Neutrosophic Systems with Applications, Vol. 15, 2024
https://doi.org/10.61356/j.nswa.2024.1512356

Unveiling Efficiency: Investigating Distance Measures in Wastewater Treatment Using Interval-Valued Neutrosophic Fuzzy Soft Set

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Abstract: To reduce the threats that wastewater poses to human health and the environment, water treatment techniques must be improved. The use of a procedure that includes preparation, testing, primary and secondary treatments, filtration, disinfection, and continuous monitoring is therefore required. The objective of this research is to create a hybrid notion that extends the idea of an interval-valued neutrosophic fuzzy soft set (IVNFSS) to an interval-valued neutrosophic fuzzy set. Operations like complement, union, and integration are included in the idea. To improve decision-making accuracy, a quality-assessment distance measure is incorporated, offering a numerical representation of the disparity between various factors. Furthermore, the IVNFSS defines distance measures, which are used in the wastewater treatment process. To monitor water quality, IVNFSS, in conjunction with a distance measure, is a potent instrument whose application extends to water waste treatment procedures. This can completely change the way that water quality management is now done by providing a methodical way to guarantee the security and quality of drinking water.

Keywords: Interval-Valued Neutrosophic Fuzzy Soft Set; Decision Making; Distance Measure; Unveiling Efficiency.

1. Introduction

Wastewater is a severe issue as it is composed of polluted water from several activities and processes. The release of this untreated water into rivers or seas has a negative impact on the ecosystem, endangering humans, animals, and plants. To avoid these problems and protect the environment and public health, it is imperative to upgrade water treatment techniques. Complete purification may be difficult to achieve since traces of impurities may remain after the first cleaning procedure. The difficult job is getting rid of these leftovers and handling any byproducts that are produced while treating water. There are several procedures involved in preparing water for human consumption. The first steps in eliminating impurities are screening and pre-treatment, which uses chemicals to help with particle clumping. Organic matter is broken down and settled by further primary and secondary treatments, and any leftover particles are removed by filtering Remaining dangerous germs are eliminated by disinfection using chemicals or UV radiation, and safety is guaranteed by pH correction. Water quality requirements are upheld by rigorous testing, and residences are supplied with purified water via pipelines. The water supply is kept secure via ongoing observation and strict conformity to the law.

Purifying water to fulfill stringent quality requirements is the goal of the painstaking process known as water treatment. It is more important to achieve predetermined standards to guarantee its safety for human use and environmental effects than to make a direct comparison to naturally clean water. The constant and trustworthy evaluation of water quality across several sources is ensured by this methodical technique. It might be challenging to implement conventional wastewater treatment in many situations due to the absence of clear, confusing, or sufficient data. Fuzzy logic and fuzzy sets are effective options for handling confusing data in wastewater treatment.

Fuzzy set theory, introduced by Zadeh [1] in 1965, is a mathematical framework that extends traditional set theory to handle uncertainty and vagueness. Fuzzy sets allow for partial membership, assigning degrees of membership between [0, 1] to elements. This enables the representation of gradual or fuzzy boundaries, capturing the inherent ambiguity and imprecision in real-world situations. Zadeh then expanded the idea of FS to interval-valued fuzzy sets in 1975 [2]. Because of this improvement, the membership function may now be divided into intervals rather than just one value, which is more appropriate for use in practical applications. Sometimes, it can be difficult to assign appropriate membership values to fuzzy sets. The use of IVFS was suggested by Turksen [3] as a solution for such scenarios. When faced with situations of anxiety and uncertainty, it is important to carefully consider the appropriate representation of an object. In such cases, the object's suitable representation values, i.e., unbiased membership and non-membership values, cannot be accurately calculated by either IVFS or fuzzy sets. In addition to making a substantial contribution to the subject of FS theory, IVFS sets have also created new opportunities for addressing and modeling uncertainty in a more thorough way [4]. IVFS addresses situations where the exact membership value is ambiguous or difficult to determine precisely, accommodating a wider range of uncertainty. These extended representations of IVFS have attracted significant research attention, particularly in the field of multi-criteria decision-making (MCDM) [5-7]. These developments offer valuable tools for handling uncertainty in decision-making processes.

Intuitionistic fuzzy set theory, introduced by Atanassov [8] in 1983, further extends fuzzy set theory by incorporating an additional function known as the non-membership degree along with the membership degree. In intuitionistic fuzzy sets, each element is characterized by its membership degree, non-membership degree, and hesitation degree. The sum of the membership degree, non-membership degree, and hesitation degree is always less than or equal to one. This property ensures that the degrees assigned to an element accurately represent the entire uncertainty spectrum without exceeding the bounds. The hesitation degree represents the lack of confidence in assigning a definite membership value and allows for a more comprehensive representation of uncertainty and imprecision in decision-making processes. By considering both membership and non-membership degrees, intuitionistic fuzzy sets provide a richer framework for modeling and reasoning with uncertain and vague information. Atanassov laid the groundwork for the voyage by first defining SM for IFS components [9], constructing on top of this. Quantifying similarity degrees for ambiguous sets was first proposed by Chen [10]. Cher measurements, according to Hong and Kim, displayed errors and indistinguishable results in severe circumstances, which called for the creation of modified SM [11]. Following this, Dengfen and Chuntian [12] concentrated on finding SMS for IFSS, particularly in the context of discrete or continuous universal sets for pattern recognition issues.

Smarandache expanded fuzzy set theory in [13] by incorporating a third element known as indeterminacy. This is known as the neutrosophic set theory. NS can more successfully enable approximation reasoning and is more prepared to maintain the fuzziness of information's contents. Traditional NS has a worse descriptive capacity than NS due to the addition of non-membership and indeterminacy-graded functions. The unit closed interval was given for each of the three uncertain NS components in Wang et al. [14] formulation of SVNS. Many researchers have contributed to NS for application in topological spaces, statistics, and the development of different hybridized frameworks to help with decision-making. In 1995, Smarandache introduced the pioneering concepts of Neutrosophic Over-/Under-/Off-Set and Logic, which were later published in 2007 [15]. These unique notions diverge from traditional sets, logic, and probabilities and have been showcased at global conferences from 1995 to 2016. Neutrosophic sets have been extended to include Neutrosophic Overset (where a component > 1) and Neutrosophic Underset (where a component < 0), allowing for the capture of intricate real-world scenarios. The framework's innovative approach, which includes neutrosophic over/under/off logic and probability, has unlocked valuable applications in technology, economics, and social sciences, making it ripe for further exploration. Pramanik et al. [16] thoroughly analyzed the Neutrosophic cubic set, which blends INS with SVNS to enable the simultaneous recording of hybrid information. Saqlain et al. [17] proposed the single and multi-valued neutrosophic hypersoft sets and the tangent similarity measure of single-valued neutrosophic hypersoft sets. In this work, we expand Said and Smarandache's [18] examination of Bhowmik and Pal's INS sets and incorporate it into the world of soft sets. The notation is because of INSS, along with the introduction of definitions, operations, and the construction of characteristics relevant to this merger. We define operations on INSS and validate assertions by bridging the gap between intuitionistic neutrosophic sets and soft sets, both of which naturally deal with imprecision. Additionally, the use of INSS in resolving a dilemma in decision-making serves to highlight how practically useful it is. Broumi [19] integrates A.A., generalizes NSS, and combines Molodtsov's soft set and Salama's neutrosophic set, introducing a new framework for handling imprecision and exploring interdisciplinary applications in engineering, mathematics, and computer science. Broumi et al. [20] propose an enhanced extension, Soft Relations IVNSS, encompassing various types of relations—soft, fuzzy, intuitionistic fuzzy, interval-valued intuitionistic fuzzy, and neutrosophic soft relations—with an exploration of reflexivity, symmetry, and transitivity, offering potential applications and prompting further research in the field. Deli [21] integrates interval-valued neutrosophic sets with soft sets, introducing the innovative concept of interval-valued neutrosophic soft, which generalizes various set types and demonstrates its efficacy in decision-making techniques [22-26].

Soft sets (SS) are a broad mathematical technique that Molodtsov [27] suggested to work with ambiguous, indeterminate, and uncertain substances. SS allow for the representation and manipulation of data without requiring crisp boundaries or precise definitions. Maji et al. [28] further on SS's work and specified a few operations and their characteristics. They also make judgments in [29-30] based on SS theory. Soft matrices with operations were introduced and their characteristics were examined by Cagman and Enginoglu [31]. They also proposed a technique for making decisions to deal with difficulties of uncertainty [32-33]. They altered the Molodtsov's SS-proposed activities in [34]. The author of [35] introduces novel methods for soft matrices, including soft difference products, to combine the characteristics of soft sets (SSs), fuzzy sets (FSs), intuitionistic fuzzy sets (IFSs), and neutrosophic sets (NSs), resulting in FS, IFSS, and NSS. Babitha and Sunil [36] further explore SSs, soft set relations, Cartesian soft set products, and related concepts, providing a foundation for addressing uncertainty in complex systems. They also investigate set theory using soft set relations, emphasizing the development of these ideas as a theoretical basis for future research and advancement in the field. By incorporating real-world applications [37-38], it expands the soft set theory. To ensure that soft set theory is correctly applied in a variety of disciplines, Broumi et al. developed the hybridized structure of NSS with SS for interval settings [39] and Das et al. [40]. In addition to talking about the basics of these models, they employed techniques to make these models applicable in varied settings. Attribute values are split up into sub-attributive values in several real-world situations.

To achieve this, an Interval-values neutrosophic fuzzy soft set environment was developed. These environments can more accurately handle uncertainty. As previously mentioned, there are some limitations to the existing studies, such as:

- Fuzzy soft sets deal with truthiness exclusively.
- An expansion of this idea is intuitionistic fuzzy soft, yet not a complete version. On the other hand, Neutrosophic Soft Sets handle three values, including truthiness, indeterminacy, and falsity, but lack sub-attributions.

- Neutrosophic fuzzy soft sets incorporate values: of truth, indeterminacy, and falsity, while also utilizing the expert value of fuzziness.
- Interval-values Neutrosophic fuzzy soft set incorporate values: of truth, indeterminacy, and falsity, while also utilizing the expert value of fuzziness.
- Some situations cannot be explained by current theories. In this thesis, we extended the theory and developed numerous techniques.

Working with neutrosophic fuzzy sets can be challenging due to their complex framework. To address this, we have developed the Interval-values neutrosophic fuzzy soft set. In this study, we aim to examine and address any issues. By doing so, we can apply all the definitions, operators, and properties of IVNFSS. These sets have been useful in various decision-making strategies. When faced with genuine logical and numerical problems, uncertainty can be helpful. To address this uncertainty, often turn to Multi-Criteria Decision-Making (MCDM). These types of problems involve various attributes, and we strive to find the best possible match. However, when faced with more complex selections like Multi-Criteria Multi Attributive Decision Problems (MCDM), then utilize tools such as Interval-valued Neutrosophic fuzzy soft sets. Additionally, researchers have dedicated significant effort towards developing distance measures (DM) to aid in these types of problems.

The proposed study has investigated and answered potential questions.

- Theoretical approach for IVNFSS.
- Development of Distance measure with the help of IVNFSS.
- Development of Algorithms (IVNFSS) to solve MCDM.

The following is how the study is set up: A few chosen preliminary definitions that are necessary for introducing IVNFSS are provided in Section 2. The idea of IVNFSS is defined in Section 3 along with the attributes and operators previously discussed. The applications of fuzzy distance measures are first discussed in Section 4, after which the development of Hamming and Normalized Hamming distance measures and the proofs necessary to support their usage are covered. An algorithm for computing similarity from an ideal solution is built in Section 5 to demonstrate the use of the distance measurements that have been constructed in the treatment of water waste. Next, using a comparison of the facilities with a reference set, this approach is employed to choose the best alternative among several. In the Conclusion section, the paper's main conclusions are summarized.

2. Preliminaries

In this section, some basic concepts are defined that are required for the development of the proposed structure.

2.1 Neutrosophic Fuzzy Set [40]

Neutrosophic fuzzy sets, which incorporate the indeterminacy dimension and combine fuzzy sets with neutrosophic set theory, are an important tool for efficiently managing uncertainty and imprecision in a variety of domains. Let °F is universal set and $\curlyvee \in$ °F and function $\ddot{\Upsilon}$ is defined as:

$$\ddot{\Upsilon} = \{\mathrm{e}, \langle \mathcal{T}_{\ddot{\Upsilon}}(\mathrm{e}, \mathfrak{Q}), \mathcal{I}_{\ddot{\Upsilon}}(\mathrm{e}, \mathfrak{Q}), \mathcal{F}_{\ddot{\Upsilon}}(\mathrm{e}, \mathfrak{Q})\rangle, \mathfrak{Q}_{\ddot{\Upsilon}}(\mathrm{e}) \; \forall \; \mathrm{e} \in {}^{\circ}\mathrm{F}\},$$

Where $\mathcal{T}_{\ddot{\Upsilon}}(\mathrm{e}, \mathfrak{Q}), \mathcal{I}_{\ddot{\Upsilon}}(\mathrm{e}, \mathfrak{Q}), \mathcal{F}_{\ddot{\Upsilon}}(\mathrm{e}, \mathfrak{Q})$ are truth, indeterminacy, and falsity membership functions of every fuzzy-membership value and $\mathcal{T}_{\ddot{\Upsilon}}, \mathcal{I}_{\ddot{\Upsilon}}$ *and* $\mathcal{F}_{\ddot{\Upsilon}}$ are the real standard or non-standard subsets of $\mathrm{I}_{0_-}^{1^+}$. Such that $\mathcal{T}_{\ddot{\Upsilon}}, \mathcal{I}_{\ddot{\Upsilon}}, \mathcal{F}_{\ddot{\Upsilon}} : {}^{\circ}\mathrm{F} \longrightarrow \mathrm{I}_{0_-}^{1^+}$ and no restrictions are there on the sum of $\mathcal{T}_{\ddot{\Upsilon}}, \mathcal{I}_{\ddot{\Upsilon}}$ and $\mathcal{F}_{\ddot{\Upsilon}}$
Hence,

$$0^{-} \leq \mathcal{T}_{\ddot{\Upsilon}}(\mathrm{e}, \mathfrak{Q}) + \mathcal{I}_{\ddot{\Upsilon}}(\mathrm{e}, \mathfrak{Q}) + \mathcal{F}_{\ddot{\Upsilon}}(\mathrm{e}, \mathfrak{Q}) \leq 3^{+}.$$

2.2 Single-Valued Neutrosophic Fuzzy Set [41]

A SVNFS $\ddot{\Upsilon}$ in universal set °F is defined as.

$$\ddot{\Upsilon} = \{e, \langle \mathcal{T}_{\ddot{Y}}(e, \mathfrak{Q}), \mathcal{I}_{\ddot{Y}}(e, \mathfrak{Q}), \mathcal{F}_{\ddot{Y}}(e, \mathfrak{Q})\rangle, \mathfrak{Q}_{\ddot{Y}}(e)\ \forall\ e \in {}^{\circ}F\},$$

where,

$$\mathcal{T}_{\ddot{Y}}(e,\mathfrak{Q}), \mathcal{I}_{\ddot{Y}}(e, \mathfrak{Q}), \mathcal{F}_{\ddot{Y}}(e, \mathfrak{Q}) \in [0,1],$$

and

$$0 \le \mathcal{T}_{\ddot{Y}}(e,\mathfrak{Q}) + \mathcal{I}_{\ddot{Y}}(e,\mathfrak{Q}) + \mathcal{F}_{\ddot{Y}}(e,\mathfrak{Q}) \le 3.$$

3. Proposed Structure based on IVNFSS

This section introduces the Interval-valued neutrosophic fuzzy soft set concept and defines its relevant operators.

3.1 Interval-Valued Neutrosophic Fuzzy Set

Interval-valued neutrosophic fuzzy sets, an extension of Zadeh's fuzzy set, effectively manage uncertainty and imprecision in a variety of disciplines by pro viding a thorough framework that combines intervals and the indeterminacy dimension. Let $^{\circ}F$ is the Universal of discourse and $e \in {}^{\circ}F$ an I-VNFS Υ in $^{\circ}F$ is define as;

$$\ddot{\Upsilon} = \{e, \langle \mathcal{T}_{\ddot{Y}}(e, \mathfrak{Q}), \mathcal{I}_{\ddot{Y}}(e, \mathfrak{Q}), \mathcal{F}_{\ddot{Y}}(e, \mathfrak{Q})\rangle, \mathfrak{Q}_{\ddot{Y}}(e)\ \forall\ e \in {}^{\circ}F\},$$

where,

$$\mathcal{T}_{\ddot{Y}}(e, \mathfrak{Q}), \mathcal{I}_{\ddot{Y}}(e, \mathfrak{Q}), \mathcal{F}_{\ddot{Y}}(e, \mathfrak{Q}) \in [0,1],$$

and

$$0 \le \mathcal{S}up\mathcal{T}_{\ddot{Y}}(e,\mathfrak{Q}) + \mathcal{S}up\mathcal{I}_{\ddot{Y}}(e,\mathfrak{Q}) + \mathcal{S}up\mathcal{F}_{\ddot{Y}}(e,\mathfrak{Q}) \le 3.$$

3.2 Neutrosophic Fuzzy Soft Set

Let $^{\circ}F$ be an initial universal set and let $\aleph$ be the set of parameters. Consider $\mathcal{A}$ to be a subset of $\aleph$ then a function $\ddot{I}$ its mapping is given by: $\ddot{I} = \mathcal{A} \to N_{fss}({}^{\circ}F)$. Then a function NFSS is characterized as:

$$Ḳ = \{e, \langle \mathcal{T}_{Ḳ}(e, \mathfrak{Q}), \mathcal{I}_{Ḳ}(e, \mathfrak{Q}), \mathcal{F}_{Ḳ}(e, \mathfrak{Q})\rangle, \mathfrak{Q}_{Ḳ}(e)\ \forall\ e \in {}^{\circ}F\},$$

$$0^{-} \le \mathcal{T}_{Ḳ}(e,\mathfrak{Q}) + \mathcal{I}_{Ḳ}(e,\mathfrak{Q}) + \mathcal{F}_{Ḳ}(e,\mathfrak{Q}) \le 3^{+}.$$

3.3 Single-Valued Neutrosophic Fuzzy Soft Set

Let $^{\circ}F$ be an initial universal set and let $\aleph$ be the set of parameters. Consider $\mathcal{A}$ to be a subset of $\aleph$ then a function $\ddot{I}$ its mapping is given by: $\ddot{I} = \mathcal{A} \to S_{VNFSS}({}^{\circ}F)$. Then a function IVNFSS is characterized as:

$$Ḳ = \{e, \langle \mathcal{T}_{Ḳ}(e, \mathfrak{Q}), \mathcal{I}_{Ḳ}(e, \mathfrak{Q}), \mathcal{F}_{Ḳ}(e, \mathfrak{Q})\rangle, \mathfrak{Q}_{Ḳ}(e)\ \forall\ e \in {}^{\circ}F\},$$

where

$$\mathcal{T}_{Ḳ}(e, \mathfrak{Q}), \mathcal{I}_{Ḳ}(e, \mathfrak{Q}), \mathcal{F}_{Ḳ}(e, \mathfrak{Q}) \subseteq I_0^1$$

and

$$0 \le \mathcal{T}_{Ḳ}(e,\mathfrak{Q}) + \mathcal{I}_{Ḳ}(e,\mathfrak{Q}) + \mathcal{F}_{Ḳ}(e,\mathfrak{Q}) \le 3.$$

3.4 Interval-Valued Neutrosophic Fuzzy Soft Set

Let $^{\circ}F$ is the universal of discourse and $\aleph$ be the set of attributes of elements in $^{\circ}F$. Take $\mathcal{A}$ to be a subset of $\aleph$ then a function F its mapping is given by: F: $\mathcal{A} \to P\ ({}^{\circ}F)$, Then the Interval-Valued Neutrosophic Fuzzy soft set can be generated as follow:

$$\mathcal{T\!D} = \{\langle \hbar, \mathfrak{f}_{\lambda}(\hbar)\rangle \mid \hbar \in \mathcal{A}, \mathfrak{f}_{\lambda}(\hbar) \in P\ ({}^{\circ}F)\},$$

where P ($^{\circ}F$) is the IVNFSS,

$$\mathfrak{f}_{\lambda}(\hbar) = \{e, \langle \mathcal{T}_{\lambda}(e, \mathfrak{Q}), \mathcal{I}_{\lambda}(e, \mathfrak{Q}), \mathcal{F}_{\lambda}(e, \mathfrak{Q})\rangle, \mathfrak{Q}_{\lambda}(e)\ \forall\ e \in {}^{\circ}F\},$$

and

$$0 \leq \mathcal{S}up\mathcal{T}_{\eth}(e, \mathfrak{Q}) + \mathcal{S}up\mathcal{I}_{\eth}(e, \mathfrak{Q}) + \mathcal{S}up\mathcal{F}_{\eth}(e, \mathfrak{Q}) \leq 3.$$

3.5 Complement of IVNFSS

The complement of an IVNFSS Ụ denoted by $Ụ^c$ defined as;

$$Ụ^c = \begin{matrix} [i_m\mathcal{F}_{Ụ}(e, \mathfrak{Q}), \mathcal{S}_m\mathcal{F}_{Ụ}(e, \mathfrak{Q})], \\ [1 - \mathcal{S}_m\mathcal{I}_{Ụ}(e, \mathfrak{Q}), 1 - i_m\mathcal{I}_{Ụ}(e, \mathfrak{Q})], \\ [i_m\mathcal{T}_{Ụ}(e, \mathfrak{Q}), \mathcal{S}_m\mathcal{T}_{Ụ}(e, \mathfrak{Q})], \\ 1 - \mathfrak{Q}_{Ụ}(e), \end{matrix}$$

$\forall\ e \in$ °F.

3.6 Example

$Ụ^c$= {⟨$\hbar_1$, (e_1 ,[0.3,0.6], [0.3,0.5] ,[0.4,0.6], 0.3), (e_2, [0.3, 0.6], [0.4, 0.6], [0.4, 0.8], 0.4), (e_3, [0.3, 0.5], [0.4, 0.6], [0.4, 0.6], 0.4)⟩, ⟨$\hbar_2$, (e_1, ([0.3, 0.6], [0.5, 0.6], [0.5, 0.8], 0.4), (e_2, ([0.5, 0.6], [0.5, 0.7], [0.5, 0.7], 0.3), (e_3, ([0.2, 0.5], [0.4, 0.7], [0.4, 0.6], 0.6)⟩, ⟨$\hbar_3$, (e_1, [0.3, 0.7], [0.4, 0.6], [0.7, 0.8], 0.2), (e_2, ([0.4, 0.6], [0.4, 0.7], [0.4, 0.8], 0.3), (e_3, ([0.2, 0.4], [0.5, 0.7], [0.6, 0.7], 0.3)⟩}.

3.7 Union of IVNFSS

The union of IVNFSS of Ọ ∪ Ụ in °F is given:

$$Ọ \cup Ụ = \begin{matrix} \left[\mathcal{M}_{\mathcal{X}}\left(i_m\mathcal{T}_{Ọ}(e, \mathfrak{Q}), i_m\mathcal{T}_{Ụ}(e, \mathfrak{Q})\right), \mathcal{M}_{\mathcal{X}}\left(\mathcal{S}_m\mathcal{T}_{Ọ}(e, \mathfrak{Q}), \mathcal{S}_m\mathcal{T}_{Ụ}(e, \mathfrak{Q})\right)\right], \\ \left[\mathcal{M}_n\left(i_m\mathcal{I}_{Ọ}(e, \mathfrak{Q}), i_m\mathcal{I}_{Ụ}(e, \mathfrak{Q})\right), \mathcal{M}_n\left(\mathcal{S}_m\mathcal{I}_{Ọ}(e, \mathfrak{Q}), \mathcal{S}_m\mathcal{I}_{Ụ}(e, \mathfrak{Q})\right)\right], \\ \left[\mathcal{M}_n\left(i_m\mathcal{F}_{Ọ}(e, \mathfrak{Q}), i_m\mathcal{F}_{Ụ}(e, \mathfrak{Q})\right), \mathcal{M}_n\left(\mathcal{S}_m\mathcal{F}_{Ọ}(e, \mathfrak{Q}), \mathcal{S}_m\mathcal{F}_{Ụ}(e, \mathfrak{Q})\right)\right], \\ \mathcal{M}_{\mathcal{X}}\left(\mathfrak{Q}_{Ọ}(e), \mathfrak{Q}_{Ụ}(e)\right), \end{matrix}$$

$\forall\ e \in$ °F.

3.8 Example

The union of Ọ ∪ Ụ is

Ọ ∪ Ụ= {⟨$\hbar_1$,(e_1, [0.6,0.7], [0.3,0.6], [0.3,0.6], 0.6), (e_2, [0.4, 0.8], [0.2, 0.6], [0.2, 0.6], 0.5), (e_3, [0.5, 0.6], [0.4, 0.7], [0.3, 0.5], 0.6)⟩, ⟨$\hbar_2$, (e_1, [0.7, 0.8], [0.4, 0.5], [0.3, 0.6], 0.6), (e_2, [0.5, 0.7], [0.3, 0.5], [0.5, 0.7], 0.7), (e_3, [0.4, 0.6], [0.3, 0.6], [0.2, 0.5], 0.7)⟩, ⟨$\hbar_3$, (e_1,[0.7, 0.8], [0.4, 0.6], [0.3, 0.6], 0.8), (e_2, [0.5, 0.8], [0.3, 0.9], [0.5, 0.7], 0.6), (e_3, [0.6, 0.7], [0.3, 0.5], [0.2, 0.4], 0.7)⟩}.

3.9 Intersection of IVNFSS

The Intersection of IVNFSS of Ọ ∩ Ụ in °F is given:

$$Ọ \cap Ụ = \begin{matrix} \left[\mathcal{M}_n\left(i_m\mathcal{T}_{Ọ}(e, \mathfrak{Q}), i_m\mathcal{T}_{Ụ}(e, \mathfrak{Q})\right), \mathcal{M}_n\left(\mathcal{S}_m\mathcal{T}_{Ọ}(e, \mathfrak{Q}), \mathcal{S}_m\mathcal{T}_{Ụ}(e, \mathfrak{Q})\right)\right], \\ \left[\mathcal{M}_{\mathcal{X}}\left(i_m\mathcal{I}_{Ọ}(e, \mathfrak{Q}), i_m\mathcal{I}_{Ụ}(e, \mathfrak{Q})\right), \mathcal{M}_{\mathcal{X}}\left(\mathcal{S}_m\mathcal{I}_{Ọ}(e, \mathfrak{Q}), \mathcal{S}_m\mathcal{I}_{Ụ}(e, \mathfrak{Q})\right)\right], \\ \left[\mathcal{M}_{\mathcal{X}}\left(i_m\mathcal{F}_{Ọ}(e, \mathfrak{Q}), i_m\mathcal{F}_{Ụ}(e, \mathfrak{Q})\right), \mathcal{M}_{\mathcal{X}}\left(\mathcal{S}_m\mathcal{F}_{Ọ}(e, \mathfrak{Q}), \mathcal{S}_m\mathcal{F}_{Ụ}(e, \mathfrak{Q})\right)\right], \\ \mathcal{M}_n\left(\mathfrak{Q}_{Ọ}(e), \mathfrak{Q}_{Ụ}(e)\right), \end{matrix}$$

$\forall\ e \in$ °F.

4. Decision Support System based on IVNFSS

The evaluation of the connections between the components of IVNFSS depends heavily on the distance measure. This metric is essential for applications like clustering and decision-making when using shaky data. It measures the degree of concordance or variance among neutrosophic memberships, fuzzy degrees of uncertainty, and memberships with interval values, providing crucial information for reliable data interpretation. To deal with the complexity of actual data sets, a variety of specialized distance metrics designed for IVNFSS are available. The Hamming and Normalized Hamming distances for interval-valued neutrosophic fuzzy soft sets IVNFSS are introduced. These measures of distance are useful in the scientific and engineering sectors and may be used in a variety of real-world situations to make analysis and comparisons easier inside the IVNFSS framework.

Let's take two IVNFSS $\check{K}$ *and* $Ғ$ a universe of discourse $°F = \{℮_1, ℮_2, ℮_3, \dots, ℮_n\}$ which are denoted by;

$$\check{K} = \{℮\,[\mathcal{T}^{L}_{\check{K}}(℮,\ \mathfrak{Q}),\ \mathcal{T}^{U}_{\check{K}}(℮,\ \mathfrak{Q})],\ [\mathcal{I}^{L}_{\check{K}}(℮,\ \mathfrak{Q}),\ \mathcal{I}^{U}_{\check{K}}(℮,\ \mathfrak{Q})],\ [\mathcal{F}^{L}_{\check{K}}(℮,\ \mathfrak{Q}),\ \mathcal{F}^{U}_{\check{K}}(℮,\ \mathfrak{Q})]\rangle, \mathfrak{Q}_{\check{K}}(℮)\ \forall:\ ℮ \in °F\}$$

$$Ғ = \{℮\,[\mathcal{T}^{L}_{Ғ}(℮,\ \mathfrak{Q}),\ \mathcal{T}^{U}_{Ғ}(℮,\ \mathfrak{Q})],\ [\mathcal{I}^{L}_{Ғ}(℮,\ \mathfrak{Q}),\ \mathcal{I}^{U}_{Ғ}(℮,\ \mathfrak{Q})],\ [\mathcal{F}^{L}_{Ғ}(℮,\ \mathfrak{Q}),\ \mathcal{F}^{U}_{Ғ}(℮,\ \mathfrak{Q})]\rangle, \mathfrak{Q}_{Ғ}(℮)\ \forall:\ ℮ \in °F\}$$

4.1 Hamming Distance

$$d^{\mathcal{H}}(\check{K}, Ғ) = \sum_{j=1}^{n}\sum_{i=1}^{m}\frac{1}{7m}\begin{cases} \hbar_J\left|\mathcal{T}^{L}_{\check{K}}(℮,\mathfrak{Q}) - \mathcal{T}^{L}_{Ғ}(℮,\mathfrak{Q})\right| + \hbar_J\left|\mathcal{T}^{U}_{\check{K}}(℮,\mathfrak{Q}) - \mathcal{T}^{U}_{Ғ}(℮,\mathfrak{Q})\right| + \\ \hbar_J\left|\mathcal{I}^{L}_{\check{K}}(℮,\mathfrak{Q}) - \mathcal{I}^{L}_{Ғ}(℮,\mathfrak{Q})\right| + \hbar_J\left|\mathcal{I}^{U}_{\check{K}}(℮,\mathfrak{Q}) - \mathcal{I}^{U}_{Ғ}(℮,\mathfrak{Q})\right| + \\ \hbar_J\left|\mathcal{F}^{L}_{\check{K}}(℮,\mathfrak{Q}) - \mathcal{F}^{L}_{Ғ}(℮,\mathfrak{Q})\right| + \hbar_J\left|\mathcal{F}^{U}_{\check{K}}(℮,\mathfrak{Q}) - \mathcal{F}^{U}_{Ғ}(℮,\mathfrak{Q})\right| + \\ \hbar_J\left|\mathfrak{Q}_{\check{K}}(℮) - \mathfrak{Q}_{Ғ}(℮)\right|. \end{cases}$$

4.2 Normalized Hamming Distance

$$d^{\mathcal{NH}}(\check{K}, Ғ) = \sum_{j=1}^{n}\sum_{i=1}^{m}\frac{1}{7nm}\begin{cases} \hbar_J\left|\mathcal{T}^{L}_{\check{K}}(℮,\mathfrak{Q}) - \mathcal{T}^{L}_{Ғ}(℮,\mathfrak{Q})\right| + \hbar_J\left|\mathcal{T}^{U}_{\check{K}}(℮,\mathfrak{Q}) - \mathcal{T}^{U}_{Ғ}(℮,\mathfrak{Q})\right| + \\ \hbar_J\left|\mathcal{I}^{L}_{\check{K}}(℮,\mathfrak{Q}) - \mathcal{I}^{L}_{Ғ}(℮,\mathfrak{Q})\right| + \hbar_J\left|\mathcal{I}^{U}_{\check{K}}(℮,\mathfrak{Q}) - \mathcal{I}^{U}_{Ғ}(℮,\mathfrak{Q})\right| + \\ \hbar_J\left|\mathcal{F}^{L}_{\check{K}}(℮,\mathfrak{Q}) - \mathcal{F}^{L}_{Ғ}(℮,\mathfrak{Q})\right| + \hbar_J\left|\mathcal{F}^{U}_{\check{K}}(℮,\mathfrak{Q}) - \mathcal{F}^{U}_{Ғ}(℮,\mathfrak{Q})\right| + \\ \hbar_J\left|\mathfrak{Q}_{\check{K}}(℮) - \mathfrak{Q}_{Ғ}(℮)\right|. \end{cases}$$

4.3 Theorem

Distance $d^{\mathcal{H}}(\check{K}, Ғ)$ is said to be distance measure if it satisfies the following properties:

1. $0 \le d^{\mathcal{H}}(\check{K}, Ғ) \le 1$;
2. $d^{\mathcal{H}}(\check{K}, Ғ) = 0$ *iff* $\check{K} = Ғ$;
3. $d^{\mathcal{H}}(\check{K}, Ғ) = d^{\mathcal{H}}(Ғ, \check{K})$;
4. If $\check{K} \subseteq Ғ \subseteq Ħ$, than $d^{\mathcal{H}}(\check{K}, Ғ) \leqslant d^{\mathcal{H}}(\check{K}, Ħ)$ and $d^{\mathcal{H}}(Ғ, Ħ) \leqslant d^{\mathcal{H}}(\check{K}, Ħ)$.

Proof:

1. By the definition of distance, it is $d^{\mathcal{H}}(\check{K}, Ғ) \geqslant 0$. For it to be valid $d^{\mathcal{H}}(\check{K}, Ғ) \leqslant 1$. By using the definition of Interval-valued neutrosophic fuzzy soft set. Such That;
   $0 \leqslant \mathcal{T}^{L}_{\check{K}}(℮,\ \mathfrak{Q}),\ \mathcal{T}^{U}_{\check{K}}(℮,\ \mathfrak{Q}) \leqslant 1$, $0 \leqslant \mathcal{I}^{L}_{\check{K}}(℮,\ \mathfrak{Q}),\ \mathcal{I}^{U}_{\check{K}}(℮,\ \mathfrak{Q}) \leqslant 1$, $0 \leqslant \mathcal{F}^{L}_{\check{K}}(℮,\ \mathfrak{Q}),\ \mathcal{F}^{U}_{\check{K}}(℮,\ \mathfrak{Q}) \leqslant 1$, $0 \leqslant \mathfrak{Q}_{\check{K}}(℮) \leqslant 1$.
   $0 \leqslant \mathcal{T}^{L}_{Ғ}(℮,\ \mathfrak{Q}),\ \mathcal{T}^{U}_{Ғ}(℮,\ \mathfrak{Q}) \leqslant 1$, $0 \leqslant \mathcal{I}^{L}_{Ғ}(℮,\ \mathfrak{Q}),\ \mathcal{I}^{U}_{Ғ}(℮,\ \mathfrak{Q}) \leqslant 1$, $0 \leqslant \mathcal{F}^{L}_{Ғ}(℮,\ \mathfrak{Q}),\ \mathcal{F}^{U}_{Ғ}(℮,\ \mathfrak{Q}) \leqslant 1$, $0 \leqslant \mathfrak{Q}_{Ғ}(℮) \leqslant 1$.

This implies that

$$0 \leqslant \left|\mathcal{T}^{L}_{\check{K}}(℮,\mathfrak{Q}) - \mathcal{T}^{L}_{Ғ}(℮,\mathfrak{Q})\right| \leqslant 1, 0 \leqslant \left|\mathcal{T}^{U}_{\check{K}}(℮,\mathfrak{Q}) - \mathcal{T}^{U}_{Ғ}(℮,\mathfrak{Q})\right| \leqslant 1$$
$$0 \leqslant \left|\mathcal{I}^{L}_{\check{K}}(℮,\mathfrak{Q}) - \mathcal{I}^{L}_{Ғ}(℮,\mathfrak{Q})\right| \leqslant 1, 0 \leqslant \left|\mathcal{I}^{U}_{\check{K}}(℮,\mathfrak{Q}) - \mathcal{I}^{U}_{Ғ}(℮,\mathfrak{Q})\right| \leqslant 1$$
$$0 \leqslant \left|\mathcal{F}^{L}_{\check{K}}(℮,\mathfrak{Q}) - \mathcal{F}^{L}_{Ғ}(℮,\mathfrak{Q})\right| \leqslant 1, 0 \leqslant \left|\mathcal{F}^{U}_{\check{K}}(℮,\mathfrak{Q}) - \mathcal{F}^{U}_{Ғ}(℮,\mathfrak{Q})\right| \leqslant 1$$
$$0 \leqslant \left|\mathfrak{Q}_{\check{K}}(℮) - \mathfrak{Q}_{Ғ}(℮)\right| \leqslant 1$$

$$0 \leqslant \begin{pmatrix} \hbar_J\left|\mathcal{T}^{L}_{\check{K}}(℮,\mathfrak{Q}) - \mathcal{T}^{L}_{Ғ}(℮,\mathfrak{Q})\right| + \hbar_J\left|\mathcal{T}^{U}_{\check{K}}(℮,\mathfrak{Q}) - \mathcal{T}^{U}_{Ғ}(℮,\mathfrak{Q})\right| + \\ \hbar_J\left|\mathcal{I}^{L}_{\check{K}}(℮,\mathfrak{Q}) - \mathcal{I}^{L}_{Ғ}(℮,\mathfrak{Q})\right| + \hbar_J\left|\mathcal{I}^{U}_{\check{K}}(℮,\mathfrak{Q}) - \mathcal{I}^{U}_{Ғ}(℮,\mathfrak{Q})\right| + \\ \hbar_J\left|\mathcal{F}^{L}_{\check{K}}(℮,\mathfrak{Q}) - \mathcal{F}^{L}_{Ғ}(℮,\mathfrak{Q})\right| + \hbar_J\left|\mathcal{F}^{U}_{\check{K}}(℮,\mathfrak{Q}) - \mathcal{F}^{U}_{Ғ}(℮,\mathfrak{Q})\right| + \\ \hbar_J\left|\mathfrak{Q}_{\check{K}}(℮) - \mathfrak{Q}_{Ғ}(℮)\right|. \end{pmatrix} \leqslant 7$$

$\Rightarrow\ 0 \le d^{\mathcal{H}}(\check{K}, Ғ) \le 1$

2. Let $d^{\mathcal{H}}(\check{K}, F) = 0$ for two IVNFSS $\check{K}$ *and* F

$$\Rightarrow \sum_{j=1}^{n}\sum_{i=1}^{m}\frac{1}{7nm}\begin{pmatrix} \hbar_j\left|\mathcal{T}_{\check{K}}^{L}(e,\mathfrak{Q}) - \mathcal{T}_{F}^{L}(e,\mathfrak{Q})\right| + \hbar_j\left|\mathcal{T}_{\check{K}}^{U}(e,\mathfrak{Q}) - \mathcal{T}_{F}^{U}(e,\mathfrak{Q})\right| + \\ \hbar_j\left|\mathcal{I}_{\check{K}}^{L}(e,\mathfrak{Q}) - \mathcal{I}_{F}^{L}(e,\mathfrak{Q})\right| + \hbar_j\left|\mathcal{I}_{\check{K}}^{U}(e,\mathfrak{Q}) - \mathcal{I}_{F}^{U}(e,\mathfrak{Q})\right| + \\ \hbar_j\left|\mathcal{F}_{\check{K}}^{L}(e,\mathfrak{Q}) - \mathcal{F}_{F}^{L}(e,\mathfrak{Q})\right| + \hbar_j\left|\mathcal{F}_{\check{K}}^{U}(e,\mathfrak{Q}) - \mathcal{F}_{F}^{U}(e,\mathfrak{Q})\right| + \\ \hbar_j\left|\mathfrak{Q}_{\check{K}}(e) - \mathfrak{Q}_{F}(e)\right|. \end{pmatrix} = 0$$

iff for all:

$$\begin{gathered} \left|\mathcal{T}_{\check{K}}^{L}(e,\mathfrak{Q}) - \mathcal{T}_{F}^{L}(e,\mathfrak{Q})\right| = 0, \left|\mathcal{T}_{\check{K}}^{U}(e,\mathfrak{Q}) - \mathcal{T}_{F}^{U}(e,\mathfrak{Q})\right| = 0 \\ \left|\mathcal{I}_{\check{K}}^{L}(e,\mathfrak{Q}) - \mathcal{I}_{F}^{L}(e,\mathfrak{Q})\right| = 0, \left|\mathcal{I}_{\check{K}}^{U}(e,\mathfrak{Q}) - \mathcal{I}_{F}^{U}(e,\mathfrak{Q})\right| = 0 \\ \left|\mathcal{F}_{\check{K}}^{L}(e,\mathfrak{Q}) - \mathcal{F}_{F}^{L}(e,\mathfrak{Q})\right| = 0, \left|\mathcal{F}_{\check{K}}^{U}(e,\mathfrak{Q}) - \mathcal{F}_{F}^{U}(e,\mathfrak{Q})\right| = 0 \\ \left|\mathfrak{Q}_{\check{K}}(e) - \mathfrak{Q}_{F}(e)\right| = 0 \end{gathered}$$

which is equivalent to

$$\begin{gathered} \mathcal{T}_{\check{K}}^{L}(e,\mathfrak{Q}) = \mathcal{T}_{F}^{L}(e,\mathfrak{Q}), \mathcal{T}_{\check{K}}^{U}(e,\mathfrak{Q}) = \mathcal{T}_{F}^{U}(e,\mathfrak{Q}) \\ \mathcal{I}_{\check{K}}^{L}(e,\mathfrak{Q}) = \mathcal{I}_{F}^{L}(e,\mathfrak{Q}), \mathcal{I}_{\check{K}}^{U}(e,\mathfrak{Q}) = \mathcal{I}_{F}^{U}(e,\mathfrak{Q}) \\ \mathcal{F}_{\check{K}}^{L}(e,\mathfrak{Q}) = \mathcal{F}_{F}^{L}(e,\mathfrak{Q}), \mathcal{F}_{\check{K}}^{U}(e,\mathfrak{Q}) = \mathcal{F}_{F}^{U}(e,\mathfrak{Q}) \\ \mathfrak{Q}_{\check{K}}(e) = \mathfrak{Q}_{F}(e). \end{gathered}$$

Thus $d^{\mathcal{H}}(\check{K}, F) = 0$

$\Rightarrow \check{K} = F$

3. For two IVNFSS $\check{K}$ and F are;

$$d^{\mathcal{H}}(\check{K}, F) = \sum_{j=1}^{n}\sum_{i=1}^{m}\frac{1}{7m}\begin{cases} \hbar_j\left|\mathcal{T}_{\check{K}}^{L}(e,\mathfrak{Q}) - \mathcal{T}_{F}^{L}(e,\mathfrak{Q})\right| + \hbar_j\left|\mathcal{T}_{\check{K}}^{U}(e,\mathfrak{Q}) - \mathcal{T}_{F}^{U}(e,\mathfrak{Q})\right| + \\ \hbar_j\left|\mathcal{I}_{\check{K}}^{L}(e,\mathfrak{Q}) - \mathcal{I}_{F}^{L}(e,\mathfrak{Q})\right| + \hbar_j\left|\mathcal{I}_{\check{K}}^{U}(e,\mathfrak{Q}) - \mathcal{I}_{F}^{U}(e,\mathfrak{Q})\right| + \\ \hbar_j\left|\mathcal{F}_{\check{K}}^{L}(e,\mathfrak{Q}) - \mathcal{F}_{F}^{L}(e,\mathfrak{Q})\right| + \hbar_j\left|\mathcal{F}_{\check{K}}^{U}(e,\mathfrak{Q}) - \mathcal{F}_{F}^{U}(e,\mathfrak{Q})\right| + \\ \hbar_j\left|\mathfrak{Q}_{\check{K}}(e) - \mathfrak{Q}_{F}(e)\right|. \end{cases}$$

$$= \sum_{j=1}^{n}\sum_{i=1}^{m}\frac{1}{7m}\begin{cases} \hbar_j\left|\mathcal{T}_{F}^{L}(e,\mathfrak{Q}) - \mathcal{T}_{\check{K}}^{L}(e,\mathfrak{Q})\right| + \hbar_j\left|\mathcal{T}_{F}^{L}(e,\mathfrak{Q}) - \mathcal{T}_{\check{K}}^{L}(e,\mathfrak{Q})\right| + \\ \hbar_j\left|\mathcal{T}_{F}^{L}(e,\mathfrak{Q}) - \mathcal{T}_{\check{K}}^{L}(e,\mathfrak{Q})\right| + \hbar_j\left|\mathcal{T}_{F}^{L}(e,\mathfrak{Q}) - \mathcal{T}_{\check{K}}^{L}(e,\mathfrak{Q})\right| + \\ \hbar_j\left|\mathcal{T}_{F}^{L}(e,\mathfrak{Q}) - \mathcal{T}_{\check{K}}^{L}(e,\mathfrak{Q})\right| + \hbar_j\left|\mathcal{T}_{F}^{L}(e,\mathfrak{Q}) - \mathcal{T}_{\check{K}}^{L}(e,\mathfrak{Q})\right| + \\ \hbar_j\left|\mathfrak{Q}_{F}(e) - \mathfrak{Q}_{\check{K}}(e)\right|. \end{cases}$$

$$= d^{\mathcal{H}}(F, \check{K})$$

Hence, $d^{\mathcal{H}}(\check{K}, F) = d^{\mathcal{H}}(F, \check{K})$

4. If $\check{K} \subseteq F \subseteq H$, than

$$\begin{gathered} [\mathcal{T}_{\check{K}}^{L}(e,\mathfrak{Q}), \mathcal{T}_{\check{K}}^{U}(e,\mathfrak{Q})] \supseteq [\mathcal{T}_{F}^{L}(e,\mathfrak{Q}), \mathcal{T}_{F}^{U}(e,\mathfrak{Q})] \subseteq [\mathcal{T}_{H}^{L}(e,\mathfrak{Q}), \mathcal{T}_{H}^{U}(e,\mathfrak{Q})] \\ [\mathcal{I}_{\check{K}}^{L}(e,\mathfrak{Q}), \mathcal{I}_{\check{K}}^{U}(e,\mathfrak{Q})] \supseteq [\mathcal{I}_{F}^{L}(e,\mathfrak{Q}), \mathcal{I}_{F}^{U}(e,\mathfrak{Q})] \supseteq [\mathcal{I}_{H}^{L}(e,\mathfrak{Q}), \mathcal{I}_{H}^{U}(e,\mathfrak{Q})] \\ [\mathcal{F}_{\check{K}}^{L}(e,\mathfrak{Q}), \mathcal{F}_{\check{K}}^{U}(e,\mathfrak{Q})] \supseteq [\mathcal{F}_{F}^{L}(e,\mathfrak{Q}), \mathcal{F}_{F}^{U}(e,\mathfrak{Q})] \supseteq [\mathcal{F}_{H}^{L}(e,\mathfrak{Q}), \mathcal{F}_{H}^{U}(e,\mathfrak{Q})] \end{gathered}$$

Also $\mathfrak{Q}_{\check{K}}(e) \geq \mathfrak{Q}_{F}(e) \geq \mathfrak{Q}_{H}(e)$

Therefore,

$$\left|\mathcal{T}^L_{\check{K}}(e,\mathfrak{Q}) - \mathcal{T}^L_{F}(e,\mathfrak{Q})\right| \leqslant \left|\mathcal{T}^L_{\check{K}}(e,\mathfrak{Q}) - \mathcal{T}^L_{H}(e,\mathfrak{Q})\right|, \left|\mathcal{T}^U_{\check{K}}(e,\mathfrak{Q}) - \mathcal{T}^U_{F}(e,\mathfrak{Q})\right| \leqslant \left|\mathcal{T}^U_{\check{K}}(e,\mathfrak{Q}) - \mathcal{T}^U_{H}(e,\mathfrak{Q})\right|,$$
$$\left|\mathcal{I}^L_{\check{K}}(e,\mathfrak{Q}) - \mathcal{I}^L_{F}(e,\mathfrak{Q})\right| \leqslant \left|\mathcal{I}^L_{\check{K}}(e,\mathfrak{Q}) - \mathcal{I}^L_{H}(e,\mathfrak{Q})\right|, \left|\mathcal{I}^U_{\check{K}}(e,\mathfrak{Q}) - \mathcal{I}^U_{F}(e,\mathfrak{Q})\right| \leqslant \left|\mathcal{I}^U_{\check{K}}(e,\mathfrak{Q}) - \mathcal{I}^U_{H}(e,\mathfrak{Q})\right|$$
$$\left|\mathcal{F}^L_{\check{K}}(e,\mathfrak{Q}) - \mathcal{F}^L_{F}(e,\mathfrak{Q})\right| \leqslant \left|\mathcal{F}^L_{\check{K}}(e,\mathfrak{Q}) - \mathcal{F}^L_{H}(e,\mathfrak{Q})\right|, \left|\mathcal{F}^U_{\check{K}}(e,\mathfrak{Q}) - \mathcal{F}^U_{F}(e,\mathfrak{Q})\right| \leqslant \left|\mathcal{F}^U_{\check{K}}(e,\mathfrak{Q}) - \mathcal{F}^U_{H}(e,\mathfrak{Q})\right|$$
$$\left|\mathfrak{Q}_{\check{K}}(e) - \mathfrak{Q}_{F}(e)\right| \leqslant \left|\mathfrak{Q}_{\check{K}}(e) - \mathfrak{Q}_{H}(e)\right|$$

$$d^{\mathcal{H}}(\check{K},F) = \sum_{j=1}^{n}\sum_{i=1}^{m}\frac{1}{7m}\begin{cases} \hbar_j\left|\mathcal{T}^L_{\check{K}}(e,\mathfrak{Q}) - \mathcal{T}^L_{F}(e,\mathfrak{Q})\right| + \hbar_j\left|\mathcal{T}^U_{\check{K}}(e,\mathfrak{Q}) - \mathcal{T}^U_{F}(e,\mathfrak{Q})\right| + \\ \hbar_j\left|\mathcal{I}^L_{\check{K}}(e,\mathfrak{Q}) - \mathcal{I}^L_{F}(e,\mathfrak{Q})\right| + \hbar_j\left|\mathcal{I}^U_{\check{K}}(e,\mathfrak{Q}) - \mathcal{I}^U_{F}(e,\mathfrak{Q})\right| + \\ \hbar_j\left|\mathcal{F}^L_{\check{K}}(e,\mathfrak{Q}) - \mathcal{F}^L_{F}(e,\mathfrak{Q})\right| + \hbar_j\left|\mathcal{F}^U_{\check{K}}(e,\mathfrak{Q}) - \mathcal{F}^U_{F}(e,\mathfrak{Q})\right| + \\ \hbar_j\left|\mathfrak{Q}_{\check{K}}(e) - \mathfrak{Q}_{F}(e)\right|. \end{cases}$$

$$\geqslant \sum_{j=1}^{n}\sum_{i=1}^{m}\frac{1}{7m}\begin{cases} \hbar_j\left|\mathcal{T}^L_{\check{K}}(e,\mathfrak{Q}) - \mathcal{T}^L_{F}(e,\mathfrak{Q})\right| + \hbar_j\left|\mathcal{T}^U_{\check{K}}(e,\mathfrak{Q}) - \mathcal{T}^U_{F}(e,\mathfrak{Q})\right| + \\ \hbar_j\left|\mathcal{I}^L_{\check{K}}(e,\mathfrak{Q}) - \mathcal{I}^L_{F}(e,\mathfrak{Q})\right| + \hbar_j\left|\mathcal{I}^U_{\check{K}}(e,\mathfrak{Q}) - \mathcal{I}^U_{F}(e,\mathfrak{Q})\right| + \\ \hbar_j\left|\mathcal{F}^L_{\check{K}}(e,\mathfrak{Q}) - \mathcal{F}^L_{F}(e,\mathfrak{Q})\right| + \hbar_j\left|\mathcal{F}^U_{\check{K}}(e,\mathfrak{Q}) - \mathcal{F}^U_{F}(e,\mathfrak{Q})\right| + \\ \hbar_j\left|\mathfrak{Q}_{\check{K}}(e) - \mathfrak{Q}_{F}(e)\right|. \end{cases}$$

$$= d^{\mathcal{H}}(\check{K},F)$$

Similarly $d^{\mathcal{H}}(F,H) \leqslant d^{\mathcal{H}}(\check{K},H)$

Hence, it is valid distance.

4.4 *Example*

Here we define example related hamming distance and normalized hamming distance:

Ǩ= {<$\hbar_1$, (e_1, [0.5,0.7], [0.6,0.8], [0.1,0.5], 0.5), (e_2, [0.7,0.8], [0.3,0.8], [0.7,0.8],0.6)>, <$\hbar_2$, (e_1, [0.6,0.9], [0.7,0.9], [0.6,0.7], 0.5), (e_2, [0.7,0.9], [0.6,0.9], [0.6,0.8], 0.6)>,< $\hbar_3$, (e_1, [0.6,0.8], [0.5,0.7], [0.4,0.6], 0.5), (e_2, [0.1,0.8], [0.5,0.9], [0.6,0.7], 0.6) >}.

Ƒ ={<$\hbar_1$,(e_1, [0.4,0.5], [0.3,0.5], [0.5,0.7], 0.6),(e_2, [0.6,0.7], [0.2,0.7], [0.6,0.8], 0.5)>, <$\hbar_2$, (e_1, [0.4, 0.5], [0.3, 0.5], [0.6, 0.7], 0.7), (e_2, [0.3, 0.4], [0.4, 0.8], [0.3, 0.6], 0.4)> , <$\hbar_3$, (e_1, [0.2, 0.5], [0.3, 0.6], [0.3, 0.7], 0.8), (e_2, [0.6, 0.8], [0.6, 0.8], [0.5, 0.7], 0.3)>}.

4.4.1 Example: Hamming Distance for IVNFSS

$$d^{\mathcal{H}}(\check{K},F) = \frac{1}{14}\begin{pmatrix} 0.1 + 0.2 + 0.3 + 0.3 + 0.4 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.2 + 0.1 + 0.1 \\ 0.2 + 0.4 + 0.4 + 0.4 + 0.1 + 0.2 + 0.4 + 0.5 + 0.2 + 0.1 + 0.3 + 0.2 + 0.2 \\ 0.4 + 0.3 + 0.2 + 0.1 + 0.1 + 0.1 + 0.3 + 0.5 + 0.1 + 0.1 + 0.1 + 0.3 \end{pmatrix}$$

= 0.5857

4.4.2 Example: Normalized Hamming Distance for IVNFSS

$$d^{\mathcal{NH}}(\check{K},F) = \frac{1}{42}\begin{pmatrix} 0.1 + 0.2 + 0.3 + 0.3 + 0.4 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.2 + 0.1 + 0.1 \\ 0.2 + 0.4 + 0.4 + 0.4 + 0.1 + 0.2 + 0.4 + 0.5 + 0.2 + 0.1 + 0.3 + 0.2 + 0.2 \\ 0.4 + 0.3 + 0.2 + 0.1 + 0.1 + 0.1 + 0.3 + 0.5 + 0.1 + 0.1 + 0.1 + 0.3 \end{pmatrix}$$

= 0.1976

5. Algorithm Design

In continuation with the previous section, this section offers a decision-making method that uses the earlier-defined distance measurements. The method is made to take full use of the pragmatic properties of the Interval-valued neutrosophic fuzzy soft set. The flow chart of the algorithm is presented in Figure 1, and methodically as follows:

Step 1. To analyze the data across universe $\mathcal{A}$, an IVNFSS element is built. It is founded on decision-makers.

Step 2. Build an IVNFSS set $(\check{K}, Ϝ)$ based on the criteria decided upon by the decision-making committee for a corporation.

Step 3. Create m IVNFSS $(C_i, Ϝ)$ sets using the decision-making team's ap praisal of the various possibilities, where i = 1, 2, ..., m.

Step 4. Calculate the distance Hamming distance and Normalized hamming distance between $(\check{K}, Ϝ)$ and $(C_i, Ϝ)$.

Step 5. Evaluate the ranking by using distance measure.

Figure 1. Flowchart of the proposed method.

5.1 *Application of IVNFSS for Decision Making*

To make sure that the water we use to drink is safe and free of toxins, water treatment is an essential procedure. Water treatment facilities strive to clean water from a variety of sources, including rivers, lakes, and groundwater, to make it appropriate for human use. Although water treatment considerably improves water quality, it's crucial to realize that the drinkability of the treated water relies on a number of factors, including the exact treatment methods employed, the quality of the source water, and the upkeep of the distribution system. In this conversation, we'll look at the key steps in water treatment as well as the variables that affect whether the water is safe to drink. For raw water from natural sources to be changed into safe and clean drinking water, the water treatment process normally entails multiple steps. The main elements of water treatment are summarized generally as follows:

- **Coagulation and Flocculation**: To destabilize particles and pollutants, chemicals like alum or ferric sulfate are added to the raw water. Due to the coagulation and flocculation that results, it is simpler to remove minute particles after they have clumped together.
- **Sedimentation:** The floc particles naturally sink to the bottom of the water while it sits in a sedimentation tank or basin. Solids and pollutants are removed from the water using this technique.
- **Filtration:** The water is filtered using several materials, including sand, gravel, and activated carbon, after sedimentation. These filters further eliminate any leftover pollutants, germs, and tiny suspended particles.
- **Disinfection**: Disinfection is an essential phase in the process of eradiating or rendering harmless microorganisms (bacteria, viruses, and parasites) inert. This objective is frequently served by the use of chlorine, chloramines, ozone, or ultraviolet (UV) radiation.
- **pH Adjustment:** An essential phase in the treatment process is adjusting the water's pH level to conform to legal requirements and make sure it is not excessively acidic or alkaline.

Now that the water has been filtered, we must compare it to pure water and determine the distance measure. The distance between the pure water and the filtered water is then calculated using the pure water as the ideal benchmark. To compute the quality control matrix for treatment, consider

{E_1, E_2, E_3, E_4} set of alternatives. Where E_1 = Ultrapure water, E_2 = Distilled water, E_3 = Deionized water, E_4 = Tap water. Let {$\hbar_1$ = Chemical Composition, $\hbar_2$ = Microbiological Quality, $\hbar_3$ = Regulatory Compliance, $\hbar_4$ = Continuous Monitoring, $\hbar_5$ = Health and Environ mental Impact Assessment} be the set of attributes.

The water treatment process encompasses crucial steps such as coagulation, sedimentation, filtration, disinfection, and pH adjustment to guarantee the production of safe drinking water. The quality control results for pure water in Table 1, comparing the treated water to different types of pure water, are detailed in Tables 2, 3, and 4. We excluded the Hamming distance and normalized Hamming for water treatment, basing our estimates on pure water as the ideal reference point. These tables' present similarities measured through Hamming Distance and Normalized Hamming Distance for alternative sets C1, C2, and C3, respectively. In Table 5, outcomes reveal that C1 exhibits the lowest similarity, followed by C3, whereas C2 demonstrates the highest similarity. We decided not to include Hamming distance or normalized Hamming values in our assessment of water treatment procedures, preferring to make pure water the gold standard. The decision to exclude Hamming distance was made due to its poor applicability in capturing fundamental aspects of water quality, whilst the decision to exclude normalized Hamming values was made to get a more contextually meaningful assessment. We attempted to determine whether consuming treated water is feasible by designating pure water as the optimal standard and assigning a value of 1 to the degree of similarity to its properties. The practical removal of some metrics supported a more accurate and pertinent assessment of water quality in the context of treatment procedures, especially as our estimations got closer to this benchmark, which is a sign of a successful treatment outcome.

Table 1. Tabular representation of pure water.

(Ǩ, Ғ)	E_1	E_2	E_3	E_4
$\hbar_1$	⟨[0.5, 0.6], [0.2, 0.3], [0.4,0.7],0.6⟩	⟨[0.2,0.7],[0.7, 0.9],[0.4,0.7], 0.4⟩	⟨[0.2,0.6], [0.3,0.7], [0.1,0.9],0.5⟩	⟨[0.4,0.6], [0.5,0.8], [0.2,0.8],0.7⟩
$\hbar_2$	⟨[0.2, 0.5], [0.4, 0.8], [0.5, 0.6],0.7⟩	⟨[0.4,0.7], [0.3, 0.8], [0.5,0.6],0.7⟩	⟨[0.3,0.7], [0.4,0.6][0.2,0.8],0.3⟩	⟨[0.4,0.7], [0.5,0.7],[0.3,0.8],0.7⟩
$\hbar_3$	⟨[0.1, 0.7], [0.3, 0.6], [0.4, 0.7], 0.8⟩	⟨[0.6,0.8], [0.1, 0.9], [0.2, 0.5], 0.5⟩	⟨[0.4,0.8], [0.5,0.8], [0.3,0.7],0.5⟩	⟨[0.4,0.8], [0.5,0.7], [0.4,0.7],0.6⟩
$\hbar_4$	⟨[0.2, 0.6], [0.7, 0.8], [0.4, 0.6], 0.7⟩	⟨[0.5,0.7], [0.7, 0.8], [0.6, 0.9], 0.2⟩	⟨[0.5,0.9], [0.6,0.7], [0.4,0.8],0.8⟩	⟨[0.2,0.7], [0.7,0.9], [0.4,0.7],0.4⟩
$\hbar_5$	⟨[0.2, 0.6], [0.7, 0.8], [0.4, 0.6], 0.7⟩	⟨[0.5,0.7], [0.7, 0.8], [0.6, 0.9], 0.2⟩	⟨[0.5,0.9], [0.6,0.7], [0.4,0.8],0.8⟩	⟨[0.2,0.7], [0.7,0.9], [0.4,0.7],0.4⟩

Table 2. Tabular representation of water treatment.

(C_1,Ғ)	**E_1**	**E_2**	**E_3**	**E_4**
$\hbar_1$	⟨[0.4,0.7],[0.3,0.8],[0.5, 0.7],0.4⟩	⟨[0.5,0.8],[0.6,0.9],[0. 1,0.7], 0.7⟩	⟨[0.4,0.7],[0.5,0.8],[0. 4,0.6],0.6⟩	⟨[0.5,0.7],[0.2,0.8], [0.3,0.7],0.6⟩
$\hbar_2$	⟨[0.4,0.6],[0.5,0.6],[0.6, 0.8],0.8⟩	⟨[0.5,0.7],[0.5,0.8],[0. 6,0.8],0.6⟩	⟨[0.4,0.8],[0.6,0.7],[0. 1,0.9],0.5⟩	⟨[0.6,0.8],[0.4,0.5], [0.3,0.8],0.6⟩
$\hbar_3$	⟨[0.2,0.8],[0.6,0.8],[0.5, 0.6],0.3⟩	⟨[0.7,0.8],[0.2,0.9],[0. 5,0.7], 0.6⟩	⟨[0.3,0.5],[0.4,0.8],[0. 2,0.8],0.6⟩	⟨[0.5,0.7],[0.7,0.9], [0.4,0.8],0.7⟩
$\hbar_4$	⟨[0.4,0.7],[0.7,0.9],[0.6, 0.8],0.5⟩	⟨[0.7,0.9],[0.5,0.6],[0. 8,0.9],0.4⟩	⟨[0.3,0.7],[0.4,0.9], [0.5,0.8],0.8⟩	⟨[0.4,0.7], [0.4,0.7], [0.5,0.8],0.6⟩
$\hbar_5$	⟨[0.3,0.7],[0.5,0.9], [0.5,0.7],0.6⟩	⟨[0.4,0.8],[0.5,0.9],[0. 3,0.7],0.6⟩	⟨[0.4,0.6],[0.5,0.8], [0.3,0.8],0.4⟩	⟨[0.3,0.6], [0.4,0.8], [0.2,0.8],0.6⟩

Table 3. Tabular representation of water treatment.

(C_2,Ғ)	E_1	E_2	E_3	E_4
$\hbar_1$	⟨[0.1,0.8], [0.7, 0.9], [0.2, 0.6], 0.3⟩	⟨[0.4,0.8],[0.5,0.7],[0.3,0.8],0.7⟩	⟨[0.5,0.9], [0.6.0.9], [0.4,0.7],0.8⟩	⟨[0.6,0.9], [0.3,0.7], [0.4,0.8],0.6⟩
$\hbar_2$	⟨[0.4,0.7], [0.6, 0.9], [0.3, 0.7], 0.5⟩	⟨[0.4,0.8],[0.5,0.8],[0.1,0.9], 0.6⟩	⟨[0.5,0.8], [0.1,0.8], [0.5,0.8],0.5⟩	⟨[0.5,0.8], [0.4,0.8], [0.4,0.9],0.6⟩
$\hbar_3$	⟨[0.4,0.8], [0.2, 0.8], [0.3, 0.5], 0.3⟩	⟨[0.3,0.7],[0.3,0.7],[0.6,0.8],0.7⟩	⟨[0.5,0.7], [0.4,0.9], [0.5,0.8],0.6⟩	⟨[0.5,0.9], [0.2,0.6], [0.2,0.8],0.3⟩
$\hbar_4$	⟨[0.4,0.7], [0.2, 0.8], [0.1, 0.9], 0.5⟩	⟨[0.3,0.8],[0.6,0.8],[0.6,0.9], 0.7⟩	⟨[0.4,0.9], [0.7,0.9], [0.5,0.8],0.5⟩	⟨[0.5,0.8], [0.4,0.8], [0.7,0.8],0.7⟩
$\hbar_5$	⟨[0.6, 0.8],[0.2, 0.7], [0.4, 0.7], 0.5⟩	⟨[0.3,0.7],[0.7,0.9],[0.2,0.8],0.5⟩	⟨[0.4,0.7], [0.5,0.8], [0.3,0.9],0.6⟩	⟨[0.7,0.8], [0.1,0.8], [0.4,0.6],0.6⟩

Table 4. Tabular representation of water treatment.

(C_3,Ғ)	E_1	E_2	E_3	E_4
$\hbar_1$	⟨[0.4,0.7], [0.1, 0.7], [0.5, 0.8], 0.6⟩	⟨[0.7,0.9],[0.3,0.6],[0.5,0.7], 0.6⟩	⟨[0.4,0.7], [0.3,0.8], [0.5,0.8],0.5⟩	⟨[0.4,0.7], [0.1,0.6], [0.6,0.9],0.7⟩
$\hbar_2$	⟨[0.6,0.9], [0.4, 0.8], [0.1,0.9],0.7⟩	⟨[0.5,0.9],[0.7,0.9],[0.2,0.5],0.3⟩	⟨[0.4,0.9], [0.4,0.7], [0.1,0.8],0.6⟩	⟨[0.6,0.9], [0.3,0.6], [0.1,0.8],0.6⟩
$\hbar_3$	⟨[0.1,0.7], [0.5, 0.8], [0.8, 0.9], 0.7⟩	⟨[0.6,0.8],[0.4,0.8],[0.1,0.9],0.6⟩	⟨[0.4,0.7], [0.3,0.8], [0.1,0.6],0.4⟩	⟨[0.2,0.8], [0.3,0.9], [0.6,0.8],0.7⟩
$\hbar_4$	⟨[0.5,0.8], [0.5, 0.9], [0.1, 0.9], 0.7⟩	⟨[0.5,0.8],[0.3,0.6],[0.4,0.7],0.4⟩	⟨[0.5,0.7], [0.1,0.6], [0.4,0.9],0.5⟩	⟨[0.3,0.7], [0.2,0.6], [0.1,0.5],0.6⟩
$\hbar_5$	⟨[0.4,0.8], [0.4, 0.6], [0.4, 0.8], 0.7⟩	⟨[0.5,0.8],[0.7,0.9],[0.2,0.6],0.7⟩	⟨[0.3,0.8], [0.1,0.6], [0.6,0.8],0.4⟩	⟨[0.4,0.7], [0.3,0.7], [0.3,0.6],0.4⟩

Table 5. Results of hamming distance and normalized hamming distance.

	Hamming Distance	Normalized Hamming
(C_1,Ғ)	0.71781	0.1435
(C_2,Ғ)	0.93571	0.18714
(C_3,Ғ)	0.8464	0.1692

5.2 Comparison and Distinctiveness

The distinctiveness of our proposed work is demonstrated in Table 6. This observation holds true for the corresponding structures as well.

Table 6. Comparing the IVNFSS with existing structures.

Structures	Membership Function	Indeterminate Function	Non-membership Function	Interval Value	Attributes	Fuzzy Value
FS [1]	Yes	No	No	No	No	No
NS [13]	Yes	Yes	Yes	No	No	No
SVNS [14]	Yes	Yes	Yes	No	No	No
IVNS [42]	Yes	Yes	Yes	Yes	No	No
IVNSS [21]	Yes	Yes	Yes	Yes	Yes	No
IVNFSS (Proposed)	Yes	Yes	Yes	Yes	Yes	Yes

6. Conclusion

Neutrosophic fuzzy sets may not be sufficient to handle complexity since every person's decision-making is unpredictable, especially when they are faced with many and split attributes under ambiguous and uncertain set tings. Due to this, the IVNFSS environments are described, and we can observe how carefully this theory handles uncertainty in a dynamically changing environment using various instances. To tackle this investigation, explore interval-valued neutrosophic fuzzy soft sets and distance measures for wastewater treatment in detail. This article determines that it has diligently built a solid theoretical framework, addressing flaws in current approaches and highlighting the requirement for IVNFSS. With the aid of a few examples, various concepts, including subsets, were presented, created a few operations, such as union, intersection, and complement. The use of IVNFSS in conjunction with distance measurements caught our attention as we looked to determine if the treated wastewater is safe to drink by accurately capturing and modeling uncertainty, ambiguity, and imprecision in data. Since IVNFSS is based on a mathematical foundation, it is easier to carry out in-depth analyses, construct algorithms, and enhance theory and practice. In the future, more distance measures can be proposed along with trigonometric similarities, and many real-life decision-making problems can be solved.

Acknowledgments
The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Data availability
The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Funding
This research was not supported by any funding agency or institute.

Conflict of interest
The authors declare that there is no conflict of interest in the research.

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Received: 02 Nov 2023, **Revised:** 28 Dec 2023,

Accepted: 16 Jan 2024, **Available online:** 25 Jan 2024.