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On Heptagonal Neutrosophic Semi-open Sets in Heptagonal Neutrosophic Topological Spaces: Testing Proofs by Examples

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Abstract: In terms of heptagonal neutrosophic topological spaces, the purpose of this paper is to present the idea of heptagonal neutrosophic semi-open sets. Additionally, we examine a few of its characterizations and heptagonal neutrosophic semi-interior and heptagonal neutrosophic semi-closure operators.

Keywords: Heptagonal Neutrosophic Topology; Heptagonal Neutrosophic Semi-open Set; Heptagonal Neutrosophic Semi-Interior and Heptagonal Neutrosophic Semi-Closure.

1. Introduction

In the year 1965, Zadeh [1] introduced and investigated fuzzy sets. An intuitionistic fuzzy set was first presented in 1986 by Atanassov [2]. Later, Coker [3] discovered intuitionistic fuzzy topological spaces in 1997. Florentin Smarandache [4] developed concepts such as neutrosophic logic and neutrosophic set in 1999. The truth, falsehood, and indeterminacy membership values are the three components on which he defined the neutrosophic set. The neutrosophic set was created in 2010 by Florentin Smarandache [5] as a generalization of intuitionistic fuzzy sets. In 2012, A.A. Salama and S.A. Albowi [6] introduced and developed the generalized neutrosophic set and generalized Neutrosophic topological spaces.

In 2014, Salama et al. [7] developed the concepts of neutrosophic closed sets and neutrosophic continuous functions. Salama [8] investigated the Basic Structure of Some Classes of Neutrosophic Crisp Nearly Open Sets & Possible Application to GIS Topology. In 2020, AL-Nafee et al. [9] explored New Types of Neutrosophic Crisp Closed Sets. In Neutrosophic Topological Spaces, Neutrosophic Semi-open sets were first introduced in 2016 by Iswarya P and K. Bageerathi [10].

Many scientists have constructed neutrosophic topological spaces on bipartitioned, quadripartitioned, and pentapartitioned neutrosophic sets. Kungumaraj et al. recently created heptagonal neutrosophic topological spaces [11]. The idea of heptagonal neutrosophic semi-open sets is introduced and its characterizations are studied in this study. Additionally, we present and investigate the heptagonal neutrosophic semi-interior and semi-closure operators.

The idea of heptagonal neutrosophic semi-open sets in heptagonal neutrosophic topological spaces is presented in this paper. The remaining part of the document is structured as follows: The preliminary information for a better comprehension of the study is contained in Section 2. In Section 3, the notion of the heptagonal neutrosophic semi-open set as well as the fundamental characteristics of these sets are introduced. The fundamental features of the heptagonal neutrosophic semi-interior operator are examined and the classical definition is presented in Section 4. The heptagonal neutrosophic semi-closure operator is defined classically and its fundamental features are examined in Section 5. The concluding Section 6 of the study contains the final results as well as some recommendations for additional research.

2. Preliminaries

Definition 2.1. [4] Let X be a non-empty fixed set. A neutrosophic set (NS) A is an object having the form A = {⟨x, α_A(x), β_A(x), γ_A(x)⟩: x ∈ X} where α_A(x), β_A(x), γ_A(x) represent the degree of membership, degree of indeterminacy and the degree of non-membership respectively of each element x ∈ X to the set A.

A Neutrosophic set A = {⟨x, α_A(x), β_A(x), γ_A(x)⟩: x ∈ X} can be identified as an ordered triple ⟨ α_A(x), β_A(x), γ_A(x)⟩in] −0, 1 +[on X.

Definition 2.2. [5] A neutrosophic topology (NT) on a non-empty set X is a family τ of neutrosophic subsets in X that satisfies the following axioms:

(NT1) 0_N, 1_N∈ τ

(NT2) $G_1 \cap G_2$∈ τ for any G_1, G_2∈ τ

(NT3)∪G_i∈ τ ∀{G_i : i∈ J} ⊆ τ

The pair (X, τ) is used to represent a neutrosophic topological space τ over X.

Definition 2.3. [11] A heptagonal neutrosophic number S is defined and described as
S = < [(p, q, r, s, t, u, v); μ], [(p ′ , q′ , r′ , s′ , t′ , u′ , v′); ℰ] , [(p ′′, q′′, r′′, s′′, t′′, u′′, v′′); η] > where μ, ℰ, η ∈ [0, 1].The truth membership function α : R ⇒ [0, μ], the indeterminacy membership function β : R⇒ [ℰ, 1], and the falsity membership function γ : R ⇒ [η, 1].

Using the ranking technique of heptagonal neutrosophic number is changed as,

$$\lambda = \frac{(p+q+r+s+t+u+v)}{7}$$

$$\mu = \frac{(p'+q'+r'+s'+t'+u'+v')}{7}$$

$$\delta = \frac{(p''+q''+r''+s''+t''+u''+v'')}{7}$$

Definition 2.4.[11] Let X be a non-empty set and A_{HN} and B_{HN} are HNS of the form A_{HN} = <x; $\lambda A_{HN}(x)$, $\mu A_{HN}(x)$, $\delta A_{HN}(x)$ >, B_{HN} = <x; $\lambda B_{HN}(x)$, $\mu B_{HN}(x)$, $\delta B_{HN}(x)$ >, then their heptagonal neutrosophic number operations may be defined as

- **Inclusive:**
  - (i) $A_{HN} \subseteq B_{HN} \Rightarrow \lambda A_{HN}(x) \le \lambda B_{HN}(x), \mu A_{HN}(x) \ge \mu B_{HN}(x), \delta A_{HN}(x) \ge \delta B_{HN}(x)$, for all x∈X.
  - (ii) $B_{HN} \subseteq A_{HN} \Rightarrow \lambda B_{HN}(x) \le \lambda A_{HN}(x), \mu B_{HN}(x) \ge \mu A_{HN}(x), \delta B_{HN}(x) \ge \delta A_{HN}(x)$, for all x∈X.
- **Union and Intersection:**
  - (iii) $A_{HN} \cup B_{HN}$ = {<x; $(\lambda A_{HN}(x) \vee \lambda B_{HN}(x), \mu A_{HN}(x) \wedge \mu B_{HN}(x), \delta A_{HN}(x) \wedge \delta B_{HN}(x))$ >}
  - (iv) $A_{HN} \cap B_{HN}$ = {<x; $(\lambda A_{HN}(x) \wedge \lambda B_{HN}(x), \mu A_{HN}(x) \vee \mu B_{HN}(x), \delta A_{HN}(x) \vee \delta B_{HN}(x))$ >}
- **Complement:**

  Let X be a non-empty set and A_{HN} be the HNS, A_{HN} = <x; $\lambda A_{HN}(x)$, $\mu A_{HN}(x)$, $\delta A_{HN}(x)$ >, then its complement is denoted by A'_{HN} and is defined by

  A'_{HN} = <x; $\delta A_{HN}(x)$, $1-\mu A_{HN}(x)$, $\lambda A_{HN}(x)$ > for all x∈X.
- **Universal and Empty set:**

  Let A_{HN} = <x; $\lambda A_{HN}(x)$, $\mu A_{HN}(x)$, $\delta A_{HN}(x)$ > be a HNS and the universal set I_A and O_A of A_{HN} is defined by
  - (v) I_{HN} = <x: (1,0,0)> for all x∈X.
  - (vi) O_{HN} = <x: (0,1,1)> for all x∈X.

Definition 2.5. [11] A Heptagonal neutrosophic topology (HNT) on a non-empty set X is a family τ of heptagonal neutrosophic subsets in X satisfies the following axioms:

(HNT1) $I_{HN}(x)$, $O_{HN}(x) \in \tau$
(HNT2) $\cup A_i \in \tau$,$\forall \{A_i : i \in J\} \subseteq \tau$
(HNT3) $A_1 \cap A_2 \in \tau$ for any A_1 , $A_2 \in \tau$
The pair (X, τ) is used to represent a heptagonal neutrosophic topological space τ over X. The sets in τ are called a heptagonal neutrosophic open set of X. The complement of heptagonal neutrosophic open sets are called heptagonal neutrosophic closed set of X.

Throughout this paper, we denote
HNS for heptagonal neutrosophic set
HNOS for heptagonal neutrosophic open set
HNCS for heptagonal neutrosophic closed set
HNTS for heptagonal neutrosophic topological space

Definition 2.6. [11] Let A be a HNS in HNTS (X, τ). Then,

- HNint(A_{HN}) = $\cup\{G_{HN}$: G_{HN} is a HNOS in X and $G_{HN} \subseteq A_{HN}\}$ is called a heptagonal neutrosophic interior of A. It is the largest HN-open subset contained in A_{HN}.
- HNcl(A_{HN}) = $\cap$ {K_{HN}: K_{HN} is a HNCS in X and $A_{HN} \subseteq K_{HN}$} is called a heptagonal neutrosophic closure of A. It is the smallest HN-closed subset containing A_{HN}.

3. HN-Semi Open Sets

Definition 3.1: Let A_{HN} be a HNS of a HNTS X. Then A_{HN} is said to be a Heptagonal Neutrosophic Semi-open [written HN-SO] set of X if there exists a heptagonal neutrosophic open set HNO such that HNO $\subseteq A_{HN} \subseteq$ HNCl (HNO).

Example 3.2: Let X = {x,y} and A_{HN}, $B_{HN} \in$ HN(X).

A_{HN} = { <x; (λ:0.85,0.65,0.55,0.78,0.92,0.63,0.38), (μ: 0.75,0.95,0.63,0.48,0.56,0.88,0.78), (δ: 0.25,0.36,0.45,0.58,0.69,0.72,0.90)>, <y; (λ:0.83,0.65,0.72,0.98,0.66,0.53,0.92), (μ:0.73,0.53,0.45,0.38,0.92,0.75,0.63), (δ:0.45,0.35,0.25,0.95,0.85,0.65,0.15)>} and

B_{HN} = { <x; (λ:0.86,0.73,0.62,0.52,0.93,0.45,1), (μ:0.43,0.39,0.26,0.75,0.58,0.93,0.88), (δ:0.55,0.73,0.62,0.52,0.95,0.89,0.44)>, <y; (λ:0.73,0.62,0.51,0.42,0.33,0.29,0.19), (μ:0.82,0.92,1,0.61,0.54,0.76,0.46), (δ:0.19,0.23,0.63,0.52,0.95,0.82,1)>}

By Ranking Technique, (Definition 2.5)

A_{HN} = { <x; (λ:0.68), (μ:0.72), (δ:0.56)>, <y; (λ:0.76), (μ:0.63), (δ:0.52)>} and
B_{HN} = { <x; (λ:0.73), (μ:0.60), (δ:0.67)>, <y; (λ:0.44), (μ:0.73), (δ:0.62)>}

For simplicity, we write the Heptagonal Neutrosophic sets after ranking technique as

A_{HN} = { <x; (0.68, 0.72, 0.56)>, <y; (0.76, 0.63, 0.52)>} and
B_{HN} = { <x; (0.73, 0.60, 0.67)>, < y; (0.44, 0.73, 0.62)>}

Let X = {x,y} and HNTS τ = {I_{HN}, O_{HN}, A_{HN}, B_{HN}, C_{HN}, D_{HN} } where
A_{HN} = { <x; (0.68, 0.72, 0.56)>, <y; (0.76, 0.63, 0.52)>}
B_{HN} = { <x; (0.73, 0.60, 0.67)>, <y; (0.44, 0.73, 0.62)>}
C_{HN} = { <x; (0.73, 0.60, 0.56)>, <y; (0.76, 0.63, 0.52)>}
D_{HN} = { <x; (0.68, 0.72, 0.67)>, <y; (0.44, 0.73, 0.62)>}

Consider the HNS after ranking technique
E_{HN} = { <x; (0.75, 0.52, 0.48)>, <y; (0.82, 0.59, 0.39)>}
F_{HN} = { <x; (0.58, 0.62, 0.75)>, <y; (0.25, 0.85, 0.75)>}
Then the HN-semi open sets of HN(X) are {I_{HN}, O_{HN}, A_{HN}, B_{HN}, C_{HN}, D_{HN}, E_{HN}, F'_{HN}}

The following theorems are the characterization of the HN-SO set in HNTS.

Theorem 3.3: A subset A_{HN} in a HNTS X is a HN-Semi open set iff $A_{HN} \subseteq$HNCl (HNInt (A_{HN})).
Proof:
Necessity: Let A_{HN} be a HN-semi open set in X. Then HNO $\subseteq$ A_{HN} $\subseteq$HNCl (HNO) for some heptagonal neutrosophic open set HNO. But HNO $\subseteq$HNInt (A_{HN}) and thus HNCl (HNO) $\subseteq$HNCl (HNInt (A_{HN})). Hence $A_{HN} \subseteq$HNCl (HNO) $\subseteq$HNCl (HNInt (A_{HN})).
Sufficiency: Let A_{HN} $\subseteq$HNCl (HNInt (A_{HN})). Since HNO = HNInt (A_{HN}), we have HNO $\subseteq$ $A_{HN} \subseteq$HNCl (HNO). Hence A_{HN} is a HN-Semi open set.

Theorem 3.4: Let (X, τ) be a HNTS. Then union of two HN-semi-open sets is again a HN-semi-open set in the HNTS X.
Proof: Let A_{HN} and B_{HN} are HN-semi open sets in X. Then A_{HN} $\subseteq$HNCl (HNInt (A_{HN})) and $B_{HN} \subseteq$HNCl (HNInt (B_{HN})). Therefore $A_{HN} \cup B_{HN} \subseteq$HNCl (HNInt ($A_{HN}$)) ∪HNCl (HNInt ($B_{HN}$)) = HNCl (HNInt ($A_{HN}$) ∪HNInt ($B_{HN}$)) $\subseteq$HNCl (HNInt ($A_{HN} \cup B_{HN}$)) [By Theorem 3.3]. Hence $A_{HN} \cup B_{HN}$ is a HN-semi open set in X.

Theorem 3.5: Let (X, τ) be a HNTS. Then union of a finite collection of HN-semi open sets is again a HN- semi open set in the HNTS X.
Proof: For each i∈∆, $(A_{HN})_i$ is a HN-semi open sets in X. Then by theorem 3.3, $(A_{HN})_i \subseteq$HNCl (HNInt($(A_{HN})_i$)). Thus, $\cup_{i\in\Delta}$ $(A_{HN})_i \subseteq \cup_{i\in\Delta}$HNCl (HNInt($(A_{HN})_i$)) $\subseteq$HNCl ($\cup_{i\in\Delta}$HNInt($(A_{HN})_i$)). Hence $\cup_{i\in\Delta}$ $(A_{HN})_i \subseteq$HNCl (HNInt($\cup_{i\in\Delta}(A_{HN})_i$)). Therefore, the union of a finite collection of HN-semi open sets is again a HN- semi-open set in the HNTS X.

Remark 3.6: The intersection of any two HN-semi open sets need not be a HN- semi-open set as shown in the following example.

Example 3.7: Let X = {x,y} and τ = {I_{HN}, O_{HN}, A_{HN}, B_{HN}, C_{HN}, D_{HN} } where
A_{HN} = { <x; (0.45,0.45,0.45,0.45,0.45,0.45,0.45)>, <y; (0.75,0.75,0.75,0.75,0.75,0.75,0.75)>}
B_{HN} = { <x; (0.95,0.95,0.95,0.95,0.95,0.95,0.95)>, <y; (0.55,0.55,0.55,0.55,0.55,0.55,0.55)>}
By ranking technique,
A_{HN} = { <x; (0.45,0.45,0.45)>, <y; (0.75,0.75,0.75)>}
B_{HN} = { <x; (0.95,0.95,0.95)>, <y; (0.55,0.55,0.55)>}
C_{HN} =$A_{HN} \cup B_{HN}$={<x; (0.95,0.45,0.45)>, <y; (0.75,0.55,0.55)>}
D_{HN} =$A_{HN} \cap B_{HN}$={<x; (0.45,0.95,0.95)>, <y; (0.55,0.75,0.75)>}
τ = {I_{HN}, O_{HN}, A_{HN}, B_{HN}, C_{HN}, D_{HN} }is a HNTS.
Then the HN-semi open sets of HN(X) are {I_{HN}, O_{HN}, A_{HN}, B_{HN}, C_{HN}, D_{HN}, B'_{HN}, C'_{HN}, D'_{HN}}.
Here $A_{HN} \cap B'_{HN}$ is not a HN-semi open set, since HNCl(HNInt($A_{HN} \cap B'_{HN}$))= C'_{HN} and $A_{HN} \cap B'_{HN} \nsubseteq C'_{HN}$.

Theorem 3.8: Let A_{HN} be a HNSO set in the HNTS X and suppose $A_{HN} \subseteq B_{HN} \subseteq$ HNCl (A_{HN}). Then B_{HN} is HNSO set in X.

Proof: There exists a heptagonal neutrosophic open set HNO such that HNO $\subseteq A_{HN} \subseteq$ HNCl (HNO). Since, $A_{HN} \subseteq B_{HN}$, HNO $\subseteq B_{HN}$. But HNCl (A_{HN}) $\subseteq$HNCl (HNO) and thus $B_{HN} \subseteq$HNCl (HNO). Hence HNO $\subseteq B_{HN} \subseteq$HNCl (HNO) and B_{HN} is HNSO set in X.

Theorem 3.9: Every heptagonal neutrosophic open set in the HNTS X is a HNSO set in X.

Proof: Let A be a heptagonal neutrosophic open set in HNTS X. Then A_{HN} = HNInt (A_{HN}). Also HNInt (A_{HN}) $\subseteq$HNCl (HNInt (A_{HN})). This implies that $A_{HN} \subseteq$HNCl (HNInt (A_{HN})). Hence by Theorem 3.3, A_{HN} is a HNSO set in X.

Remark 3.10: The converse of the above theorem need not be true as shown in the following example.

Example 3.11: From Example 3.7, B'_{HN}, C'_{HN}, D'_{HN} are HN-semi open sets, but not HN-open sets.

4. Heptagonal Neutrosophic Semi-Interior In Heptagonal Neutrosophic Topological Spaces

In this section, we introduce the heptagonal neutrosophic semi-interior operator and their properties in the heptagonal neutrosophic topological space.

Definition 4.1: Let (X, τ) be a HNTS. Then for a heptagonal neutrosophic subset A_{HN} of X, the heptagonal neutrosophic semi-interior of A_{HN} [HN-SInt (A_{HN}) for short] is the union of all heptagonal neutrosophic semi-open sets of X contained in A_{HN}.

HN-SInt (A_{HN}) = $\bigcup\{ S_{HN} : S_{HN}$ is a HNSO set in X and $S_{HN} \subseteq A_{HN}\}$

Proposition 4.2: Let (X, τ) be a HNTS. Then for any heptagonal neutrosophic subsets A_{HN} and B_{HN} of a HNTS X we have

(i) HN-SInt (A_{HN}) $\subseteq A_{HN}$
(ii) A_{HN} is HNSO set in X $\Leftrightarrow$ HN-SInt (A_{HN}) = A_{HN}
(iii) HN-SInt (HN-SInt (A_{HN})) = HN-SInt (A_{HN})
(iv) If $A_{HN} \subseteq B_{HN}$ then HN-SInt (A_{HN}) $\subseteq$ HN-SInt (B_{HN})
(v) HN-SInt($A_{HN} \cap B_{HN}$) = HN-SInt (A_{HN}) $\cap$ HN-SInt (B_{HN})
(vi) HN-SInt (A_{HN}) $\cup$ HN-SInt (B_{HN}) $\subseteq$ HN-SInt ($A_{HN} \cup B_{HN}$)

Proof:

(i) Follows from Definition 4.1.
(ii) Let A_{HN} be a HNSO set in X. Then $A_{HN} \subseteq$ HN-SInt(A_{HN}). By using (i) we get A_{HN} = HN-SInt(A_{HN}). Conversely assume that A_{HN} = HN-SInt(A_{HN}). By using Definition 4.1, A_{HN} is NSO set in X. Thus (ii) is proved.
(iii) By using (ii), HN-SInt(HN-SInt(A_{HN})) = HN-SInt(A_{HN}). This proves (iii). Since $A_{HN} \subseteq B_{HN}$, by using (i), HN-SInt(A_{HN}) $\subseteq A_{HN} \subseteq B_{HN}$. That is HN-SInt($A_{HN}$) $\subseteq B_{HN}$. Thus (iii) is proved
(iv) By (iii), HN-SInt(HN-SInt(A_{HN})) $\subseteq$ HN-SInt(B_{HN}). Thus HN-SInt(A_{HN}) $\subseteq$ HN-SInt(B_{HN}). Thus (iv) is proved.
(v) Since $A_{HN} \cap B_{HN} \subseteq A_{HN}$ and $A_{HN} \cap B_{HN} \subseteq B_{HN}$, by using (iv), HN-SInt ($A_{HN} \cap B_{HN}$) $\subseteq$ HN-SInt (A_{HN}) and HN-SInt($A_{HN} \cap B_{HN}$)$\subseteq$HN-SInt(B_{HN}). This implies that
HN-SInt($A_{HN} \cap B_{HN}$) $\subseteq$ HN-SInt(A_{HN})$\cap$HN-SInt(B_{HN}) ---(1).

By(i), HN-SInt(A_{HN})⊆A_{HN} and HN-SInt(B_{HN})⊆B_{HN}. This implies that HN-SInt(A_{HN})⋂HN-SInt(B_{HN}) ⊆ A_{HN}⋂B_{HN}.

Now by (iv), HN-SInt ((HN-SInt(A_{HN})⋂HN-SInt(B_{HN})) ⊆ HN-SInt(A_{HN}⋂ B_{HN}).

By (1), HN-SInt(HN-SInt (A_{HN}))⋂HN-SInt(HN-SInt(B_{HN}))⊆HN-SInt(A_{HN}⋂ B_{HN}).

By (iii), HN-SInt(A_{HN})⋂HN-SInt (B_{HN})⊆HN-SInt(A_{HN}⋂ B_{HN}) -----(2).

From (1) and (2), HN-SInt (A_{HN}⋂ B_{HN}) = HN-SInt(A_{HN})⋂HN-SInt(B_{HN}). Thus (v) is proved.

(vi) Since A_{HN}⊆ A_{HN}⋃B_{HN} and B_{HN}⊆ A_{HN}⋃ B_{HN}, by (iv), HN-SInt (A_{HN}) ⊆ HN-SInt (A_{HN}⋃ B_{HN}) and HN-SInt (B_{HN}) ⊆ HN-SInt (A_{HN}⋃ B_{HN}). This implies that,

HN-SInt (A_{HN}) ⋃ HN-SInt (B_{HN}) ⊆ HN-SInt (A_{HN}⋃ B_{HN}). Thus (vi) is proved.

The following example shows that the equality need not be held in Theorem 4.2 (vi).

Example 4.3: Let X = {x,y} and

A_{HN} = { <x; (0.45,0.45,0.45,0.45,0.45,0.45,0.45)>, <y; (0.75,0.75,0.75,0.75,0.75,0.75,0.75)>}

B_{HN} = { <x; (0.95,0.95,0.95,0.95,0.95,0.95,0.95)>, <y; (0.55,0.55,0.55,0.55,0.55,0.55,0.55)>}

By ranking technique,

A_{HN} = { <x; (0.45,0.45,0.45)>, <y; (0.75,0.75,0.75)>}

B_{HN} = { <x; (0.95,0.95,0.95)>, <y; (0.55,0.55,0.55)>}

C_{HN} =A_{HN}⋃B_{HN}={<x; (0.95,0.45,0.45)>, <y; (0.75,0.55,0.55)>}

D_{HN} =A_{HN}⋂B_{HN}={<x; (0.45,0.95,0.95)>, <y; (0.55,0.75,0.75)>}

Then, τ = {I_{HN}, O_{HN}, A_{HN}, B_{HN}, C_{HN}, D_{HN} }is a HNTS

Consider the HNS after the ranking technique,

E_{HN} = { <x; (0.75,0.52,0.48)>, <y; (0.82,0.59,0.39)>}

Then the HN-semi open sets of HN(X) are {I_{HN}, O_{HN}, A_{HN}, B_{HN}, C_{HN}, D_{HN}, B'_{HN}, C'_{HN}, D'_{HN}}.

Here, HN-SInt (A'_{HN}) ⋃ HN-SInt (E_{HN}) = C'_{HN}⋃ D_{HN} = C'_{HN}

HN-SInt (A'_{HN}⋃ E_{HN}) = D_{HN}

Hence, HN-SInt (A'_{HN}) ⋃ HN-SInt (E_{HN}) ≠ HN-SInt (A'_{HN}⋃ E_{HN}).

5. Heptagonal Neutrosophic Semi-Closure In Heptagonal Neutrosophic Topological Spaces

In this section, we introduce the heptagonal neutrosophic semi-closure operator and its properties in the heptagonal neutrosophic topological space.

Definition 5.1: Let (X,τ) be a HNTS. Then for a heptagonal neutrosophic subset A_{HN} of X, the heptagonal neutrosophic semi-closure of A_{HN} [HN-SCl (A_{HN}) for short] is the intersection of all heptagonal neutrosophic semi-closed sets of X contained in A_{HN}.

HN-SCl (A_{HN}) = ⋃{ K_{HN} : K_{HN} is a HNSC set in X and A_{HN}⊆ K_{HN}}.

Proposition 5.2: Let (X, τ) be a HNTS. Then for any heptagonal neutrosophic subsets A_{HN} and B_{HN} of a HNTS X we have

(i) A_{HN}⊆ HN-SCl (A_{HN})

(ii) A_{HN} is HNSC set in X ⇔ HN-SCl (A_{HN}) = A_{HN}

(iii) HN-SCl (HN-SCl (A_{HN})) = HN-SCl (A_{HN})

(iv) If A_{HN}⊆ B_{HN} then HN-SCl (A_{HN}) ⊆ HN-SCl (B_{HN})

(v) HN-SCl (A_{HN}⋂ B_{HN}) ⊆ HN-SCl (A_{HN}) ⋂ HN-SCl (B_{HN})

(vi) HN-SCl (A_{HN}) ⋃ HN-SCl (B_{HN}) = HN-SCl (A_{HN}⋃ B_{HN})

Proof:

(i) Follows from Definition 5.1.

(ii) Let A_{HN} be a HNSC set in X. Then A_{HN} contains HN-SCl(A_{HN}). Now by using (i), we get A_{HN} = HN-SCl(A_{HN}). Conversely assume that A_{HN} = HN-SCl(A_{HN}). By using Definition 5.1, A_{HN} is a HNSC set in X. Thus (ii) is proved.

(iii) By using (ii), HN-SCl(HN-SCl(A_{HN})) = HN-SCl(A_{HN}). This (iii) is proved.

(iv) Since $A_{HN} \subseteq B_{HN}$, by using (i), $B_{HN} \subseteq$ HN-SCl(B_{HN}) implies $A_{HN} \subseteq$ HN-SCl(B_{HN}) . But HN-SCl(A_{HN}) is the smallest closed set containing A_{HN}, hence HN-SCl(A_{HN}) $\subseteq$ HN-SCl(B_{HN}). Thus (iv) is proved.

(v) Since $A_{HN} \cap B_{HN} \subseteq A_{HN}$ and $A_{HN} \cap B_{HN} \subseteq B_{HN}$, by using (iv), HN-SCl ($A_{HN} \cap B_{HN}$) $\subseteq$ HN-SCl (A_{HN}) and HN-SCl($A_{HN} \cap B_{HN}$)$\subseteq$HN-SCl(B_{HN}). This implies that
HN-SCl($A_{HN} \cap B_{HN}$) $\subseteq$ HN-SCl(A_{HN})$\cap$HN-SCl(B_{HN}). Thus (v) is proved.

(vi) Since $A_{HN} \subseteq A_{HN} \cup B_{HN}$ and $B_{HN} \subseteq A_{HN} \cup B_{HN}$, by (iv), HN-SCl ($A_{HN}$) $\subseteq$ HN-SCl ($A_{HN} \cup B_{HN}$) and HN-SCl (B_{HN}) $\subseteq$ HN-SCl ($A_{HN} \cup B_{HN}$). This implies that,
HN-SCl (A_{HN}) $\cup$ HN-SCl (B_{HN}) $\subseteq$ HN-SCl ($A_{HN} \cup B_{HN}$) ------(1)
By(i), $A_{HN} \subseteq$ HN-SCl(A_{HN}) and $B_{HN} \subseteq$ HN-SCl(B_{HN}). This implies that
$A_{HN} \cup B_{HN} \subseteq$HN-SCl($A_{HN}$) $\cup$ HN-SCl(B_{HN}).
Now by (iv), HN-SCl($A_{HN} \cup B_{HN}$)$\subseteq$HN-SCl ((HN-SCl(A_{HN})$\cup$HN-SCl(B_{HN})).
By (1), HN-SCl($A_{HN} \cup B_{HN}$)$\subseteq$HN-SCl(HN-SCl (A_{HN}))$\cup$HN-SCl(HN-SCl(B_{HN})).
By (iii), HN-SCl($A_{HN} \cup B_{HN}$)$\subseteq$HN-SCl(A_{HN})$\cup$HN-SCl (B_{HN})-----(2).
From (1) and (2), HN-SCl ($A_{HN} \cup B_{HN}$) = HN-SCl(A_{HN})$\cup$HN-SCl(B_{HN}).
Thus (vi) is proved.

The following example shows that equality need not be held in Theorem 5.2 (vi).

Example 5.3: Let X = {x,y} and
A_{HN} = { <x; (0.45,0.45,0.45,0.45,0.45,0.45,0.45)>, <y; (0.75,0.75,0.75,0.75,0.75,0.75,0.75)>}
B_{HN} = { <x; (0.95,0.95,0.95,0.95,0.95,0.95,0.95)>, <y; (0.55,0.55,0.55,0.55,0.55,0.55,0.55)>}
By ranking technique,
A_{HN} = { <x; (0.45,0.45,0.45)>, <y; (0.75,0.75,0.75)>}
B_{HN} = { <x; (0.95,0.95,0.95)>, <y; (0.55,0.55,0.55)>}
$C_{HN} = A_{HN} \cup B_{HN}$={<$x$; (0.95,0.45,0.45)>, <$y$; (0.75,0.55,0.55)>}
$D_{HN} = A_{HN} \cap B_{HN}$={<$x$; (0.45,0.95,0.95)>, <$y$; (0.55,0.75,0.75)>}
Then, τ = {I_{HN}, O_{HN}, A_{HN}, B_{HN}, C_{HN}, D_{HN}} is a HNTS.
Consider the HNS after the ranking technique,
E_{HN} = { <x; (0.75,0.52,0.48)>, <y; (0.82,0.59,0.39)>}
Then the HN-semi open sets of HN(X) are {I_{HN}, O_{HN}, A_{HN}, B_{HN}, C_{HN}, D_{HN}, B'_{HN}, C'_{HN}, D'_{HN}}.
Here, HN-SCl (A'_{HN}) $\cup$ HN-SCl (E_{HN}) = $C'_{HN} \cup D_{HN} = C'_{HN}$
HN-SCl ($A'_{HN} \cup E_{HN}$) = D_{HN}
Hence, HN-SCl (A'_{HN}) $\cup$ HN-SCl (E_{HN}) ≠ HN-SCl ($A'_{HN} \cup E_{HN}$)

Proposition 5.4: Let (X, τ) be a HNTS. Then for any heptagonal neutrosophic subsets $A_{HN}$of a HNTS X, we have

(i) (HN-SInt(A_{HN}))′ =HN-SCl(A'_{HN})

(ii) (HN-SCl(A_{HN}))′ =HN-SInt(A'_{HN})

Proof:

(i) By definition 4.1, HN-SInt (A_{HN}) = ⋃ { S_{HN} : S_{HN} is a HNSO set in X and $S_{HN} \subseteq A_{HN}$}

Taking the complement on both sides,

(HN-SInt(A_{HN}))′ = ⋂ { S'_{HN} : S'_{HN} is a HNSC set in X and $A'_{HN} \subseteq S'_{HN}$}

Now, replace S'_{HN} with K_{HN}, we get

(HN-SInt(A_{HN}))′ = ⋂ {K_{HN} : K_{HN} is a HNSC set in X and $A'_{HN} \subseteq K_{HN}$}

By definition 5.1, (HN-SInt(A_{HN}))′ =HN-SCl(A'_{HN}). Thus (i) is proved.

(ii) From (i) for the HNS A'_{HN}

We write, (HN-SInt(A'_{HN}))′ =HN-SCl(A_{HN})

Taking the complement on both sides we get

HN-SInt(A'_{HN}) = (HN-SCl(A_{HN}))′. Thus (ii) is proved.

6. Conclusion

The notion of heptagonal neutrosophic semi-open sets and their characterization were presented and examined in this paper. It can also be expanded upon in the areas of quotient, continuous, and contra-continuous mappings. It is possible to investigate the set's homeomorphism, connectedness, and compactness in further detail.

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Author Contributions

All authors contributed equally to this research.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

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Conflict of interest

The authors declare that there is no conflict of interest in the research.

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