Neutrosophic Systems with Applications, Vol. 17, 2024
https://doi.org/10.61356/j.nswa.2024.17243

MAGDM Model Using Single-Valued Neutrosophic Credibility Matrix Energy and Its Decision-Making Application

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Abstract: This paper aims to develop a MAGDM model using single-valued neutrosophic credibility matrix (SVNCM) energy in a SVNCM scenario. To do it, first, SVNCM energy and its score function are presented as a conceptual extension of existing single-valued neutrosophic matrix (SVNM) energy. Then, a MAGDM model is developed in terms of SVNCM energy and its score function in a SVNCM scenario and also its decision algorithm is provided to solve MAGDM problems with SVNCMs. Finally, the developed MAGDM model is applied in the school site selection problem as an actual example, then the comparative investigation of the decision results in the SVNM and SVNCM scenarios indicates the superiority of the developed model over existing MAGDM model.

Keywords: Single-Valued Neutrosophic Credibility Matrix; Single-Valued Neutrosophic Credibility Matrix Energy; Score Function; Group Decision Making.

1. Introduction

Matrix energy (ME) is one of important mathematical tools in the representation and processing of collective data, it is usually used in group decision making (GDM) applications. Bravo et al. [1] introduced ME as a generalization of graph energy and provided the upper and lower bounds of ME. Donbosco et al. [2] introduced rough neutrosophic ME as a generalization of ME and established its MAGDM method for handling multiple attribute group decision making (MAGDM) problems with rough neutrosophic matrix information, and then applied it to the optimal choice of building sites. After that, Li and Ye [3] proposed intuitionistic fuzzy matrix (IFM) energy and its MAGDM model for the best selection of hospital sites in a complete IFM scenario. Yong et al. [4] further presented the linguistic neutrosophic ME and its MAGDM model to solve the MAGDM problems in the scenario of full linguistic neutrosophic matrices. Jeni Seles Martina and Deepa [5] gave the concepts of multi-valued neutrosophic ME and neutrosophic hesitant ME and used them for MAGDM problems. However, the aforementioned neutrosophic ME lacks the credibility measures of true, false, and uncertain membership values in inconsistent and uncertain scenarios so that it is difficult to guarantee its decision credibility level in uncertain and ambiguous MAGDM environments.

In general, neutrosophic sets (NSs) [6] are not only the extended form of fuzzy sets (FSs) [7] and intuitionistic FSs [8], but also independently depict inconsistent, uncertain, and incomplete information though the true, false, and uncertain membership values, which FSs and intuitionistic FSs cannot do. Although existing fuzzy, intuitionistic fuzzy, and neutrosophic decision making methods and applications [9-20] have contained a lot of studies in existing literature, but they do not consider the credibility measures of various evaluation values in uncertain and ambiguous setting. To guarantee the credibility degrees of fuzzy values in uncertain and ambiguous environments, Ye et al. [21] first proposed fuzzy credibility values and their aggregation operators to perform the multiple attribute decision making (MADM) application in the selection of slope design schemes. Then, Ye et al. [22] further introduced intuitionistic fuzzy credibility sets and their similarity measures and applied them to the performance assessment of industrial robots. Ye et al. [23] also proposed single-valued neutrosophic credibility sets/values (SVNCSs/SVNCVs) to ensure the credibility degrees of true, false and uncertain membership values, and then developed their trigonometric aggregation operators and their MADM application in the selection of slope design schemes, but the MADM model [23] cannot tackle MAGDM problems in the scenario of full single-valued neutrosophic credibility matrices (SVNCMs). In this case, the existing MADM model [23] implies its obvious insufficiency and research gap in full SVNCM setting. Therefore, it is necessary to develop a MAGDM model using the SVNCM energy and score function in a SVNCM scenario to fill the research gap.

In general, this study mainly contains the following original contributions:

- SVNCM energy is defined as a generalization of neutrosophic ME.
- A score function for the SVNCM energy is presented to rank the SVNCM energy.
- A MAGDM model using the SVNCM energy and score function is developed to solve MAGDM problems in the full SVNCM scenario.
- The developed MAGDM model is applied in the actual example on the selection of primary school sites in Shaoxing, China.

The rest of the paper includes the following content. Section 2 introduces some concepts of SVNCSs, SVNCVs, and single-valued neutrosophic matrix (SVNM) energy as the preliminaries of this study. Section 3 proposes SVNCM energy and the score function and ranking rules of SVNCM energy. In Section 4, we develop a MAGDM model based on the SVNCM energy and score function. A MAGDM example on the selection of primary school sites and a comparative investigation are provided in Section 5. Section 6 remarks conclusions and future work.

2. Preliminaries

2.1 Some Concepts of SVNCSs and SVNCVs

Wang et al. [8] introduced the SVNS $N_S = \{<y, V_T(y), V_U(y), V_F(y)> \mid y \in Y\}$ in a universe set Y, where $V_T(y), V_U(y), V_F(y) \in [0, 1]$ for $y \in Y$ are the true, uncertain, and false membership values. Then, each element $<y, V_T(y), V_U(y), V_F(y)>$ in N_S can be simply denoted by the single-valued neutrosophic value (SVNV) $n_S = <V_T, V_U, V_F>$.

To measure the credibility level of SVNV, Ye et al. [23] proposed a SVNCS in Y, which is represented by

$$N_C = \left\{\left\langle y, \left(V_T(y), C_T(y)\right), \left(V_U(y), C_U(y)\right), \left(V_F(y), C_F(y)\right)\right\rangle \mid y \in Y\right\}, \tag{1}$$

where $(V_T(y), C_T(y))$, $(V_U(y), C_U(y))$ and $(V_F(y), C_F(y))$ are the true, false and uncertain fuzzy credibility values, then their true, false and uncertain membership values and their corresponding credibility values are $V_T(y)$, $V_U(y)$, $V_F(y) \in [0, 1]$ and $C_T(y)$, $C_U(y)$, $C_F(y) \in [0, 1]$, respectively, such that $0 \le V_T(y) + V_U(y) + V_F(y) \le 3$ and $0 \le C_T(y) + C_U(y) + C_F(y) \le 3$ for $y \in Y$. For ease of expression, any element $<y, (V_T(y), C_T(y)), (V_U(y), C_U(y)), (V_F(y), C_F(y))>$ in N_C can be expressed as a simplified form of the SVNCV $n_c = <(V_T, C_T), (V_U, C_U), (V_F, C_F)>$.

It is worth noting that when one does not consider the credibility values in the SVNCV n_c, n_C becomes SVNV. Therefore, the credibility values contained in the SVNCV n_C can guarantee the credibility degree of SVNV.

For any two SVNCVs $n_{C1} = <(V_{T1}, C_{T1}), (V_{U1}, C_{U1}), (V_{F1}, C_{F1})>$ and $n_{C2} = <(V_{T2}, C_{T2}), (V_{U2}, C_{U2}), (V_{F2}, C_{F2})>$, their operation laws are presented as follows:

(1) $n_{C1} \subseteq n_{C2} \Leftrightarrow V_{T1} \le V_{T2}, C_{T1} \le C_{T2}, V_{U1} \ge V_{U2}, C_{U1} \ge C_{U2}, V_{F1} \ge V_{F2}, C_{F1} \ge C_{F2}$;

(2) $n_{C1} = n_{C2} \Leftrightarrow n_{C1} \subseteq n_{C2}, n_{C2} \subseteq n_{C1}$;

(3) $n_{C1} \cup n_{C2} = \langle (V_{T1} \vee V_{T2}, C_{T1} \vee C_{T2}), (V_{U1} \wedge V_{U2}, C_{U1} \wedge C_{U2}), (V_{F1} \wedge V_{F2}, C_{F1} \wedge C_{F2}) \rangle$;

(4) $n_{C1} \cap n_{C2} = \langle (V_{T1} \wedge V_{T2}, C_{T1} \wedge C_{T2}), (V_{U1} \vee V_{U2}, C_{U1} \vee C_{U2}), (V_{F1} \vee V_{F2}, C_{F1} \vee C_{F2}) \rangle$;

(5) $(n_{C1})^c = \langle (V_{F1}, C_{F1}), (1 - V_{U1}, 1 - C_{U1}), (V_{T1}, C_{T1}) \rangle$ (Complement of n_{C1});

(6) $n_{C1} \oplus n_{C2} = \left\langle \begin{matrix} (V_{T1} + V_{T2} - V_{T1}V_{T2}, C_{T1} + C_{T2} - C_{T1}C_{T2}), \\ (V_{U1}V_{U2}, C_{U1}C_{I2}), (V_{F1}V_{F2}, C_{F1}C_{F2}) \end{matrix} \right\rangle$;

(7) $n_{C1} \otimes n_{C2} = \left\langle \begin{matrix} (V_{T1}V_{T2}, C_{T1}C_{T2}), (V_{U1} + V_{U2} - V_{U1}V_{U2}, C_{U1}C_{U2} - C_{U1}C_{U2}), \\ (V_{F1} + V_{F2} - V_{F1}V_{F2}, C_{F1} + C_{F2} - C_{F1}C_{F2}) \end{matrix} \right\rangle$;

(8) $\zeta n_{C1} = \left\langle \begin{matrix} \left(1 - (1 - V_{T1})^{\zeta}, 1 - (1 - C_{T1})^{\zeta}\right), \\ (V_{U1}^{\zeta}, C_{U1}^{\zeta}), (V_{F1}^{\zeta}, C_{F1}^{\zeta}) \end{matrix} \right\rangle, \zeta > 0$;

(9) $n_{C1}^{\zeta} = \left\langle \begin{matrix} (V_{T1}^{\zeta}, C_{T1}^{\zeta}), \left(1 - (1 - V_{U1})^{\zeta}, 1 - (1 - C_{U1})^{\zeta}\right), \\ \left(1 - (1 - V_{F1})^{\zeta}, 1 - (1 - C_{F1})^{\zeta}\right) \end{matrix} \right\rangle, \zeta > 0$.

2.1 Matrix Energy

Set $M(d_{jl})$ for $d_{jl} \in \Re$ (all real numbers) (j, l = 1, 2, …, b) as a $b \times b$ matrix, which is represented as

$$M(d_{jl}) = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1b} \\ d_{21} & d_{22} & \cdots & d_{2b} \\ \vdots & \vdots & \vdots & \vdots \\ d_{b1} & d_{b2} & \cdots & d_{bb} \end{bmatrix}. \quad (2)$$

Then, ME of $M(d_{jl})$ is introduced below [1]:

$$E(M(d_{jl})) = \sum_{j=1}^{b} \left| \delta_j - \frac{1}{b} \sum_{j=1}^{b} \delta_j \right|, \quad (3)$$

where δ_j (j = 1, 2, …, b) are the eigenvalues of $M(d_{jl})$.

(4)

Set the SVNM $M(n_{Sjl})$ (j, l = 1, 2, …, b) as a $b \times b$ matrix [5]:

$$M(n_{Sjl}) = \begin{bmatrix} n_{S11} & n_{S12} & \cdots & n_{S1b} \\ n_{S21} & n_{S22} & \cdots & n_{S2b} \\ \vdots & \vdots & \vdots & \vdots \\ n_{Sb1} & n_{Sb2} & \cdots & n_{Sbb} \end{bmatrix}, \quad (5)$$

where n_{Sjl} is the SVNV $n_{Sjl} = \langle V_{Tjl}, V_{Ujl}, V_{Fjl} \rangle$ (j, l = 1, 2, …, b) that consists of the true, uncertain, and false membership values V_{Tjl}, V_{Ujl}, $V_{Fjl} \in [0, 1]$. Then, the SVNM $M(n_{Sjl})$ can be divided into the true matrix $M(V_{Tjl})$, the uncertain matrix $M(V_{Ujl})$, and the false matrix $M(V_{Fjl})$, which is also represented as the following SVNM form:

$$M(n_{Sjl}) = \langle M(V_{Tjl}), M(V_{Ujl}), M(V_{Fjl}) \rangle = \left\langle \begin{bmatrix} V_{T11} & V_{T12} & \cdots & V_{T1b} \\ V_{T21} & V_{T22} & \cdots & V_{T2b} \\ \vdots & \vdots & \vdots & \vdots \\ V_{Tb1} & V_{Tb2} & \cdots & V_{Tbb} \end{bmatrix}, \begin{bmatrix} V_{U11} & V_{U12} & \cdots & V_{U1b} \\ V_{U21} & V_{U22} & \cdots & V_{U2b} \\ \vdots & \vdots & \vdots & \vdots \\ V_{Ub1} & V_{Ub2} & \cdots & V_{Ubb} \end{bmatrix}, \begin{bmatrix} V_{F11} & V_{F12} & \cdots & V_{F1b} \\ V_{F21} & V_{F22} & \cdots & V_{F2b} \\ \vdots & \vdots & \vdots & \vdots \\ V_{Fb1} & V_{Tb2} & \cdots & V_{Fbb} \end{bmatrix} \right\rangle. \quad (6)$$

In terms of the concepts of true, uncertain and false ME, the energy of the SVNM $M(n_{Sjk})$ is introduced below [5]:

$$E(M(n_{Sjl})) = \langle E[M(V_{Tjl})], E[M(V_{Ujl})], E[M(V_{Fjl})] \rangle = \left\langle \sum_{j=1}^{b} |\mu_{Tj} - \mu_{MT}|, \sum_{j=1}^{b} |\mu_{Uj} - \mu_{MU}|, \sum_{j=1}^{b} |\mu_{Fj} - \mu_{MF}| \right\rangle, \quad (7)$$

where μ_{Tj}, μ_{Uj}, and μ_{Fj} ($j = 1, 2, ..., b$) are the eigenvalues corresponding to the three matrices $M(V_{Tjl})$, $M(V_{Ujl})$, and $M(V_{Fjl})$ and μ_{MT}, μ_{MU}, and μ_{MF} are the average values corresponding to the eigenvalues μ_{Tj}, μ_{Uj}, and μ_{Fj} ($j = 1, 2, ..., b$). Then, there are the following equations [5]:

(1) $\sum_{j=1}^{b} (\mu_{Tj} - \mu_{MT}) = \sum_{j=1}^{b} (V_{Tjj} - \mu_{MT}) = 0$;

(2) $\sum_{j=1}^{b} (\mu_{Uj} - \mu_{MU}) = \sum_{j=1}^{b} (V_{Ujj} - \mu_{MU}) = 0$;

(3) $\sum_{j=1}^{b} (\mu_{Fj} - \mu_{MF}) = \sum_{j=1}^{b} (V_{Fjj} - \mu_{MF}) = 0$;

(4) $\sum_{j=1}^{b} (\mu_{Tj} - \mu_{MT})^2 = \sum_{j=1}^{b} V_{Tjj}^2 + 2 \sum_{1 \le j < l \le b} V_{Tjl} V_{Tlj} - b\mu_{MT}^2$;

(5) $\sum_{j=1}^{b} (\mu_{Uj} - \mu_{MU})^2 = \sum_{j=1}^{b} V_{Ujl}^2 + 2 \sum_{1 \le j < l \le b} V_{Ujl} V_{Ulj} - b\mu_{MU}^2$;

(6) $\sum_{j=1}^{b} (\mu_{Fj} - \mu_{MF})^2 = \sum_{j=1}^{b} V_{Fjj}^2 + 2 \sum_{1 \le j < l \le b} V_{Fjl} V_{Flj} - b\mu_{MF}^2$.

The lower and upper bounds of the true, uncertain, and false MEs and the true, uncertain, and false credibility MEs are implied below [5]:

(1) $$\sqrt{\left(\sum_{j=1}^{b} |\mu_{Tj} - \mu_{MT}|\right)^2 - 2 \sum_{1 \le j < l \le b} |\mu_{Tj} - \mu_{MT}| |\mu_{Tl} - \mu_{MT}| + b(b-1) |M(V_{Tjl}) - \mu_{MT}|^{2/b}} \le E[M(V_{Tjl})] \le \sqrt{b\left(\left(\sum_{j=1}^{b} |\mu_{Tj} - \mu_{MT}|\right)^2 - 2 \sum_{1 \le j < l \le b} |\mu_{Tj} - \mu_{MT}| |\mu_{Tl} - \mu_{MT}|\right)};$$

(2) $$\sqrt{\left(\sum_{j=1}^{b}\left|\mu_{Uj}-\mu_{MU}\right|\right)^2-2\sum_{1\le j<l\le b}\left|\mu_{Uj}-\mu_{MU}\right|\left|\mu_{Ul}-\mu_{MU}\right|+b(b-1)\left|M(V_{Ujl})-\mu_{MU}\right|^{2/b}} \le E[M(V_{Ujl})] \le \sqrt{b\left(\left(\sum_{j=1}^{b}\left|\mu_{Uj}-\mu_{MU}\right|\right)^2-2\sum_{1\le j<l\le b}\left|\mu_{Uj}-\mu_{MU}\right|\left|\mu_{Ul}-\mu_{MU}\right|\right)};$$

(3) $$\sqrt{\left(\sum_{j=1}^{b}\left|\mu_{Fj}-\mu_{MF}\right|\right)^2-2\sum_{1\le j<l\le b}\left|\mu_{Fj}-\mu_{MF}\right|\left|\mu_{Fl}-\mu_{MF}\right|+b(b-1)\left|M(V_{Fjl})-\mu_{MF}\right|^{2/b}} \le E[M(V_{Fjl})] \le \sqrt{b\left(\left(\sum_{j=1}^{b}\left|\mu_{Fj}-\mu_{MF}\right|\right)^2-2\sum_{1\le j<l\le b}\left|\mu_{Fj}-\mu_{MF}\right|\left|\mu_{Fl}-\mu_{MF}\right|\right)}.$$

To compare SVNM energy magnitudes, the ranking values are given by a SVNME score function [5]:

$$H\left\{E\left[M\left(n_{Sjk}\right)\right]\right\}=2E\left[M\left(V_{Tjk}\right)\right]+E\left[M\left(V_{Ujk}\right)\right]-E\left[M\left(V_{Fjk}\right)\right]. \quad (8)$$

In view of the score values of Eq. (8), the ranking rules between $E[M(n_{S2l})]$ and $E[M(n_{S1l})]$ are presented below:

(a) If $H\{E[M(n_{S1k})]\} > H\{E[M(n_{S2k})]\}$, then $E[M(n_{S1k})] > E[M(n_{S2k})]$;

(b) If $H\{E[M(n_{Sj1})]\} < H\{E[M(n_{Sj2})]\}$, then $E[M(n_{Sj1})] < E[M(n_{Sj2})]$;

(c) If $H\{E[M(n_{Sj1})]\} = H\{E[M(n_{Sj2})]\}$, then $E[M(n_{Sj1})] = E[M(n_{Sj2})]$.

3. SVNCM Energy

This section presents the concepts of SVNCM and SVNCM energy based on the energy of the true, false, and uncertain fuzzy credibility matrices in the setting of SVNCMs.

Definition 1. Set the SVNCM $M(n_{Cjl})$ (j, l = 1, 2, …, b) as a $b \times b$ matrix:

$$M(n_{Cjl})=\begin{bmatrix} n_{C11} & n_{C12} & \cdots & n_{C1b} \\ n_{C21} & n_{C22} & \cdots & n_{C2b} \\ \vdots & \vdots & \vdots & \vdots \\ n_{Cb1} & n_{Cb2} & \cdots & n_{Cbb} \end{bmatrix}, \quad (9)$$

where n_{Cjl} is the SVNCV n_{Cjl} = <(V_{Tjl}, C_{Tjl}), (V_{Ujl}, C_{Ujl}), (V_{Fjl}, C_{Fjl})> (j, l = 1, 2, …, b) that consists of the true, uncertain, and false membership values V_{Tjl}, V_{Ujl}, V_{Fjl} ∈ [0, 1] and the true, uncertain, and false credibility values C_{Tjl}, C_{Ujl}, C_{Fjl} ∈ [0, 1]. Then, the SVNCM $M(n_{Sjl})$ can be divided into the true matrix $M(V_{Tjl})$, the uncertain matrix $M(V_{Ujl})$, and the false matrix $M(V_{Fjl})$ and the true credibility matrix $M(C_{Tjl})$, the uncertain credibility matrix $M(C_{Ujl})$, and the false credibility matrix $M(C_{Fjl})$, which is also represented as the following SVNCM form:

$$M(n_{Cjl})=\langle (M(V_{Tjl}),M(C_{Tjl})),(M(V_{Ujl}),M(C_{Ujl})),(M(V_{Fjl}),M(C_{Fjl}))\rangle$$
$$=\left\langle \begin{array}{l} \left(\begin{bmatrix} V_{T11} & V_{T12} & \cdots & V_{T1b} \\ V_{T21} & V_{T22} & \cdots & V_{T2b} \\ \vdots & \vdots & \vdots & \vdots \\ V_{Tb1} & V_{Tb2} & \cdots & V_{Tbb} \end{bmatrix}, \begin{bmatrix} C_{T11} & C_{T12} & \cdots & C_{T1b} \\ C_{T21} & C_{T22} & \cdots & C_{T2b} \\ \vdots & \vdots & \vdots & \vdots \\ C_{Tb1} & C_{Tb2} & \cdots & C_{Tbb} \end{bmatrix}\right), \\ \left(\begin{bmatrix} V_{U11} & V_{U12} & \cdots & V_{U1b} \\ V_{U21} & V_{U22} & \cdots & V_{U2b} \\ \vdots & \vdots & \vdots & \vdots \\ V_{Ub1} & V_{Ub2} & \cdots & V_{Ubb} \end{bmatrix}, \begin{bmatrix} C_{U11} & C_{U12} & \cdots & C_{U1b} \\ C_{U21} & C_{U22} & \cdots & C_{U2b} \\ \vdots & \vdots & \vdots & \vdots \\ C_{Ub1} & C_{Ub2} & \cdots & C_{Ubb} \end{bmatrix}\right), \\ \left(\begin{bmatrix} V_{F11} & V_{F12} & \cdots & V_{F1b} \\ V_{F21} & V_{F22} & \cdots & V_{F2b} \\ \vdots & \vdots & \vdots & \vdots \\ V_{Fb1} & V_{Tb2} & \cdots & V_{Fbb} \end{bmatrix}, \begin{bmatrix} C_{F11} & C_{F12} & \cdots & C_{F1b} \\ C_{F21} & C_{F22} & \cdots & C_{F2b} \\ \vdots & \vdots & \vdots & \vdots \\ C_{Fb1} & C_{Tb2} & \cdots & C_{Fbb} \end{bmatrix}\right) \end{array} \right\rangle. \quad (10)$$

Definition 2. Let the SVNCM $M(n_{Cjl})$ (j, l = 1, 2, ..., b) be a $b\times b$ matrix, which can be expressed as $M(n_{Cjl}) = \langle(M(V_{Tjl}), M(C_{Tjl})), (M(V_{Ujl}), M(C_{Ujl})), (M(V_{Fjl}), M(C_{Fjl}))\rangle$, including the true, uncertain and false matrices $M(V_{Tjl})$, $M(V_{Ujl})$ and $M(V_{Fjl})$ and the true, uncertain and false credibility matrices $M(C_{Tjl})$, $M(C_{Ujl})$ and $M(C_{Fjl})$. Then ME of $M(n_{Cjl})$ can be represented below:

$$E\left[M\left(n_{Cjk}\right)\right]=\left\langle \begin{array}{l} \left(E\left[M\left(V_{Tjk}\right)\right],E\left[M\left(C_{Tjk}\right)\right]\right), \\ \left(E\left[M\left(V_{Ujk}\right)\right],E\left[M\left(C_{Ujk}\right)\right]\right), \\ \left(E\left[M\left(V_{Fjk}\right)\right],E\left[M\left(C_{Fjk}\right)\right]\right) \end{array}\right\rangle$$
$$=\left\langle \begin{array}{l} \left(\sum_{j=1}^{b}\left|\mu_{Tj}-\frac{1}{b}\sum_{j=1}^{b}\mu_{Tj}\right|,\sum_{j=1}^{b}\left|\rho_{Tj}-\frac{1}{b}\sum_{j=1}^{b}\rho_{Tj}\right|\right), \\ \left(\sum_{j=1}^{b}\left|\mu_{Uj}-\frac{1}{b}\sum_{j=1}^{b}\mu_{Uj}\right|,\sum_{j=1}^{b}\left|\rho_{Uj}-\frac{1}{b}\sum_{i=1}^{b}\rho_{Uj}\right|\right), \\ \left(\sum_{j=1}^{b}\left|\mu_{Fj}-\frac{1}{b}\sum_{j=1}^{b}\mu_{Fj}\right|,\sum_{j=1}^{b}\left|\rho_{Fj}-\frac{1}{b}\sum_{i=1}^{b}\rho_{Fj}\right|\right) \end{array}\right\rangle=\left\langle \begin{array}{l} \left(\sum_{j=1}^{b}\left|\mu_{Tj}-\mu_{MT}\right|,\sum_{j=1}^{b}\left|\rho_{Tj}-\rho_{MT}\right|\right), \\ \left(\sum_{j=1}^{b}\left|\mu_{Uj}-\mu_{MU}\right|,\sum_{j=1}^{b}\left|\rho_{Uj}-\rho_{MU}\right|\right), \\ \left(\sum_{j=1}^{b}\left|\mu_{Fj}-\mu_{MF}\right|,\sum_{j=1}^{b}\left|\rho_{Fj}-\rho_{MF}\right|\right) \end{array}\right\rangle, \quad (11)$$

where μ_{Tj}, μ_{Uj}, and μ_{Fj} (j = 1, 2, ..., b) are the eigenvalues corresponding to the three matrices $M(V_{Tjl})$, $M(V_{Ujl})$, $M(V_{Fjl})$; ρ_{Tj}, ρ_{Uj} and ρ_{Fj} (j = 1, 2, ..., b) are the eigenvalues corresponding to the three credibility matrices $M(C_{Tjl})$, $M(C_{Ujl})$, $M(C_{Fjl})$; μ_{MT}, μ_{MU}, and μ_{MF} are the average values corresponding to the eigenvalues μ_{Tj}, μ_{Uj}, and μ_{Fj} (j = 1, 2, ..., b) and ρ_{MT}, ρ_{MU} and ρ_{MF} are the average values corresponding to the eigenvalues ρ_{Tj}, ρ_{Uj} and ρ_{Fj} (j = 1, 2, ..., b).

Especially when one does not consider the credibility values in the SVNCM $M(n_{Cjl})$, $E[M(n_{Cjl})]$ is reduced to the SVNM energy of Eq. (3).

In terms of similar properties corresponding to SVNM [5], the SVNCM $M(n_{Cjl})$ (j, l = 1, 2, ..., b) also contains the following equations:

(1) $\sum_{j=1}^{b}(\mu_{Tj}-\mu_{MT})=\sum_{j=1}^{b}(V_{Tjj}-\mu_{MT})=0$;

(2) $\sum_{j=1}^{b}(\mu_{Uj}-\mu_{MU})=\sum_{j=1}^{b}(V_{Ujj}-\mu_{MU})=0$;

(3) $\sum_{j=1}^{b}(\mu_{Fj}-\mu_{MF})=\sum_{j=1}^{b}(V_{Fjj}-\mu_{MF})=0$;

(4) $\sum_{j=1}^{b}(\mu_{Tj}-\mu_{MT})^2=\sum_{j=1}^{b}V_{Tjj}^2+2\sum_{1\le j<l\le b}V_{Tjl}V_{Tlj}-b\mu_{MT}^2$;

(5) $\sum_{j=1}^{b}(\mu_{Uj}-\mu_{MU})^2=\sum_{j=1}^{b}V_{Ujl}^2+2\sum_{1\le j<l\le b}V_{Ujl}V_{Ulj}-b\mu_{MU}^2$;

(6) $\sum_{j=1}^{b}(\mu_{Fj}-\mu_{MF})^2=\sum_{j=1}^{b}V_{Fjj}^2+2\sum_{1\le j<l\le b}V_{Fjl}V_{Flj}-b\mu_{MF}^2$;

(7) $\sum_{j=1}^{b}(\rho_{Tj}-\rho_{MT})=\sum_{j=1}^{b}(C_{Tjj}-\rho_{MT})=0$;

(8) $\sum_{j=1}^{b}(\rho_{Uj}-\rho_{MU})=\sum_{j=1}^{b}(C_{Ujj}-\rho_{MU})=0$;

(9) $\sum_{j=1}^{b}(\rho_{Fj}-\rho_{MF})=\sum_{j=1}^{b}(C_{Fjj}-\rho_{MF})=0$;

(10) $\sum_{j=1}^{b}(\rho_{Tj}-\rho_{MT})^2=\sum_{j=1}^{b}C_{Tjj}^2+2\sum_{1\le j<l\le b}C_{Tjl}C_{Tlj}-b\rho_{MT}^2$;

(11) $\sum_{\mu=1}^{e}(\rho_{Uj}-\rho_{MU})^2=\sum_{j=1}^{b}C_{Ujl}^2+2\sum_{1\le j<l\le b}C_{Ujl}C_{Ulj}-b\rho_{MU}^2$;

(12) $\sum_{j=1}^{b}(\rho_{Fj}-\rho_{MF})^2=\sum_{j=1}^{b}C_{Fjj}^2+2\sum_{1\le j<l\le b}C_{Fjl}C_{Flj}-b\rho_{MF}^2$.

Furthermore, the lower and upper bounds of the true, uncertain, and false MEs are introduced below:

(1) $$\sqrt{\left(\sum_{j=1}^{b}\left|\mu_{Tj}-\mu_{MT}\right|\right)^2-2\sum_{1\le j<l\le b}\left|\mu_{Tj}-\mu_{MT}\right|\left|\mu_{Tl}-\mu_{MT}\right|+b(b-1)\left|M(V_{Tjl})-\mu_{MT}\right|^{2/b}}\le E[M(V_{Tjl})] \le\sqrt{b\left(\left(\sum_{j=1}^{b}\left|\mu_{Tj}-\mu_{MT}\right|\right)^2-2\sum_{1\le j<l\le b}\left|\mu_{Tj}-\mu_{MT}\right|\left|\mu_{Tl}-\mu_{MT}\right|\right)};$$

(2) $$\sqrt{\left(\sum_{j=1}^{b}\left|\mu_{Uj}-\mu_{MU}\right|\right)^2-2\sum_{1\le j<l\le b}\left|\mu_{Uj}-\mu_{MU}\right|\left|\mu_{Ul}-\mu_{MU}\right|+b(b-1)\left|M(V_{Ujl})-\mu_{MU}\right|^{2/b}}\le E[M(V_{Ujl})] \le\sqrt{b\left(\left(\sum_{j=1}^{b}\left|\mu_{Uj}-\mu_{MU}\right|\right)^2-2\sum_{1\le j<l\le b}\left|\mu_{Uj}-\mu_{MU}\right|\left|\mu_{Ul}-\mu_{MU}\right|\right)};$$

(3) $$\begin{array}{l}\sqrt{\left(\sum_{j=1}^{b}\left|\mu_{Fj}-\mu_{MF}\right|\right)^2-2\sum_{1\le j<l\le b}\left|\mu_{Fj}-\mu_{MF}\right|\left|\mu_{Fl}-\mu_{MF}\right|+b(b-1)\left|M(V_{Fjl})-\mu_{MF}\right|^{2/b}}\le E[M(V_{Fjl})]\\ \le\sqrt{b\left(\left(\sum_{j=1}^{b}\left|\mu_{Fj}-\mu_{MF}\right|\right)^2-2\sum_{1\le j<l\le b}\left|\mu_{Fj}-\mu_{MF}\right|\left|\mu_{Fl}-\mu_{MF}\right|\right)}\end{array};$$

(4) $$\begin{array}{l}\sqrt{\left(\sum_{j=1}^{b}\left|\rho_{Tj}-\rho_{MT}\right|\right)^2-2\sum_{1\le j<l\le b}\left|\rho_{Tj}-\rho_{MT}\right|\left|\rho_{Tl}-\rho_{MT}\right|+b(b-1)\left|M(C_{Tjl})-\rho_{MT}\right|^{2/b}}\le E[M(C_{Tjl})]\\ \le\sqrt{b\left(\left(\sum_{j=1}^{b}\left|\rho_{Tj}-\rho_{MT}\right|\right)^2-2\sum_{1\le j<l\le b}\left|\rho_{Tj}-\rho_{MT}\right|\left|\rho_{Tl}-\rho_{MT}\right|\right)}\end{array};$$

(5) $$\begin{array}{l}\sqrt{\left(\sum_{j=1}^{b}\left|\rho_{Uj}-\rho_{MU}\right|\right)^2-2\sum_{1\le j<l\le b}\left|\rho_{Uj}-\rho_{MU}\right|\left|\rho_{Ul}-\rho_{MU}\right|+b(b-1)\left|M(C_{Ujl})-\rho_{MU}\right|^{2/b}}\le E[M(C_{Ujl})]\\ \le\sqrt{b\left(\left(\sum_{j=1}^{b}\left|\rho_{Uj}-\rho_{MU}\right|\right)^2-2\sum_{1\le j<l\le b}\left|\rho_{Uj}-\rho_{MU}\right|\left|\rho_{Ul}-\rho_{MU}\right|\right)}\end{array};$$

(6) $$\begin{array}{l}\sqrt{\left(\sum_{j=1}^{b}\left|\rho_{Fj}-\rho_{MF}\right|\right)^2-2\sum_{1\le j<l\le b}\left|\rho_{Fj}-\rho_{MF}\right|\left|\rho_{Fl}-\rho_{MF}\right|+b(b-1)\left|M(C_{Fjl})-\rho_{MF}\right|^{2/b}}\le E[M(C_{Fjl})]\\ \le\sqrt{b\left(\left(\sum_{j=1}^{b}\left|\rho_{Fj}-\rho_{MF}\right|\right)^2-2\sum_{1\le j<l\le b}\left|\rho_{Fj}-\rho_{MF}\right|\left|\rho_{Fl}-\rho_{MF}\right|\right)}\end{array}.$$

To compare two SVNCM energy magnitudes, we present the score function of the SVNCM energy $E(M(n_{Cijl}))$ ($j, l = 1, 2, ..., b$; $i = 1, 2$):

$$Z\left\{E\left(M\left(n_{Cijk}\right)\right)\right\}=2E\left[M\left(V_{Tijk}\right)\right]E\left[M\left(C_{Tijk}\right)\right]+E\left[M\left(V_{Uijk}\right)\right]E\left[M\left(C_{Uijk}\right)\right]-E\left[M\left(V_{Fijk}\right)\right]E\left[M\left(C_{Fijk}\right)\right]. \quad (12)$$

In view of the score values of Eq. (12), the ranking rules between $E(M(n_{C1jl}))$ and $E(M(n_{C2jl}))$ are presented below:

(a) If $Z\{E[M(n_{C1jl})]\} > Z\{E[M(n_{C2jl})]\}$, then $E[M(n_{C1jl})] > E[M(n_{C2jl})]$;

(b) If $Z\{E[M(n_{C1jl})]\} < Z\{E[M(n_{C2jl})]\}$, then $E[M(n_{C1jl})] < E[M(n_{C2jl})]$;

(c) If $Z\{E[M(n_{C1jl})]\} = Z\{E[M(n_{C2jl})]\}$, then $E[M(n_{C1jl})] = E[M(n_{C2jl})]$.

Example 1. Assume that there are two SVNCMs:

$$M(n_{C1jl})=\begin{bmatrix}<(0.6,0.7),(0.3,0.7),(0.2,0.7)> & <(0.5,0.6),(0.5,0.8),(0.3,0.6)> & <(0.7,0.6),(0.1,0.5),(0.3,0.9)>\\ <(0.8,0.7),(0.2,0.8),(0.1,0.8)> & <(0.8,0.8),(0.2,0.8),(0.4,0.6)> & <(0.3,0.8),(0.2,0.6),(0.1,0.6)>\\ <(0.7,0.9),(0.1,0.9),(0.3,0.8)> & <(0.7,0.5),(0.2,0.6),(0.1,0.9)> & <(0.8,0.5),(0.3,0.6),(0.5,0.8)>\end{bmatrix},$$

$$M(n_{C2jl})=\begin{bmatrix}<(0.5,0.6),(0.2,0.8),(0.3,0.8)> & <(0.6,0.7),(0.6,0.8),(0.2,0.8)> & <(0.6,0.6),(0.1,0.7),(0.2,0.8)>\\ <(0.7,0.7),(0.2,0.7),(0.2,0.9)> & <(0.7,0.7),(0.1,0.8),(0.3,0.7)> & <(0.2,0.7),(0.4,0.7),(0.3,0.7)>\\ <(0.6,0.8),(0.1,0.7),(0.1,0.8)> & <(0.6,0.6),(0.1,0.6),(0.1,0.7)> & <(0.7,0.6),(0.2,0.8),(0.4,0.5)>\end{bmatrix}.$$

Then, their SVNCM energy and ranking order are given by the following results:

Using Eq. (11), there are $E[M(n_{C1jl})]$ = <(2.4559, 2.7161), (0.8413, 2.8041), (0.8193, 3.1193)> and $E[M(n_{C2jl})]$ = (2.1708, 2.7372), (0.9355, 2.8000), (0.6916, 3.1601)>.

Using Eq. (12), since $Z\{E[M(n_{C1jl})]\}$ = 13.1444 > $Z\{E[M(n_{C2jl})]\}$ = 12.3177, there is $E[M(n_{C1jl})] > E[M(n_{C2jl})]$.

4. MAGDM Model

This section establishes a MAGDM model based on the SVNCM energy and score function in the setting of SVNCMs.

Considering a MADM problem, there are a group of alternatives and a group of attributes, denoted respectively by $Gs = \{Gs_1, Gs_2, \ldots, Gs_a\}$ and $Cs = \{Cs_1, Cs_2, \ldots, Cs_b\}$. A group of decision makers/experts, denoted as $Es = \{Es_1, Es_2, \ldots, Es_r\}$, is invited to assess the satisfiability levels of each alternative over the attributes and the weight vector of the decision makers/experts is specified as $\theta_{Cj} = \langle(\theta_{Tj}, \theta_{CTj}), (\theta_{Uj}, \theta_{CUj}), (\theta_{Fj}, \theta_{CFj})\rangle$ (j = 1, 2, …, r).

In this MADM problem, the SVNCM energy can be used to build a MADM model in the following steps:

Step 1: The decision makers/experts specify the SVNCV weights of the attributes by $\lambda_{Cjk} = \langle(\lambda_{Tjk}, \lambda_{CTjk}), (\lambda_{Ujk}, \lambda_{CUjk}), (\lambda_{Fjk}, \lambda_{CFjk})\rangle$ (j = 1, 2, …, r; k = 1, 2, …, b) for $\lambda_{Tjk}, \lambda_{CTjk}, \lambda_{Ujk}, \lambda_{CUjk}, \lambda_{Fjk}, \lambda_{CFjk} \in [0, 1]$, and then they are constructed as the weight matrix of the attributes:

$$M(\lambda_{Cjk}) = \begin{matrix} & Cs_1 & Cs_2 & \cdots & Cs_b \\ Es_1 \\ Es_2 \\ \vdots \\ Es_r \end{matrix} \begin{bmatrix} \lambda_{C11} & \lambda_{C12} & \cdots & \lambda_{C1b} \\ \lambda_{C21} & \lambda_{C22} & \cdots & \lambda_{C2b} \\ \vdots & \vdots & \vdots & \vdots \\ \lambda_{Cr1} & \lambda_{Cr2} & \cdots & \lambda_{Crb} \end{bmatrix}. \quad (13)$$

Step 2: Decision makers/experts evaluate their satisfiability levels of each alternative Gs_i over attributes Cs_k by providing the SVNCVs $n_{Cijk} = \langle(V_{Tijk}, C_{Tijk}), (V_{Uijk}, C_{Uijk}), (V_{Fijk}, C_{Fijk})\rangle$ (i = 1, 2, …, a; j = 1, 2, …, r; k = 1, 2, …, b), and then the i-th SVNCM for Gs_i can be built below:

$$M(n_{Cijk}) = \begin{bmatrix} n_{Ci11} & n_{Ci12} & \ldots & n_{Ci1b} \\ n_{Ci21} & n_{Ci22} & \ldots & n_{Ci2b} \\ \vdots & \vdots & \vdots & \vdots \\ n_{Cir1} & n_{Cir2} & \ldots & n_{Cirb} \end{bmatrix}. \quad (14)$$

Step 3: In view of the influence of the decision makers/experts' weights θ_{Cj} on the i-th SVNCM for Gs_i, the weighted SVNCM can be obtained below:

$$
M_E\left(\theta_{Cj}\otimes n_{Cijk}\right)=
\begin{bmatrix}
\left\langle \begin{matrix}(\theta_{T1}V_{Ti11},\theta_{CT1}C_{Ti11}),\\ (\theta_{U1}+V_{Ui11}-\theta_{U1}V_{Ui11},\theta_{CU1}+C_{Ui11}-\theta_{CU1}C_{Ui11}),\\ (\theta_{F1}+V_{Fi11}-\theta_{F1}V_{Fi11},\theta_{CF1}+C_{Fi11}-\theta_{CF1}C_{Fi11})\end{matrix}\right\rangle &
\left\langle \begin{matrix}(\theta_{T1}V_{Ti12},\theta_{CT1}C_{Ti12}),\\ (\theta_{U1}+V_{Ui12}-\theta_{U1}V_{Ui12},\theta_{CU1}+C_{Ui12}-\theta_{CU1}C_{Ui12}),\\ (\theta_{F1}+V_{Fi12}-\theta_{F1}V_{Fi12},\theta_{CF1}+C_{Fi12}-\theta_{CF1}C_{Fi12})\end{matrix}\right\rangle &
\cdots &
\left\langle \begin{matrix}(\theta_{T1}V_{Ti1b},\theta_{CT1}C_{Ti1b}),\\ (\theta_{U1}+V_{Ui1b}-\theta_{U1}V_{Ui1b},\theta_{CU1}+C_{Ui1b}-\theta_{CU1}C_{Ui1b}),\\ (\theta_{F1}+V_{Fi1b}-\theta_{F1}V_{Fi1b},\theta_{CF1}+C_{Fi1b}-\theta_{CF1}C_{Fi1b})\end{matrix}\right\rangle \\
\left\langle \begin{matrix}(\theta_{T2}V_{Ti21},\theta_{CT2}C_{Ti21}),\\ (\theta_{U2}+V_{Ui21}-\theta_{U2}V_{Ui21},\theta_{CU2}+C_{Ui21}-\theta_{CU2}C_{Ui21}),\\ (\theta_{F2}+V_{Fi21}-\theta_{F2}V_{Fi21},\theta_{CF2}+C_{Fi21}-\theta_{CF2}C_{Fi21})\end{matrix}\right\rangle &
\left\langle \begin{matrix}(\theta_{T2}V_{Ti21},\theta_{CT2}C_{Ti21}),\\ (\theta_{U2}+V_{Ui22}-\theta_{U2}V_{Ui22},\theta_{CU2}+C_{Ui22}-\theta_{CU2}C_{Ui22}),\\ (\theta_{F2}+V_{Fi22}-\theta_{F2}V_{Fi22},\theta_{CF2}+C_{Fk22}-\theta_{CF2}C_{Fi22})\end{matrix}\right\rangle &
\cdots &
\left\langle \begin{matrix}(\theta_{T2}V_{Ti2b},\theta_{CT2}C_{Ti2b}),\\ (\theta_{U2}+V_{Ui2b}-\theta_{U2}V_{Ui2b},\theta_{CU2}+C_{Ui2b}-\theta_{CU2}C_{Ui2b}),\\ (\theta_{F2}+V_{Fi2b}-\theta_{F2}V_{Fi2b},\theta_{CF2}+C_{Fi2b}-\theta_{CF2}C_{Fi2b})\end{matrix}\right\rangle \\
\vdots & \vdots & \vdots & \vdots \\
\left\langle \begin{matrix}(\theta_{Tr}V_{Tir1},\theta_{CTr}C_{Tir1}),\\ (\theta_{Ur}+V_{Uir1}-\theta_{Ur}V_{Uir1},\theta_{CUr}+C_{Uir1}-\theta_{CUr}C_{Uir1}),\\ (\theta_{Fr}+V_{Fir1}-\theta_{Fr}V_{Fir1},\theta_{CFr}+C_{Fir1}-\theta_{CFr}C_{Fir1})\end{matrix}\right\rangle &
\left\langle \begin{matrix}(\theta_{Tr}V_{Tir2},\theta_{CTr}C_{Tir2}),\\ (\theta_{Ur}+V_{Uir2}-\theta_{Ur}V_{Uir2},\theta_{CUr}+C_{Uir2}-\theta_{CUr}C_{Uir2}),\\ (\theta_{Fr}+V_{Fir2}-\theta_{Fr}V_{Fir2},\theta_{CFr}+C_{Fir2}-\theta_{CFr}C_{Fir2})\end{matrix}\right\rangle &
\cdots &
\left\langle \begin{matrix}(\theta_{Tr}V_{Tirb},\theta_{CTr}C_{Tirb}),\\ (\theta_{Ur}+V_{Uirb}-\theta_{Ur}V_{Uirb},\theta_{CUr}+C_{Uirb}-\theta_{CUr}C_{Uirb}),\\ (\theta_{Fr}+V_{Firb}-\theta_{Fr}V_{Firb},\theta_{CFr}+C_{Firb}-\theta_{CFr}C_{Firb})\end{matrix}\right\rangle
\end{bmatrix}. \quad (15)
$$

Step 4: In view of the influence of the attribute weights λ_{Cjk} on the i-th SVNCM for Gs_i, the weighted SVNCM can be obtained below:

$$M_C\left(\lambda_{Cjk}\otimes n_{Cijk}\right)=$$

$$\begin{bmatrix} \left\langle \begin{array}{l} (\lambda_{T11}V_{Ti11},\lambda_{CT11}C_{Ti11}),\\ (\lambda_{U11}+V_{Ui11}-\lambda_{U11}V_{Ui11},\lambda_{CU11}+C_{Ui11}-\lambda_{CU11}C_{Ui11}),\\ (\lambda_{F11}+V_{Fi11}-\lambda_{F1}V_{Fi11},\lambda_{CF11}+C_{Fi11}-\lambda_{CF11}C_{Fi11}) \end{array} \right\rangle & \left\langle \begin{array}{l} (\lambda_{T12}V_{Ti12},\lambda_{CT12}C_{Ti12}),\\ (\lambda_{U12}+V_{Ui12}-\lambda_{U12}V_{Ui12},\lambda_{CU12}+C_{Ui12}-\lambda_{CU1}C_{Ui12}),\\ (\lambda_{F12}+V_{Fi12}-\lambda_{F12}V_{Fi12},\lambda_{CF12}+C_{Fi12}-\lambda_{CF12}C_{Fi12}) \end{array} \right\rangle & \cdots & \left\langle \begin{array}{l} (\lambda_{T1b}V_{Ti1b},\lambda_{CT1b}C_{Ti1b}),\\ (\lambda_{U1b}+V_{Ui1b}-\lambda_{U1b}V_{Ui1b},\lambda_{CU1b}+C_{Ui1b}-\lambda_{CU1b}C_{Ui1b}),\\ (\lambda_{F1b}+V_{Fi1b}-\lambda_{F1b}V_{Fi1b},\lambda_{CF1b}+C_{Fi1b}-\lambda_{CF1b}C_{Fi1b}) \end{array} \right\rangle \\ \left\langle \begin{array}{l} (\lambda_{T21}V_{Ti21},\lambda_{CT2}C_{Ti21}),\\ (\lambda_{U21}+V_{Ui21}-\lambda_{U21}V_{Ui21},\lambda_{CU21}+C_{Ui21}-\lambda_{CU21}C_{Ui21}),\\ (\lambda_{F21}+V_{Fi21}-\lambda_{F21}V_{Fi21},\lambda_{CF21}+C_{Fi21}-\lambda_{CF21}C_{Fi21}) \end{array} \right\rangle & \left\langle \begin{array}{l} (\lambda_{T22}V_{Ti22},\lambda_{CT22}C_{Ti22}),\\ (\lambda_{U22}+V_{Ui22}-\lambda_{U22}V_{Ui22},\lambda_{CU22}+C_{Ui22}-\lambda_{CU22}C_{Ui22}),\\ (\lambda_{F22}+V_{Fi22}-\lambda_{F22}V_{Fi22},\lambda_{CF22}+C_{Fi22}-\lambda_{CF22}C_{Fi22}) \end{array} \right\rangle & \cdots & \left\langle \begin{array}{l} (\lambda_{T2b}V_{Ti2b},\lambda_{CT2b}C_{Ti2b}),\\ (\lambda_{I2b}+V_{Ui2b}-\lambda_{U2b}V_{Ui2b},\lambda_{CU2b}+C_{Ui2b}-\lambda_{CU2b}C_{Ui2b}),\\ (\lambda_{F2b}+V_{Fi2b}-\lambda_{F2b}V_{Fi2b},\lambda_{CF2b}+C_{Fi2b}-\lambda_{CF2b}C_{Fi2b}) \end{array} \right\rangle \\ \vdots & \vdots & & \vdots \\ \left\langle \begin{array}{l} (\lambda_{Tr1}V_{Tir1},\lambda_{CTr1}C_{Tir1}),\\ (\lambda_{Ur1}+V_{Uir1}-\lambda_{Ur1}N_{Uir1},\lambda_{CUr1}+C_{Uir1}-\lambda_{CUr1}C_{Uir1}),\\ (\lambda_{Fr1}+V_{Fir1}-\lambda_{Fr1}N_{Fir1},\lambda_{CFr1}+C_{Fir1}-\lambda_{CFr1}C_{Fir1}) \end{array} \right\rangle & \left\langle \begin{array}{l} (\lambda_{Tr2}V_{Tir2},\lambda_{CTr2}C_{Tir2}),\\ (\lambda_{Ur2}+V_{Uir2}-\lambda_{Ur2}V_{Uir2},\lambda_{CUr2}+C_{Uir2}-\lambda_{CUr2}C_{Uir2}),\\ (\lambda_{Fr2}+V_{Fir2}-\lambda_{Fr2}V_{Fir2},\lambda_{CFr2}+C_{Fir2}-\lambda_{CFr2}C_{Fir2}) \end{array} \right\rangle & \cdots & \left\langle \begin{array}{l} (\lambda_{Trb}V_{Tirb},\lambda_{CTrb}C_{Tirb}),\\ (\lambda_{Urb}+V_{Uirb}-\lambda_{Urb}V_{Uirb},\lambda_{CUrb}+C_{Uirb}-\lambda_{CUrb}C_{Uirb}),\\ (\lambda_{Frb}+V_{Firb}-\lambda_{Frb}V_{Firb},\lambda_{CFrb}+C_{Firb}-\lambda_{CFrb}C_{Firb}) \end{array} \right\rangle \end{bmatrix}. \quad (16)$$

Step 5: Based on the above weighted SVNCMs, we obtain the collective SVNCMs $M(n_{Cijk})$ = <($M(V_{Tijl})$, $M(C_{Tijl})$), ($M(V_{Uijl})$, $M(C_{Uijl})$), ($M(V_{Fijl})$, $M(C_{Fijl})$)> (j, l = 1, 2, …, r; i = 1, 2, …, a) by calculating the true, false and uncertain squire matrices and the true, false and uncertain credibility squire matrices:

$$M\left(V_{Tijl}\right)=M_C\left(\lambda_{Tjk}V_{Tijk}\right)\times\left[M_E\left(\theta_{Tj}V_{Tijk}\right)\right]^T$$
$$=\begin{bmatrix}\lambda_{T11}V_{Ti11} & \lambda_{T12}V_{Ti12} & \cdots & \lambda_{T1b}V_{Ti1b}\\ \lambda_{T21}V_{Ti21} & \lambda_{T22}V_{Ti22} & \cdots & \lambda_{T2b}V_{Ti2b}\\ \vdots & \vdots & \vdots & \vdots\\ \lambda_{Tr1}V_{Tir1} & \lambda_{Tr2}V_{Tir2} & \cdots & \lambda_{Trb}V_{Tirb}\end{bmatrix}\times\begin{bmatrix}\theta_{T1}V_{Ti11} & \theta_{T2}V_{Ti21} & \cdots & \theta_{Tr}V_{Tir1}\\ \theta_{T1}V_{Ti12} & \theta_{T2}V_{Ti22} & \cdots & \theta_{Tr}V_{Tir2}\\ \vdots & \vdots & \vdots & \vdots\\ \theta_{T1}V_{Ti1b} & \theta_{T2}V_{Ti2b} & \cdots & \theta_{Tr}V_{Tirb}\end{bmatrix}, \quad (17)$$

$$M\left(V_{Uijl}\right)=M_{C}\left(\lambda_{Ujk}+V_{Uijk}-\lambda_{Ujk}V_{Uijk}\right)\times\left[M_{E}\left(\theta_{Ui}+V_{Uijk}-\theta_{Uj}V_{Uijk}\right)\right]^{T}$$
$$=\begin{bmatrix}\lambda_{U11}+V_{Ui11}-\lambda_{U11}V_{Ui11} & \lambda_{U12}+V_{Ui12}-\lambda_{U12}V_{Ui12} & \cdots & \lambda_{U1b}+V_{Ui1b}-\lambda_{U1b}V_{Ui1b}\\ \lambda_{U21}+V_{Ui21}-\lambda_{U21}V_{Ui21} & \lambda_{U22}+V_{Ui22}-\lambda_{U22}V_{Ui22} & \cdots & \lambda_{U2b}+V_{Ui2b}-\lambda_{U2b}V_{Ui2b}\\ \vdots & \vdots & \vdots & \vdots\\ \lambda_{Ur1}+V_{Uir1}-\lambda_{Ur1}V_{Uir1} & \lambda_{Ur2}+V_{Uir2}-\lambda_{Ur2}V_{Uir2} & \cdots & \lambda_{Urb}+V_{Uirb}-\lambda_{Urb}V_{Uirb}\end{bmatrix}, \quad (18)$$
$$\times\begin{bmatrix}\theta_{U1}+V_{Ui11}-\theta_{U1}V_{Ui11} & \theta_{U2}+V_{Ui21}-\theta_{U2}V_{Ui21} & \cdots & \theta_{Ur}+V_{Uir1}-\theta_{Ur}V_{Uir1}\\ \theta_{U1}+V_{Ui12}-\theta_{U1}V_{Ui12} & \theta_{U2}+V_{Ui22}-\theta_{U2}V_{Ui22} & \cdots & \theta_{Ur}+V_{Uir2}-\theta_{Ur}V_{Uir2}\\ \vdots & \vdots & \vdots & \vdots\\ \theta_{U1}+V_{Ui1b}-\theta_{I1}V_{Ui1b} & \theta_{U2}+V_{Ui2b}-\theta_{I2}V_{Ui2b} & \cdots & \theta_{Ur}+V_{Uirb}-\theta_{Ur}V_{Uirb}\end{bmatrix}$$

$$M\left(V_{Fijl}\right)=M_{C}\left(\lambda_{Fjk}+V_{Fijk}-\lambda_{Fjk}V_{Fijk}\right)\times\left[M_{E}\left(\theta_{Fj}+V_{Fijk}-\theta_{Fj}V_{Fijk}\right)\right]^{T}$$
$$=\begin{bmatrix}\lambda_{F11}+V_{Fi11}-\lambda_{F11}V_{Fi11} & \lambda_{F12}+V_{Fi12}-\lambda_{F12}V_{Fi12} & \cdots & \lambda_{F1b}+V_{Fi1b}-w_{F1b}N_{Fi1b}\\ \lambda_{F21}+V_{Fi21}-\lambda_{F21}V_{Fi21} & \lambda_{F22}+V_{Fi22}-\lambda_{F22}V_{Fi22} & \cdots & \lambda_{F2b}+V_{Fi2b}-w_{F2b}N_{Fi2b}\\ \vdots & \vdots & \vdots & \vdots\\ \lambda_{Fr1}+V_{Fir1}-\lambda_{Fr1}V_{Fir1} & \lambda_{Fr2}+V_{Fir2}-\lambda_{Fr2}V_{Fir2} & \cdots & \lambda_{Frb}+V_{Firb}-\lambda_{Frb}V_{Firb}\end{bmatrix}, \quad (19)$$
$$\times\begin{bmatrix}\theta_{F1}+V_{Fi11}-\theta_{F1}V_{Fi11} & \theta_{F2}+V_{Fi21}-\theta_{F2}V_{Fi21} & \cdots & \theta_{Fr}+V_{Fir1}-\theta_{Fr}V_{Fir1}\\ \theta_{F1}+V_{Fi12}-\theta_{F1}V_{Fi12} & \theta_{F2}+V_{Fi22}-\theta_{F2}V_{Fi22} & \cdots & \theta_{Fr}+V_{Fir2}-\theta_{Fr}V_{Fir2}\\ \vdots & \vdots & \vdots & \vdots\\ \theta_{F1}+V_{Fi1b}-\theta_{F1}V_{Fi1b} & \theta_{F2}+V_{Fi2b}-\theta_{F2}N_{Fi2b} & \cdots & \theta_{Fr}+V_{Firb}-\theta_{Fr}V_{Firb}\end{bmatrix}$$

$$M\left(C_{Tijl}\right)=M_{C}\left(\lambda_{Tjk}C_{Tijk}\right)\times\left[M_{E}\left(\theta_{Tj}C_{Tijk}\right)\right]^{T}$$
$$=\begin{bmatrix}\lambda_{T11}C_{Ti11} & \lambda_{T12}C_{Ti12} & \cdots & \lambda_{T1b}C_{Ti1b}\\ \lambda_{T21}C_{Ti21} & \lambda_{T22}C_{Ti22} & \cdots & \lambda_{T2b}C_{Ti2b}\\ \vdots & \vdots & \vdots & \vdots\\ \lambda_{Tr1}C_{Tir1} & \lambda_{Tr2}C_{Tir2} & \cdots & \lambda_{Trb}C_{Tirb}\end{bmatrix}\times\begin{bmatrix}\theta_{T1}C_{Ti11} & \theta_{T2}C_{Ti21} & \cdots & \theta_{Tr}C_{Tir1}\\ \theta_{T1}C_{Ti12} & \theta_{T2}C_{Ti22} & \cdots & \theta_{Tr}C_{Tir2}\\ \vdots & \vdots & \vdots & \vdots\\ \theta_{T1}C_{Ti1b} & \theta_{T2}C_{Ti2b} & \cdots & \theta_{Tr}C_{Tirb}\end{bmatrix}, \quad (20)$$

$$M\left(C_{Uijl}\right)=M_{C}\left(\lambda_{Ujk}+C_{Uijk}-\lambda_{Ujk}C_{Uijk}\right)\times\left[M_{E}\left(\theta_{Uj}+C_{Uijk}-\theta_{Uj}C_{Uijk}\right)\right]^{T}$$
$$=\begin{bmatrix}\lambda_{I11}+C_{Ui11}-\lambda_{U11}C_{Ui11} & \lambda_{U12}+C_{Ui12}-\lambda_{U12}C_{Ui12} & \cdots & \lambda_{U1b}+C_{Ui1b}-\lambda_{U1b}C_{Ui1b}\\ \lambda_{U21}+C_{Ui21}-\lambda_{U21}C_{Ui21} & \lambda_{U22}+C_{Ui22}-\lambda_{U22}C_{Ui22} & \cdots & \lambda_{U2b}+C_{Ui2b}-\lambda_{U2b}C_{Ui2b}\\ \vdots & \vdots & \vdots & \vdots\\ \lambda_{Ur1}+C_{Uir1}-\lambda_{Ur1}C_{Uir1} & \lambda_{Ur2}+C_{Uir2}-\lambda_{Ur2}C_{Uir2} & \cdots & \lambda_{Urb}+C_{Uirb}-\lambda_{Urb}C_{Uirb}\end{bmatrix}, \quad (21)$$
$$\times\begin{bmatrix}\theta_{U1}+C_{Ui11}-\theta_{U1}C_{Ui11} & \theta_{U2}+C_{Ui21}-\theta_{U2}C_{Ui21} & \cdots & \theta_{Ur}+C_{Uir1}-\theta_{Ur}C_{Uir1}\\ \theta_{U1}+C_{Ui12}-\theta_{U1}C_{Ui12} & \theta_{U2}+C_{Ui22}-\theta_{U2}C_{Ui22} & \cdots & \theta_{Ur}+C_{Uir2}-\theta_{Ur}C_{Uir2}\\ \vdots & \vdots & \vdots & \vdots\\ \theta_{U1}+C_{Ui1b}-\theta_{U1}C_{Ui1b} & \theta_{U2}+C_{Ui2b}-\theta_{U2}C_{Ui2b} & \cdots & \theta_{Ur}+C_{Uirb}-\theta_{Ur}C_{Uirb}\end{bmatrix}$$

$$M\left(C_{Fijl}\right)=M_C\left(\lambda_{Fjk}+C_{Fijk}-\lambda_{Fjk}C_{Fijk}\right)\times\left[M_E\left(\theta_{Fj}+C_{Fijk}-\theta_{Fj}C_{Fijk}\right)\right]^T$$
$$=\begin{bmatrix}\lambda_{F11}+C_{Fi11}-\lambda_{F11}C_{Fk11} & \lambda_{F12}+C_{Fi12}-\lambda_{F12}C_{Fi12} & \cdots & \lambda_{F1b}+C_{Fi1b}-\lambda_{F1b}C_{Fi1b}\\ \lambda_{F21}+C_{Fi21}-\lambda_{F21}C_{Fk21} & \lambda_{F22}+C_{Fi22}-\lambda_{F22}C_{Fi22} & \cdots & \lambda_{F2b}+C_{Fi2b}-\lambda_{F2b}C_{Fi2b}\\ \vdots & \vdots & \vdots & \vdots\\ \lambda_{Fr1}+C_{Fir1}-\lambda_{Fr1}C_{Fir1} & \lambda_{Fr2}+C_{Fir2}-\lambda_{Fr2}C_{Fir2} & \cdots & \lambda_{Frb}+C_{Firb}-\lambda_{Frb}C_{Firb}\end{bmatrix}. \quad (22)$$
$$\times\begin{bmatrix}\theta_{F1}+C_{Fi11}-\theta_{F1}C_{Fi11} & \theta_{F2}+C_{Fi21}-\theta_{F2}C_{Fi21} & \cdots & \theta_{Fr}+C_{Fir1}-\theta_{Fr}C_{Fir1}\\ \theta_{F1}+C_{Fi12}-\theta_{F1}C_{Fi12} & \theta_{F2}+C_{Fi22}-\theta_{F2}C_{Fi22} & \cdots & \theta_{Fr}+C_{Fir2}-\theta_{Fr}C_{Fir2}\\ \vdots & \vdots & \vdots & \vdots\\ \theta_{F1}+C_{Fi1b}-\theta_{F1}C_{Fi1b} & \theta_{F2}+C_{Fi2b}-\theta_{F2}C_{Fi2b} & \cdots & \theta_{Fr}+C_{Firb}-\theta_{Fr}C_{Firb}\end{bmatrix}$$

Step 6: The respective SVNCV matrix energy values for each alternative can be obtained by Eq. (11).
Step 7: The SVNCM energy score values of for each alternative G_{Si} (i = 1, 2, …, a) are calculated by Eq. (12).
Step 8: According to the score values, all alternatives are ranked in descending order and the alternative with the largest value is the best.

5. MAGDM Application in Primary School Site Selection

5.1 Actual Example of Primary School Site Selection

In recent years, Shaoxing's level of economic development has risen in China, and as the city's framework has been further expanded, the city's population has dispersed to multiple centers. It is necessary to build a new primary school in a suitable position of Shaoxing City in China. In this section, the feasibility and validity of the MAGDM model in a SVNCM environment are verified through an actual example of primary school site selection in Shaoxing.

By analyzing the city framework and population distribution in Shaoxing, the decision department provides four potential locations as a set of alternatives $G_S = \{G_{S1}, G_{S2}, G_{S3}, G_{S4}\}$. In the assessment issue of the alternatives, the four main requirements/attributes of the school site can be considered by construction cost (C_{S1}), regional population (C_{S2}), transport facilities (C_{S3}) and regional environment (C_{S4}). For this siting decision problem, a group of three experts $E_S = \{E_{S1}, E_{S2}, E_{S3}\}$ is invited to evaluate the best alternative among them, and then the three experts' SVNCV weights are specified as $\theta_{C1} = \langle(0.8, 0.7), (0.1, 0.8), (0.2, 0.7)\rangle$, $\theta_{C2} = \langle(0.7, 0.6), (0.2, 0.7), (0.3, 0.7)\rangle$, and $\theta_{C3} = \langle(0.6, 0.8), (0.2, 0.6), (0.1, 0.9)\rangle$.

The MAGDM model based on the SVNCM energy proposed in the above section can be applied to the site selection problem of this school in the following steps:
Step 1: The three experts specify the SVNCV weights of the attributes by $\lambda_{Cjk} = \langle(\lambda_{Tjk}, \lambda_{CTjk}), (\lambda_{Ujk}, \lambda_{CUjk}), (\lambda_{Fjk}, \lambda_{CFjk})\rangle$ (j = 1, 2, 3; k = 1, 2, 3, 4) for $\lambda_{Tjk}, \lambda_{CTjk}, \lambda_{Ujk}, \lambda_{CUjk}, \lambda_{Fjk}, \lambda_{CFjk} \in [0, 1]$, and then they are constructed as the weight matrix of the attributes:

$$M(\lambda_{Cjk})=\begin{bmatrix}\langle(0.8,0.8),(0.1,0.7),(0.3,0.8)\rangle & \langle(0.6,0.9),(0.2,0.8),(0.1,0.7)\rangle & \langle(0.8,0.6),(0.4,0.9),(0.3,0.8)\rangle & \langle(0.7,0.7),(0.1,0.7),(0.2,0.6)\rangle\\ \langle(0.7,0.7),(0.2,0.6),(0.1,0.7)\rangle & \langle(0.6,0.8),(0.2,0.7),(0.2,0.6)\rangle & \langle(0.6,0.7),(0.1,0.8),(0.1,0.9)\rangle & \langle(0.9,0.6),(0.1,0.8),(0.2,0.9)\rangle\\ \langle(0.8,0.7),(0.3,0.7),(0.2,0.6)\rangle & \langle(0.7,0.9),(0.2,0.7),(0.2,0.7)\rangle & \langle(0.9,0.9),(0.2,0.6),(0.3,0.8)\rangle & \langle(0.8,0.8),(0.2,0.7),(0.1,0.8)\rangle\end{bmatrix}.$$

Step 2: Decision makers/experts evaluate their satisfiability levels of each alternative Gs_i over attributes Cs_k by providing the SVNCVs n_{Cijk} = <(V_{Tijk}, C_{Tijk}), (V_{Uijk}, C_{Uijk}), (V_{Fijk}, C_{Fijk})> (i, k = 1, 2, 3, 4; j = 1, 2, 3), and then SVNCMs for Gs_i for i = 1, 2, 3, 4 can be built below:

$$M\left(n_{C1jk}\right)=\begin{bmatrix}\langle(0.7,0.8),(0.2,0.7),(0.1,0.8)\rangle & \langle(0.6,0.7),(0.1,0.8),(0.3,0.7)\rangle & \langle(0.8,0.8),(0.1,0.8),(0.3,0.8)\rangle & \langle(0.9,0.8),(0.3,0.7),(0.2,0.6)\rangle\\ \langle(0.6,0.7),(0.1,0.8),(0.2,0.6)\rangle & \langle(0.7,0.9),(0.2,0.7),(0.3,0.8)\rangle & \langle(0.7,0.8),(0.2,0.7),(0.3,0.7)\rangle & \langle(0.8,0.7),(0.1,0.7),(0.2,0.6)\rangle\\ \langle(0.8,0.8),(0.4,0.7),(0.2,0.7)\rangle & \langle(0.8,0.7),(0.3,0.7),(0.2,0.8)\rangle & \langle(0.8,0.7),(0.3,0.7),(0.2,0.8)\rangle & \langle(0.6,0.8),(0.2,0.8),(0.1,0.9)\rangle\end{bmatrix},$$

$$M\left(n_{C2jk}\right)=\begin{bmatrix}\langle(0.7,0.7),(0.2,0.7),(0.3,0.6)\rangle & \langle(0.7,0.8),(0.2,0.7),(0.3,0.7)\rangle & \langle(0.8,0.9),(0.2,0.8),(0.3,0.7)\rangle & \langle(0.6,0.7),(0.1,0.7),(0.3,0.6)\rangle\\ \langle(0.8,0.6),(0.3,0.6),(0.2,0.7)\rangle & \langle(0.9,0.7),(0.3,0.7),(0.2,0.8)\rangle & \langle(0.9,0.7),(0.4,0.6),(0.3,0.7)\rangle & \langle(0.8,0.7),(0.1,0.8),(0.1,0.9)\rangle\\ \langle(0.7,0.8),(0.2,0.7),(0.3,0.8)\rangle & \langle(0.8,0.7),(0.1,0.8),(0.2,0.9)\rangle & \langle(0.8,0.6),(0.1,0.7),(0.2,0.8)\rangle & \langle(0.7,0.8),(0.2,0.8),(0.3,0.7)\rangle\end{bmatrix},$$

$$M\left(n_{C3jk}\right)=\begin{bmatrix}\langle(0.7,0.8),(0.2,0.6),(0.1,0.7)\rangle & \langle(0.9,0.8),(0.2,0.7),(0.3,0.6)\rangle & \langle(0.7,0.8),(0.2,0.7),(0.3,0.7)\rangle & \langle(0.6,0.8),(0.2,0.8),(0.3,0.8)\rangle\\ \langle(0.6,0.9),(0.2,0.7),(0.3,0.8)\rangle & \langle(0.8,0.8),(0.2,0.7),(0.1,0.9)\rangle & \langle(0.9,0.8),(0.2,0.6),(0.1,0.8)\rangle & \langle(0.8,0.9),(0.1,0.9),(0.3,0.7)\rangle\\ \langle(0.8,0.7),(0.1,0.8),(0.2,0.7)\rangle & \langle(0.7,0.9),(0.1,0.8),(0.3,0.8)\rangle & \langle(0.8,0.7),(0.1,0.7),(0.2,0.7)\rangle & \langle(0.7,0.8),(0.3,0.7),(0.1,0.8)\rangle\end{bmatrix},$$

$$M\left(n_{C4jk}\right)=\begin{bmatrix} \langle(0.9,0.7),(0.2,0.7),(0.1,0.8)\rangle & \langle(0.8,0.8),(0.2,0.8),(0.2,0.7)\rangle & \langle(0.7,0.8),(0.1,0.7),(0.3,0.7)\rangle & \langle(0.6,0.8),(0.2,0.8),(0.3,0.6)\rangle \\ \langle(0.7,0.8),(0.3,0.7),(0.2,0.9)\rangle & \langle(0.8,0.8),(0.3,0.7),(0.1,0.8)\rangle & \langle(0.9,0.7),(0.2,0.8),(0.1,0.8)\rangle & \langle(0.8,0.7),(0.4,0.8),(0.2,0.7)\rangle \\ \langle(0.8,0.9),(0.1,0.8),(0.1,0.8)\rangle & \langle(0.9,0.7),(0.1,0.8),(0.3,0.7)\rangle & \langle(0.7,0.8),(0.2,0.9),(0.2,0.9)\rangle & \langle(0.7,0.8),(0.3,0.7),(0.1,0.9)\rangle \end{bmatrix}.$$

Step 3: In view of the influence of the decision makers/experts' weights θ_{Cj} on the four SVNCMs for G_{Si} for i = 1, 2, 3, 4, the weighted SVNCMs using Eq. (15) can be obtained below:

$$M_E\left(\theta_{Cj}\otimes n_{C1jk}\right)=\begin{bmatrix} \langle(0.56,0.56),(0.28,0.94),(0.28,0.94)\rangle & \langle(0.48,0.49),(0.19,0.96),(0.44,0.91)\rangle & \langle(0.64,0.56),(0.19,0.96),(0.44,0.94)\rangle & \langle(0.72,0.56),(0.37,0.94),(0.36,0.88)\rangle \\ \langle(0.42,0.42),(0.28,0.94),(0.44,0.88)\rangle & \langle(0.49,0.54),(0.36,0.91),(0.51,0.94)\rangle & \langle(0.49,0.48),(0.36,0.91),(0.51,0.91)\rangle & \langle(0.56,0.42),(0.28,0.91),(0.44,0.88)\rangle \\ \langle(0.48,0.64),(0.52,0.88),(0.28,0.97)\rangle & \langle(0.48,0.56),(0.44,0.88),(0.28,0.98)\rangle & \langle(0.48,0.56),(0.44,0.88),(0.28,0.98)\rangle & \langle(0.36,0.64),(0.36,0.92),(0.19,0.99)\rangle \end{bmatrix},$$

$$M_E\left(\theta_{Cj}\otimes n_{C2jk}\right)=\begin{bmatrix} \langle(0.56,0.49),(0.28,0.94),(0.44,0.88)\rangle & \langle(0.56,0.56),(0.28,0.94),(0.44,0.91)\rangle & \langle(0.64,0.63),(0.28,0.96),(0.44,0.91)\rangle & \langle(0.48,0.49),(0.19,0.94),(0.44,0.88)\rangle \\ \langle(0.56,0.36),(0.44,0.88),(0.44,0.91)\rangle & \langle(0.63,0.42),(0.44,0.91),(0.44,0.94)\rangle & \langle(0.63,0.42),(0.52,0.88),(0.51,0.91)\rangle & \langle(0.56,0.42),(0.28,0.94),(0.37,0.97)\rangle \\ \langle(0.42,0.64),(0.36,0.88),(0.37,0.98)\rangle & \langle(0.48,0.56),(0.28,0.92),(0.28,0.99)\rangle & \langle(0.48,0.48),(0.28,0.88),(0.28,0.98)\rangle & \langle(0.42,0.64),(0.36,0.92),(0.37,0.97)\rangle \end{bmatrix},$$

$$M_E\left(\theta_{Cj}\otimes n_{C3jk}\right)=\begin{bmatrix} \langle(0.56,0.56),(0.28,0.92),(0.28,0.91)\rangle & \langle(0.72,0.56),(0.28,0.94),(0.44,0.88)\rangle & \langle(0.56,0.56),(0.28,0.94),(0.44,0.91)\rangle & \langle(0.48,0.56),(0.28,0.96),(0.44,0.94)\rangle \\ \langle(0.42,0.54),(0.36,0.91),(0.51,0.94)\rangle & \langle(0.56,0.48),(0.36,0.91),(0.37,0.97)\rangle & \langle(0.63,0.48),(0.36,0.88),(0.37,0.94)\rangle & \langle(0.56,0.54),(0.28,0.97),(0.51,0.91)\rangle \\ \langle(0.48,0.56),(0.28,0.92),(0.28,0.97)\rangle & \langle(0.42,0.72),(0.28,0.92),(0.37,0.98)\rangle & \langle(0.48,0.56),(0.28,0.88),(0.28,0.97)\rangle & \langle(0.42,0.64),(0.44,0.88),(0.19,0.98)\rangle \end{bmatrix},$$

$$M_E\left(\theta_{Cj}\otimes n_{C4jk}\right)=\begin{bmatrix} \langle(0.72,0.49),(0.28,0.94),(0.28,0.94)\rangle & \langle(0.64,0.56),(0.28,0.96),(0.36,0.91)\rangle & \langle(0.56,0.56),(0.19,0.94),(0.44,0.91)\rangle & \langle(0.48,0.56),(0.28,0.96),(0.44,0.88)\rangle \\ \langle(0.49,0.48),(0.44,0.91),(0.44,0.97)\rangle & \langle(0.56,0.48),(0.44,0.91),(0.37,0.94)\rangle & \langle(0.63,0.42),(0.36,0.94),(0.37,0.94)\rangle & \langle(0.56,0.42),(0.52,0.94),(0.44,0.91)\rangle \\ \langle(0.48,0.72),(0.28,0.92),(0.19,0.98)\rangle & \langle(0.54,0.56),(0.28,0.92),(0.37,0.97)\rangle & \langle(0.42,0.64),(0.36,0.96),(0.28,0.99)\rangle & \langle(0.42,0.64),(0.44,0.88),(0.19,0.99)\rangle \end{bmatrix}.$$

Step 4: In terms of the influence of the attribute weights λ_{Cjk} on the four SVNCMs for G_{Si} for i = 1, 2, 3, 4, the weighted SVNCMs using Eq. (16) can be obtained below:

$$M_C\left(\lambda_{Cjk}\otimes n_{C1jk}\right)=\begin{bmatrix}\langle(0.56,0.64),(0.28,0.91),(0.37,0.96)\rangle & \langle(0.36,0.63),(0.28,0.96),(0.37,0.91)\rangle & \langle(0.64,0.48),(0.46,0.98),(0.51,0.96)\rangle & \langle(0.63,0.56),(0.37,0.91),(0.36,0.84)\rangle\\ \langle(0.42,0.49),(0.28,0.92),(0.28,0.88)\rangle & \langle(0.42,0.72),(0.36,0.91),(0.44,0.92)\rangle & \langle(0.42,0.56),(0.28,0.94),(0.37,0.97)\rangle & \langle(0.72,0.42),(0.19,0.94),(0.36,0.96)\rangle\\ \langle(0.64,0.56),(0.58,0.91),(0.36,0.88)\rangle & \langle(0.56,0.63),(0.44,0.91),(0.36,0.94)\rangle & \langle(0.72,0.63),(0.44,0.88),(0.44,0.96)\rangle & \langle(0.48,0.64),(0.36,0.94),(0.19,0.98)\rangle\end{bmatrix},$$

$$M_C\left(\lambda_{Cjk}\otimes n_{C2jk}\right)=\begin{bmatrix}\langle(0.56,0.56),(0.28,0.91),(0.51,0.92)\rangle & \langle(0.42,0.72),(0.36,0.94),(0.37,0.91)\rangle & \langle(0.64,0.54),(0.52,0.98),(0.51,0.94)\rangle & \langle(0.42,0.49),(0.19,0.91),(0.44,0.84)\rangle\\ \langle(0.56,0.42),(0.44,0.84),(0.28,0.91)\rangle & \langle(0.54,0.56),(0.44,0.91),(0.36,0.92)\rangle & \langle(0.54,0.49),(0.46,0.92),(0.37,0.97)\rangle & \langle(0.72,0.42),(0.19,0.96),(0.28,0.99)\rangle\\ \langle(0.56,0.56),(0.44,0.91),(0.44,0.92)\rangle & \langle(0.56,0.63),(0.28,0.94),(0.36,0.97)\rangle & \langle(0.72,0.54),(0.28,0.88),(0.44,0.96)\rangle & \langle(0.56,0.64),(0.36,0.94),(0.37,0.94)\rangle\end{bmatrix},$$

$$M_C\left(\lambda_{Cjk}\otimes n_{C3jk}\right)=\begin{bmatrix}\langle(0.56,0.64),(0.28,0.88),(0.37,0.94)\rangle & \langle(0.54,0.72),(0.36,0.94),(0.37,0.88)\rangle & \langle(0.56,0.48),(0.52,0.97),(0.51,0.94)\rangle & \langle(0.42,0.56),(0.28,0.94),(0.44,0.92)\rangle\\ \langle(0.42,0.63),(0.36,0.88),(0.37,0.94)\rangle & \langle(0.48,0.64),(0.36,0.91),(0.28,0.96)\rangle & \langle(0.54,0.56),(0.28,0.92),(0.19,0.98)\rangle & \langle(0.72,0.54),(0.19,0.98),(0.44,0.97)\rangle\\ \langle(0.64,0.49),(0.37,0.94),(0.36,0.88)\rangle & \langle(0.49,0.81),(0.28,0.94),(0.44,0.94)\rangle & \langle(0.72,0.63),(0.28,0.88),(0.44,0.94)\rangle & \langle(0.56,0.64),(0.44,0.91),(0.19,0.96)\rangle\end{bmatrix},$$

$$M_C\left(\lambda_{Cjk}\otimes n_{C4jk}\right)=\begin{bmatrix}\langle(0.72,0.56),(0.28,0.91),(0.37,0.96)\rangle & \langle(0.48,0.72),(0.36,0.96),(0.28,0.91)\rangle & \langle(0.56,0.48),(0.46,0.97),(0.51,0.94)\rangle & \langle(0.42,0.56),(0.28,0.94),(0.44,0.84)\rangle\\ \langle(0.49,0.56),(0.44,0.88),(0.28,0.97)\rangle & \langle(0.48,0.64),(0.44,0.91),(0.28,0.92)\rangle & \langle(0.54,0.49),(0.28,0.96),(0.19,0.98)\rangle & \langle(0.72,0.42),(0.46,0.96),(0.36,0.97)\rangle\\ \langle(0.64,0.63),(0.37,0.94),(0.28,0.92)\rangle & \langle(0.63,0.63),(0.28,0.94),(0.44,0.91)\rangle & \langle(0.63,0.72),(0.36,0.96),(0.44,0.98)\rangle & \langle(0.56,0.64),(0.44,0.91),(0.19,0.98)\rangle\end{bmatrix}.$$

Step 5: Using Eqs. (17)–(22), we obtain the collective SVNCMs $M(n_{Cijk})$ = <($M(V_{Tijl})$, $M(C_{Tijl})$), ($M(V_{Uijl})$, $M(C_{Uijl})$), ($M(V_{Fijl})$, $M(C_{Fijl})$)> (j, l = 1, 2, 3; i = 1, 2, 3, 4), where $M(V_{Tijl})$, $M(C_{Tijl})$, ($M(V_{Uijl})$, $M(C_{Uijl})$, and ($M(V_{Fijl})$, $M(C_{Fijl})$ are given as follows:

$$M\left(V_{T1jl}\right)=\begin{bmatrix}1.3496 & 1.0780 & 0.9756\\ 1.2240 & 1.9912 & 0.8640\\ 1.4336 & 1.1648 & 1.0944\end{bmatrix},\ M\left(V_{T2jl}\right)=\begin{bmatrix}1.1600 & 1.2166 & 0.9204\\ 1.3072 & 1.3972 & 1.0560\\ 1.3568 & 1.4336 & 1.0848\end{bmatrix},$$

$$M\left(V_{T3jl}\right)=\begin{bmatrix}1.2176 & 1.1256 & 0.9408\\ 1.2288 & 1.1886 & 0.9648\\ 1.3832 & 1.3104 & 1.0938\end{bmatrix},\ M\left(V_{T4jl}\right)=\begin{bmatrix}1.3408 & 1.2096 & 1.0164\\ 1.3080 & 1.2523 & 1.0236\\ 1.4856 & 1.3769 & 1.1472\end{bmatrix},$$

$$M\left(V_{U1jl}\right)=\begin{bmatrix}0.3559 & 0.4484 & 0.6044\\ 0.2703 & 0.3620 & 0.4956\\ 0.4628 & 0.5800 & 0.8184\end{bmatrix},\ M\left(V_{U2jl}\right)=\begin{bmatrix}0.3609 & 0.6052 & 0.4156\\ 0.4113 & 0.6796 & 0.4788\\ 0.3484 & 0.5632 & 0.4448\end{bmatrix},$$

$$M\left(V_{U3jl}\right)=\begin{bmatrix}0.4032 & 0.4960 & 0.4480\\ 0.3332 & 0.4132 & 0.3636\\ 0.3836 & 0.4580 & 0.4540\end{bmatrix},\ M\left(V_{U4jl}\right)=\begin{bmatrix}0.3450 & 0.5928 & 0.4680\\ 0.4284 & 0.7272 & 0.5496\\ 0.3736 & 0.6444 & 0.5052\end{bmatrix},$$

$$M\left(V_{F1jl}\right)=\begin{bmatrix}0.6204 & 0.7700 & 0.4184\\ 0.5644 & 0.6947 & 0.3736\\ 0.5212 & 0.6500 & 0.3609\end{bmatrix},\ M\left(V_{F2jl}\right)=\begin{bmatrix}0.8052 & 0.8101 & 0.5979\\ 0.5676 & 0.5739 & 0.4116\\ 0.7084 & 0.7133 & 0.5237\end{bmatrix},$$

$$M\left(V_{F3jl}\right)=\begin{bmatrix}0.6844 & 0.7387 & 0.4669\\ 0.5040 & 0.5870 & 0.3440\\ 0.5716 & 0.6061 & 0.4229\end{bmatrix},\ M\left(V_{F4jl}\right)=\begin{bmatrix}0.6224 & 0.6487 & 0.4003\\ 0.4212 & 0.4555 & 0.2784\\ 0.5140 & 0.5324 & 0.3753\end{bmatrix},$$

$$M\left(C_{T1jl}\right)=\begin{bmatrix}1.2495 & 1.0746 & 1.3896\\ 1.1760 & 1.0398 & 1.2992\\ 1.3335 & 1.1466 & 1.4736\end{bmatrix},\ M\left(C_{T2ii}\right)=\begin{bmatrix}1.2579 & 0.9366 & 1.3344\\ 1.0339 & 0.7686 & 1.0864\\ 1.2810 & 0.9618 & 1.3800\end{bmatrix},$$

$$M\left(C_{T3jl}\right)=\begin{bmatrix}1.3440 & 1.2240 & 1.5040\\ 1.3272 & 1.2078 & 1.4728\\ 1.4392 & 1.3014 & 1.6200\end{bmatrix},\ M\left(C_{T4jl}\right)=\begin{bmatrix}1.2600 & 1.0512 & 1.4720\\ 1.1424 & 0.9582 & 1.3440\\ 1.4231 & 1.1760 & 1.6768\end{bmatrix},$$

$$M\left(C_{U1jl}\right)=\begin{bmatrix}3.5732 & 3.4489 & 3.3452\\ 3.5244 & 3.4037 & 3.3024\\ 3.4574 & 3.3397 & 3.2408\end{bmatrix},\ M\left(C_{U2jl}\right)=\begin{bmatrix}3.5352 & 3.3740 & 3.3652\\ 3.4306 & 3.2793 & 3.2692\\ 3.4674 & 3.3142 & 3.3048\end{bmatrix},$$

$$M\left(C_{U3jl}\right)=\begin{bmatrix}3.5074 & 3.4216 & 3.3552\\ 3.4706 & 3.3891 & 3.3188\\ 3.4492 & 3.3679 & 3.3048\end{bmatrix},\ M\left(C_{U4jl}\right)=\begin{bmatrix}3.5912 & 3.4971 & 3.4788\\ 3.5248 & 3.4337 & 3.4132\\ 3.5620 & 3.4686 & 3.4520\end{bmatrix},$$

$$M\left(C_{F1jl}\right)=\begin{bmatrix}3.3721 & 3.3130 & 3.5954\\ 3.4210 & 3.3667 & 3.6562\\ 3.4474 & 3.3940 & 3.6858\end{bmatrix},\ M\left(C_{F2jl}\right)=\begin{bmatrix}3.2323 & 3.3628 & 3.5385\\ 3.3919 & 3.5359 & 3.7135\\ 3.3931 & 3.5344 & 3.7145\end{bmatrix},$$

$$M\left(C_{F3jl}\right)=\begin{bmatrix}3.3500 & 3.4580 & 3.5876\\ 3.5038 & 3.6187 & 3.7538\\ 3.3858 & 3.4962 & 3.6274\end{bmatrix},\ M\left(C_{F4jl}\right)=\begin{bmatrix}3.3251 & 3.4346 & 3.5857\\ 3.4944 & 3.6096 & 3.7735\\ 3.4471 & 3.5608 & 3.7247\end{bmatrix}.$$

Step 6: Using Eq. (11), the respective SVNCM energy values for all alternatives can be obtained below:

$E[M(n_{C1jl})] = $<(4.4771, 4.9817), (2.0120, 13.6191), (2.2225, 13.8893)>;

$E[M(n_{C2jl})] = $<(4.8330, 4.5180), (1.9188, 13.4854), (2.5293, 13.9709)>;

$E[M(n_{C3jl})] = $<(4.5915, 5.5376), (1.6398, 13.5913), (2.1478, 14.1255)>;

$E[M(n_{C4jl})] = $<(4.9048, 5.1673), (2.0910, 13.9646), (1.8518, 14.2063)>.

Step 7: Using Eq. (12), the SVNCM energy score values for each alternative G_{Si} (i = 1, 2, 3, 4) is calculated and given as follows:

$Z\{E[M(n_{C1jk})]\}$ = 41.1393, $Z\{E[M(n_{C2jk})]\}$ = 34.2103, $Z\{E[M(n_{C3jk})]\}$ = 42.8003, and $Z\{E[M(n_{C4jk})]\}$ = 53.5819.

Step 8: According to the score values, the ranking order of the four alternatives is $G_{S4} > G_{S3} > G_{S1} > G_{S2}$ and the best one is G_{S4}.

5.2 Comparative Investigation of the Decision Results Between SVNM and SVNCM Scenarios

Since the existing MAGDM model [5] introduced in the SVNM scenario cannot perform the school site selection problem in the SVNCM scenario, we must ignore all the credibility values in SVNCMs as a special case of the site selection problem. Thus, we can apply the existing MAGDM model based on SVNM energy in the above site section problem to compare the proposed model with the existing model in the SVNM and SVNCM scenarios.

Based on the MAGDM algorithm in [5], we can obtain the respective SVNM energy values for all alternatives G_{Si} (i = 1, 2, 3, 4):

$E[M(n_{1jk})]$ = <4.4771, 2.0120, 2.2225>, $E[M(n_{2jk})]$ = <4.8330, 1.9188, 2.5293>, $E[M(n_{3jk})]$ = <4.5915, 1.6398, 2.1478>, and $E[M(n_{4jk})]$ = <4.9048, 2.0910, 1.8518>.

Using Eq. (8) [5], the SVNM energy score values for all alternative G_{Si} (i = 1, 2, 3, 4) are calculated and given as follows:

$H\{E[M(n_{1jk})]\}$ = 8.7436, $H\{E[M(n_{2jk})]\}$ = 9.0555, $H\{E[M(n_{3jk})]\}$ = 8.6750, and $H\{E[M(n_{4jk})]\}$ = 9.4150.

According to the score values, the ranking order of the four alternatives is $G_{S4} > G_{S2} > G_{S1} > G_{S3}$ and the best one is G_{S4}.

For the comparative convenience of the decision results in the SVNM and SVNCM scenarios, all results are shown in Table 1.

Table 1. Decision results between SVNM and SVNCM scenarios

MAGDM model	Ranking	Best one	Information environment
Proposed model	$G_{S4} > G_{S3} > G_{S1} > G_{S2}$	G_{S4}	SVNCMs
Existing model [5]	$G_{S4} > G_{S2} > G_{S1} > G_{S3}$	G_{S4}	SVNMs

In terms of the decision results in Table 1, the ranking orders of the four alternatives between the SVNM and SVNCM scenarios are different, then the best one G_{S4} is the same in the school site selection problem. It is clear that the credibility measures with respect to true, false, and uncertain evaluation values reveal their importance in the neutrosophic MAGDM problem because they can affect the ranking order and decision credibility of the four alternatives. Furthermore, the proposed model is the generalization of the existing model [5] and more general and creditable than the existing model in neutrosophic MAGDM problems under uncertain and inconsistent environments.

6. Conclusions

Regarding an extension of SVNM energy, this study presented SVNCM energy and its properties. Then, a MAGDM model using the SVNCM energy was established in the SVNCM scenario, which can solve MAGDM problems and fill a research gap of MAGDM in the SVNCM scenario. Finally, the proposed MAGDM model was applied to the school site selection problem, then the comparative investigation of the decision results in the SVNM and SVNCM scenarios indicated that the proposed model was more general and creditable than the existing model in neutrosophic MAGDM problems under uncertain and inconsistent environments. Furthermore, the credibility measures with respect to true, false and uncertain evaluation values revealed their importance and necessity in the neutrosophic MAGDM problem and affected the ranking of the alternatives, then the decision credibility of the proposed model in the SVNCM scenario is significantly better than the existing model in the SVNM scenario.

However, the proposed SVNCM energy and MAGDM model can be further applied in image processing, clustering analysis, project risk evaluation, slope stability analysis/assessment, and so on in engineering fields, which are future research directions.

Acknowledgments

The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Funding

This research received no external funding.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

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Received: 05 Jan 2024, **Revised:** 26 Mar 2024,

Accepted: 27 Apr 2024, **Available online:** 01 May 2024.