)$ and $B\in SVNS(Y)$. Let $P(x,y)=\mathcal{R}(\mathcal{R}(A(x),B(y)),\mathcal{R}(A^*(x),1^*))$, and $\mathrm{B}(A,B,A^*)=\{C\in SVNS(Y)\mid \mathcal{R}(\mathcal{R}(A(x),B(y)),\mathcal{R}(A^*(x),C(y)))=P(x,y), x\in X, y\in Y\}$.

If there exist the smallest element of the set $\mathrm{B}(A,B,A^*)$ (denoted by B^*), then B^* is called the single-valued neutrosophic fuzzy inference triple I solution for FMP.

Definition 3.2. (Single valued neutrosophic fuzzy inference triple I principle for FMT) Suppose that $\mathcal{R}$ is a single-valued neutrosophic residual implication induced by a left-continuous single-valued neutrosophic t-representable t-norm $\mathcal{T}$. $A\in SVNS(X)$ and $B,B^*\in SVNS(Y)$. Let $Q(x,y)=\mathcal{R}(\mathcal{R}(A(x),B(y)),\mathcal{R}(0^*,B^*(x)))$, and $\mathrm{A}(A,B,B^*)=\{D\in SVNS(X)\mid \mathcal{R}(\mathcal{R}(A(x),B(y)),\mathcal{R}(D(x),B^*(x)))=Q(x,y), x\in X, y\in Y\}$.

If there exists the greatest element of the set $\mathrm{A}(A,B,B^*)$ (denoted by A^*), then A^* is called the single-valued neutrosophic fuzzy inference triple I solution for FMT.

After introducing the single-valued neutrosophic fuzzy inference triple I principle for FMP and FMT, we can now derive the single-valued neutrosophic fuzzy inference triple I solution of FMP and FMT.

Theorem 3.1. Let $A,A^*\in SVNS(X)$, $B\in SVNS(Y)$, $\mathcal{R}$ be single valued neutrosophic residual implication induced by a left-continuous single valued neutrosophic t-representable t-norm $\mathcal{T}$, then the single-valued neutrosophic fuzzy inference triple I solution B^* of FMP is as follows:

$$B^*(y) = \sup_{x\in X}\mathcal{T}(A^*(x), \mathcal{R}(A(x), B(y)))(\forall y \in Y) \tag{1}$$

Proof:

Firstly, we prove $B^* \in \mathrm{B}(A, B, A^*)$. It follows from equation (1), we have $\mathcal{T}(A^*(x), \mathcal{R}(A(x), B(y))) \leq B^*$ (y). By the residuation property, we obtain $\mathcal{R}(A(x), B(y))) \leq \mathcal{R}(A^*(x), B^*(y)))$. Therefore, $\mathcal{R}(\mathcal{R}(A(x), B(y)), \mathcal{R}(A^*(x), B^*(y))) = 1^*$, i.e., $B^* \in \mathrm{B}(A, B, A^*)$.

Secondly, we prove that B^* is the smallest single valued neutrosophic fuzzy subset of $\mathrm{B}(A, B, A^*)$. Suppose C is an arbitrary single-valued neutrosophic fuzzy subset in $\mathrm{B}(A, B, A^*)$, i.e.$\mathcal{R}(\mathcal{R}(A(x), B(y)), \mathcal{R}(A^*(x), C(y))) = 1^*$.

By the residuation property, then $\mathcal{R}(A(x), B(y)) \leq \mathcal{R}(A^*(x), C(y))$. we have $\mathcal{T}(A^*(x), \mathcal{R}(A(x), B(y)) \leq C(y)$, hence $B^* \leq C$, i.e., B^* is the smallest single valued neutrosophic fuzzy subset of $\mathrm{B}(A, B, A^*)$, and B^* is the single-valued neutrosophic fuzzy inference triple I solution for FMP.

After obtaining the solution for single valued neutrosophic fuzzy inference triple I solution of FMP, we can now obtain the single-valued neutrosophic residual implication induced by a left-continuous single valued neutrosophic t-representable t-norm $\mathcal{T}$ triple I solution for FMP.

Corollary 3.1. Let $\mathcal{R}$ be single valued neutrosophic residual implication induced by a left-continuous single-valued neutrosophic t-representable t-norm $\mathcal{T}$, then the single-valued neutrosophic fuzzy inference triple I solution $B^* = \{\langle y, t_{B^*}(y), i_{B^*}(y), f_{B^*}(y)\rangle \mid y \in Y\}$ for FMP can be shown as follows:

$t_{B^*}(y) = \vee_{x\in X} T\,(t_{A^*}(x), R_T(t_A(x), t_B(y)))(\forall y \in Y)$,

$i_{B^*}(y) = \wedge_{x\in X} S\,(i_{A^*}(x), R_S(i_B(y), i_A(x)))(\forall y \in Y)$,

$f_{B^*}(y) = \wedge_{x\in X} S\,(f_{A^*}(x), R_S(f_B(y), f_A(x)))(\forall y \in Y)$.

Corollary 3.2. Let $\mathcal{R}$ be the single-valued neutrosophic Łukasiewicz residual implication $\mathcal{R}_{\mathcal{T}_L}$, then the single-valued neutrosophic fuzzy inference triple I solution $B^* = \{\langle y, t_{B^*}(y), i_{B^*}(y), f_{B^*}(y)\rangle \mid y \in Y\}$ of FMPas follows:

$t_{B^*}(y) = \vee_{x\in X}\{\,[t_{A^*}(x) + ((1 - t_A(x) + t_B(y)) \wedge 1) - 1] \vee 0\}(\forall y \in Y)$,

$i_{B^*}(y) = \wedge_{x\in X}\{\,[i_{A^*}(x) + ((i_B(y) - i_A(x)) \vee 0)] \wedge 1\}(\forall y \in Y)$,

$f_{B^*}(y) = \wedge_{x\in X}\{\,[f_{A^*}(x) + ((f_B(y) - f_A(x)) \vee 0)] \wedge 1\}(\forall y \in Y)$.

Corollary 3.3. Let $\mathcal{R}$ be the single-valued neutrosophic Gougen residual implication $\mathcal{R}_{\mathcal{T}_{Go}}$, then the single-valued neutrosophic fuzzy inference triple I solution $B^* = \{\langle y, t_{B^*}(y), i_{B^*}(y), f_{B^*}(y)\rangle \mid y \in Y\}$ of FMP as follows:

$t_{B^*}(y) = \vee_{x\in X}\{\,t_{A^*}(x)\cdot(\frac{t_{B(y)}}{t_{A(x)}} \wedge 1)\}(\forall y \in Y)$,

$i_{B^*}(y) = \wedge_{x\in X}\{\,i_{A^*}(x) + [\frac{i_B(y) - i_A(x)}{1 - i_A(x)} \vee 0] - i_{A^*}(x)\cdot[\frac{i_B(y) - i_A(x)}{1 - i_A(x)} \vee 0]\}(\forall y \in Y)$,

$f_{B^*}(y) = \wedge_{x\in X}\{\,f_{A^*}(x) + [\frac{f_B(y) - f_A(x)}{1 - f_A(x)} \vee 0] - f_{A^*}(x)\cdot[\frac{f_B(y) - f_A(x)}{1 - f_A(x)} \vee 0]\}(\forall y \in Y)$.

Corollary 3.4. Let $\mathcal{R}$ be the single-valued neutrosophic $Gödel$ residual implications $\mathcal{R}_{\mathcal{T}_G}$, then the single-valued neutrosophic fuzzy inference triple I solution $B^* = \{\langle y, t_{B^*}(y), i_{B^*}(y), f_{B^*}(y)\rangle \mid y \in Y\}$ of FMP as follows:

$t_{B^*}(y) = \vee_{x\in X}\{\,(t_{A^*}(x) \wedge R_{T_G}(t_A(x), t_B(y)))\}(\forall y \in Y)$,

$i_{B^*}(y) = \wedge_{x\in X}\{\,(i_{A^*}(x) \vee R_{S_G}(i_B(y), i_A(x)))\}(\forall y \in Y)$,

$f_{B^*}(y) = \wedge_{x\in X}\{\,(f_{A^*}(x) \vee R_{S_G}(f_B(y), f_A(x)))\}(\forall y \in Y)$.

Theorem 3.2. Let $A \in SVNS(X)$, $B, B^* \in SVNS(Y)$, $\mathcal{R}$ be single valued neutrosophic residual implication induced by a left-continuous single-valued neutrosophic t-representable t-norm $\mathcal{T}$, then the single-valued neutrosophic fuzzy inference triple I solution A^* of FMT is as follows:

$$A^*(x) = \bigwedge_{y \in Y} \mathcal{R}\,(\mathcal{R}(A(x), B(y)), B^*(y))(\forall x \in X) \tag{2}$$

Proof:

Firstly, we prove $A^* \in \mathrm{A}(A, B, B^*)$. It follows from equation (2), we obtain $A^*(x) \leq \mathcal{R}(\mathcal{R}(A(x), B(y)), B^*(y))$. By the residuation property, we have $\mathcal{T}(A^*, \mathcal{R}(A(x), B(y))) \leq B^*$, and $\mathcal{R}(A(x), B(y)) \leq \mathcal{R}(A^*(x), B^*(y))$. Therefore, $\mathcal{R}(\mathcal{R}(A(x), B(y)), \mathcal{R}(A^*(x), B^*(y))) = 1^*$, i.e., $A^* \in \mathrm{A}(A, B, B^*)$.

Secondly, we show that A^* is the greatest single valued neutrosophic fuzzy subset of $\mathrm{A}(A, B, B^*)$. Suppose D is an arbitrary single-valued neutrosophic fuzzy subset in $\mathrm{A}(A, B, B^*)$, i.e.,$\mathcal{R}(\mathcal{R}(A(x), B(y)), \mathcal{R}(D(x), B^*(y))) = 1^*$, then $\mathcal{R}(A(x), B(y)) \leq \mathcal{R}(D(x), B^*(y))$ by the residuation property. We have $\mathcal{T}(D(x), \mathcal{R}(A(x), B(y)) \leq B^*(y)$ and $D(x) \leq \mathcal{R}(\mathcal{R}(A(x), B(y)), B^*(y))$ by Proposition 2.3, hence $D \leq A^*$, i.e., A^* is the greatest single valued neutrosophic fuzzy subset of $\mathrm{A}(A, B, B^*)$, and A^* is the single-valued neutrosophic fuzzy inference triple I solution for FMT.

After obtaining the solution for single valued neutrosophic fuzzy inference triple I solution of FMT, we can now obtain the single-valued neutrosophic residual implication induced by a left-continuous single valued neutrosophic t-representable t-norm $\mathcal{T}$ triple I solution for FMT.

Corollary 3.5. Let $\mathcal{R}$ be a single-valued neutrosophic residual implication induced by a left-continuous single-valued neutrosophic t-representable t-norm $\mathcal{T}$, then the single-valued neutrosophic fuzzy inference triple I solution $A^* = \{\langle x, t_{A^*}(x), i_{A^*}(x), f_{A^*}(x)\rangle \mid x \in X\}$ for FMT can be shown as follows:

$t_{A^*}(x) = \bigwedge_{y \in Y} R_T\,(R_T(t_A(x), t_B(y)), t_{B^*}(y))(\forall x \in X)$,
$i_{A^*}(x) = \bigvee_{y \in Y} R_S\,(i_{B^*}(y), R_S(i_B(y), i_A(x)))(\forall x \in X)$,
$f_{A^*}(x) = \bigvee_{y \in Y} R_S\,(f_{B^*}(y), R_S(f_B(y), f_A(x)))(\forall x \in X)$.

Corollary 3.6. Let $\mathcal{R}$ be the single-valued neutrosophic Łukasiewicz residual implication $\mathcal{R}_{\mathcal{T}_L}$, then the single-valued neutrosophic fuzzy inference triple I solution $A^* = \{\langle x, t_{A^*}(x), i_{A^*}(x), f_{A^*}(x)\rangle \mid x \in X\}$ for FMT as follows:

$t_{A^*}(x) = \bigwedge_{y \in Y}\{\,[1 - ((1 - t_A(x) + t_B(y)) \wedge 1) + t_{B^*}(y)] \wedge 1\}(\forall x \in X)$,
$i_{A^*}(x) = \bigvee_{y \in Y}\{\,[i_{B^*}(y) - ((i_B(y) - i_A(x)) \vee 0)] \vee 0\}(\forall x \in X)$,
$f_{A^*}(x) = \bigvee_{y \in Y}\{\,[f_{B^*}(y) - ((f_B(y) - f_A(x)) \vee 0)] \vee 0\}(\forall x \in X)$.

Corollary 3.7. Let $\mathcal{R}$ be the single-valued neutrosophic Gougen residual implication $\mathcal{R}_{\mathcal{T}_{Go}}$, then the single-valued neutrosophic fuzzy inference triple I solution $A^* = \{\langle x, t_{A^*}(x), i_{A^*}(x), f_{A^*}(x)\rangle \mid x \in X\}$ for FMT as follows:

$t_{A^*}(x) = \bigwedge_{y \in Y}\{\frac{t_{B^*}(y)}{(\frac{t_B(y)}{t_A(x)} \wedge 1)} \wedge 1\}(\forall x \in X)$,

$i_{A^*}(x) = \bigvee_{y \in Y}\{\frac{i_{B^*}(y) - (\frac{i_B(y) - i_A(x)}{1 - i_A(x)} \vee 0)}{1 - (\frac{i_B(y) - i_A(x)}{1 - i_A(x)} \vee 0)} \vee 0\}(\forall x \in X)$,

$f_{A^*}(x) \bigvee_{y \in Y}\{\frac{f_{B^*}(y) - (\frac{f_B(y) - f_A(x)}{1 - f_A(x)} \vee 0)}{1 - (\frac{f_B(y) - f_A(x)}{1 - f_A(x)} \vee 0)} \vee 0\}(\forall x \in X)$.

Corollary 3.8. Let B be the single-valued neutrosophic $Gödel$ residual implications $\mathcal{R}_{\mathcal{T}_G}$, then the single-valued neutrosophic fuzzy inference triple I solution $A^* = \{\langle x, t_{A^*}(x), i_{A^*}(x), f_{A^*}(x)\rangle \mid x \in X\}$ for FMT as follows:

$t_{A^*}(x) = \wedge_{x\in X}\{ R_{T_G}(R_{T_G}(t_A(x), t_B(y)), t_{B^*}(y))\}(\forall x \in X),$

$i_{A^*}(x) = \vee_{x\in X}\{ R_{S_G}(i_{B^*}(y), R_{S_G}(i_B(y), i_A(x)))\}(\forall x \in X),$

$f_{A^*}(x) = \vee_{x\in X}\{ R_{S_G}(f_{A^*}(x), R_{S_G}(f_B(y), f_A(x)))\}(\forall x \in X).$

To prove the single-valued neutrosophic fuzzy inference triple I method is recoverable, we define reducibility.

Definition 3.3. [4] A method for FMP is called recoverable if $A^* = A$ implies $B^* = B$. similarly, a method for FMT is called recoverable if $B^* = B$ implies $A^* = A$.}

Theorem 3.3. The single-valued neutrosophic fuzzy inference triple I method for FMP is reductive if A is a normal single-valued neutrosophic set.

Proof:
Suppose $A^* = A$ and there exists an element $x_0 \in X$ such that $A(x_0) = A^*(x_0) = \langle 1,0,0\rangle = 1^*$. Then we have

$$\begin{aligned} B^*(y) = & \bigvee_{x\in X} \mathcal{T}\,(A^*(x), \mathcal{R}(A(x), B(y))) \\ \geq & \quad \mathcal{T}(A^*(x_0), \mathcal{R}(A(x_0), B(y))) \\ = & \quad \mathcal{T}(1^*, \mathcal{R}(1^*, B(y))) = B(y). \end{aligned}$$

On the other hand, by Proposition 2.6 (5) for any $y \in Y$,

$$\mathcal{R}(B^*(y), B(y)) = \mathcal{R}(\bigvee_{y\in Y} \mathcal{T}\,(\mathcal{R}(A(x), B(y)), A^*(x)), B(y)) = \bigwedge_{y\in Y} \mathcal{R}\,(\mathcal{T}(\mathcal{R}(A(x), B(y)), A(x)), B(y)) = 1^*,$$

we have, $B^*(y) \leq B(y)$.
Therefore, $B^* = B$. This shows that the single-valued neutrosophic fuzzy inference triple I method for FMP is recoverable.

Theorem 3.4. The single-valued neutrosophic fuzzy inference triple I method for FMT is reductive if single-valued neutrosophic residual implication $\mathcal{R}$ satisfies $\mathcal{R}(\mathcal{R}(A, 0^*), 0^*) = A$, and B is a co-normal single-valued neutrosophic set.
Proof:
Suppose $B^* = B$ is a co-normal single-valued neutrosophic set, i.e. there exists an element $y_0 \in Y$ such that $B^*(y_0) = B(y_0) = \langle 0,1,1\rangle = 0^*$, then we have:

$$\begin{aligned} A^*(x) = & \bigwedge_{y\in Y} \mathcal{R}\,(\mathcal{R}(A(x), B(y)), B^*(y)) \\ \leq & \quad \mathcal{R}(\mathcal{R}(A(x), B(y_0)), B^*(y_0)) \\ = & \quad \mathcal{R}(\mathcal{R}(A(x), 0^*), 0^*) = A(x). \end{aligned}$$

On the other hand, by Proposition 2.5(3) and (4) for any $x \in X$,

$$\mathcal{R}(A(x), A^*(x)) = \mathcal{R}(A(x), \bigwedge_{y\in Y} \mathcal{R}\,(\mathcal{R}(A(x), B(y)), B^*(y))) = \bigwedge_{y\in Y} \mathcal{R}\,(A(x), \mathcal{R}(\mathcal{R}(A(x), B(y)), B(y))) = 1^*,$$

we have $A(x) \leq A^*(x)$.
Therefore, $A^* = A$. This shows that the single-valued neutrosophic fuzzy inference triple I method for FMT is recoverable.

4. Robustness of Single-Valued Neutrosophic Fuzzy Inference Triple I Method

In this section, we introduce a new distance between single-valued neutrosophic sets. Through this distance, we can prove the robustness of the single-valued neutrosophic fuzzy inference triple I method. We study the robustness of the single-valued neutrosophic fuzzy inference triple I method based on left-continuous single-valued neutrosophic t-representable t-norms with this new distance.

Theorem 4.1. Let $X=\{x_1,x_2,\ldots,x_n\}$, for all $A,B\in SVNS(X)$, then

$$d(A,B)=\max\{\bigvee_{x_i\in X}|t_A(x_i)-t_B(x_i)|,\bigvee_{x_i\in X}|i_A(x_i)-i_B(x_i)|,\bigvee_{x_i\in X}|f_A(x_i)-f_B(x_i)|\}$$

is a metric on $SVNS(X)$ and $(SVNS(X),d)$ is a metric space. d is called a distance on $SVNS(X)$.

Proof: By Definition 2.7, (1) (2) are obvious for any $A,B\in SVNS(X)$. Therefore, we only prove (3). For any $A,B,C\in SVNS(X)$

$$\begin{aligned}
&d(A,B)\\
&=\max\{\bigvee_{x_i\in X}|t_A(x_i)-t_B(x_i)|,\bigvee_{x_i\in X}|i_A(x_i)-i_B(x_i)|,\bigvee_{x_i\in X}|f_A(x_i)-f_B(x_i)|\}\\
&=\max\{\bigvee_{x_i\in X}|t_A(x_i)-t_C(x_i)+t_C(x_i)-t_B(x_i)|,\\
&\bigvee_{x_i\in X}|i_A(x_i)-i_C(x_i)+i_C(x_i)-i_B(x_i)|,\bigvee_{x_i\in X}|f_A(x_i)-f_C(x_i)+f_C(x_i)-f_B(x_i)|\}\\
&\le\max\{\bigvee_{x_i\in X}|t_A(x_i)-t_C(x_i)|,\bigvee_{x_i\in X}|i_A(x_i)-i_C(x_i)|,\bigvee_{x_i\in X}|f_A(x_i)-f_C(x_i)|\}\\
&+\max\{\bigvee_{x_i\in X}|t_C(x_i)-t_B(x_i)|,\bigvee_{x_i\in X}|i_C(x_i)-i_B(x_i)|,\bigvee_{x_i\in X}|f_C(x_i)-f_B(x_i)|\}\\
&\le d(A,C)+d(C,B)
\end{aligned}$$

Therefore, d is a metric on $SVNS(X)$, and $(SVNS(X),d)$ is a metric space.

Definition 4.1. Suppose that $\mathfrak{F}$ is a n-tuple mapping form to $SVNN^n$ to $SVNN$, $\forall\varepsilon\in(0,1)$. For any $\langle \mathrm{t,i,f}\rangle=(\langle t_1,i_1,f_1\rangle,\langle t_2,i_2,f_2\rangle,\ldots,\langle t_n,i_n,f_n\rangle)\in SVNN^n$,

$$\Delta_{\mathfrak{F}}(\langle \mathrm{t,i,f}\rangle,\varepsilon)=\vee\{d(\mathfrak{F}\langle \mathrm{t,i,f}\rangle,\mathfrak{F}\langle \mathrm{t',i',f'}\rangle)|\langle \mathrm{t',i',f'}\rangle\in SVNN^n,d(\langle \mathrm{t,i,f}\rangle,\langle \mathrm{t',i',f'}\rangle)\le\varepsilon\}$$

is called the sensitivity of the point $\langle \mathrm{t,i,f}\rangle$, where $d(\langle \mathrm{t,i,f}\rangle,\langle \mathrm{t',i',f'}\rangle)=\max\{\vee_j|t_j-t_j'|,\vee_j|i_j-i_j'|,\vee_j|f_j-f_j'|\}$.

Definition 4.2. The biggest ε sensitivity of $\mathfrak{F}$ denoted by $\Delta_{\mathfrak{F}}(\varepsilon)=\vee_{\langle \mathrm{i,t,f}\rangle\in SVNN^n}\Delta_{\mathfrak{F}}(\langle \mathrm{t,i,f}\rangle,\varepsilon)$ is called sensitivity of $\mathfrak{F}$.

Definition 4.3. Let $\mathfrak{F}$ and $\mathfrak{F}'$ be two n-tuple single-valued neutrosophic fuzzy connectives. We say that $\mathfrak{F}$ at least as robust as $\mathfrak{F}'$ at point $\langle \mathrm{t,i,f}\rangle$, if $\forall\varepsilon\in(0,1)$, $\Delta_{\mathfrak{F}}(\langle \mathrm{t,i,f}\rangle,\varepsilon)\le\Delta_{\mathfrak{F}'}(\langle \mathrm{t,i,f}\rangle,\varepsilon)$. We say that $\mathfrak{F}$ is more robust than $\mathfrak{F}'$ at point $\langle \mathrm{t,i,f}\rangle$, if there exists $\varepsilon>0$ such that $\Delta_{\mathfrak{F}}(\langle \mathrm{t,i,f}\rangle,\varepsilon)<\Delta_{\mathfrak{F}'}(\langle \mathrm{t,i,f}\rangle,\varepsilon)$.

Definition 4.4. Let $\mathfrak{F}$ and $\mathfrak{F}'$ be two n-tuple single-valued neutrosophic fuzzy connectives. We say that $\mathfrak{F}$ at least as robust as $\mathfrak{F}'$, if $\forall\varepsilon\in(0,1)$, $\Delta_{\mathfrak{F}}(\varepsilon)\le\Delta_{\mathfrak{F}'}(\varepsilon)$. We say that $\mathfrak{F}$ is more robust than $\mathfrak{F}'$ if there exists $\varepsilon>0$ such that $\Delta_{\mathfrak{F}}(\varepsilon)<\Delta_{\mathfrak{F}'}(\varepsilon)$.

Proposition 4.1. For a binary single valued neutrosophic fuzzy connectives $\mathfrak{F}$: $SVNN\times SVNN\to SVNN$, we can obtain:

(i). Let $\mathfrak{F}$ be a left-continuous single valued neutrosophic t-representable t-norm on $SVNN$, $\mathcal{T}(\alpha,\beta)=\langle T(t_\alpha,t_\beta),S(i_\alpha,i_\beta),S(f_\alpha,f_\beta)\rangle$ for all $\alpha=\langle t_\alpha,i_\alpha,f_\alpha\rangle$, $\beta=\langle t_\beta,i_\beta,f_\beta\rangle\in SVNN$, then

$$
\begin{aligned}
\triangle_{\mathcal{T}}(\varepsilon) &= \bigvee_{(\alpha,\beta)\in SNVS^2} \triangle_{\mathcal{T}}((\alpha,\beta),\varepsilon) \\
&= \bigvee_{(\alpha,\beta)\in SNVS^2} \{\vee\{|T(t_\alpha,t_\beta)-T(t_\alpha^{'},t_\beta^{'})|,\ |S(i_\alpha,i_\beta)-S(i_\alpha^{'},i_\beta^{'})|,\ |S(f_\alpha,f_\beta)-S(f_\alpha^{'},f_\beta^{'})|\ \ |d(\mathcal{T}(\alpha,\beta),\mathcal{T}(\alpha^{'},\beta^{'}))\le\varepsilon\}\} \\
&= \bigvee_{(\alpha,\beta)\in SNVS^2} \left\{\begin{matrix}
T(t_\alpha,t_\beta)-T(t_\alpha+\varepsilon,t_\beta+\varepsilon)|,|S(i_\alpha,i_\beta)-S(i_\alpha+\varepsilon,i_\beta+\varepsilon)|,|S(f_\alpha,f_\beta)-S(f_\alpha+\varepsilon,f_\beta+\varepsilon)|, \\
|T(t_\alpha,t_\beta)-T(t_\alpha+\varepsilon,t_\beta-\varepsilon)|,|S(i_\alpha,i_\beta)-S(i_\alpha+\varepsilon,i_\beta-\varepsilon)|,|S(f_\alpha,f_\beta)-S(f_\alpha+\varepsilon,f_\beta-\varepsilon)|, \\
|T(t_\alpha,t_\beta)-T(t_\alpha+\varepsilon,t_\beta)|,|S(i_\alpha,i_\beta)-S(i_\alpha+\varepsilon,i_\beta)|,|S(f_\alpha,f_\beta)-S(f_\alpha+\varepsilon,f_\beta)|, \\
|T(t_\alpha,t_\beta)-T(t_\alpha,t_\beta+\varepsilon)|,|S(i_\alpha,i_\beta)-S(i_\alpha,i_\beta+\varepsilon)|,|S(f_\alpha,f_\beta)-S(f_\alpha,f_\beta+\varepsilon)|, \\
|T(t_\alpha,t_\beta)-T(t_\alpha,t_\beta-\varepsilon)|,|S(i_\alpha,i_\beta)-S(i_\alpha,i_\beta-\varepsilon)|,|S(f_\alpha,f_\beta)-S(f_\alpha,f_\beta-\varepsilon)|, \\
|T(t_\alpha,t_\beta)-T(t_\alpha-\varepsilon,t_\beta+\varepsilon)|,|S(i_\alpha,i_\beta)-S(i_\alpha-\varepsilon,i_\beta+\varepsilon)|,|S(f_\alpha,f_\beta)-S(f_\alpha-\varepsilon,f_\beta+\varepsilon)|, \\
|T(t_\alpha,t_\beta)-T(t_\alpha-\varepsilon,t_\beta-\varepsilon)|,|S(i_\alpha,i_\beta)-S(i_\alpha-\varepsilon,i_\beta-\varepsilon)|,|S(f_\alpha,f_\beta)-S(f_\alpha-\varepsilon,f_\beta-\varepsilon)|, \\
|T(t_\alpha,t_\beta)-T(t_\alpha-\varepsilon,t_\beta)|,|S(i_\alpha,i_\beta)-S(i_\alpha-\varepsilon,i_\beta)|,|S(f_\alpha,f_\beta)-S(f_\alpha-\varepsilon,f_\beta)|
\end{matrix}\right\}
\end{aligned}
$$

(ii). Let $\mathfrak{F}$ be single valued neutrosophic residuated implication $\mathcal{R}_{\mathcal{T}}$ induced by left-continuous single valued neutrosophic t-representable t-norm $\mathcal{T}$, $\mathcal{R}_{\mathcal{T}}(\alpha,\beta)$ = $\langle R_T(t_\alpha,t_\beta)),R_S(i_\beta,i_\alpha),R_S(f_\beta,f_\alpha)\rangle$ for all $\alpha=\langle t_\alpha,i_\alpha,f_\alpha\rangle$, $\beta=\langle t_\beta,i_\beta,f_\beta\rangle\in SVNN$, then

$$
\begin{aligned}
\triangle_{\mathcal{R}_{\mathcal{T}}}(\varepsilon) &= \bigvee_{(\alpha,\beta)\in SNVS^2} \triangle_{\mathcal{R}_{\mathcal{T}}}((\alpha,\beta),\varepsilon) \\
&= \bigvee_{(\alpha,\beta)\in SNVS^2} \left\{\bigvee\{|R_T(t_\alpha,t_\beta)-R_T(t_\alpha^{'},t_\beta^{'})|,|R_S(i_\beta,i_\alpha)-R_S(i_\beta^{'},i_\alpha^{'})|,|R_S(f_\beta,f_\alpha)-R_S(f_\beta^{'},f_\alpha^{'})||d(\mathcal{T}(\alpha,\beta),\mathcal{T}(\alpha^{'},\beta^{'}))\le\varepsilon\}\right\} \\
&= \bigvee_{(\alpha,\beta)\in SNVS^2} \left\{\begin{matrix}
|R_T(t_\alpha,t_\beta)-R_T(t_\alpha+\varepsilon,t_\beta+\varepsilon)|,|R_S(i_\beta,i_\alpha)-R_S(i_\beta+\varepsilon,i_\alpha+\varepsilon)|,|R_S(f_\beta,f_\alpha)-R_S(f_\beta+\varepsilon,f_\alpha+\varepsilon)|, \\
|R_T(t_\alpha,t_\beta)-R_T(t_\alpha+\varepsilon,t_\beta-\varepsilon)|,|R_S(i_\beta,i_\alpha)-R_S(i_\beta+\varepsilon,i_\alpha-\varepsilon)|,|R_S(f_\beta+\varepsilon,f_\alpha)-R_S(f_\beta,f_\alpha-\varepsilon)|, \\
|R_T(t_\alpha,t_\beta)-R_T(t_\alpha+\varepsilon,t_\beta)|,|R_S(i_\beta,i_\alpha)-R_S(i_\beta+\varepsilon,i_\alpha)|,|R_S(f_\beta,f_\alpha)-R_S(f_\beta+\varepsilon,f_\alpha)|, \\
|R_T(t_\alpha,t_\beta)-R_T(t_\alpha,t_\beta+\varepsilon)|,|R_S(i_\beta,i_\alpha)-R_S(i_\beta,i_\alpha+\varepsilon)|,|R_S(f_\beta,f_\alpha)-R_S(f_\beta,f_\alpha+\varepsilon)|, \\
|R_T(t_\alpha,t_\beta)-R_T(t_\alpha,t_\beta-\varepsilon)|,|R_S(i_\beta,i_\alpha)-R_S(i_\beta,i_\alpha-\varepsilon)|,|R_S(f_\beta,f_\alpha)-R_S(f_\beta,f_\alpha-\varepsilon)|, \\
|R_T(t_\alpha,t_\beta)-R_T(t_\alpha-\varepsilon,t_\beta+\varepsilon)|,|R_S(i_\beta,i_\alpha)-R_S(i_\beta-\varepsilon,i_\alpha+\varepsilon)|,|R_S(f_\beta,f_\alpha)-R_S(f_\beta-\varepsilon,f_\alpha+\varepsilon)|, \\
|R_T(t_\alpha,t_\beta)-R_T(t_\alpha-\varepsilon,t_\beta-\varepsilon)|,|R_S(i_\beta,i_\alpha)-R_S(i_\beta-\varepsilon,i_\alpha-\varepsilon)|,|R_S(f_\beta,f_\alpha)-R_S(f_\beta-\varepsilon,f_\alpha-\varepsilon)|, \\
|R_S(i_\beta,i_\alpha)-R_S(i_\beta-\varepsilon,i_\alpha)|,|R_S(f_\beta,f_\alpha)-R_S(f_\beta-\varepsilon,f_\alpha)|
\end{matrix}\right\}
\end{aligned}
$$

where R_T is residual implication induced by left-continuous t-norm T, R_S is coresidual implication induced by right-continuous t-conorm S.

Corollary 4.1. The ε sensitivity of the single-valued neutrosophic Łukasiewicz t-representable t-norm is $\triangle_{\mathcal{T}_L}(\varepsilon)=2\varepsilon\wedge 1$.

Corollary 4.2. The ε sensitivity of the single-valued neutrosophic Łukasiewicz residual implication is $\triangle_{\mathcal{R}_{\mathcal{T}_L}}=2\varepsilon\wedge 1$.

Definition 4.5. Let A and A' be two single valued neutrosophic fuzzy sets on universal X. If $\| A-A' \|=\vee_{x\in X} d\,(A(x),A'(x))\ \le\varepsilon$ for all $x\in X$, then A' is called ε-perturbation of A denoted by $A'\in O(A,\varepsilon)$.

Theorem 4.2. Let $A,\ A',\ B,\ B',\ A^*$ and A'^* be single-valued neutrosophic fuzzy sets. If $\| A-A' \|\le\varepsilon$, $\| B-B' \|\le\varepsilon$, $\| A^*-A'^* \|\le\varepsilon$, B^* and B'^* are the single-valued neutrosophic fuzzy inference triple I solutions of $FMP(A,\ B,\ A^*)$ and $FMP(A',\ B',\ A'^*)$ given in Theorem 3.1 respectively, then the ε sensitivity of the single-valued neutrosophic fuzzy inference triple I solution B^* for FMP is

$$\triangle_{B^*}(\varepsilon)=\| B^*-B'^* \|\le\triangle_{\mathcal{T}}(\triangle_{\mathcal{R}}(\varepsilon)).$$

Proof: Let $A,A',A^*,A'^*\in SNVS(X)$, $B,B'\in SNVS(Y)$. If $\| A-A' \|\le\varepsilon$, $\| B-B' \|\le\varepsilon$, $\| A^*-A'^* \|\le\varepsilon$, then we have,

$$\begin{aligned}\Delta_{B^*}(\varepsilon) &= \| B^* - B'^* \| \\ &= \bigvee_{y\in Y} d\,(B^*(y), B'^*(y)) \\ &= \bigvee_{y\in Y} d\,(\bigvee_{x\in X} \mathcal{T}\,(\mathcal{R}(A(x),B(y)),A^*(x)), \bigvee_{x\in X} \mathcal{T}\,(\mathcal{R}(A'(x),B'(y)),A'^*(x))) \\ &\le \bigvee_{y\in Y}\bigvee_{x\in X} d\,(\mathcal{T}(\mathcal{R}(A(x),B(y)),A^*(x)),\mathcal{T}(\mathcal{R}(A'(x),B'(y)),A'^*(x))) \\ &\le \Delta_{\mathcal{T}}\,(\Delta_{\mathcal{R}}\,(\varepsilon))\end{aligned}$$

Corollary 4.3. Suppose $\mathcal{R}$ is residuated implication induced by single valued neutrosophic Łukasiewicz t-representable t-norm $\mathcal{T}$, then $\Delta_{B^*}(\varepsilon) = 3\varepsilon \wedge 1$.

Proof: Let $A^*(x) = \langle t_1, i_1, f_1\rangle$, $A(x) = \langle t_2, i_2, f_2\rangle$, $B(y) = \langle t_3, i_3, f_3\rangle$, $A'^*(x) = \langle t_1', i_1', f_1'\rangle$, $A'(x) = \langle t_2', i_2', f_2'\rangle$, $B'(y) = \langle t_3', i_3', f_3'\rangle$. Suppose that $\| A - A' \| \le \varepsilon$, $\| B - B' \| \le \varepsilon$, $\| A^* - A'^* \| \le \varepsilon$, according to Proposition 4.1, then we have:

$$\begin{aligned}& d(\mathcal{T}(A^*(x),\mathcal{R}(A(x),B(y))),\mathcal{T}(A'^*(x),\mathcal{R}(A'(x),B'(y)))) \\ &= max\{|(0 \vee (t_1 + R_T(t_2,t_3) - 1)) - (0 \vee (t_1' + R_T(t_2',t_3') - 1))|, \\ &|((i_1 + R_S(i_3,i_2)) \wedge 1) - ((i_1' + R_S(i_3',i_2')) \wedge 1)|, \\ &|((f_1 + R_S(f_3,f_2)) \wedge 1) - ((f_1' + \mathcal{R}_S(f_3',f_2')) \wedge 1)|\} \\ &\le max\{|(0 \vee (t_1 + R_T(t_2,t_3) - 1)) - (0 \vee ((t_1 + \varepsilon) + R_T(t_2,t_3) + \Delta_{\mathcal{R}}(\varepsilon) - 1))|, \\ &|((i_1 + R_S(i_3,i_2)) \wedge 1) - ((i_1 + \varepsilon + R_S(i_3,i_2) + \Delta_{\mathcal{R}}(\varepsilon)) \wedge 1)|, \\ &|((f_1 + R_S(f_3,f_2)) \wedge 1) - ((f_1 + \varepsilon + R_S(f_3,f_2) + \Delta_{\mathcal{R}}(\varepsilon)) \wedge 1)|\} \\ &\le \varepsilon + \Delta_{\mathcal{R}}(\varepsilon)\end{aligned}$$

For Łukasiewicz implication, for all $A^*(x) = \langle t_1, i_1, f_1\rangle$, we can tack $A'^*(x) = \langle t_1 + \varepsilon, i_1 + \varepsilon, f_1 + \varepsilon\rangle$, $\mathcal{R}(\langle t_2, i_2, f_2\rangle, \langle t_3, i_3, f_3\rangle) = \langle 1,0,0\rangle$, $\mathcal{R}(\langle t_2', i_2', f_2'\rangle, \langle t_3', i_3', f_3'\rangle) = \langle 1 - \Delta_{\mathcal{R}}(\varepsilon), \Delta_{\mathcal{R}}(\varepsilon), \Delta_{\mathcal{R}}(\varepsilon)\rangle$ satisfy the above equation, i.e. $\Delta_{B^*}(\varepsilon) = \varepsilon + \Delta_{\mathcal{R}}(\varepsilon)$. Therefore, $\Delta_{B^*}(\varepsilon) = 3\varepsilon \wedge 1$ by Corollary 4.2.

Corollary 4.4. Suppose $\mathcal{R}$ is a residuated implication induced by single valued neutrosophic Goguen t-representable t-norm $\mathcal{T}$, then $\Delta_{B^*}(\varepsilon) = \varepsilon + (1 - \varepsilon)\,\Delta_{\mathcal{R}}(\varepsilon)$.

Proof: Let $A^*(x) = \langle t_1, i_1, f_1\rangle$, $A(x) = \langle t_2, i_2, f_2\rangle$, $B(y) = \langle t_3, i_3, f_3\rangle$, $A'^*(x) = \langle t_1', i_1', f_1'\rangle$, $A'(x) = \langle t_2', i_2', f_2'\rangle$, $B'(y) = \langle t_3', i_3', f_3'\rangle$. Suppose that $\| A - A' \| \le \varepsilon$, $\| B - B' \| \le \varepsilon$, $\| A^* - A'^* \| \le \varepsilon$, according to Proposition 4.1, then we have:

$$\begin{aligned}& d(\mathcal{T}(A^*(x),\mathcal{R}(A(x),B(y))),\mathcal{T}(A'^*(x),\mathcal{R}(A'(x),B'(y)))) \\ &= |t_1 \cdot \mathcal{R}_T(t_2,t_3) - t_1' \cdot \mathcal{R}_T(t_2',t_3')| \\ &\vee |(i_1 + \mathcal{R}_S(i_3,i_2) - i_1 \cdot \mathcal{R}_S(i_3,i_2)) - (i_1' + \mathcal{R}_S(i_3',i_2') - i_1' \cdot \mathcal{R}_S(i_3',i_2'))| \\ &\vee |(f_1 + \mathcal{R}_S(f_3,f_2) - f_1 \cdot \mathcal{R}_S(f_3,f_2)) - (f_1' + \mathcal{R}_S(f_3',f_2') - f_1' \cdot \mathcal{R}_S(f_3',f_2')| \\ &\le |t_1 \cdot \mathcal{R}_T(t_2,t_3) - (t_1 - \varepsilon) \cdot (\mathcal{R}_T(t_2,t_3) - \Delta_{\mathcal{R}}(\varepsilon))| \\ &\vee |(i_1 + \mathcal{R}_S(i_3,i_2) - i_1 \cdot \mathcal{R}_S(i_3,i_2)) - ((i_1 + \varepsilon) + (\mathcal{R}_S(i_3,i_2) - \Delta_{\mathcal{R}}(\varepsilon)) - ((i_1 + \varepsilon) \cdot (\mathcal{R}_S(i_3,i_2) - \Delta_{\mathcal{R}}(\varepsilon)))| \\ &\vee |(f_1 + \mathcal{R}_S(f_3,f_2) - f_1 \cdot \mathcal{R}_S(f_3,f_2)) - ((f_1 + \varepsilon) + (\mathcal{R}_S(f_3,f_2) - \Delta_{\mathcal{R}}(\varepsilon)) - ((f_1 + \varepsilon) \cdot (\mathcal{R}_S(f_3,f_2) - \Delta_{\mathcal{R}}(\varepsilon)))| \\ &\le \varepsilon + (1 - \varepsilon)\,\Delta_{\mathcal{R}}(\varepsilon)\end{aligned}$$

For Goguen implication, we can take $A^*(x) = \langle t_1, i_1, f_1\rangle = \langle 1,0,0\rangle$, $A'^*(x) = \langle 1 - \varepsilon, \varepsilon, \varepsilon\rangle$, $\mathcal{R}(\langle t_2, i_2, f_2\rangle, \langle t_3, i_3, f_3\rangle) = \langle 1,0,0\rangle$, satisfy the above equation, i.e. $\Delta_{B^*}(\varepsilon) = \varepsilon + (1 - \varepsilon)\,\Delta_{\mathcal{R}}(\varepsilon)$.

Corollary 4.5. Suppose $\mathcal{R}$ is residuated implication induced by single valued neutrosophic $Gödel$ t-representable t-norm $\mathcal{T}$, then $\Delta_{B^*}(\varepsilon) = \Delta_{\mathcal{R}}(\varepsilon)$.

Proof: According to Theorem 4.1, we have $\Delta_{B^*}(\varepsilon) \le \Delta_{\mathcal{T}}(\Delta_{\mathcal{R}}(\varepsilon))$, since $\mathcal{T}$ is single-valued neutrosophic $Gödel$ t-norm, then we have $\Delta_{B^*}(\varepsilon) \le \Delta_{\mathcal{R}}(\varepsilon)$. Let $A^*(x) = 1^*$, then $B^*(y) = \bigvee_{x\in X} \mathcal{T}\,(1^*, \mathcal{R}(A(x),B(y))) = \bigvee_{x\in X} \mathcal{R}\,(A(x),B(y))$, i.e., $\Delta_{B^*}(\varepsilon) \ge \Delta_{\mathcal{R}}(\varepsilon)$. Therefore, $\Delta_{B^*}(\varepsilon) = \Delta_{\mathcal{R}}(\varepsilon)$.

Theorem 4.3. Let $A,\ A',\ B,\ B',\ B^*$ and B'^* be single-valued neutrosophic fuzzy sets. If $\| A - A' \| \leq \varepsilon$, $\| B - B' \| \leq \varepsilon$, $\| B^* - B'^* \| \leq \varepsilon$, A^* and A'^* are single-valued neutrosophic $\mathcal{R}$-type triple I solutions of FMT $(A,\ B,\ B^*)$ and FMT$(A',\ B',\ B'^*)$ given in Theorem 3.2 respectively, then the ε sensitivity of the single-valued neutrosophic $\mathcal{R}$-type triple I solution A^* for FMT is

$$\triangle_{A^*}(\varepsilon) = \| A^* - A'^* \| \leq \triangle_{\mathcal{R}}(\triangle_{\mathcal{R}}(\varepsilon)).$$

Proof: Let $A, A' \in SNVS(X),\ B, B', B^*, B'^* \in SNVS(Y)$. If $\| A - A' \| \leq \varepsilon,\ \| B - B' \| \leq \varepsilon,\ \| B^* - B'^* \| \leq \varepsilon$, then we have,

$$\begin{aligned} \triangle_{A^*}(\varepsilon) &= \| A^* - A'^* \| \\ &= \bigvee_{x\in X} d\,(A^*(x), A'^*(x)) \\ &= \bigvee_{x\in X} d\,(\bigwedge_{y\in Y} \mathcal{R}\,(\mathcal{R}(A(x), B(y)), B^*(y)), \bigwedge_{y\in Y} \mathcal{R}\,(\mathcal{R}(A'(x), B'(y)), B'^*(y))) \\ &\leq \bigvee_{x\in X}\bigvee_{y\in Y} d\,(\mathcal{R}(\mathcal{R}(A(x), B(y)), B^*(y)), \mathcal{R}(\mathcal{R}(A'(x), B'(y)), B'^*(y))) \\ &\leq \triangle_{\mathcal{R}}(\triangle_{\mathcal{R}}(\varepsilon)) \end{aligned}$$

Corollary 4.6. Suppose $\mathcal{R}$ is residuated implication induced by single valued neutrosophic Łukasiewicz t-representable t-norm $\mathcal{T}$, then $\triangle_{A^*}(\varepsilon) = 3\varepsilon \wedge 1$.

Proof: Let $B^*(y) = \langle t_1, i_1, f_1\rangle$, $A(x) = \langle t_2, i_2, f_2\rangle$, $B(y) = \langle t_3, i_3, f_3\rangle$, $B'^*(y) = \langle t_1', i_1', f_1'\rangle$, $A'(x) = \langle t_2', i_2', f_2'\rangle$, $B'(y) = \langle t_3', i_3', f_3'\rangle$. Suppose that $\| A - A' \| \leq \varepsilon,\ \| B - B' \| \leq \varepsilon,\ \| A^* - A'^* \| \leq \varepsilon$, according to Proposition 4.1, then we have:

$$\begin{aligned} & d(\mathcal{R}(\mathcal{R}(A(x), B(y)), B^*(y)), \mathcal{R}(\mathcal{R}(A'(x), B'(y)), B'^*(y))) \\ &= max\{|(1 \wedge (1 - R_T(t_2, t_3) + t_1)) - (1 \wedge (1 - R_T(t_2', t_3') + t_1'))|, \\ &\quad |((i_1 - R_S(i_3, i_2)) \vee 0) - ((i_1' - R_S(i_3', i_2')) \vee 0)|, \\ &\quad |((f_1 - R_S(f_3, f_2)) \vee 0) - ((f_1' - \mathcal{R}_S(f_3', f_2')) \vee 0)|\} \\ &\leq max\{|(1 \wedge (1 - R_T(t_2, t_3) + t_1)) - (1 \wedge (1 - (R_T(t_2, t_3) - \triangle_{\mathcal{R}}(\varepsilon)) + (t_1 + \varepsilon)))|, \\ &\quad |((i_1 - R_S(i_3, i_2)) \vee 0) - ((i_1 - \varepsilon - (R_S(i_3, i_2) + \triangle_{\mathcal{R}}(\varepsilon))) \vee 0)|, \\ &\quad |((f_1 - R_S(f_3, f_2)) \vee 0) - ((f_1 - \varepsilon - (R_S(f_3, f_2) + \triangle_{\mathcal{R}}(\varepsilon))) \vee 0)|\} \\ &\leq \varepsilon + \triangle_{\mathcal{R}}(\varepsilon) \end{aligned}$$

For the single-valued neutrosophic Lukasiewicz implication, for all $A^*(x) =$ Žt_1, i_1, f_1ž, we can tack $A'^*(x) = \langle t_1 + \varepsilon, i_1 - \varepsilon, f_1 - \varepsilon\rangle$, $\mathcal{R}(\langle t_2, i_2, f_2\rangle, \langle t_3, i_3, f_3\rangle) = \langle 1,0,0\rangle$, $\mathcal{R}(\langle t_2', i_2', f_2'\rangle, \langle t_3', i_3', f_3'\rangle) = \langle 1 - \triangle_{\mathcal{R}}(\varepsilon), \triangle_{\mathcal{R}}(\varepsilon), \triangle_{\mathcal{R}}(\varepsilon)\rangle$ satisfy the above equation, i.e. $\triangle_{A^*}(\varepsilon) = \varepsilon + \triangle_{\mathcal{R}}(\varepsilon)$. Therefore, $\triangle_{A^*}(\varepsilon) = 3\varepsilon \wedge 1$ by Corollary 4.2.

Corollary 4.7. Suppose $\mathcal{R}$ is a residuated implication induced by single valued neutrosophic Goguen t-representable t-norm $\mathcal{T}$, then $\triangle_{A^*}(\varepsilon) = \frac{\varepsilon}{1 - \triangle_{\mathcal{R}}(\varepsilon)}$.

Proof: Let $B^*(y) = \langle t_1, i_1, f_1\rangle$, $A(x) = \langle t_2, i_2, f_2\rangle$, $B(y) = \langle t_3, i_3, f_3\rangle$, $B'^*(y) = \langle t_1', i_1', f_1'\rangle$, $A'(x) = \langle t_2', i_2', f_2'\rangle$, $B'(y) = \langle t_3', i_3', f_3'\rangle$. Suppose that $\| A - A' \| \leq \varepsilon,\ \| B - B' \| \leq \varepsilon,\ \| A^* - A'^* \| \leq \varepsilon$, according to Proposition 4.1, then we have:

$$d(\mathcal{R}(\mathcal{R}(A(x),B(y)),B^*(y)),\mathcal{R}(\mathcal{R}(A'(x),B'(y)),B'^*(y)))$$
$$= max\{|(1 \wedge (\frac{t_1}{R_T(t_2,t_3)})) - (1 \wedge (\frac{t'_1}{R_T(t'_2,t'_3)})|,$$
$$|\frac{(i_1 - R_S(i_3,i_2)) \vee 0}{1 - R_S(i_3,i_2)} - \frac{(i'_1 - R_S(i'_3,i'_2)) \vee 0}{1 - R_S(i'_3,i'_2)}|,$$
$$|\frac{(f_1 - R_S(f_3,f_2)) \vee 0}{1 - R_S(f_3,f_2)} - \frac{(f'_1 - R_S(f'_3,f'_2)) \vee 0}{1 - R_S(f'_3,f'_2)}|,$$
$$\leq max\{|(1 \wedge (\frac{t_1}{R_T(t_2,t_3)})) - (1 \wedge (\frac{t_1 - \varepsilon}{R_T(t_2,t_3) + \Delta_{\mathcal{R}}(\varepsilon)}))|,$$
$$|\frac{(i_1 - R_S(i_3,i_2)) \vee 0}{1 - R_S(i_3,i_2)} - \frac{((i_1 - \varepsilon) - (R_S(i_3,i_2) + \Delta_{\mathcal{R}}(\varepsilon))) \vee 0}{1 - (R_S(i_3,i_2) + \Delta_{\mathcal{R}}(\varepsilon)}|,$$
$$|\frac{(f_1 - R_S(f_3,f_2)) \vee 0}{1 - R_S(f_3,f_2)} - \frac{((f_1 - \varepsilon) - (R_S(f_3,f_2) + \Delta_{\mathcal{R}}(\varepsilon))) \vee 0}{1 - (R_S(f_3,f_2) + \Delta_{\mathcal{R}}(\varepsilon)}\}$$
$$\leq \frac{\varepsilon}{1 - \Delta_{\mathcal{R}}(\varepsilon)}$$

For the single-valued neutrosophic Goguen implication, we can tack $B^*(y) = \langle \varepsilon,1,1 \rangle$, $B'^*(y) = \langle 0,1-\varepsilon,1-\varepsilon \rangle$, $\mathcal{R}(\langle t_2,i_2,f_2 \rangle, \langle t_3,i_3,f_3 \rangle) = \langle 1 - \Delta_{\mathcal{R}}(\varepsilon),0,0 \rangle$, satisfy the above equation, i.e. $\Delta_{A^*}(\varepsilon) = \frac{\varepsilon}{1-\Delta_{\mathcal{R}}(\varepsilon)}$.

Corollary 4.8. Suppose $\mathcal{R}$ is residuated implication induced by single valued neutrosophic *Gödel* t-representable t-norm $\mathcal{T}$, then $\Delta_{A^*}(\varepsilon) = 1$.

Proof: Let $B^*(y) = \langle t_1,i_1,f_1 \rangle$, $A(x) = \langle t_2,i_2,f_2 \rangle$, $B(y) = \langle t_3,i_3,f_3 \rangle$, $B'^*(y) = \langle t'_1,i'_1,f'_1 \rangle$, $A'(x) = \langle t'_2,i'_2,f'_2 \rangle$, $B'(y) = \langle t'_3,i'_3,f'_3 \rangle$. Suppose that $\parallel A - A' \parallel \leq \varepsilon$, $\parallel B - B' \parallel \leq \varepsilon$, $\parallel A^* - A'^* \parallel \leq \varepsilon$, according to Proposition 4.1, then we have:

$$d(\mathcal{R}(\mathcal{R}(A(x),B(y)),B^*(y)),\mathcal{R}(\mathcal{R}(A'(x),B'(y)),B'^*(y)))$$
$$= max\{|R_T(R_T(t_2,t_3),t_1) - R_T(R_T(t'_2,t'_3),t'_1))|,$$
$$|R_S(i_1,R_S(i_3,i_2)) - R_S(i'_1,R_S(i'_3,i'_2))|,$$
$$|R_S(f_1,R_S(f_3,f_2)) - R_S(f'_1,\mathcal{R}_S(f'_3,f'_2))|\}$$
$$\leq 1$$

For the single-valued neutrosophic *Gödel* implication, we can tack $B^*(y) = \langle \varepsilon,1,1 \rangle$, $A(x) = \langle \frac{\varepsilon}{2},1-\varepsilon,1-\varepsilon \rangle$, $B(y) = \langle \frac{\varepsilon}{4},1-\frac{\varepsilon}{2},1-\frac{\varepsilon}{2} \rangle$, $B'^*(y) = \langle 0,1-\varepsilon,1-\varepsilon \rangle$, $A'(x) = \langle \varepsilon,1-2\varepsilon,1-2\varepsilon \rangle$, $B'(y) = \langle \frac{\varepsilon}{2},1-\varepsilon,1-\varepsilon \rangle$, then $\mathcal{R}(A(x),B(y)) = \langle \frac{\varepsilon}{4},0,0 \rangle$, $\mathcal{R}(A'(x),B'(y)) = \langle \frac{\varepsilon}{2},0,0 \rangle$, satisfy the above equation, i.e., $\Delta_{A^*}(\varepsilon) = 1$.

5. Conclusions

In this paper, we extend the fuzzy inference triple I method on single-valued neutrosophic sets. Single valued neutrosophic fuzzy inference triple I Principle for and are proposed. Moreover, the single-valued neutrosophic fuzzy inference triple I solutions for and are given respectively. The reductivity and the robustness of the single-valued neutrosophic fuzzy inference triple I methods are studied.

This article only conducts research on fuzzy reasoning algorithms at the theoretical level and has not been applied in databases; when using t-representable t-norm, this article only considers the case of $R_T = R_S$, without analyzing and demonstrating the case of $R_T \neq R_S$.

The logical basis of a fuzzy inference method is very important. In the future, we will consider building the strict logic foundation for the triple I method based on left-continuous single-valued neutrosophic t-representable t-norms, and bring the single-valued neutrosophic fuzzy inference method within the framework of logical semantic. Not only that, analyze and discuss the case of $R_T \neq R_S$ for the algorithm, and apply the algorithm to pattern recognition in the database.

Acknowledgments
The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Data availability
The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Funding
This work was supported by the National Natural Science Foundation of China (No.12171445).

Conflict of interest
The authors declare that there is no conflict of interest in the research.

Ethical approval
This article does not contain any studies with human participants or animals performed by any of the authors.

References

1. Wang, G. (1999). The full implication triple I method for fuzzy reasoning. Science in China (Series E), 29(1), 43-53.
2. Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning-I. Information Sciences, 8(3). https://doi.org/10.1016/0020-0255(75)90036-5
3. Zadeh, L. A. (1973). Outline of a New Approach to the Analysis of Complex Systems and Decision Processes. IEEE Transactions on Systems, Man and Cybernetics, SMC-3(1). https://doi.org/10.1109/TSMC.1973.5408575
4. Wang, G. J. (1999). On the logic foundation of fuzzy reasoning. Information Sciences, 117(1). https://doi.org/10.1016/S0020-0255(98)10103-2
5. Wang, G. J., & Fu, L. (2005). Unified forms of Triple I method. Computers and Mathematics with Applications, 49(5–6). https://doi.org/10.1016/j.camwa.2004.01.019
6. Pei, D. (2008). Unified full implication algorithms of fuzzy reasoning. Information Sciences, 178(2). https://doi.org/10.1016/j.ins.2007.09.003
7. Pei, D. (2012). Formalization of implication based fuzzy reasoning method. International Journal of Approximate Reasoning, 53(5). https://doi.org/10.1016/j.ijar.2012.01.007
8. Liu, H. W., & Wang, G. J. (2007). Unified forms of fully implicational restriction methods for fuzzy reasoning. Information Sciences, 177(3). https://doi.org/10.1016/j.ins.2006.08.012
9. Luo, M., & Yao, N. (2013). Triple I algorithms based on Schweizer–Sklar operators in fuzzy reasoning. International Journal of Approximate Reasoning, 54(5), 640-652. https://doi.org/10.1016/j.ijar.2013.01.008
10. K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20(1986)87-96. https://doi.org/10.1016/S0165-0114(86)80034-3
11. B. Gorzalczany, Approximate inference with interval-valued fuzzy sets-an outline, in: Proc. Polish Symp. On Interval and Fuzzy Mathematics, Poznan, 1983, pp. 89-95.
12. Turksen, I. B. (1986). Interval valued fuzzy sets based on normal forms. Fuzzy Sets and Systems, 20(2). https://doi.org/10.1016/0165-0114(86)90077-1
13. Atanassov, K., & Gargov, G. (1989). Interval valued intuitionistic fuzzy sets. Fuzzy Sets and Systems, 31(3). https://doi.org/10.1016/0165-0114(89)90205-4
14. Zheng, M., Shi, Z., & Liu, Y. (2014). Triple i method of approximate reasoning on Atanassov's intuitionistic fuzzy sets. International Journal of Approximate Reasoning, 55(6). https://doi.org/10.1016/j.ijar.2014.01.001

15. Li, D. C., Li, Y. M., & Xie, Y. J. (2011). Robustness of interval-valued fuzzy inference. Information Sciences, 181(20). https://doi.org/10.1016/j.ins.2011.06.015.
16. Luo, M., & Zhang, K. (2015). Robustness of full implication algorithms based on interval-valued fuzzy inference. International Journal of Approximate Reasoning, 62. https://doi.org/10.1016/j.ijar.2015.05.006
17. Luo, M., & Zhou, X. (2015). Robustness of reverse triple i algorithms based on interval-valued fuzzy inference. International Journal of Approximate Reasoning, 66. https://doi.org/10.1016/j.ijar.2015.07.004
18. Luo, M., Cheng, Z., & Wu, J. (2016). Robustness of interval-valued universal triple i algorithms 1. Journal of Intelligent and Fuzzy Systems, 30(3). https://doi.org/10.3233/IFS-151870
19. Luo, M., & Liu, B. (2017). Robustness of interval-valued fuzzy inference triple I algorithms based on normalized Minkowski distance. Journal of Logical and Algebraic Methods in Programming, 86(1). https://doi.org/10.1016/j.jlamp.2016.09.006
20. Luo, M., & Wang, Y. (2020). Interval-valued fuzzy reasoning full implication algorithms based on the t-representable t-norm. International Journal of Approximate Reasoning, 122. https://doi.org/10.1016/j.ijar.2020.03.009
21. Smarandache, F. (1999). A Unifying Field in Logics: Neutrosophic Logic, Neutrosophy, Neutrosophic Set, Neutrosophic Probability. In American Research Press.
22. Smarandache, F. (1998). Neutrosophy: neutrosophic probability, set, and logic: analytic synthesis & synthetic analysis. In Rehoboth: American Research Press (Issue October).
23. H. Wang, F. Smarandache, Y.Q. Zhang, R. Sunderraman, Single valued neutrosophic sets, Multispace and Multistructure 4(2010)410-413.
24. Smarandache, F. (2009). N-norm and N-conorm in Neutrosophic Logic and Set, and the Neutrosophic Topologies. arXiv preprint arXiv:0901.1289.
25. Rivieccio, U. (2008). Neutrosophic logics: Prospects and problems. Fuzzy Sets and Systems, 159(14). https://doi.org/10.1016/j.fss.2007.11.011
26. Alkhazaleh, S. (2015). More on neutrosophic norms and conorms. Neutrosophic Sets Syst, 9, 23-30.
27. Zhang, X., Bo, C., Smarandache, F., & Dai, J. (2018). New inclusion relation of neutrosophic sets with applications and related lattice structure. International Journal of Machine Learning and Cybernetics, 9(10). https://doi.org/10.1007/s13042-018-0817-6
28. Hu, Q., & Zhang, X. (2019). Neutrosophic triangular norms and their derived residuated lattices. Symmetry, 11(6). https://doi.org/10.3390/sym11060817
29. Das, S., Poulik, S., & Ghorai, G. (2023). Picture fuzzy ϕ-tolerance competition graphs with its application. Journal of Ambient Intelligence and Humanized Computing, 1-13.
30. Poulik, S., & Ghorai, G. (2022). Connectivity Concepts in Bipolar Fuzzy Incidence Graphs. Thai Journal of Mathematics, 20(4), 1609-1619.
31. Poulik, S., & Ghorai, G. (2022). Estimation of most effected cycles and busiest network route based on complexity function of graph in fuzzy environment. Artificial Intelligence Review, 55(6). https://doi.org/10.1007/s10462-021-10111-2
32. Zhao, R., Luo, M., & Li, S. (2020). Reverse triple I method based on single valued neutrosophic fuzzy inference. Journal of Intelligent & Fuzzy Systems, 39(5), 7071-7083. https://doi.org/10.3233/jifs-200265
33. E. P. Klement, R. Mesiar, E. Pap, Triangular Norms, Springer Netherlands, 2000.
34. Edison, L. A., & Duncan, J. (1971). The Elements of Complex Analysis. The American Mathematical Monthly, 78(3). https://doi.org/10.2307/2317549

Received: 27 Nov 2023, **Revised:** 28 Nov 2023,

Accepted: 06 Jan 2024, **Available online:** 07 Jan 2024.