Neutrosophic Systems with Applications, Vol. 14, 2024
https://doi.org/10.61356/j.nswa.2024.116

Extended Event Calculus using Neutrosophic Logic: Method, Implementation, Analysis, Recent Progress and Future Directions

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Abstract: Brains do not reason as digital computers do. Computers reason in clear steps with statements that are either true or false, while humans reason with vague terms of common sense. Neutrosophy is a new branch of philosophy and machine intelligence that deals with neutralities, specifically the idea of indeterminacy that is evident and experienced in our everyday lives. Indeterminacy is interpreted as everything that falls between a concept, idea, statement, declaration, etc. and its opposite. The fundamental thesis of neutrosophy is to employ neutrosophic logic, an extension of fuzzy logic, to incorporate fuzzy truth into complex schemes of formal reasoning. Event calculus is a logical formalism used to describe and reason about events and their consequences over time. It is considered a valuable mathematical tool in the field of artificial intelligence (AI) for depicting dynamic systems where events occur and have temporal relationships with each other. However, previous studies in AI have neglected to adequately address the complexity of time. In this context, our work aims to introduce a neutrosophic event-based calculus as a logic formalism to handle situations where there is insufficient knowledge or ambiguity regarding the occurrence or consequences of certain events in a system. In particular, neutrosophic event calculus examines causality between ideas and the connection between tasks and actions in the presence of time. Due to the lack of related studies in the existing literature, we believe that our work will contribute to the field of knowledge representation by proposing an alternative to current forms of logic. We aim to demonstrate the capacity of neutrosophic event calculus in the context of knowledge representation and reasoning.

Keywords: Neutrosophy; Neutrosophic Logic; Event Calculus; Logical Formalism; Knowledge Representation and Reasoning; Artificial Intelligence.

1. Introduction

Fuzziness originated as vagueness in the late 19th century. A concept is considered vague if its boundaries are blurred, meaning not all statements can be categorized as true or false to the same extent. Logician Bertrand Russell was the first to identify vagueness in symbolic logic [1]. Concept A is vague if it violates Aristotle's law of excluded middle, meaning A or not-A does not hold. Russell realized in his 1923 article "Vagueness" that we may need to relax Aristotle's law to handle paradoxes and account for the vagueness of factual statements. This article marks the beginning of formal fuzzy logic.

Polish logician Jan Lukasiewicz made the next major breakthrough after Russell. In the 1920s, he developed the first fuzzy or multivalued logic [2]. In a 1937 article in Philosophy of Science, quantum philosopher Max Black applied multivalued logic to lists or sets of objects and drew the first fuzzy curves [3]. These sets A are such that each object x partially belongs to A and not-A, making them properly vague or fuzzy. Kaplan and Schott [4], along with other logicians in the 1950s [5], introduced the min and max operations to define a fuzzy set algebra.

In 1965, Zadeh published his influential paper "Fuzzy Sets" [6], which introduced the term "fuzzy" to mean "vague" in technical literature for the first time. Zadeh's paper applied Lukasiewicz's logic to each object in a set to establish a complete fuzzy set algebra and extend the convex separation theorem of pattern recognition. Zadeh introduced the concept of objects in a class being seen as a continuum of grades of membership. He explained the grade of membership function, including its union and intersection operations. When the nodes and edges of a linked graphical or network system are unclear, fuzzy graphs (FG) can provide intuition. Within this framework, the determination of vertex degree and membership values is always necessary to determine the strengths of vertices in a FG. The Randic index can be used to identify the most significant vertices and the most loaded pathways [7]. Bipolar FGs, which represent two opposite ways of thinking, such as forward and backward, effect and side effect, cooperation and competition, gain and loss etc, can be used to provide qualitative solutions in decision-making problems in real life. The article of [8] introduces the concept of a bipolar fuzzy incidence graph (BFIG) and its matrix representation and it also discusses the characteristics of a bipolar fuzzy incidence subgraph. Researchers in [9] have applied the concept of competitive graphs (CG) using ϕ-tolerances (TCG) in a picture fuzzy (PF) environment which is not well studied in the literature. PF-TCG models are more successful than other models in solving specific scheduling and resource allocation problems in operations research. Three special types of picture fuzzy ϕ-tolerance CGs are introduced and applied to two real-life applications in railway network and medical science, using ϕ as max, min, and sum functions.

In the mid-90s, Smarandache began utilizing non-standard analysis with a tri-component logic/set/probability theory, starting from a philosophical exploration of multi-valued logics. As a result, he developed neutrosophic logic, as fuzzy logic alone is believed to be unable to demonstrate indeterminacy [10]. According to the definition provided in [10], "Neutrosophic logic is a logic variety that generalizes fuzzy logic, paraconsistent logic, intuitionistic logic, and other logic variants. The degree of membership (T) of each set element is the first part of neutrosophic logic, indeterminacy (I) is the middle part, and falsehood (F) is the third part, respectively."

Neutrosophic logic is significant and has been applied in various research areas in recent years. Within our research framework, particularly in the field of knowledge representation, scholars in [11] have explored how neutrosophic logic can be integrated into situation analysis to propose a framework that addresses the multiple aspects of uncertainty and information inherent in the situation analysis environment, effectively dealing with the ontological and epistemological challenges of situation analysis. Additionally, the work in [12] deserves mention as the first study to introduce neutrosophic modal logic in the related literature. Neutrosophic modal logic is a formal logic that incorporates neutrosophic modalities and is governed by a set of neutrosophic axioms and rules.

It is evident from our understanding that the natural world is constantly evolving or changing. Therefore, processes, whether natural or technical, are dynamic, and abstract concepts must embrace change to be useful. Consequently, the concept of evolving representations over time is crucial. The notion of time and its explanation within the limits of our perception has been a concern for humanity since ancient times. The study of temporal logic originated with Aristotle and the Megarian and Stoic schools in ancient Greece. It is worth noting that as early as 350 B.C., Aristotle argued that actions are justified by a logical connection between goals and knowledge of the outcome of the action. In the modern era, Findley [13] was the first to propose a standardized calculus for reasoning based on time, but the most significant impact is attributed to the seminal work of Arthur Prior published in 1967 [14].

Event calculus is a logical formalism used to reason about dynamic systems and events in the fields of AI and philosophy. Kowalski and Sergot introduced the event calculus as a logic programming paradigm for modeling events and their consequences, particularly in database systems [15]. Shanahan [16] proposed further improvements based on first-order predicate calculus, which can describe a wide range of phenomena, including acts with indirect consequences, activities with non-deterministic effects, compound actions, concurrent actions, and continuous change.

1.1 Motivation

- The main consideration that we had in mind concerning the motivation of our study was to propose a proper formalism that would integrate and manipulate the concepts of uncertainty and incomplete information in the context of event calculus. Event calculus's constraints, in terms of uncertainty, stem from its fundamental character as a deterministic logic-based formalism. Traditional event calculus lacks explicit techniques for dealing with uncertainty or partial or probabilistic information. This is achieved by proposing a hybrid logical framework that incorporates neutrosophic logic with event calculus.
- Another concern that led us to our current research work is that time representation and comprehension in AI have been identified as areas that require further study. While there are several temporal formalisms, they frequently simplify time in ways that do not convey its entire complexity in real-world circumstances. In everyday life, time is sometimes uncertain, and events may have fuzzy or probabilistic temporal features. Furthermore, the dynamics of systems fluctuate over time, and describing such changing temporal dynamics is difficult. Within this concern, developing formalisms that explicitly account for uncertain or probabilistic temporal information, enabling AI systems to reason effectively in the face of incomplete or uncertain temporal knowledge, is considered crucial.
- At this point, one could possibly ask why we should use event calculus instead of, for example, situation calculus [17], which is also a prominent logic-based formalism that deals with similar situations. Our answer comes with the observation that when there is a single agent doing instantaneous and discrete actions, situation analysis formalism works effectively. But in cases when actions have durations and can overlap, situation analysis becomes fairly cumbersome. As a result, we address these concerns using an alternative formulation based on event calculus, which could be considered a time-based formalism rather than a state-based one. Furthermore, it allows reasoning in terms of time intervals instead of states, which is a more realistic approach when dealing with real-world problems because being able to handle temporal aspects and causal relationships makes it useful in modeling and reasoning about dynamic environments.
- Lastly, we should remark that, in the framework of classical event calculus, formulating inference rules can be complex and cumbersome, especially for domains with a large number of interconnected occurrences. Developing rules that correctly capture all conceivable time links can be difficult. In response to this, we chose to integrate in our model a simpler representation of inference rules in the form of IF…THEN… that approaches human intuition and allows the modification or updating of rules as new information becomes available, allowing systems to react to changing situations or uncertainty in real-time.

1.2 Novelties

In this paper, we suggest the Neutrosophic Event Calculus (NEC), which is a temporal reasoning framework that integrates neutrosophic logic to manage uncertainty, indeterminacy, and inadequate knowledge. The main characteristics of the NEC framework include the following:

- Inference rules: Our model enables the user to generate inference rules, aiding decision-makers in understanding and resolving difficulties by facilitating efficient problem-solving in complex and unpredictable contexts.

- Handling Uncertainty: In the presence of uncertainty, NEC provides a depiction of events and temporal connections. It is appropriate for circumstances in which the precise result or temporal relationships between events are unknown.
- Indeterminate Events and Consequences: NEC can describe events and their consequences when the truth value or result is unknown, expressing scenarios in which the repercussions of events are uncertain or not completely known.
- Inadequate Knowledge Expressiveness: It provides a framework for reasoning about events and their temporal linkages, even when knowledge is inadequate or the truth value of assertions is uncertain.

1.3 Contributions

The main contributions of the current manuscript can be summarized as follows:

- To the best of our knowledge, this is the first study in the related literature that integrates neutrosophic logic with event calculus. In this way, we seek to suggest a new formalism that will operate as a solid theoretical basis in the field of knowledge representation, especially in the way that a logical agent should make decisions or act in response to the effects of actions.
- In the field of neutrosophic logic, we introduce for the first time the neutrosophic event calculus to enrich the related mathematical toolbox, considering the advances of neutrosophic theories and their applications first discussed in [18]. We consider that our study will aid towards direction of the intersection of computer science and neutrosophic calculus/logic.
- In this light, we hope to spark research interest in the academic community, aiming at their need to comprehend not only logic-based formalisms in the process of designing complex computer programs based on sound engineering principles but also defining a mathematical framework for examining, on a logical basis, research problems in various fields.

1.4 Structure of the paper

The remainder of the article is as follows: The current article was written with the intention of being as self-contained as possible. As a result, section 2 summarizes the fundamental concepts and ideas required to understand the basic concepts of neutrosophic logic and event calculus in order to build our theory and propose our logic formalism, namely neutrosophic event calculus (NEC). In section 3, we present an illustrative example to examine NEC's applicability and expressiveness in a real-world situation. Next, in section 4, we discuss why and where our formalism could find fertile research ground, and in the last section, we highlight, from a scientific perspective, NEC's usefulness and importance, which could pave the way for academics and practitioners. Lastly, we propose future research work in which NEC could play a pivotal role in the context of knowledge representation and reasoning.

2. Materials and Methods

In this section we firstly present the basic concepts and definitions of neutrosophic logic and event calculus that will provide the necessary knowledge needed so as to describe our proposed formalism, namely neutrosophic event calculus (NEC). For a deeper investigation on neutrosophic logic the interested reader is referred to the works of [19-20] and for a detailed review on event calculus and its extensions we propose the works of [15-16].

As a first step, and beginning with the consideration of what kind of reasoning we will adopt in our paper, we made a decision to extend first order predicate logic and specifically neutrosophic predicate logic by providing appropriate predicates and functions for describing the type of action-related information we're interested in, as well as offering a set of axioms restricting the set of models we desire.

2.1 Neutrosophic logic

Neutrosophic logic, which is presented as a general framework for logical approaches, is a branch of classical and fuzzy logic that deals with the idea of indeterminacy, allowing for the representation of three different types of components: truth, falsity, and indeterminacy. More specifically, truth component (T) represents the degree to which a statement or proposition is true. Falsehood component (F) denotes the degree to which a statement or proposition is false and indeterminacy component (I) represents the degree of indeterminacy, uncertainty, or incompleteness associated with a statement or proposition.

In this framework, a formula φ is characterized by a triplet of truth-values, called the *neutrosophical value* defined as [11]:

NL(φ) = (T(φ), I(φ), F(φ)) (1)

where (T(φ), I(φ), F(φ)) ⊂ ||-0, 1+||3 , ||-0, 1+|| being an interval of hyperreals.

2.1.1 Neutrosophic predicate logic

Neutrosophic predicate is a generalization of neutrosophic propositional logic and of classical predicate logic. As a neutrosophic formal syntax, neutrosophical predicate logic addresses *neutrosophic predicates*, *neutrosophic variables*, and *neutrosophic quantifiers*, which are predicates, variables and quantifiers respectively that deal with indeterminacy [18]. In other words, instead of the classical binary true or false values, neutrosophic predicates allow for truth-membership, false-membership, and indeterminacy-membership degrees.

Let us consider the following simple example:

$P(\Theta)$ = "Θ is a logician academic", where Θ is a human being. The neutrosophic truth-value of $P(\Theta)$ is (T, I, F) where T, I, F are subsets of the interval [0, 1]. Then we say that the predicate "is a logician academic" takes one variable, namely "Θ".

2.2 Event calculus

In this subsection and throughout our paper we will use the basic event calculus version which has all the characteristics of a full version and is considered efficacious for the scope of our current work. Different formulations of event calculus have been proposed in the literature that are suitably established to cope with specific research problems such as continuous change and mathematical modelling [21], with ramifications [22], with representing agent beliefs [23] and to deal with programming constructs and compound events [24].

The event calculus is a logical system that deduces what is true given what happens when and what actions take place. The "what happens when" section provides a timeline of events, while the "what actions do" segment outlines the outcomes of acts [25]. The basic ontology of the event calculus includes *actions or events*[1], *fluents* and *time points*. A fluent is anything whose value fluctuates with time.

In the event calculus, fluents apply to points in time, rather than states, and the calculus is designed to allow reasoning in terms of time intervals. The event calculus axiom says that a fluent is true at a point in time if the fluent was initiated by an event at some past time and not terminated by an intervening event. Table 1 depicts the basic predicates used in the simple event calculus.

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[1] we use the terms action and event interchangeably.

Table 1. Event calculus predicates [15].

Formula	Meaning
Initiates (a, b, t)	Fluent b starts to hold after action a at time t
Terminates(a, b, t)	Fluent b ceases to hold after action a at time t
$Initially_P$(b)	Fluent b holds from time 0
$t_1 < t_2$	Time point t_1 is before time point t_2
Happens(a, t)	Action a occurs at time t
HoldsAt(b, t)	Fluent b holds at time t
Clipped(t_1,b,t_2)	Fluent b is terminated between times t_1 and t_2

Fluents are reified in the event calculus, as is apparent from Table 1. That is, fluents are first-class objects that may be quantified and appear as parameters to predicate statements.

Typically the axiom of the event calculus consists of the following:

$$T(f, t_2) \Leftrightarrow \exists\ e, t\ Happens(e, t) \wedge Initiates(e, f, t) \wedge (t < t_2) \wedge Clipped(t, f, t_2)$$

$$Clipped(t, f, t_2) \Leftrightarrow \exists\ e, t_1\ Happens(e, t_1) \wedge Terminates(e, f, t_1) \wedge (t<t_1) \wedge (t_1<t_2)$$

The above axiom gives us functionality similar to that of calculus of states but with the ability to talk about time points and intervals. In this manner we can say for example *Clipped(10:00, TurnOff(TV),11:00)* so as to indicate that the TV appliance switched off at some time between 10:00 and 11:00.

It is worth noting that, according to the above axiom, a fluent does not hold during the event that originates it but does hold during the event that ends it. In other words, fluents retain open intervals on the left and closed intervals on the right.

2.3 Neutrosophic event calculus

In this subsection we introduce for the first time in the literature the Neutrosophic Event Calculus (NEC) as an extension of the Event Calculus, which integrates neutrosophic logic into its logical framework. In this context, NEC enables the modelling and reasoning of systems containing aspects of uncertainty, imprecision, or missing knowledge. In order to achieve the latter, it expands standard event calculus by including neutrosophic features, allowing for the representation and manipulation of ambiguous, uncertain, or conflicting information inside the logical framework used for thinking about events and their consequences throughout time.

Like in classical event calculus, our ontology includes actions or events, fluents and time points which are considered the basic concepts of our framework but this time they are enriched and applied in neutrosophic logic based environment. This means that they are allowed to be T% true, F% false, and I% indeterminate. This leads us to adopt the notation presented in [16] which indicates that in a neutrosophic model each neutrosophic proposition $\mathcal{P}$ has a neutrosophic truth-value (T_{wn}, I_{wn}, F_{wn}) respectively to each neutrosophic world $w_N \in G_N$, where T_{wn}, I_{wn} F_{wn} are subsets of [0, 1] and G_N is a neutrosophic frame which is a non-empty neutrosophic set, whose elements are called possible neutrosophic worlds. In order to capture the aforesaid information and based on the definition of the neutrosophic formula given with Eq. (1), we add a parameter, namely *neutrosophic degree (nd)*, in the predicates so as to define its neutrosophical value. Parameter *neutrosophic degree*, which is based on expert's knowledge, expertise and available or historical data, could be expressed as a neutrosophic numerical value or as a linguistic variable or even phrase to ease human intuition. For example, we could assign the term "*very low*" with a neutrosophic degree such as *(0.1, 0.8, 0.9)*, thus indicating that the examined concept has 10% chance to occur, 90% chance not to occur and 80% indeterminate chance to happen. Furthermore, we could have replaced the argument *nd* with an expression such as "possibly 30 units "so as to refer to the possible number of units that we could place for a specific order. So it is obvious that this degree reflects the level of ambiguity or lack of clarity about the truth value of the statement or event in question.

Another key feature that is introduced in our model, as opposed to classical event calculus, is that we can generate several inference rules based on the predicates according to the problem domain in a much simpler form aiming to offer a significantly more flexible version. This is achieved by using the form of IF…THEN… to derive new facts or conclusions based on existing facts and rules in the knowledge base. This feature is really useful as it enables us to insights into potential relationships between events or states, thus aiding in decision-making by capturing possible cause-and-effect relationships amidst uncertainties. In this manner, it also provides us with a significant advantage over the notation adopted in the classical event calculus. It enables us to reason in a high level language which is remarkably akin to human language that is easily understood by non-experts in the field.

The selection of basic predicates goes together in hand with the selection of ontology. Table 2 shows the predicates used in NEC.

Table 2. Neutrosophic event calculus predicates.

Formula	Meaning
$\boldsymbol{Initiates_N}$(a, b, t, nd)	Fluent *b* *neutrosophically* starts to hold after action *a* at time *t*
$\boldsymbol{Terminates_N}$(a, b, t, nd)	Fluent *b* *neutrosophically* ceases to hold after action *a* at time *t*
$\boldsymbol{InitiallyP_N}$(b, nd)	Fluent *b* *neutrosophically* holds from time 0
$t_1 < t_2$	Time point t_1 is before time point t_2
$\boldsymbol{Happens_N}$(a, t, nd)	Action a *neutrosophically* occurs at time *t*
$\boldsymbol{HoldsAt_N}$(b, t, nd)	Fluent *b* *neutrosophically* holds at time *t*
$\boldsymbol{Clipped_N}$(t_1,b,t_2, nd)	Fluent *b* will *neutrosophically* be terminated between times t_1 and t_2

Within our context the above predicates have the following meaning:

- *Initiates*: interpreting this statement requires admitting that once action α happens, there is a transition or commencement of fluent *b*'s truth value from a possibly false or indeterminate state to a state where it begins to hold or becomes true. Because of the statement's neutrosophic character, the degree of certainty or truthfulness about this transition may fluctuate, incorporating degrees of truth, falsity, and indeterminacy at the same time.
- *Terminates:* this relationship might encompass various degrees of termination, acknowledging uncertainty or indeterminacy regarding the exact impact or timing of action 'a' on the termination of fluent 'b'. The neutrosophic truth values associated with this predicate would capture the degrees of truth, falsehood, and indeterminacy regarding the termination of fluent *b* by action *a* at time *t*.
- *Initially:* the truth value associated with *b* holding at time 0 could encompass elements of truth, falsehood, and indeterminacy simultaneously. The statement indicates that, at the onset (time 0), the truth status of *b* is considered, taking into account any uncertainties or degrees of indeterminacy associated with its truth value.
- *Happens:* while the statement indicates the occurrence of action 'a' at time 't' in classical event calculus, the neutrosophic interpretation accounts for the uncertainty or imprecision surrounding the actual occurrence of the action at that specific time, allowing for varying degrees of truth or falsehood associated with this event-fluent relation.
- *HoldsAt:* the statement fluent *b* holds at time *t* signifies the status or truth value of fluent *b* at a specific time point denoted by *t*. Therefore, it accommodates the inherent uncertainty or indeterminacy, allowing for different degrees of truth, falsehood, or indeterminacy associated with the fluent's state at that moment in the temporal domain.

- *Clipped:* in a neutrosophic context, it allows for degrees of truth, falsehood, and indeterminacy regarding the certainty of this termination within that specific time range, i.e. between time t_1 and t_2. It acknowledges that it might not be definitively true or false; instead, it could have varying levels of certainty or uncertainty regarding the fluent's termination within the specified time interval.

Following the above terminology we can now re formulate the simple example that we examined in subsection 2.2. This would showcase the core idea behind neutrosophic event calculus and how this could be applied in more complex problems as shown in the next section. We would like to indicate that a TV appliance *might* be switched off at some time between 10:00 and 11:00 with a neutrosophic degree (0.8, 0.2, 0.1), i.e. 80% chance that the TV appliance will switch off, 10% chance not to switch off and 20% indeterminate chance to happen. In this context this could be expressed as: $Clipped_N$(10.00, *TurnOff(TV)*,11.00, (0.8, 0.2, 0.1))

From the above it can be concluded that when combined with neutrosophic truth values, these predicates enable a more nuanced representation of uncertainty, imprecision, and indeterminacy within the neutrosophic event calculus, allowing for more flexible and realistic modelling of dynamic systems where complete information may not be available or certain. In the next section we will showcase the robustness of our method by examining an illustrative example.

3. Implementation of Neutrosophic Event Calculus

In order to study the effectiveness and usefulness of our proposed formalism (NEC), let us examine the following example taken from the logistics/supply chain domain which is based in a real world scenario. For the sake of brevity we will restrict the solution of the given example to the inventory and supply chain management levels. However, our approach will be efficiently being demonstrated

Example 1. Due to unknown circumstances like as weather, transportation delays, and various customs processes across nations, a global corporation confronts difficulty in properly anticipating delivery schedules. This ambiguity has an impact on inventory management, production planning, and customer satisfaction.

Step 1. Let us first explain and list the NEC's predicates used in the above example.

- $\boldsymbol{HoldsAt_N}$ **(InventoryLevel, t, neutrosophic_ degree):** Represents the uncertain inventory level of a product at a specific time.
- $\boldsymbol{Initiates_N}$ **(OrderPlacement, Product_X, t, neutrosophic_ degree):** Indicates the initiation of an uncertain order for a specific quantity of a product at a specific time.
- $\boldsymbol{Happens_N}$ **(WeatherImpact, t, neutrosophic_ degree):** Represents the potential impact of weather on transportation at a specific time.
- $\boldsymbol{HoldsAt_N}$ **(CustomsProcessDelay, t, neutrosophic_ degree):** Indicates the potential delay in customs processes at a specific time.
- $\boldsymbol{Terminates_N}$ **(DelayedDelivery, Product_X, t, neutrosophic_degree):** Represents the uncertain termination or delay in delivery of a certain quantity of a product at a specific time.
- $\boldsymbol{Clipped_N}$ **(t_1, DelayedDeliver, t_2, neutrosophic_ degree):** Refers to delay in delivery of a certain quantity of a product clipped between minimum and maximum time values.

We should note the key role of the argument *neutrosophic_degree* in our model which allows the representation of uncertainty, permitting to reason about prospective outcomes or occurrences without the need for precise, deterministic values.

Step 2. We proceed by giving possible numerical values or linguistic expressions where appropriate to the above predicates based on expert(s) judgement(s).

$HoldsAt_N$(InventoryLevel, ProductX, 11:00 a.m., possibly 100 units).

$Initiates_N$(OrderPlacement, 11:00 a.m., ProductX, possibly 70 units).
$Happens_N$(WeatherImpact, 11:00 a.m., possibly moderate).
$HoldsAt_N$(CustomsProcessDelay, 11:00 a.m., possibly 3 days).
$Terminates_N$(DelayedDelivery, ProductX, 11:00 am, possibly 25 units).
$Clipped_N$(1 day, DelayedDeliver, 4 days, possibly 25 units).

Step 3. Now we are ready to write the inference rules that best accommodate our example in the inventory and supply chain management levels.

1. Inventory Level:
   - **IF** $HoldsAt_N$ (InventoryLevel, ProductX, 11:00 a.m., possibly 100 units) **THEN** $Initiates_N$(OrderPlacement, 11:00 a.m., ProductX, possibly 70 units).
   - **IF** $Happens_N$ (WeatherImpact, 11:00 a.m., possibly moderate) **THEN** $HoldsAt_N$(CustomsProcessDelay, 11:00 a.m., possibly 3 days).
   - **IF** $Terminates_N$ (DelayedDelivery, ProductX, 11:00 a.m., possibly 25 units) **THEN** $Clipped_N$(1 day, DelayedDelivery, 4 days, possibly 25 units).

According to the first rule, a given inventory level signals the prospective beginning of an order placement, showing that observed stock levels impact the choice to initiate an order. The second rule provides a probable relationship between moderate weather impacts and predicted delays in customs processes, meaning that moderate weather may cause customs delays. The third rule leverages the 'Clipped' predicate to guarantee that the inferred delivery delay remains within a reasonable range (between 1 and 4 days), while acknowledging the uncertainty indicated by the possibility of terminating delayed delivery for Product X.

2. Supply chain management Level:
   - **IF** $HoldsAt_N$ (InventoryLevel, ProductX, 11:00 a.m., possibly *100* units) **THEN** $Initiates_N$(OrderPlacement, 11:00 a.m., ProductX, possibly *60* units)// if there is a probability of InventoryLevel being high then initiate OrderPlacement conservatively.
   - **IF** $Happens_N$ (WeatherImpact, 11:00 a.m., (0.8, 0.3, 0.2)) **THEN** $HoldsAt_N$(CustomsProcessDelay, 11:00 a.m., possibly *4* days) // If weather conditions show a likelihood of impact then anticipate CustomsProcessDelay prudently.
   - **IF** $Terminates_N$(DelayedDelivery, ProductX, 11:00 a.m., possibly 70 units) **THEN**

$Clipped_N$ (1 day, DelayedDelivery, 4 days, possibly 30 units)// if there is a chance of DelayedDelivery then prepare for potential clipping.

The usefulness of the NEC formalism in the context of the given real world case study could be summarized as follows:

- Inventory Management: It aids in order placement decisions based on unknown inventory levels, guaranteeing appropriate supply without relying on accurate information.
- Supply Chain Management: Assists in simulating the influence of unknown occurrences such as weather or customs delays on supply schedules, allowing for proactive management and planning.
- Clipped Predicate Utility: The usage of predicates such as 'Clipped' enables the definition of realistic limitations for inferred delays, ensuring they remain within practical and reasonable ranges.

4. Discussion

It is the first time in related literature that an extended formalism like ours has been proposed. The aim of establishing a new logical approach, known as NEC, stems from the need to propose a suitable formalism that can effectively represent relationships between events, fluents, and their properties, considering the inherent indeterminacy encountered in real-world problems. This indeterminacy is addressed using neutrosophic logic.

NEC provides a versatile means of representing events, their initiation, termination, and attributes, while accounting for the uncertainty inherent in complex systems. This adaptability enables more accurate modeling of real-world settings with unpredictable and imprecise occurrences. In this way, NEC's ability to accommodate and reason about uncertain information is particularly valuable in scenarios where traditional logic-based approaches struggle to provide accurate representations.

We believe that our suggested approach could act as a useful mathematical toolbox when dealing with the following real situations:

- Modelling uncertainty: It is advantageous when capturing and reasoning about systems where uncertainty exists. Many real-world circumstances contain partial or unclear knowledge, which NEC enables for a more realistic description of similar situations.
- Decision-making under uncertainty: It is valuable for decision-making processes when information is inadequate or uncertain. It aids in making informed judgments even in uncertain contexts by concurrently recording degrees of truth, untruth, and indeterminacy.
- Dynamic system analysis: It analyzes and predicts the behavior of dynamic systems that are influenced by events and changes over time. This is especially important in industries such as engineering, finance, logistics, and artificial intelligence, where understanding dynamic relationships is critical.
- Risk assessment and management: It aids in the assessment and management of risks in systems with a high degree of uncertainty. It can provide a more thorough risk assessment by taking into account varying degrees of truth and falsity.
- Artificial intelligence and Robotics: It can be used in AI and robotics to represent settings with noisy or unclear sensor input, allowing these systems to make more nuanced judgments.

5. Conclusions

Neutrosophic Event Calculus offers a logic-based framework that extends traditional event-based reasoning to include indeterminacy, uncertainty, and imprecision. This research study has demonstrated the promise of this approach in various research disciplines. We suggest that this integrated approach could offer a more realistic portrayal of dynamic systems, where events, fluents, and their interactions are susceptible to different degrees of truth, untruth, and indeterminacy. Its capacity to deal with inadequate or ambiguous data offers possibilities for more complex and accurate modeling of real-world systems.

Future research objectives for expanding the NEC include refining neutrosophic logic to strengthen the basis of NEC, particularly in dealing with degrees of truth, falsehood, and indeterminacy, as well as reasoning from effects to causes [26, 27]. To ensure the formalism's soundness and completeness, it is necessary to examine examples, counterexamples, and logical arguments to establish its sufficiency and consistency. Additionally, validation through real-world case studies is required to demonstrate its practical application. Furthermore, the combination of NEC with other computational models, such as fuzzy logic, probabilistic methodology, or machine learning techniques, may result in more adaptive and robust modeling approaches, thereby extending its applicability across other domains.

Acknowledgments
The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

References

1. Rolf, B. (1982). Russell's theses on vagueness. Taylor & Francis. http://dx.doi.org/10.1080/01445348208837031
2. Bělohlávek, R., Dauben, J. W., & Klir, G. J. (2017). Fuzzy logic and mathematics: a historical perspective. Oxford University Press. https://doi.org/10.1093/oso/9780190200015.001.0001
3. Bohan, J. C. (1971). On Black's "Loose" Concepts. Dialogue: Canadian Philosophical Review,10, 332–336. http://dx.doi.org/10.1017/S0012217300050083
4. Schutz, W.C. (1959). Some implications of the logical calculus for empirical classes for social science methodology. Psychometrika, 24, 69–87. https://doi.org/10.1007/BF02289764
5. Dubois, D., Prade, H., Yager, R.R. (Eds.) (1993). Readings in fuzzy sets for intelligent systems, Morgan Kaufmann. https://doi.org/10.1016/C2013-0-08304-1
6. Zadeh, L.A. (1965). Fuzzy sets. Information and Control, 8, 338-353. http://dx.doi.org/10.1016/S0019-9958(65)90241-X
7. Poulik, S., Ghorai, G., & Xin, Q. (2023). Explication of crossroads order based on Randic index of graph with fuzzy information. Soft Computing, 1-14. https://doi.org/10.1007/s00500-023-09453-6
8. Poulik, S., & Ghorai, G. (2022). Connectivity Concepts in Bipolar Fuzzy Incidence Graphs. Journal of Mathematics, 20, 1609–1619. Retrieved from https://thaijmath2.in.cmu.ac.th/index.php/thaijmath/article/view/1425
9. Das, S., Poulik, S., & Ghorai, G. (2023). Picture fuzzy ϕ-tolerance competition graphs with its application. Journal of Ambient Intelligence and Humanized Computing,1-13. https://doi.org/10.1007/s12652-023-04704-8
10. Rivieccio, U. (2008). Neutrosophic logics: Prospects and problems. Elsevier, 159,1860–1868. https://doi.org/10.1016/j.fss.2007.11.011
11. Jousselme, A.L., & Maupin, P. (2004). Neutrosophy in situation analysis [Review of Neutrosophy in situation analysis]. In fusion (pp. 400–406). Retrieved from https://fs.unm.edu/
12. Smarandache, F. (2017). Neutrosophic modal logic. Neutrosophic Sets and Systems, 15, 90-96. https://doi.org/10.5281/zenodo.570945
13. Findlay, J. N. (1941). Time: A treatment of some puzzles. The Australasian Journal of Psychology and Philosophy, 19, 216-235. https://doi.org/10.1080/00048404108541170
14. Prior, A. (1967). Past, present and future. Oxford University Press, Oxford, UK. https://doi.org/10.1093/acprof:oso/9780198243113.001.0001
15. Kowalski, R.A.; Sergot, M.J. (1986). A Logic-Based Calculus of Events. New Generation Computing, 4, 67–95. https://doi.org/10.1007/978-3-642-83397-7_2
16. Shanahan, M. (1999). The Event Calculus Explained. In Lecture Notes in Computer Science (pp. 409–430). https://doi.org/10.1007/3-540-48317-9_17
17. McCarthy, J., & Hayes, P. J. (1981). Some philosophical problems from the standpoint of artificial intelligence. In Readings in artificial intelligence, (pp.431-450), Morgan Kaufmann. https://doi.org/10.1016/B978-0-934613-03-3.50033-7

18. Smarandache, F. (2017). Neutrosophic Perspectives: Triplets, Duplets, Multisets, Hybrid Operators, Modal Logic, Hedge Algebras and Applications. Infinite Study. Retrieved from https://digitalrepository.unm.edu/math_fsp/27
19. Smarandache, F. (2010). Neutrosophic logic-a generalization of the intuitionistic fuzzy logic. Multispace & multistructure. Neutrosophic transdisciplinarity (100 collected papers of science), 4, 396. http://dx.doi.org/10.2139/ssrn.2721587
20. Smarandache, F. (1999). A unifying field in Logics: Neutrosophic Logic. American Research Press. Retrieved from https://arxiv.org/ftp/math/papers/0101/0101228.pdf
21. Miller, R., & Shanahan, M. (1999). The event calculus in classical logic-alternative axiomatisations. Linköping University Electronic Press. Retrieved from http://www.ep.liu.se/ej/etai/1999/016/
22. Miller, R.; Shanahan, M. (1996). Reasoning about discontinuities in the event calculus. KR, 96, 63-74. Retrieved from https://www.doc.ic.ac.uk/~mpsha/cont_changeKR.pdf
23. Denecker, M., Theseider-Dupré, D., & Van Belleghem, K. (1998). An inductive definition approach to ramifications. Linkoping Electronic Articles in Computer and Information Science, 3(7), 1-43. Retrieved from http://www.ep.liu.se/ea/cis/1998/007/
24. Lévy, F.; Quantz, J. (1998). Representing Beliefs in a Situated Event Calculus. In ECAI, (pp. 547-551).
25. Shanahan, M. (1990). Representing Continuous Change in the Event Calculus. In ECAI, 90, 598-603. Retrieved from https://www.doc.ic.ac.uk/~mpsha/cont_change.pdf
26. Van Harmelen, F.; Lifschitz, V.; Porter, B. (Eds.). (2008). Handbook of knowledge representation. Elsevier. Retrieved from https://dai.fmph.uniba.sk/~sefranek/kri/handbook/handbook_of_kr.pdf
27. McCarthy, J. (1980). Circumscription—a form of non-monotonic reasoning. Artificial intelligence,13, 27-39. http://dx.doi.org/10.1016/0004-3702(80)90011-9

Received: 10 Dec 2023, **Revised:** 09 Jan 2024,

Accepted: 10 Jan 2024, **Available online:** 12 Jan 2024.