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Choosing Optimal Supply Radius of Transformer Substations (TSs) in Iraq's Cities Using Geometric Programming with Neutrosophic Coefficients

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Abstract: Numerous uncertainties exist in various electricity power system problems due to the size, complexity, geographical distribution, and influence of unforeseen events in these systems, making it difficult for traditional mathematics tools based on crisp set theory to have an impact on and solve many power system problems. As a new branch of mathematical uncertainty techniques, the neutrosophic expert systems approach has therefore emerged with the development of electric power systems and has proven successful when correctly linked. The expert typically uses ambiguous or neutrosophic language to describe their empirical knowledge, such as "very likely," "quite likely," "if x is large, then y is very likely to occur," "x should not be less than a," etc. To design an optimal radius of power supply in the electrical transformer substation, this article presents a new method for creating primal and dual neutrosophic geometric programming problems. It also provides a numerical example to evaluate the approximate optimal economic power supply radius.

Keywords: Optimal Supply Radius; Neutrosophic Geometric Programming; Neutrosophic Coefficients; Transformer Substations.

NOMENCLATURES	
TS	Transformer Substation
GPP	Geometric Programming Problem
SPP	Signomial Programming Problem
NGPP	Neutrosophic Geometric Programming Problem
$\boldsymbol{H}$	An annual extraction coefficient is the annual running cost of the entire extracted investment.
$\boldsymbol{a_1}$	Unconnected Part of TS's investment capacity $(unit, IQ)$
$\boldsymbol{a_2}$	Investment in $(11kV)$ Line for Every Kilometre of Unconnected Wire $(unit, IQ/km)$
$\boldsymbol{b_1}$	Coefficient of the Section Associated with the TS's Capacity in Investing $(unit, IQ/kV\ A)$
$\boldsymbol{b_2}$	Coefficient of the Wire Section-Connected Part of the $(11\ kV)$ Line Investment $(unit, IQ/km \cdot mm^2)$
$\boldsymbol{E}$	Coefficient of Terrain Correction

$\boldsymbol{\sigma}$	Average Load Density $(unit, kW/km^2)$
$\boldsymbol{K_c}$	Hold Ratio $(the\ 132k\ V - TS\ takes\ 2.2 \sim 2.5)$
$\boldsymbol{P_{av}}$	Mean Line Load Every Time $(unit, kW)$
$\boldsymbol{U_N}$	The Middle Voltage Distribution Network's Rated Voltage $(11k\ V)$
$\boldsymbol{j}$	Economical Current Density Wire $(unit, A/mm^2)$
$\boldsymbol{\rho}$	Wire in Resistivity $(unit,\ \Omega/km\ \cdot\ mm^2)$
$\boldsymbol{\tau}$	Annually Wasted Mean Maximum Load Hours
$\boldsymbol{C_0}$	Price of Each Wasted Watt-Hour $(unit, IQ/\ k\ W.h\ )$
$\boldsymbol{K_{Fe}}$	Coefficient of Transformer Iron Loss$(\approx\ 0.0085kW/(kV\ A)^{3/4})$
$\boldsymbol{\tau_b}$	Equivalent Hours of Transformer Copper Loss
$\boldsymbol{K_{cu}}$	Transformer Copper Loss Coefficient $(\approx\ 0.055kW/(kV\ A)^{3/4})$
$\boldsymbol{S_{NL}}$	Rated Load of Transformer $(unit, kV\ A)$
$\boldsymbol{S_N}$	Rated Capacity of Transformer $(unit, kV\ A)$
$\boldsymbol{Q}$	Voltage Range in Electrically-Distributed Cities $(unit, km^2)$.
$\boldsymbol{r}$	Mean Radius in the Supply Area, or it is the economic power supply radius
IQ	Iraqi Dinar

1. Introduction

The building of an optimal mathematical model for the annual cost function needs a geometric programming model because its variables have negative rational powers.

It is well known that any classical geometric programming problem neglects the uncertainty part of the sophisticated practical environment. So, the neutrosophic model will be more flexible having more information. In Iraq, the services of electricity supply are very poor in almost all cities for many reasons, such as the cascade wars, for example, the Arab Gulf War in 1990, where the Iraqi community suffered from many setbacks:

1. The economy block in the nineties of the last century,
2. Replace the system of the government with a parliamentary system of government,
3. In the twenty-one century the events of the sectarian and ethnic civil wars in Iraqi society.

All previous reasons and more, lead to infrastructural destruction.

This essay comes as an attempt to shed light on some important problems involving how can rebuild the Iraqi power electricity supply substations. The necessary point that this paper focused on is how to choose an optimal radius (r) of the electric power supply in the substations of Iraqi cities. The reader should pay attention that the monetary currency of the country is the Iraqi dinar (IQ).

2. Classical Geometric Programming Problem (CGPP)

To handle a class of non-linear optimization problems that are commonly seen in engineering design, an optimization technique known as geometric programming (GP) was developed. The GP technique was inspired by the work of Zener. Zener tried a novel approach in 1961 to solve a class of unconstrained non-linear optimization problems with polynomial terms in the objective function. To answer these problems, he used the well-known inequality between arithmetic and geometric means, which states that the arithmetic mean is greater than or equal to the geometric mean. Because the arithmetic-geometric mean inequality is applied, this method is referred to as the "GP technique". Only in cases when the objective function was unconstrained and the number of polynomial terms was one more than the number of variables did Zener employ this technique. Later in 1962, Duffin expanded the application of this method to resolve issues when the objective function's number of polynomial terms is arbitrary. In 1967, Peterson enhanced the application of this technique to issues that also incorporate inequality restrictions expressed as posynomials with the aid of Zener and Duffin [1]. Additionally, Passy and Wilde (1967) extended this method to tackle issues where some of the posynomial terms have negative coefficients [2].

For instance, Duffin demonstrated that a function "duality gap" could not develop in geometric programming by condensing the posynomial functions to monomial form (1970) and converting them to linear form using a logarithmic transformation. Posynomial programming with (posy) monomial objective and constraint functions is equivalent to linear programming. Duffin and Peterson (1972) demonstrate that each of these posynomial programs GPP can be reformulated so that every constraint function is a (posy) binomial in that it includes at most two posynomial terms. An effective and extremely flexible way of solution was desired since geometric programming has become a popular optimization methodology. Several factors became crucial as the complexity of the sample geometric programs to be solved grew: Canonically, the problem's level of difficulty and inactive constraints [3] reported an algorithm that might take these factors into account. Later, in 1976, Mcnamara suggested a method for solving geometric programming problems that involved formulating an augmented problem with a degree of complexity zero. As a result, many algorithms have been proposed for solving GP, the majority of which were made before (1980); these algorithms are somewhat more efficient and reliable when applied to the convex problem and also avoid problems with derivative singularities as variables raised to fractional powers approach zero because logs of such variables will approach$-\infty$, There should be significant negative lower bounds on those variables. A significant amount of interior point (IP) algorithms for general purpose (GP) were developed in the 1990s. An effective method for solving posynomial geometric programming was developed by Rajgopal and Bricker in 2002. Condensation was a concept that was employed in the method, which was integrated into an algorithm for the more broad (Signomial) GP issue. The reformulation's constraint structure sheds light on why this algorithm is effective in avoiding every computing issue usually connected to dual-based algorithms. A method for addressing (positive, zero, or negative) variables in SPP was put out by Li and Tsai in 2005. A linear or convex relaxation of the original problem is computed using the majority of current global optimization methods for SPP. These methods might occasionally offer an impractical answer, or they might constitute the genuine optimum to get around these restrictions. Shen, Ma, and Chen (2008) proposed a robust solution algorithm for the global algorithm optimization of SPP. This algorithm adequately ensures the achievement of a robust optimal solution that is both feasible and close to the actual optimal solution and is stable under small perturbations of the constraints [4].

Huda E. Khalid [5] suggested an innovative GPP algorithm for discovering the ranging analysis by examining the impact of perturbations in the coefficients without solving the issue, as this proposed procedure had been caught on two coefficients at once. Additionally, the reference [6] investigated an original GPP employing substitution effects in a sensitivity analysis for two coefficients at once based on a new extended theorem and adjusted new constants. Huda E. Khalid [7] proposed one of the incremental procedures required to appropriately compare the results obtained from the incremental analysis methodology and the sensitivity analysis approach. Additionally, there was an effort to develop a novel computational approach that could be used to examine the sensitivity analysis of the geometric programming problem (GPP) and the signomial geometric programming problem (SPP), both of which had a difficulty level larger than zero. The studies [8, 9] cast a bad light on multi-objective geometric programming's degree of challenges.

Definition 2.1: [10]: The vector x^* that makes the constraint inequalities $g_j(x) \leq 1,\ j = 1,2 \dots\dots, p \ and \ x > 0$ into exact equalities is the optimal solution to a GPP. We refer to the constraint set in such issues as being tight or active. As a result, we can assess any inactive constraints by assessing those for which $g_j(x) > 1$ or $g_j(x) < 1$.

Definition 2.2: [11] Let R_{++}^{m} stand for the set of real m-vectors with positive component values. Let there be m real positive vectors, $x_1, \dots, x_m$. A function with the definition $f: R_{++}^{m} \to R$ is called a monomial, and is defined as $f(x) = c\prod_{j=1}^{m} x_j^{a_j}$, $c > 0$ where $a_j \in R$. A polynomial is a sum of monomials or a function of the type $(x) = \sum_{i=1}^{n} c_k \prod_{j=1}^{m} x_j^{a_{ij}}$, with $c_k > 0$ and $a_{jk} \in R$.

3. Neutrosophic Geometric programming problems (NGPP) [4]

In 2016, Florentin Smarandache and Huda E. Khalid developed unconstrained neutrosophic geometric programming, where the models were constructed as posynomials. For this subject, the following definitions are thought to be fundamental:

3.1 **Definition:** Let $h(x)$ be any linear or non-linear neutrosophic function, where $x_i \in [0,1] \cup [0, nI]$ and $x = (x_1, x_2, \dots, x_m)^{T}$ an m-dimensional fuzzy neutrosophic variable vector.

We have the inequality

$$h(x) < ₦1 \quad (1)$$

where "$< ₦$" denotes the neutrosophic version of "$\leq$" with the linguistic interpretation being "less than (the original claimed), greater than (the anti-claim of the original less than), equal (neither the original claim nor the anti-claim)".

The inequality (1) can be redefined as follows:

$$\left.\begin{array}{r} h(x) < 1 \\ \text{anti}\,(h(x)) > 1 \\ \text{neut}(\,h(x)) = 1 \end{array}\right\} \quad (2)$$

3.2 **Definition:** Let

$$\left.\begin{array}{ll} \overset{N}{(P)} \quad \overset{N}{\min} & h(x) \\ \quad x_i \in FN_s & \end{array}\right\} \quad (3)$$

The neutrosophic unconstrained posynomial geometric programming, where $x = (x_1, x_2, \dots, x_m)^{T}$ is an m-dimensional fuzzy neutrosophic variable vector, "T" represents a transpose symbol and $h(x) = \sum_{k=1}^{J} c_k \prod_{l=1}^{m} x_l^{\gamma_{kl}}$ is a neutrosophic posynomial GP function of x, $c_k \geq 0$ a constant, γ_{kl} an arbitrary real number, $h(x) < ₦\, z \to \overset{N}{\min} g(x)$; the objective function $h(x)$ can be written as a minimizing goal to consider z as an upper bound; z is an expectation value of the objective function $h(x)$,"$< ₦$" denotes the neutrosophic version of "$\leq$" with the linguistic interpretation (see Definition 3.1), and $d_o > 0$ denotes a flexible index of $h(x)$.

Note that the above program is undefined and has no solution in the case of $\gamma_{kl} < 0$ with some x_l's taking indeterminacy value, for example,

$$\overset{N}{\min} \quad h(x) = 2x_1^{-.2}x_2^{3}x_4^{1.5} + 7x_1^{3}x_2^{-.5}x_3, \quad (4)$$

where $x_i \in FN_s, i = 1,2,3,4$. This program is not defined at $x = (.2I, .3, .25, I)^{T}$, $h(x) = 2(.2I)^{-.2}(.3)^{.3}I^{1.5} + 7(.2I)^{3}(.3)^{-.5}(.25)$ is undefined at $x_1 = .2I$ with $\gamma_1 = -0.2$.

3.3 **Definition:** Let A_0 be the set of all neutrosophic non-liner functions $h(x)$ that are neutrosophically less than or equal to z, i.e.

$A_0 = \{x_i \in FN_m,\ h(x) < ₦\, z\}$.

The membership functions of $h(x)$ and $\text{anti}(h(x))$ are:

$$\mu_{A_o}(\,h(x)) = \begin{cases} 1 & 0 \leq h(x) \leq z \\ \left(e^{\frac{-1}{d_o}(g(x)-z)} + e^{\frac{-1}{d_o}(\text{anti}\,(g(x))-z)} - 1\right), & z < h(x) \leq z - d_o \ln 0.5 \end{cases} \quad (5)$$

$$\mu_{A_o}(\text{anti}(h(x))) = \begin{cases} 0 & 0 \leq h(x) \leq z \\ \left(1 - e^{\frac{-1}{d_o}(\text{anti}\,(h(x))-z)} - e^{\frac{-1}{d_o}(h(x)-z)}\right), & z - d_o \ln 0.5 \leq h(x) \leq z + d_o \end{cases} \quad (6)$$

Eq. (6) can be changed into

$$h(x) < ₦\, z, \quad x = (x_1, x_2, \dots, x_m), x_i \in FN_s \quad (7)$$

The above program can be redefined as follows:

$$\left.\begin{array}{l} h(x) < z \\ anti(h(x)) > z \\ neut(h(x)) = z \\ x = (x_1, x_2, \dots, x_m), x_i \in FN_s \end{array}\right\} \quad (8)$$

It is clear that $\mu_{A_o}(neut(h(x)))$ consists of the intersection of the following functions:

$$e^{\frac{-1}{d_o}(h(x)-z)} \quad \& \quad 1 - e^{\frac{-1}{d_o}(anti(h(x))-z)} \quad (9)$$

$$\mu_{A_o}(neut(h(x))) = \begin{cases} 1 - e^{\frac{-1}{d_o}(anti(\,h(x))-z)} & z \le h(x) \le z - d_o \ln 0.5 \\ e^{\frac{-1}{d_o}(h(x)-z)} & z - d_o \ln 0.5 < h(x) \le z + d_o \end{cases} \quad (10)$$

3.4 **Definition:** Let $\tilde{N}$ be a fuzzy neutrosophic set defined on $[0,1] \cup [0, nI], n \in [0,1]$; if there exists a fuzzy neutrosophic optimal point set A_o^* of $h(x)$ such that

$$\tilde{N}(x) = \begin{array}{c} \min\{\mu(neut\ h(x))\} \\ x = (x_1, x_2, \dots, x_m), x_i \in FN_s \end{array} \quad (11)$$

$$\tilde{N}(x) = e^{\frac{-1}{d_o}\left(\sum_{k=1}^{J} c_k \prod_{l=1}^{m} x_l^{\gamma_{kl}} - z\right)} \wedge\ 1 - e^{\frac{-1}{d_o}\left(anti\left(\sum_{k=1}^{J} c_k \prod_{l=1}^{m} x_l^{\gamma_{kl}}\right) - z\right)}, \quad (12)$$

Then $\max \tilde{N}(x)$ is said to be a neutrosophic geometric programming (the unconstrained case) concerning $\tilde{N}(x)$ of $h(x)$.

3.5 Definition: Let x^* be an optimal solution to $\tilde{N}(x)$, i.e. $\tilde{N}(x^*) = \max \tilde{N}(x)$, $x = (x_1, x_2, \dots, x_m), x_i \in FN_s$, and the fuzzy neutrosophic set $\tilde{N}$ satisfying Eq. (11) is a fuzzy neutrosophic decision in Eq. (8).

4. Designing an Optimal Radius of Power Supply in Transformers Substation Using Classical Geometric Programming Problems GPP [12]

The annual-cost way had been built with consideration to the following assumptions:

1. 132 KV-TS power supply to its consumers in a city by the direct-step-down method of 11 KV.
2. The load density is even over the whole electrified wire netting cover. Therefore, a static model is built using an annual cost as follows:

   $$F = \frac{Z}{N} + \mu \quad (13)$$

   Where Z denotes the cost of total investment, μ is the annual cost of investment operational under a certain load level, $N(8 - 10\ years)$ is an investment-recovery deadline, i.e., the total investment is returned within $(8 - 10)$ redeemable years.

   The annual cost function in a unit capacity is denoted by

   $$F_o = \frac{F}{S} = \frac{\frac{(Z_b + Z_l)}{N} + \mu_1 + \mu_2 + \mu_3}{S} \quad (14)$$

   Where,

   $$Z_b = a_1 + b_1 S \quad (IQ) \quad (15)$$

   Is the investment in the construction of 132 KV-TS [14].

   $$Z_l = L(Ma_2 + b_2 S_l) \quad (IQ) \quad (16)$$

Denotes the construction investment in the main-supply lines of the (11 KV) middle voltage distribution network, where $M = S\cos\phi / P_{av}$ is a circle line of $(11KV)$ middle voltage distribution network, $L = Er\ (km)$ is each circle-line length of it, and $S_l = \frac{S}{\sqrt{3}U_{Nj}}\ (mm^2)$ denotes the wire total selection in the main-supply lines of all $(11\ KV)$ middle voltage distribution net-work in $(132\ KV) - TS$; S is the capacity in the $TS\ (unit, KVA)$.

$$\mu_1 = H(Z_b + Z_l) \quad (17)$$

Behaves as a direct proportion function of the unchangeable part in operations cost (large repair, small repair, and depreciation charge) and in the total investment of $(132\,KV - TS)$ and $(11\,KV)$ middle voltage distribution net line.

$$\mu_2 = \Delta P_\tau C_o = 7.26 \frac{E_{\rho j \tau} C_o}{U_N cos^2\phi} \frac{1}{\sqrt{\sigma}} S^{3/2}\, 10^{-5} \quad (18)$$

Stands for the depreciation charge of $(11\,KV)$ line in a year, while by [15], the depreciation charge to transformers of

$(132\,K\,V - TS)$ is $\mu_3 = (C_0 K_{Fe} 8760 + C_0 \tau_b K_{cu}) \times \gamma S^{3/4}\, NL$ (19)

Where γ denotes the number of transformers. When the rated capacity as S_N hours for a chosen transformer is 11 kV-TS, γ is taken to denote an average number of transformers S/S_N in the$11k\,V - TS$.

Substitute (15)-(19) for (14), then

$$K_0 = \left(\frac{1}{N} + H\right)\left(\frac{a_1}{S} + b_1\right) + E\left[\left(\frac{1}{N} + H\right)\sqrt{\frac{1}{\pi\sigma K_c}}\left(\frac{a_2 \cos\emptyset}{P_{av}} + \frac{b_2}{\sqrt{3}\, U_{Nj}}\right) + 7.26\, \frac{p_j \tau C_0}{U_N \cos^2\emptyset} \frac{10^{-5}}{\sqrt{\sigma}}\right] S^{\frac{1}{2}} + (8760 C_0 K_{Fe} + C_0 \tau_b K_{cu}) \frac{S^{3/4}\, NL}{S_N} \quad (20)$$

The determination of the objective function of a static (classical) model aims at making unit capacity annual cost minimum, i.e., $in\, K_0$, with the limits to constraint $S > 0$, such that we have the following model:

$$\begin{aligned} &\min K_0 \\ &s.t. \quad S > 0 \end{aligned} \quad (21)$$

Where K_0 is illustrated in Eqs. (20), and (21) are called classical geometric programming models, concerned with the model I.

5. Geometric Programming with Neutrosophic Coefficients

In this section, the objective function K_0 will be reformulated from classical annual coast in unit capacity into minimum neutrosophic function. We expect that there are many vague or incomplete factors in load, investment process, and electricity prices, where they hold many neutrosophic phenomena.

It is well known that the transformer substations' capacity is non-negative (i.e. $S > 0$), also K_0 is an exponential polynomial function having neutrosophic coefficients with respect to S , therefore the classical model can be changed into finding an answer to a neutrosophic minimum of annual cost under the capacity $S > 0$ in constraint, such that the geometric programming with neutrosophic coefficients can be written as a forthcoming model (22).

5.1 Building a Neutrosophic GPP Model in an Iraqi's Transformer Substation Depending Upon Neutrosophic Truth Membership Function Related to the Coefficient C_2

Solve the following problem

$$\begin{aligned} &min \quad K_0 \\ &s.t. \quad S > 0 \end{aligned} \quad (22)$$

Where,

$$K_0 = 91800 S^{-1} + 0.77\sigma^{-0.5} S^{0.5} \quad (23)$$

Having the first coefficient $c_1 = 91800$ as an ordinary real number, while the second coefficient $c_2 = 0.77\sigma^{-0.5}$ is a neutrosophic number varying in a certain interval. Suppose the truth membership function μ_A, indeterminacy membership function σ_A, and falsity of membership function v_A for the coefficient c_2 are defined as follows:

Let $A = [0.008, 0.03]$ be a certain neutrosophic interval in which $c_2 \in A$,

$$\mu_A(c_2) = \begin{cases} 0 & c_2 \le 0.008 \\ (\frac{c_2 - 0.008}{0.022})^2 & 0.008 < c_2 < 0.03 \\ 1 & c_2 > 0.03 \end{cases} \quad (24)$$

$$\sigma_A(c_2) = \begin{cases} (\frac{c_2-0.008}{0.011})^2 & 0.008 < c_2 < 0.011 \\ \frac{1}{2} - (\frac{c_2-0.011}{0.03})^2 & 0.011 \le c_2 \le 0.03 \\ 0 & 0.008 \ge c_2 \ge 0.03 \end{cases} \quad (25)$$

$$v_A(c_2) = \begin{cases} 1 & c_2 \le 0.008 \\ 1 - (\frac{c_2-0.008}{0.022})^2 & 0.008 < c_2 < 0.03 \\ 0 & c_2 > 0.03 \end{cases} \quad (26)$$

It is easy to draw the above truth, indeterminacy, and falsity membership functions.

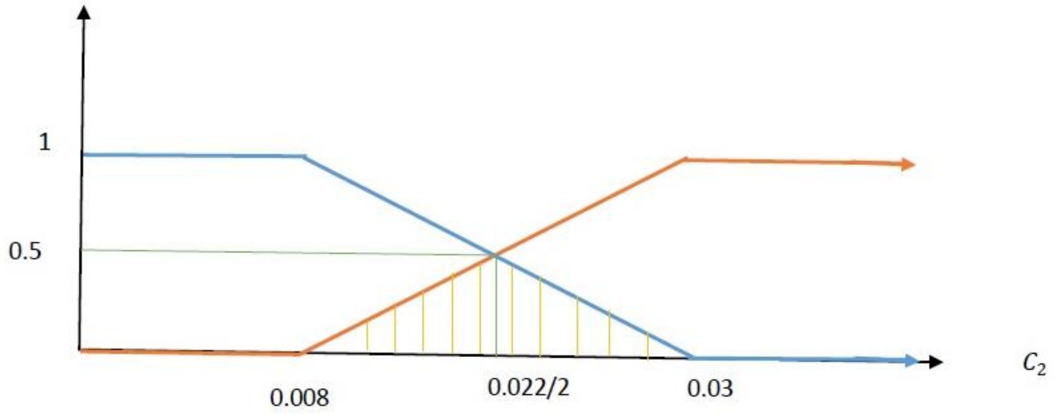

Figure 1: The blue linear function represents the region covered by $\mu_A(c_2)$, while the orange function represents the region covered by $v_A(c_2)$, and finally, the intersection region (yellow-shaded color) is the intersection region between $\mu_A(c_2)$ and $v_A(c_2)$ represents the region covered by $\sigma_A(c_2)$.

On the other hand, and depending upon [16], we have

$$(\frac{c_2-0.008}{0.022})^2 = 1-\alpha \Rightarrow c_2 = 0.008 + 0.022\sqrt{1-\alpha} \quad (27)$$

Here,α denotes the $\alpha - cut$ related to the truth membership functions $\mu_A(c_2)$.

Consequently, the program (22) is turned into

$$\min K_0 = \{91800S^{-1} + (0.008 + 0.022\sqrt{1-\alpha})S^{0.5}\} \quad (28)$$
$$s.t. \; S > 0$$

Program (28) is called neutrosophic polynomial geometric programming problems (NPGPP), we should not forget that the degree of the dual program for eq. (28) is equal to zero, so it is immediate to find the solution by means of the dual program. To solve this problem more easily way, we try to rewrite this program (i.e. eq. 28) in its dual form as follows:

$$\max \quad D = (\frac{91800}{\omega_0})^{\omega_0}(\frac{0.008+0.022\sqrt{1-\alpha}}{\omega_1})^{\omega_1}$$
$$s.t. \quad \omega_0 + \omega_1 = 1 \quad (29)$$
$$-\omega_0 + 0.5\omega_1 = 0$$
$$\omega_0, \omega_1 > 0, \;\; \alpha \in [0,1]$$

By solving the above (normality and orthogonality conditions), the values $\omega_0 = \frac{1}{3}, \omega_1 = \frac{2}{3}$ are obtained.

The well-known relationship between the primal of the GPP and its dual program is concluded by the below theorem:

Theorem [13]: If (P) is canonical and there exists a $\bar{x}$ such that $g_k(\bar{x}) < 1$ for $= 1,2,\dots,p$, then the following mathematical phrases are true:

i. The dual program (D) has a maximizing point δ^*,

ii. The maximum value $v(\delta^*)$ of problem(D) equals to the minimum value $g_\circ(x^*)$ of (P), {i.e. the primal and the dual problems are related through the fact that, for the optimal solution, $\min g_\circ(x) = \max v(\delta)$}.

iii. Each minimizing point x for (P) for the optimal solution $\min g_0(x) = \max v(\delta)$ satisfies

$$u_i(x) = \begin{cases} \delta_i^* v(\delta^*) & \text{for} \quad i = 0 \\ \dfrac{\delta_i^*}{\lambda_k(\delta^*)} & \forall\, i = 1,2, \dots k \text{ and } \lambda_k(\delta^*) \neq 0 \end{cases}$$

However, there is a good opportunity to study the various values of the dual objective function D depending on the values of α. The following formulas are:

- r is the mean radius in the supply area, or it is the economic power supply radius,

$$r = \sqrt{\frac{S_{av}}{\pi\sigma K_c}} \tag{30}$$

Suppose that the mean load density of Telafer township / Nineveh province/ Iraq country will raised to $\sigma = 5195\ kW/km^2$ by the year 2025, suppose the economic capacity of the specific transformer substation as S_{av}.

Average item N_b (which denotes the amount of 132 kV-TS that needs to be built up in Telafer district to meet the growing demand for electricity) given by the following formulas

$$N_b = \frac{\sigma Q K_c}{S} = \frac{\sigma Q K_c}{r_i^2 \pi\sigma K_c} = \frac{Q}{\pi {r_i}^2} \tag{31}$$

Keep in mind, the fact that, the area of the electrical supply on Telafer township station cover $Q = 1743.69\ km^2$.

- The numbers $n_b$in a unit area in the specific transformers substation TS,

$$n_b = \frac{N_b}{Q} = \frac{\sigma Q K_c}{S} = \frac{1}{\pi r_i^2} \tag{32}$$

- Note that $c_2 = 0.008 + 0.022\sqrt{1-\alpha}$, $\alpha \in [0,1]$, at $\alpha = 0 \Longrightarrow c_2 = 0.03$, while at $\alpha = 1 \Longrightarrow c_2 = 0.008$, as $0 < \alpha < 1 \Longrightarrow 0.008 < c_2 < 0.03$.

The following Table 1 illustrates the $\max D$ are the approximate optimal solutions and optimal values for (29),

Table 1. The range values of D.

Index i	$\boldsymbol{\alpha_i \in [0,1]}$	$\max \boldsymbol{D} = (\frac{\mathbf{91800}}{\boldsymbol{\omega_0}})^{\omega_0}(\frac{\mathbf{0.008 + 0.022}\sqrt{\mathbf{1}-\boldsymbol{\alpha_i}}}{\boldsymbol{\omega_1}})^{\omega_1}$	$\boldsymbol{r_i = \sqrt{\frac{S_{av_i}}{\pi\sigma K_c}}}$ $\boldsymbol{K_c = 2.2\ kV\ ;\ \sigma = 5915\ kW/km^2}$
1	0	$d_1 = 8.23119687$	$0.641439553\ km$
2	0.1	$d_2 = 8.02337386$	$0.649671351\ km$
3	0.3	$d_3 = 7.56001993$	$0.66923184\ km$
4	0.5	$d_4 = 7.00574202$	$0.695134289\ km$
5	0.7	$d_5 = 6.29202210$	$0.733404644\ km$
6	0.8	$d_6 = 5.82046397$	$0.762466041\ km$
7	0.9	$d_7 = 5.17541713$	$0.808481231\ km$
8	1	$d_8 = 3.41016566$	$0.995585028\ km$

Note that, the values of α's were selected depending on the author's decision. To evaluate the values of r_i. Now we need to determine the average values S_{av_i}, by the previous theorem. we have the following relationship between the terms of the primal program and its corresponding terms of dual program:

$91800\, S^{-1} = \frac{1}{3}\, d_i \quad \forall\, i = 1,2,\ldots,8$ (33)

$\left(0.008 + 0.022\sqrt{1-\alpha_i}\right)S^{0.5} = \frac{2}{3} d_i \quad \forall\, i = 1,2,\ldots,8$ (34)

Suppose that all evaluated S through Eq. (33) are symbolized by S_1, and all computed S through eq. (34) symbolized by S_2. The following Table 2 illustrates all values of S_1 and S_2, this is for each $i = 1,2,\ldots 8$:

Table 2. The Average Values of S_1 and S_2 Denoted by S_{av} .

index i	$S_1 = \frac{91800}{1/3\, d_i} = \frac{275400}{d_i}$	$S_2 = \left(\frac{\frac{2}{3} d_i}{0.008 + 0.022\sqrt{1-\alpha_i}}\right)^2$	$S_{av_i} = \frac{S_1 + S_2}{2}$
1	33458.0747	182.915482	16820.4951
2	34324.7124	185.269297	17254.9909
3	36428.4754	190.862453	18309.669
4	39310.6111	198.269037	19754.4401
5	43769.7128	209.212124	21989.4625
6	47315.8156	217.52199	23766.6688
7	53213.1021	230.679653	26721.8909
8	80758.54	284.180472	40521.3603

Substituting all amounts of S_{av_i}, in the Table 1 to enumerate the values of r_i. Now, the last duty is to determine the optimal value of S_{av_i} that gives the optimal solution for problem (28), tracking the concluded values of the below Table 3:

Table 3. The approximate values of the primal program.

index i	$S_{av_i} = \frac{S_1 + S_2}{2}$	$\min K_0 = \left\{91800(S_{avi})^{-1} + \left(0.008 + 0.022\sqrt{1-\alpha}\right)(S_{avi})^{0.5}\right\}$
1	16820.4951	9.34844324
2	17254.9909	9.11264828
3	18309.669	8.58689587
4	19754.4401	7.95791296
5	21989.4625	7.14789479
6	23766.6688	6.61264497
7	26721.8909	5.88038304
8	40521.3603	3.87586527

From Tables 2 and 3, and in terms of Eqs. (33) and (34), the approximate optimal value of $K_0 \cong 3.8758627$ which holds at the optimal economic capacity $S_{av8} = 40521.3603$ of 132 kV- TS. Moreover, by the results of Table 1, we can see that the value of the approximate optimal economic power supply radius is $r_8 = 0.995585028\ km$, as well as the optimal value of $N_b \cong 559.96332$ occurs at r_8 (i.e., about (560) $132\ kV - TS$ needs to build up in Telafer township).

5.2 *Building a Neutrosophic Model Depending Upon Either Falsity or Indeterminacy Membership Functions Related to the Coefficient* C_2

The fact that many researchers in the field of neutrosophic optimization may ignore or may have been unaware of it, is that the indeterminacy membership function exactly represents the intersection region of the truth membership function and the falsity membership function as we sighted it by the representing diagram (1). Hence, for this manuscript, we took the vagueness for the second coefficientc_2, so the following mathematical intersection is completely true:

$$\sigma_A(c_2) = \mu_A(c_2) \cap v_A(c_2) \quad (35)$$

For solving the program (22) with respect to its falsity membership function and depending upon the thoughts presented in [ ], we can suppose:

$$1 - \left(\frac{c_2 - 0.008}{0.022}\right)^2 = \beta \Rightarrow \left(\frac{c_2 - 0.008}{0.022}\right)^2 = 1 - \beta = \alpha \quad (36)$$

Here, β is the β- cut related to the falsity membership function $v_A(c_2)$, eq. (29) is exactly given the same solution as Eq. (27), which means Eq. (36) is somehow considered a dual form to Eq. (27). Also, there is another unfathomable program that gives the solution of neutrosophic polynomial geometric programming (22) by using the indeterminacy membership function $\sigma_A(c_2)$:

$$\frac{1}{2} - \left(\frac{c_2 - 0.008}{0.022}\right)^2 = \gamma \quad (37)$$

Where $\gamma \in [0,1]$ is the γ- cut corresponding to $\sigma_A(.)$, Eq. (30) implies that :$\left(\frac{c_2 - 0.008}{0.022}\right)^2 = \frac{1}{2} - \gamma \Rightarrow c_2 - 0.011 = 0.0009\sqrt{0.5 - \gamma} \Rightarrow$

$c_2 = 0.011 + 0.0009\sqrt{0.5 - \gamma}$.

Consequently, the program (15) can be reformulated as:

$$\min K_0 = \{91800 S^{-1} + (0.011 + 0.0009\sqrt{0.5 - \gamma})S^{0.5}\}$$
$$s.t.\ S > 0 \quad (38)$$

Program (38) is defined as a neutrosophic posynomial geometric programming problem, and the main difference between program (28) and program (38): is that program (28) is NPGP related to the truth-membership function of the coefficientc_2, while the program (38) is NPGP related to the indeterminacy membership function of the same coefficientc_2. Also, the same previous analysis tables (1, 2, 3) that were for $\alpha, \max D,\ r_i$ can be re-written and re-analysis for $\gamma, \max D, r_i$.

6. Conclusions

This manuscript discussed an innovative trying to analyze the annual cost of investment in the power supply systems with respect to the validity of some uncertainty by assuming the neutrosophic coefficient c_2, to be able to calculate the mean of the economic capacity and economic supply radius of 132 kV- transformer substation in Telafer township, all these issues held by building mathematical non-linear programming named Neutrosophic Geometric Programming Problems (NGPP) this was for the first time, also there more analyzing techniques can be made for the same geometric programming problems with the neutrosophic point of view, either by a truth membership function, or by a falsity membership function, or by an indeterminacy membership function, to their corresponding α, β, γ- cuts.

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Author Contributions
All authors contributed equally to this research.

Data availability
The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

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Conflict of interest
The authors declare that there is no conflict of interest in the research.

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